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International trade and net investment: theory and evidence

Lucas Bretschger · Simone Valente

Abstract The theory of welfare accounting shows that comprehensive measures of net investment can be used to test whether an economy is following unsustainable paths of consumption. However, the notion of net investment used in most applied studies rules out technological progress and terms-of-trade gains from international trade. This paper considers an augmented expression of net investment derived from a dynamic growth model featuring international trade in different types of resource inputs, exogenous productivity growth in final sectors, and cost-reducing progress in resource extraction. Calculating augmented net investment for the world's top twenty oil producers, we show that the difference with standard non-augmented measures can be large and may even revert some established conclusions regarding sustainability: prospects are more favorable than previously thought in oil-exporting countries endowed with large reserves like Angola, Azerbaijan, Kuwait, Saudi Arabia and Venezuela. In oil-importing economies, future consumption possibilities are limited by the lack of expected rental incomes from future resource exports.

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1 Introduction

The production process of modern economies still relies heavily on the use of non-renewable natural resources like minerals and fossil fuels. Over the last three decades, the growing concern for the issue of sustainability has prompted economists to recognize that the resource base is a fundamental capital asset—an appreciation which stimulated researchers and institutions to develop new systems of "green national accounts" whereby we can measure the value of the depletion of several types of natural capital (Heal and Kriström 2006).

The idea of building green accounts takes its inspiration from the theories of welfare accounting pioneered by Weitzman (1976). This literature studies how national accounting aggregates can be used to measure differences in welfare over time. A central result is that comprehensive measures of aggregate *Net Investment* (henceforth NI) allow us to calculate the present discounted value of the future consumption increases that the economy can attain. In this context, the term 'comprehensive' has an important qualification: NI is defined as the sum of the values of the net increases in *all* the productive assets of the economy. In other words, the notion of net investment that is relevant for measuring future consumption gains is not just the value of fixed capital formation: we need to add the value of human capital increases and subtract the value of depleted natural resources. This result paved the way for several applications in the analysis of economies that exploit non-renewable primary inputs. Dixit et al. (1980), Solow (1986) and Hartwick (1990) studied the properties of net investments in this specific context, and Pearce and Atkinson (1993) made the first attempt to calculate the NI indicator in practice for several resource-rich economies. Nowadays, the World Bank publishes yearly estimations of aggregate net investments—also termed adjusted net savings, or 'genuine savings'—for a large set of countries and macro-regions (World Bank 2011). The interest in the NI indicator hinges on its relationship with sustainability conditions: an important theorem establishes that a necessary condition for an economy to exhibit non-declining consumption in the future is to exhibit positive net investment in the present. Put simply, negative current net investment implies unsustainable development (Pezzey 2004).

In this paper, we analyze in detail how international trade and technological progress affect the future consumption possibilities of resource-dependent economies, and how these effects can be captured by comprehensive measures of net investment at the operational level. Our contribution is twofold. First, we use a dynamic model of optimal growth to derive a theory-consistent expression of net investment that takes into account (1) international trade in different types of primary inputs, (2) productivity growth in final sectors, and (3) cost-reducing technological progress in resource extraction. Second, we apply our model-based formula to calculate net investment for the world's top twenty oil producers.

The main motivation for our analysis is the observed discrepancy between the definitions of net investment suggested by theoretical models and those currently used in applied analysis. Specifically, the theories of welfare

accounting show that the *basic notion* of net investment—that is, the NI indicator defined in the context of closed economies with static technology—is not a valid indicator for testing unsustainability if the economy is open to international trade and displays technological progress. In this more general setting, the frontier of future consumption possibilities is affected by total factor productivity growth, by the dynamics of the world interest rate (which induces capital gains/losses from holding foreign assets), and by the dynamics of the world prices of traded goods (that induce real income gains/losses via terms of trade). Consequently, the notion of net investment that should be used for testing unsustainability in real-world open economies is an expanded measure called *Augmented Net Investment* (henceforth ANI), given by the sum of two terms. The first term is the basic NI measure. The second term is called *value of time* and equals the present value of the future improvements in consumption possibilities generated by exogenous productivity growth, capital gains over net foreign assets, and terms-of-trade gains (Sefton and Weale 1996; Weitzman 1997). The discrepancy between theory and practice arises because most if not all applied studies estimate non-augmented net investment—i.e., they calculate net investment in real-world economies without estimating the value of time.[1]

The lack of estimates for augmented net investments is probably due to the inherent difficulties in measuring the future shifts in consumption possibilities: while the basic NI indicator is expressed in terms of current variables, calculating the augmented measure ANI requires using projections of the future growth rates of productivity and of world commodity prices. However, if we only consider the non-augmented NI indicator we neglect the role of international trade and technological progress. We argue that, using a definition of Augmented Net Investment that exhibits sound theoretical foundations, we can obtain important insights on how the prospects for sustainability change depending on whether a resource-rich country is a net importer or a net exporter of the resource. From the empirical standpoint, our argument is particularly relevant when considering natural resources that are extensively traded at the world level and are produced by a set of countries within which some are net importers. Of particular interest is the case of oil: more than one fifth of the world's total merchandise trade consists of oil products (WTO 2010) and, among the world's top producers, six countries—Brazil, China, India, Indonesia, the United Kingdom and the United States—are net oil importers. Based on this evidence, in the applied part of our analysis we estimate ANI for the world's top twenty oil producers and show that net trade positions indeed have a significant impact on the results.

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[1]Apart from Weitzman's (1997) calculation of the technological time premium for the United States, and the analysis of 'natural capital gains' for Indonesia in Vincent et al. (1997), we do not know of any published work conducting a systematic analysis of augmented measures of net investment in real-world economies.

The plan of the paper is as follows. Section 2 reviews the theoretical foundations of net investment indicators. Section 3 proposes a model with international trade, resources and technological progress and derives an explicit expression for augmented net investment. Section 4 applies our model-based formula to real data and estimates ANI for the world's top-twenty oil producers. Section 5 offers some concluding remarks.

2 Theoretical foundations

2.1 Net investment and sustainability

As noted in the Introduction, much of the appeal of the NI indicator comes from its links with sustainability conditions—a point which deserves a formal discussion. In the literature on economic growth, sustainable development is typically defined as a path along which private instantaneous utility is non-declining over time (Barbier 1999; Groth and Schou 2002; Bretschger and Smulders 2006, 2007; Di Maria and Valente 2008).[2] For the sake of clarity, in this paper we assume that private utility depends on consumption and identify sustainability with development paths along which consumption never declines. Hence, sustainability requires

$$\dot{c}(t) \geqslant 0 \text{ in each future instant } t, \tag{1}$$

where $c(t)$ is consumption and the dot indicates its total time-derivative, $\dot{c}(t) \equiv dc(t)/dt$. Now consider an economy in which production requires the use of n types of assets: the quantities of the productive stocks are denoted by $(k_1, ..., k_n)$. For example, k_1 is conventional man-made capital, k_2 represents oil reserves, k_3 is the stock of copper, *etcetera*. In this economy, aggregate net investment at time t equals

$$NI(t) \equiv p_1(t)\,\dot{k}_1(t) + p_2(t)\,\dot{k}_2(t) + ... + p_n(t)\,\dot{k}_n(t), \tag{2}$$

where $(p_1, ..., p_n)$ is a vector of prices associated to the productive stocks. In general the sign of NI is ambiguous: it is positive (negative) when the value of the increase in assets that are being accumulated exceeds (falls short of) the value of the decline in assets that are being depleted. Considering a closed economy with constant population and no technological progress, a crucial

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[2] See Pezzey (1992) for an extensive discussion of sustainability concepts. The notion of sustainability that we employ in this paper corresponds to that of "sustained development" in Pezzey (1992). An alternative definition of sustainable path is that of a development path along which the economy's level of consumption never exceeds the maximum constant level that could be sustained forever given the available technology, endowments and resource constraints. As noted below, the sustainability properties of the NI indicator remain valid under this alternative definition.

result of the theory of welfare accounting (Weitzman 1976; Dixit et al. 1980; Asheim 1994; Asheim and Weitzman 2001) is the following[3]

Proposition 1 *Suppose that a closed economy with constant population produces a consumption good by means of a static technology and maximizes present-value welfare*

$$W_0 \equiv \int_0^\infty u\left(c\left(t\right)\right) e^{-\rho t} dt \tag{3}$$

subject to the accumulation constraints determining the dynamics of all productive stocks, $(\dot{k}_1, ..., \dot{k}_n)$. Then, evaluating Eq. 2 along the optimal path—with each price p_i given by the marginal net benefit of accumulating k_i for each capital type $i = (1, ..., n)$—the net investment at time t coincides with the present-discounted value of future consumption variations

$$\text{NI}\left(t\right) = \int_t^\infty \dot{c}\left(v\right) \cdot e^{-\int_t^v r(v')dv'} dv, \tag{4}$$

where r is the real consumption rate of interest along the optimal path. (Proof: see Appendix).

The economic intuition for result (4) is as follows. At each point in time, future production possibilities are enhanced by the accumulation of some assets and reduced by the depletion of other productive stocks. Hence, NI (t) represents the shift occurring at instant t in the frontier of future production possibilities, which coincides with the present-value stream of all future consumption gains. An important consequence is that the sign of net investment can be used to perform empirical tests of unsustainability (Pezzey 2004):[4]

Proposition 2 *Negative net investments at time t imply unsustainability in the future. Positive net investments at time t are necessary for, but do not guarantee sustainability in the future.*

The proof of Proposition 2 is intuitive. Suppose that we correctly estimate net investment by calculating the right hand side of expression (2) on the basis of observed data. If we obtain a strictly negative value, the economy under study violates sustainability: result (4) implies that there must be an interval of

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[3]Result (4) underlies most of the results of the theory of welfare accounting but is somewhat neglected in this literature—if not hidden between the lines of several theorems' proofs—because the vast majority of contributions focus (with the notable exception of Pezzey 2004) on the welfare significance of Net National Product rather than on the predictive power of Net Investment. The proof of Proposition 1 is based on Asheim and Weitzman (2001) but the general result can be attributed to Weitzman (1976) and Dixit et al. (1980).

[4]Proposition 2 is a variant of Pezzey (2004: Proposition 1). Pezzey's (2004) definition of sustainability is slightly different: a sustainable path is one along which consumption never exceeds the maximum sustainable level. However, the basic property of the NI indicator is unchanged: positive current net investment is necessary but not sufficient for sustainability.

time in the future during which consumption declines.[5] In other words, NI $(t) \geqslant$ 0 is a necessary condition for sustainable development and we can use the sign of estimated net investment to test *un*sustainability.

However, we cannot use the same test to ascertain sustainability: observing positive net investment at a given point in time does not guarantee sustainable development. To verify this statement, suppose that we observe NI $(t) > 0$. From Eq. 4, the present value of future consumption gains is strictly positive. However, this does not mean that consumption will always be sustained: it is possible that consumption will decline (e.g., it will shrink to zero in the long run) but current net investments are strictly positive because there is a sufficiently wide interval (e.g., in the close future) during which consumption increases sufficiently fast. As an example, imagine that the future consumption time path followed by the economy is *single-peaked*: consumption increases until time τ and then declines forever. In this case, expression (4) can be decomposed as

$$
\text{NI} \, (t) = \underbrace{\int_{t}^{\tau} \dot{c} \, (v) \cdot e^{-\int_{t}^{v} r(v') dv'} dv}_{\text{Positive (growing consumption)}} + \underbrace{\int_{\tau}^{\infty} \dot{c} \, (v) \cdot e^{-\int_{\tau}^{v} r(v') dv'} dv}_{\text{Negative (declining consumption)}} \, , \qquad (5)
$$

where the first integral refers to the increasing phase and takes a positive value whereas the second integral refers to the declining phase and takes a negative value. By construction, this economy violates sustainability in the long run. Despite this, NI (t) can be strictly positive as long as the increasing phase that the economy will experience in the close future is sufficiently long and/or 'intense'—that is, consumption initially grows at fast rates. This is a stark example of what Pezzey (2004) calls *the false message of genuine savings*: an economy may well exhibit positive current net investment while being along an unsustainable consumption path.[6] In line with these considerations, we will interpret the NI indicator as a method to test unsustainability without claiming that positive net investments imply sustainable development.

2.2 Net national product and consumption possibilities

Before extending the definition of Net Investment to more general environments, it is necessary to clarify the links between Net National Product (NNP)—i.e., the standard measure appearing in national accounts—and Consumption Net National Product (CNNP)—i.e., the notion of income used in

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[5]Formally, if we correctly estimate the right hand side of Eq. 2 and we observe NI $(t) < 0$, the right hand side of Eq. 4 has to be strictly negative so that there must be an interval of time in the future during which $\dot{c} \, (v) < 0$.

[6]The possibility of observing positive net investment in unsustainable economies was first noted by Asheim (1994) and Vellinga and Withagen (1996). Building on these results, Valente (2008) shows that model-specific estimations of the rates of resource regeneration and augmentation may provide an additional criterion for testing sustainability in economies where current genuine savings appear to be positive.

the theoretical literature on welfare accounting (Weitzman 1976; Sefton and Weale 1996; Heal and Kriström 2008).[7]

Suppose that, within the set of all productive stocks of the economy $(k_1, ..., k_n)$, the quantity k_1 indicates conventional man-made capital, whereas $(k_2, ..., k_n)$ are the quantities of human and natural assets. Further assume that k_1 is expressed in terms of the consumption good: normalizing $p_1 = 1$, and keeping the assumption of a closed economy, the standard accounting measure of NNP equals

$$\text{NNP}(t) \equiv c(t) + \dot{k}_1(t). \tag{6}$$

Now define CNNP as the sum of current consumption and aggregate net investment—that is, the conventional NNP plus the value of the net change in the stock of human-and-natural wealth,

$$\text{CNNP}(t) \equiv c(t) + \text{NI}(t) =$$

$$\equiv \underbrace{c(t) + \dot{k}_1(t)}_{NNP(t)} + \underbrace{p_2(t)\dot{k}_2(t) + ... + p_n(t)\dot{k}_n(t)}_{\text{Accumulation of human and natural assets}}. \tag{7}$$

The crucial difference between NNP and CNNP is that the latter is a comprehensive indicator of wealth-equivalent income, that is, a measure of *present-and-future consumption possibilities*. In fact, if we sum current consumption and net investment (i.e., the present-discounted value of future consumption gains by Proposition 1), we obtain a weighted average of future consumption levels,

$$\text{CNNP}(t) = \int_t^\infty r(v) \cdot c(v) \cdot e^{-\int_t^v r(v')dv'} dv. \tag{8}$$

This result clarifies that CNNP is an indicator of future consumption possibilities in terms of present consumption. The conventional measure of net national product (6) does not have this property because it neglects the fact that human and natural assets contribute to the determination of future consumption possibilities. Indeed, the fundamental claim of the green-accounting literature—i.e., that conventional measures of national income and/or investment should be adjusted for the value of natural resources depletion—hinges on the idea that welfare changes are captured by changes in future consumption possibilities and hence by changes in CNNP. This view is in line with

> "[Samuelson's (1961) intuition that] the rigorous search for a meaningful welfare concept leads to a rejection of all current

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[7]In resource economics, CNNP is called "Green National Product" because it equals Net National Product minus the value of the depletion of the stocks of natural resources and environmental amenities. In this section, we use the term CNNP as it is more generally referred to the frontier of future consumption possibilities.

income concepts and ends up with something closer to a wealth-like magnitude, such as the present discounted value of future consumption" (Weitzman 1976: p.156).

Beyond this, the notion of CNNP is crucial to understanding how aggregate net investment should be computed in more general models where the economy is open to international trade and productivity grows over time, as shown below.

2.3 Technological progress and international trade

Proposition 2 provides the theoretical legitimation for using aggregate net investment in empirical analysis: if we observe negative net investment, sustainability is violated. However, this conclusion hinges on Proposition 1, which assumes static technology, no trade and constant population. Relaxing each of these assumptions implies a specific change in the notion of net investment that is relevant for sustainability analysis—i.e., we must modify the definition of net investment (2) in order to obtain an indicator that satisfies property (4).

In the present paper, we abstract from the case of growing population—the analysis of which is rather complicated and requires making *ad-hoc* assumptions that we avoid here for the sake of generality.[8] The following subsections describe the effects of technical progress and international trade separately. Later in Section 3, we will derive a generalized formula of net investment that takes into account (1) international trade in primary inputs, (2) international mobility of financial assets and two different types of technological progress—namely, (3) total factor productivity growth, and (4) cost-reducing technical progress in resource extraction.

2.3.1 Technological progress

Considering technological progress, the re-definition of net investment follows an intuitive logic. If total factor productivity grows, the frontier of future consumption possibilities is expanded at each point in time by an exogenous process that is independent of the investment choices of economic agents. As a consequence, the sustainability-relevant notion of net investment must include the 'value of time'—i.e., a measure of the autonomous shift generated by technological progress on the consumption possibility frontier.

Building on this idea, originally put forward by Weitzman (1997), Pezzey (2004) suggests a general procedure to augment the basic measure of net investment: in any environment in which the consumption frontier shifts autonomously over time, all the exogenous shifts are captured by the partial time derivative of the various components of net national product. Denoting

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[8]The re-definition of net investments in the case of population growth is studied in Arrow et al. (2003) and Asheim (2004).

the autonomous shift by $q(t)$, the present-discounted value at time t of all future exogenous improvements in the consumption frontier is

$$Q(t) \equiv \int_t^\infty q(v) \cdot e^{-\int_t^v r(v')dv'} dv. \tag{9}$$

As may be construed, we need to assume specific types of technological progress in order to obtain explicit forms of expression (9). In general, however, this definition suffices to define a new aggregate measure called Augmented Net Investment (ANI), equal to net investment (NI) plus the *value of time* (Q)—i.e., the present-value of the future gains generated by autonomous dynamic processes:

$$\underbrace{\text{ANI}(t)}_{\text{Augmented Net Investment}} \equiv \underbrace{\text{NI}(t)}_{\text{Net Investment}} + \underbrace{Q(t)}_{\text{Value of Time}}. \tag{10}$$

Using the augmented definition of net investment (10), we can replicate Propositions 1–2 for the case of economies displaying exogenous productivity growth. That is, it can be shown that along the optimal path that maximizes welfare (3), the augmented measure ANI (t) equals the present-discounted value of all future consumption variations. Consequently, if we can estimate augmented net investment in practice, we can test the hypothesis of unsustainability empirically.

2.3.2 International trade

If we introduce international trade in primary inputs and perfectly integrated financial markets, the sustainability-relevant notion of net investment is modified in two ways. First, if the economy is endowed with a stock of natural resource and sells (part of) the extracted resource flow on the world market, net investment must be augmented to include all the 'terms-of-trade gains' induced by increases in the world resource price. Second, if the economy is small with respect to the world financial market—i.e., the world interest rate is taken as given—net investment must also be augmented to include all the 'foreign-capital gains' induced by increases in the world interest rate. These results are formally proved by Sefton and Weale (1996). As noted by Pezzey (2004: pp.620–623), the same conclusions can be equivalently established by applying the procedure of time-augmentation described above: along the optimal path followed by a small open economy, the terms-of-trade gains and the foreign-capital gains will both appear in the expressions for $q(t)$ and $Q(t)$. The reason is that small open economies take the whole *time paths* of resource prices and the world interest rate as given.[9] This implies that we can treat these paths as exogenous dynamic processes in the same way as total factor

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[9]The assumption of perfect foresight is obviously implicit in the optimal paths studied here—defined as paths chosen at time $t = 0$ by economies that maximize present-value welfare (3).

productivity, and extend the notion of net investment accordingly. A concrete application of this procedure is described below.

3 A model with trade, resources and technological progress

In this section, we derive an explicit formula for augmented net investment that has three desirable characteristics. First, being explicitly *model-based*, our notion of ANI satisfies the same fundamental welfare properties that the non-augmented measure, NI, exhibits in closed economies with static technology. Second, it takes into account two different types of technological progress—i.e., total factor productivity growth in final production and cost-reducing technical progress in resource extraction. Third, it includes international trade in primary inputs. In particular, since the geographic distribution of different types of natural resource endowments is far from being uniform in reality, we consider a *representative small open economy* which uses two different exhaustible resources at the same time: one is extracted from a domestic reserve and partly exported, whereas the other only exists abroad and has to be imported.

In the remainder of the analysis, we concentrate on conventional man-made physical capital and natural assets that represent essential primary inputs for the economy: in order to emphasize the mechanism of substitution between physical and natural capital, we abstract from human capital accumulation and endogenous productivity growth.

3.1 The representative small open economy

Consider a small open economy, called Home, where final production requires the use of three inputs: conventional man-made capital (denoted by k), an imported resource (denoted by g) and an exhaustible resource extracted from a domestic stock. Total domestic extraction (m) is partly used by Home producers (m_h) and partly exported for use by foreign firms (m_f). Denoting by s the domestic resource stock, the variation of Home reserves at time t equals

$$\dot{s}(t) = -m(t) = -m_h(t) - m_f(t).\qquad(11)$$

Domestic final output (x) is given by the technology

$$x(t) = a(t) \cdot \mathcal{F}(k(t), g(t), m_h(t)) = x_h(t) + x_f(t) + \dot{k}(t) + w(t) \cdot m(t)\quad(12)$$

where $a(t)$ is total factor productivity and $\mathcal{F}(\cdot)$ is a well-behaved production function.[10] The right-hand-side term in Eq. 12 shows that final output is

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[10]By well-behaved production we mean that $\mathcal{F}(k, g, m_h)$ is, with respect to each argument, twice continuously differentiable, strictly increasing, strictly concave, and satisfying the Inada conditions. We also assume that all inputs are essential, i.e., $\mathcal{F}(k, g, m_h) = 0$ if at least one argument is zero. All our results hold for $\mathcal{F}(k, g, m_h)$ displaying non-increasing returns to scale.

tradable and has four competing uses: consumption of domestic residents (x_h), consumption of foreign residents (x_f), accumulation in the form of homogeneous capital ($\dot{k}$) and use in the extractive sector: the total extraction cost is $w \cdot m$, where the marginal cost w is independent of resource use. The rest of the world produces and exports a homogeneous final good. Hence, consumption of Home residents equals

$$c(t) = x_h(t) + z_h(t) \tag{13}$$

where z_h is the quantity imported from abroad. We assume perfectly integrated financial markets. Denoting by r the world interest rate and by b the stock of Home's net foreign assets, the current account identity reads

$$\dot{b}(t) = r(t) b(t) + \left[x_f(t) + p_m(t) m_f(t) - z_h(t) - p_g(t) g(t) \right] \tag{14}$$

where p_m and p_g are the world prices of the exported and the imported resources, respectively, and the term in square brackets equals Home's trade surplus. There are five autonomous dynamic processes that Home takes as given, i.e.

$$\left\{ r(t), p_m(t), p_g(t), a(t), w(t) \right\}.$$

In particular, the real interest rate $r(t)$ is determined by the equilibrium in the world financial market; the world prices $p_m(t)$ and $p_g(t)$ are generally time-varying and determined by the equilibrium in the world commodity markets; total factor productivity $a(t)$ grows over time due to *Hicks-neutral technological progress* in final production; the marginal cost of extraction $w(t)$ may decline over time by virtue of *cost-reducing technological progress* in the extraction sector.

3.2 Augmented net investment

The economy under study exploits three types of stocks: domestic physical capital k, net foreign assets b, and oil reserves s. Since domestic and foreign capital are both expressed in terms of final output, the price representing the marginal benefit of accumulating both types of assets is normalized to unity. The marginal rent from domestic extraction is ($p_m - w$), and augmented net investment is given by

$$\text{ANI}(t) = \dot{k}(t) + \dot{b}(t) + (p_m(t) - w(t)) \cdot \dot{s}(t) + Q(t), \tag{15}$$

where the value of time $Q(t)$ is defined in Eq. 9. In the present model, the autonomous shift in the consumption frontier at time t is given by

$$q(t) = \underbrace{\dot{a}(t)\,\mathcal{F}(k(t), g(t), m_h(t)) - \dot{w}(t)\,m(t)}_{\text{Technological Improvements}} +$$

$$+ \underbrace{\dot{p}_m(t)\,m_f(t) - \dot{p}_g(t)\,g(t) + \dot{r}(t)\,b(t)}_{\text{Trade-related Gains}}. \tag{16}$$

Expression (16) includes five effects. The first is the productivity gain generated by Hicks-neutral progress in final production ($\dot{a}\mathcal{F}$). The second is the benefit generated by cost-reducing progress in domestic resource extraction ($-\dot{w}m$). The third and fourth effects are terms-of-trade gains that the economy realizes when the price of exported resources rises ($\dot{p}_m m_f$) and the price of imported resources declines ($-\dot{p}_g g$). The fifth effect is the foreign-capital gain that a country holding positive net foreign assets would obtain from an increase in the world interest rate ($\dot{r}b$).

We can now establish, by analogy with Propositions 1 and 2, the fundamental property of ANI in the economy under study:

Proposition 3 *Suppose that economy Home maximizes present-value welfare (3) subject to the constraints (11)–(14). Along the optimal path, current augmented net investments (15) coincide with the present-discounted value of future consumption variations*

$$\text{ANI}(t) = \int_t^\infty \dot{c}(v) \cdot e^{-\int_t^v r(v')dv'} dv,$$

where r is the world rate of interest in terms of domestic consumption. Consequently, observing $\text{ANI}(t) < 0$ *implies a violation of sustainability in the future. (Proof: see Appendix).*

Proposition 3 legitimates the use of ANI for testing unsustainability. In particular, it implies that the frontier of future consumption possibilities is now given by an augmented notion of CNNP, which includes the value of time.[11]

From an operational perspective, an important difference between augmented and non-augmented measures of net investment is that NI(t) contains variables that are directly observable[12] whereas ANI(t) contains forward

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[11]Formally, the weighted present value of future consumption levels is now given by current consumption plus augmented net investment: $c(t) + \text{ANI}(t) = \int_t^\infty r(v) \cdot c(v) \cdot e^{-\int_t^v r(v')dv'} dv$.

[12]Clearly, we are implicitly assuming that current prices reflect to a good extent the supporting prices of the optimal path—i.e., the prices that would hold in a welfare-maximizing economy. This assumption is necessary and is in fact made by virtually all studies that calculate net investment on the basis of real data.

variables: the value of time $Q(t)$ is the present discounted value of all *future* autonomous shifts in the consumption frontier. As these shifts are not directly observable at time t, it is necessary to approximate, if not estimate them: using expression (16) to calculate ANI (t) in practice bears the cost of making predictions concerning the future behavior of prices and technological developments. However, as shown below, it is possible to re-express several components of the value of time so as to minimize the informational requirements.

3.3 Components of the value of time

In expression (16), the value of time contains two types of forward-looking components: the future gains generated by technological progress and the future gains generated by international trade. In order to obtain an estimable expression, we now rewrite the technological components as a weighted average of autonomous productivity gains (Weitzman 1997) and the trade-related components as current rental income from domestic resources targeted for export (Sefton and Weale 1996).

In order to express the value of time in terms of an *observable constant-equivalent at time t*, we assume that the future values taken by the six crucial variables—namely, the interest rate, the rates of Hicks-neutral and cost-reducing technological progress, the growth rate of the world price of the imported resource and the growth rates of the levels of domestic resource use m_h and g—can be approximated by their future average values predicted at time t. Denoting by $\bar{r}$ the predicted average interest rate and by γ_j the projected average growth rate of the generic variable j, we posit[13]

$$r(\tau) \approx \bar{r}, \quad \frac{\dot{a}(\tau)}{a(\tau)} \approx \gamma_a, \quad \frac{\dot{w}(\tau)}{w(\tau)} \approx \gamma_w, \quad \frac{\dot{p}_g(\tau)}{p_g(\tau)} \approx \gamma_{p_g},$$

$$\frac{\dot{m}_h(\tau)}{m_h(\tau)} \approx \gamma_{m_h}, \quad \frac{\dot{g}(\tau)}{g(\tau)} \approx \gamma_g.$$

As regards preferences, we assume that the utility function appearing in Eq. 3 takes the logarithmic form $u(c(t)) = \ln c(t)$. This implies that, considering growth paths where consumption and output grow at the same rate in the

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[13]With respect to technical progress in extraction, if the projected parameter γ_w is strictly negative—that is, technical progress in the oil sector is actually cost-reducing—the assumption of constant exponential decline in costs is not as optimistic as it may appear at first sight. On the one hand, marginal extraction costs would approach zero in the long run. On the other hand, this would not solve the problem of resource scarcity because sustainability in consumption is far from being guaranteed even when extraction costs are zero at each point in time: as explained in detail by Dasgupta and Heal (1974) and Schulze (1974), the sustainability problem does not arise from extraction costs but from the dynamic productivity loss implied by the use of non-renewable inputs.

long run, the difference between the average rates of interest and of output growth $(\bar{r} - \gamma_x)$ is approximated by the utility discount rate ρ. Given these definitions, the value of time in the representative small open economy equals (see Appendix)

$$
Q(t) \approx \underbrace{\frac{\gamma_a}{\rho} \cdot x(t)}_{Q_1(t)} + \underbrace{\bar{r} \cdot (p_m(t) - w(t)) \cdot \int_t^\infty m_f(v)\, dv}_{Q_2(t)} +
$$

$$
\underbrace{- \frac{\gamma_{p_g}}{\bar{r} - \gamma_{p_g} - \gamma_g} \cdot p_g(t)\, g(t)}_{Q_3(t)} - \underbrace{\frac{\gamma_w}{\bar{r} - \gamma_w - \gamma_{m_h}} \cdot w(t)\, m_h(t).}_{Q_4(t)} \tag{17}
$$

The four terms appearing in the right hand side of Eq. 17 have the following interpretation. The first term (Q_1) is the *technological progress premium* (Weitzman 1997)—that is, the present discounted value of all future improvements in consumption possibilities due to total factor productivity growth—and equals the average future rate of exogenous progress in final production, γ_a, weigthed by ρ, times current final output at time t.

The second term (Q_2) is *current rental income from domestic resources targeted for export* (Sefton and Weale 1996). In particular, recalling that optimal extraction requires that the current reserve $s(t)$ must equal the sum of all future extracted units, we can define the new variable $s_f(t)$ as the amount of the present oil stock earmarked for export,

$$
s_f(t) \equiv \int_t^\infty m_f(v)\, dv = s(t) - \int_t^\infty m_h(v)\, dv.
$$

As suggested by Sefton and Weale (1996: p.42), if the export-import shares of total domestic oil production observed in the past are relatively constant, it is possible to obtain a projection of the targeted stock $s_f(t)$ by multiplying current reserves by the average export share. That is, denoting by $\varphi(v) \equiv m_f(v)/m(v)$ the share of exported oil production and by $\bar{\varphi}$ the average value of this share observed in the past, the current stock targeted for export can be estimated as $s_f(t) \approx \bar{\varphi} s(t)$. Consequently, the second component of the value of time reads

$$
Q_2(t) = \bar{r} \cdot (p_m(t) - w(t)) \cdot \bar{\varphi} \cdot s(t), \tag{18}
$$

where all the variables are directly observable at time t.

The third term (Q_3) in the right hand side of Eq. 17 is the *terms-of-trade gain* (*loss*) realized by the Home economy when the price of imported resources follows a declining (increasing) trend and does not require further comment. The last term (Q_4) is the technological progress premium generated

by *cost reductions* in the domestic extractive sector. In particular, exploiting the definition of export share $\varphi(t)$, we can re-write it as

$$Q_4(t) = \frac{\gamma_w}{\bar{r} - \gamma_w - \gamma_{m_h}} \cdot (1 - \varphi(t)) \cdot w(t) m(t), \tag{19}$$

where $w(t) m(t)$ is total current production cost in the domestic oil sector.

We now have all the elements for calculating augmented net investment in practice. In the next section, we apply our model-based formulation to real data following a step-by-step procedure which emphasizes the relative importance of each component of ANI in determining the overall gap with conventional Net National Savings.

4 Evidence

In the theoretical model of the previous section, we have treated exported and imported resources as two distinct inputs for the sake of generality. If we restrict our attention to oil production and consumption, result (15) remains perfectly valid: we just have to reinterpret p_g as the price of imported oil and g as the quantity of imported oil. In this environment, the components of the value of time that are relevant for a given country depend on whether the economy under study is a net exporter or a net importer. The advantage of focusing on one specific natural resource is twofold. First, we use the same reference equation for all countries. Second, this analysis is capable of showing how the prospects for sustainability change depending on whether a resource-rich country is a net importer or a net exporter. This point is particularly relevant in that many real world economies—namely, Brazil, China, India, Indonesia, the United Kingdom and the United States—are large oil producers and nonetheless import oil from abroad.

4.1 Net investment of top oil producers

Suppose that the imported resource analyzed in the theoretical model is oil. For each open economy, $p_g g$ represents the value of oil imports, the price p_g being the oil price on the world market. The only component of augmented net investment for which there is a fundamental lack of data is the technological premium yielded by cost reductions—that is, Q_4 defined in expression (19). The time series of the unit cost of oil production whereby the World Bank calculates unit oil rents is actually based on a single observation (i.e., unit costs in the various countries in 1993). In other words, the World Bank data implicitly assume $\gamma_w = 0$ in each country (see Bolt et al. 2002). As a consequence, we do not include the term Q_4 in our analysis.

Considering the top-20 world producers of oil, indexed by $i = 1, ..., 20$, we can estimate augmented net investment for each country i by means of the same equation

$$\text{ANI}^i \equiv \text{NNS}^i - \left(p_m^i - w^i\right) \cdot m^i + \left[Q_1^i + Q_2^i - Q_3^i\right], \tag{20}$$

where NNS^i is observed net national savings and the term in square brackets equals the value of time Q^i, with

$$Q_1^i \equiv \frac{\gamma_a^i}{\rho} \cdot \mathrm{GDP}^i, \quad Q_2^i \equiv \bar{r} \cdot \bar{\varphi}^i \cdot \left(p_m - w^i\right) \cdot s^i, \quad Q_3^i \equiv \frac{\gamma_{p_g}}{\bar{r} - \gamma_{p_g} - \gamma_g^i} \cdot p_g g^i,$$

where GDP^i is observed gross domestic product. Having ruled out cost-reducing technological progress for reasons of data availability, expression (20) exactly matches the formula derived in our theoretical model: estimated augmented net investment equal net national savings (i.e., the standard accounting measure for $\dot{k} + \dot{b}$) minus the net rents from domestic oil production (i.e., the negative of the value of the variation in current reserves) plus the value of time decomposed in three terms. In particular, if country i is a net oil exporter, we have $Q_2^i > 0$ and $Q_3^i = 0$. If country i is a net oil importer, instead, we have $Q_2^i = 0$ and $Q_3^i > 0$. In fact, the term Q_2 represents the cash flow of rental income from reserves targeted for export whereas Q_3 is the terms-of-trade loss determined by future increases in the price of imported oil.

Before calculating the full expression for augmented net investment, it is instructive to measure the extent by which the non-augmented measure (NI) differs from net national savings. That is, we temporarily neglect the value of time and calculate the difference between net national savings and the value of domestic oil depletion,

$$\mathrm{NI}^i \equiv \mathrm{NNS}^i - \left(p_m^i - w^i\right) \cdot m^i.$$

The country sample includes the top-20 world oil producers for which we have data. Specifically, the following countries have been excluded because there is no data on conventional savings: Iran, Iraq, Libya, Malaysia, Nigeria, Oman, Qatar, and United Arab Emirates. The twenty countries for which we can calculate net investment are listed in the first column of Table 1. The second column shows which countries are net exporters and which are net importers. Data for net national savings (NNS), unit net oil rents ($p_m^i - w^i$) and domestic oil extraction (m^i) are directly obtained from the components of the "adjusted net savings" estimated by the World Bank for the year 2008. Both NNS and NI are expressed as ratios to Gross National Income (GNI) for each country.

The results show that net investments are negative for seven countries: Angola, Azerbaijan, Kuwait, Saudi Arabia, the United Kingdom, the United States and Venezuela. In the United Kingdom and the United States, the role of resource extraction is not particularly important: net investments are negative mainly because national savings are very low. In Angola, Azerbaijan, Kuwait, Saudi Arabia and Venezuela, the value of domestic oil extraction is high and more than compensates for the value of net national savings. However, these five countries are net exporters and have huge oil reserves: these circumstances are not taken into account in the non-augmented measure NI^i. In fact, considering augmented net investment we obtain radically different results, as shown below.

Table 1 Net national savings and net investment relative to gross national income in 2008 for top oil producers

		Net national savings and net investment relative to GNI in 2008		
	Exp/Imp	$\dfrac{\text{NNS}^i}{\text{GNI}^i}$	$\dfrac{(p_m - w^i) \cdot m^i}{\text{GNI}^i}$	$\dfrac{\text{NI}^i}{\text{GNI}^i}$
Algeria	Exp	0.4791	0.2583	0.2338
Angola	Exp	0.1120	0.8090	−0.6969
Argentina	Exp	0.1375	0.0626	0.0749
Azerbaijan	Exp	0.5070	0.6490	−0.1421
Brazil	Imp	0.0579	0.0353	0.0226
Canada	Exp	0.0940	0.0415	0.0525
China	Imp	0.4382	0.0261	0.4121
Colombia	Exp	0.0882	0.0810	0.0072
Egypt	Exp	0.1416	0.0980	0.0436
India	Imp	0.2968	0.0175	0.2793
Indonesia	Imp	0.1159	0.0532	0.0627
Kazakhstan	Exp	0.3275	0.3186	0.0089
Kuwait	Exp	0.4532	0.5807	−0.1275
Mexico	Exp	0.1334	0.0850	0.0483
Norway	Exp	0.2619	0.1380	0.1239
Russian Federation	Exp	0.2039	0.1776	0.0262
Saudi Arabia	Exp	0.3588	0.6366	−0.2778
United Kingdom	Imp	0.0115	0.0149	−0.0034
United States	Imp	−0.0136	0.0093	−0.0229
Venezuela	Exp	0.2269	0.2687	−0.0418

4.2 The value of time for top oil producers

In order to calculate augmented net investment, we first need to estimate the parameters appearing in (Q_1^i, Q_2^i, Q_3^i). In this respect, the world interest rate, the utility discount rate and the projected growth rate of oil prices for importers are set equal to

$$\rho = 4\%, \quad \bar{r} = 6.23\%, \quad \gamma_{p_g} = 3.53\%$$

for all countries. In general, the logic behind imposing the same values of $\bar{r}$ and γ_{p_g} for all countries is that we want to abstract from specific differences in the discounting factors applied to each country and measure, instead, cross-country differences in augmented net investments generated by total factor productivity growth, GDP levels, size of domestic oil reserves and propensity to export/import. In particular, the value $\bar{r} = 6.23\%$ reflects the average real lending interest rate observed across all the countries over the period 1990–2008 according to World Bank data, and the value $\gamma_{p_g} = 3.53\%$ is the average growth rate in the world oil price (denominated in dollars per barrel and deflated by the US deflator) over the period 1990–2008.

The country-specific variables and parameters appearing in the value of time in expression (20) are reported in Table 2 and are obtained as follows. The projected rate of Hicks-neutral progress in final production, γ_a^i, is set equal to the average growth rate of total factor productivity in country i over the period 1990–2008 according to the calculations of the Conference

Table 2 Country-specific parameters used to calculate the components of the value of time

	Country-specific parameters					
	γ_a^i	$\dfrac{GDP^i}{GNI^i}$	$\bar{\varphi}^i$	$\dfrac{(p_m - w^i) \cdot s^i}{GNI^i}$	γ_g^i	$\dfrac{p_g g^i}{GNI^i}$
Algeria	−0.25%	1.01	0.88	6.31		
Angola	7.22%	1.21	0.96	10.51		
Argentina	1.70%	1.02	0.35	0.69		
Azerbaijan	8.84%	1.13	0.77	14.39		
Brazil	−0.21%	1.02		0.63	−10.08%	0.0010
Canada	−0.05%	1.01	0.29	9.10		
China	2.47%	0.99		0.30	22.40%[a]	0.0298
Colombia	−0.45%	1.04	0.51	0.55		
Egypt	1.75%	0.99	0.14	1.97		
India	1.76%	1.00		0.40	8.47%[a]	0.0603
Indonesia	1.30%	1.11		0.76	−5.83%	0.0160
Kazakhstan	4.31%	1.17	0.83	23.03		
Kuwait	0.98%	0.94	0.88	59.13		
Mexico	−0.16%	1.02	0.42	0.92		
Norway	0.78%	1.01	0.92	1.30		
Russian Federation	2.69%	1.03	0.71	3.12		
Saudi Arabia	−0.04%	0.98	0.81	50.14		
United Kingdom	0.92%	0.98		0.11	−12.67%	0.0215
United States	0.53%	1.00		0.11	2.41%	0.0273
Venezuela	0.96%	1.00	0.77	24.51		

[a]The values of the growth rate of imports obtained for China and India are not consistent with intertemporal solvency: see the main text for discussion

Board (2011). The value of gross domestic product over gross national income in 2008 is taken from World Bank (2011). The projected export share of domestic oil production, $\bar{\varphi}^i$, is set equal to the 2003-2008 average of the ratio between Net Oil Exports and Total Oil Supply in physical terms according to the international energy statistics published by the US Energy Information Administration (EIA 2011). The net value of reserves $(p_m - w^i) \cdot s^i$ is set equal to the unit net oil rent calculated by the World Bank (2011) times the physical amount of proven oil reserves estimated by EIA (2011). The projected growth rate of oil imports in physical terms, γ_g^i, is set equal to the average growth rate of Net Oil Imports over the period 1991–2008. The value of current oil imports in 2008, $p_g g^i$, is equal to the oil price (World Bank 2011) times Net Oil Imports calculated from EIA (2011).

The results reported in Table 2 suggest two main remarks. First, Angola, Azerbaijan and Kazakhstan display fast growth in total factor productivity as well as high current value of domestic reserves relative to gross national income: these characteristics will have a strong positive impact on the value of time and thereby on augmented net investment. Second, looking at the projected growth rate of imports, China and India exhibit unsustainable values: if we take the average growth rates of oil imports observed in the past (22% and 8%, respectively) as projected future values, the two countries would not fulfil the intertemporal budget constraint with the rest of the world because the

Table 3 Components of the value of time

	Components of the value of time			
	$\dfrac{Q_1}{\mathrm{GNI}^i}$	$\dfrac{Q_2}{GNI^i}$	$\dfrac{Q_3}{\mathrm{GNI}^i}$	$\dfrac{Q}{\mathrm{GNI}^i}$
Algeria	−0.06	0.34		0.28
Angola	2.18	0.63		2.81
Argentina	0.43	0.02		0.45
Azerbaijan	2.49	0.69		3.18
Brazil	−0.05		0.0003	−0.05
Canada	−0.01	0.17		0.15
China	0.61[a]		-	-
Colombia	−0.12	0.02		−0.10
Egypt	0.43	0.02		0.45
India	0.44[a]		-	-
Indonesia	0.36		0.0066	0.35
Kazakhstan	1.26	1.18		2.45
Kuwait	0.23	3.23		3.46
Mexico	−0.04	0.02		−0.02
Norway	0.20	0.07		0.27
Russian Federation	0.69	0.14		0.83
Saudi Arabia	−0.01	2.53		2.52
United Kingdom	0.23		0.0049	0.22
United States	0.13		0.3316	−0.20
Venezuela	0.24	1.18		1.42

[a]For China and India, the value of time is virtually minus infinity (see the main text for discussion)

sequence of trade deficits would yield unbounded growth in foreign debt.[14] Clearly, this problem arises because China and India experienced dramatic growth accelerations in the 1991–2008 period, one consequence being that oil imports increased at exceptionally high rates. Still, if we project these growth rates into the future, the stream of the value of future imports would be unbounded and would result in $Q_3 = \infty$, an infinite terms-of-trade loss that necessarily implies unsustainability. We will later show that removing this component of the value of time yields rather high values of ANI for both China and India (cf. Table 5 below).

Using the values reported in Table 2, we calculate the value of time $Q \equiv Q_1 + Q_2 - Q_3$ on the basis of expression (20). The results are reported in Table 3 and suggest the following remarks. In six countries—Angola, Azerbaijan, Kazakhstan, Kuwait, Saudi Arabia and Venezuela—the value of time exceeds the value of current gross national income in 2008. The reasons are however different. In Kuwait, Saudi Arabia and Venezuela, the gain mostly comes from the size of proven reserves which guarantees high rental incomes from resources targeted for exports (that is, high values of Q_2). In Angola,

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[14]If we take the average growth rates of oil imports observed in the past (22 and 8%, respectively) as projected future values for China and India, we have $r - \gamma_{p_g}^i - \gamma_g^i < 0$ and thereby an infinite value of the integral representing the present-value loss from terms of trade (that is, insolvency in the long run): see the derivation of equation (17) in Appendix.

Azerbaijan and Kazakhstan, instead, huge reserves are combined with fast growth in total factor productivity, which yields substantial progress premia (that is, high values of Q_1). At the other extreme, we observe a negative value of time in Brazil, Colombia, Mexico—where the result is due to negative growth in total factor productivity—and the United States—where the result is due to the terms-of-trade loss implied by a projected increase in physical oil imports around 2% per annum (cf. Table 2 above).

4.3 Augmented net investment of top oil producers

The value of augmented net investment for the world's top 20 oil producers is directly obtained from the previous calculations. Table 4 reports a general summary of our final results. These figures suggest two general remarks. First, the difference between augmented and non-augmented measures of net investment can be large and may even revert the conclusions regarding sustainability—in the favorable or unfavorable sense. Angola, Azerbaijan, Kuwait, Saudi Arabia and Venezuela exhibit a favorable reversal: net investment is negative but augmented net investment is positive. The reason for this result differs across countries. In Kuwait, Saudi Arabia and Venezuela, augmented net investment is high because domestic reserves are huge and the propensity to export is high: the projected value of the stock targeted for exports is very high and more than compensates for the value of current

Table 4 Net national savings, net investment and augmented net investment in 2008 for top oil producers

		Net investment and augmented net investment (relative to GNI) in 2008		
	Exp/Imp	$\dfrac{NNS^i}{GNI^i}$	$\dfrac{NI^i}{GNI^i}$	$\dfrac{ANI^i}{GNI^i}$
Algeria	Exp	0.4791	0.2338	0.5148
Angola	Exp	0.1120	−0.6969	2.1121
Argentina	Exp	0.1375	0.0749	0.5236
Azerbaijan	Exp	0.5070	−0.1421	3.0405
Brazil	Imp	0.0579	0.0226	−0.0323
Canada	Exp	0.0940	0.0525	0.2059
China	Imp	0.4382	0.4121	
Colombia	Exp	0.0882	0.0072	−0.0918
Egypt	Exp	0.1416	0.0436	0.4939
India	Imp	0.2968	0.2793	
Indonesia	Imp	0.1159	0.0627	0.4160
Kazakhstan	Exp	0.3275	0.0089	2.4563
Kuwait	Exp	0.4532	−0.1275	3.3283
Mexico	Exp	0.1334	0.0483	0.0328
Norway	Exp	0.2619	0.1239	0.3971
Russian Federation	Exp	0.2039	0.0262	0.8570
Saudi Arabia	Exp	0.3588	−0.2778	2.2395
United Kingdom	Imp	0.0115	−0.0034	0.2169
United States	Imp	−0.0136	−0.0229	−0.2225
Venezuela	Exp	0.2269	−0.0418	1.3774

domestic extraction (which makes non-augmented net investments negative in all cases). In Angola and Azerbaijan, instead, augmented net investment are high mainly due to the technological progress premium. Also, we observe a reversal of conclusions in the unfavorable direction: Brazil and Colombia exhibit positive net investment but negative augmented net investment. In both cases, the main problem is that total factor productivity has been stagnating if not declining in the past: if we project the same development path in the future, there is no positive technological progress premium. Moreover, the size of reserves in Colombia is relatively limited (which implies low rental income from future exports) and Brazil is a net importer (which makes this country subject to terms-of-trade losses generated by future increases in the world oil price).

The second general remark is that international trade matters for these results. Recalling Table 3, the present value rental income from future exports is estimated to be above 60% of current gross national income for six countries (Angola, Azerbaijan, Kazakhstan, Kuwait, Saudi Arabia and Venezuela) and represents a substantial fraction of the calculated value of time. The other side of the coin is that net importers tend to exhibit low values of augmented net investment because the value of time does not include rental income for future exports and, instead, includes terms-of-trade losses due to future increases in the world oil price.

It may be objected that the estimates for net importers are highly sensitive to the projected rates of growth of future imports and oil prices. This issue can be addressed in quantitative terms as follows. Suppose that, differently from the estimates reported in Table 2, the projected growth rates of oil imports are the same for all net importers. In Table 5, we consider three scenarios in which Brazil, China, India, Indonesia, the United Kingdom and the United States satisfy intertemporal solvency and, in particular, exhibit a declining trend in the quantity of imported oil. Calculating augmented net investment under these scenarios—where all parameters are as before except for the growth rate of imports—we see that imposing $\gamma_g = -1\%$ does not yield substantial differences with respect to $\gamma_g = -10\%$. In Brazil, augmented net investment is negative because of the combination of low national savings and stagnating total factor productivity: the terms-of-trade loss due to imports plays at best a

Table 5 Augmented net investment in 2008 for net oil importers: alternative scenarios

| | ANI relative to GNI for net importers in 2008 | | |
| | Alternative scenarios | | |
	$\gamma_g = -0.01$	$\gamma_g = -0.05$	$\gamma_g = -0.10$
Brazil	−0.0329	−0.0324	−0.0323
China	0.9972	1.0119	1.0173
India	0.6636	0.6935	0.7043
Indonesia	0.4074	0.4153	0.4182
United Kingdom	0.2013	0.2120	0.2159
United States	0.0830	0.0965	0.1014

minor role. China and India display high values of ANI because, in Table 5, we remove from the computation the strong growth in oil imports observed in the past (cf. Table 2). Again, these figures do not change much if we let the assumed rate of decline in oil imports range from one to ten percentage points. The results for the United Kingdom are substantially unaltered with respect to our previous calculations, whereas the United States now exhibit a positive value of ANI. Still, augmented net investments in the United States are relatively low due to very low levels of national savings: this inevitably scales down all measures of net investment.

5 Conclusion

In the controversial public debate about sustainable development, it is natural to ask whether the current patterns of economic activity are in fact sustainable in the long run. As the topic includes complex economic and ecological relationships, the answer is not easy to find. Nevertheless, to derive concrete results for specific countries, the calculation of adjusted investment rates has emerged as a promising tool. However, in applied studies, the most used framework assumes static technologies and no international trade. Relaxing these two assumptions implies big differences, especially for the case of resource-rich countries. Accordingly, the conclusion that many resource-exporting countries are developing unsustainably because of negative net investments has to be reconsidered with an appropriate approach.

Based on earlier theoretical contributions, this paper develops a formal rule for calculating augmented net investment, which explicitly refers to trade and technical progress. Methodologically, this leads to a separate calculation of the value of time. In particular, we stress that future consumption growth due to technical progress and the rental income from exported resources entail major corrections of the investment rates.

In the second part of the paper, the rule is applied to the world's top 20 oil producers. We find two remarkable results. First, the difference between augmented and non-augmented measures of net investment can be large and may even revert previous conclusions on sustainability. Prominently, in Kuwait, Saudi Arabia and Venezuela, augmented net investments are high because domestic reserves are large and the propensity to export is high, which compensates current domestic extraction. In Angola and Azerbaijan, augmented net investments are high due to the technological progress premium. Thus, according to our rule, these countries cannot be qualified as unsustainable, although their net investment rate is negative. On the contrary, countries with limited reserves, poor productivity growth, and considerable resource imports are more likely to have negative augmented net investment. Accordingly, Brazil and Colombia exhibit positive net investment but are not sustainable according to the augmented net investment criterion.

Second, international trade is a major factor driving the results. For six countries, the present value rental income from future exports is estimated

to be above 60% of current gross national income and thus represents a substantial fraction of the calculated value of time. Net oil importers however have lower values of augmented net investment because there is no rental income from future exports and future increases in the world oil price entail negative terms-of-trade effects.

It seems rewarding to extend the present analysis to additional natural resources and to include data on resource extraction cost, which would complement the calculation of the value of time. Moreover, the predicted values for the different parameters could be connected more closely to macroeconomic forecasting models. Also, for the study of the single countries, institutional factors and sensitivity analyses would be useful to derive sustainability conclusions. These issues are left for future research.

Appendix

Proof of Proposition 1 As shown by Asheim and Weitzman (2001: p. 237, Eq. 9), along the optimal path we have

$$\frac{d}{dt}\text{NI}(t) = r(t)\,\text{NI}(t) - \dot{c}(t)$$

in each instant t. Integrating this expression forward and imposing the transversality condition

$$\lim_{T\to\infty} \text{NI}(T) \cdot e^{-\int_t^T r(v)dv} = 0,$$

which must hold along the optimal path, we obtain Eq. 4. For further details, see Asheim and Weitzman (2001). The same proof can be equivalently obtained as a special case of Proposition 3 below by excluding trade and technological progress from the model of Section 3. □

Proof of Proposition 2 See the main text. □

Derivation of Eq. 8 Substituting result (4) in definition (7), and integrating by parts, we have

$$\text{CNNP}(t) = c(t) + \int_t^\infty \dot{c}(v) \cdot e^{-\int_t^v r(v')dv'} dv =$$

$$= c(t) + \left[\lim_{v\to\infty} c(v) \cdot e^{-\int_t^v r(v')dv'}\right] - c(t) + \int_t^\infty r(v)\,c(v) \cdot e^{-\int_t^v r(v')dv'} dv,$$

where the limit in square brackets is zero by the transversality condition that must be satisfied along optimal paths. Hence, the above expression reduces to Eq. 8.

Proof of Proposition 3 Economy Home maximizes welfare (3) subject to Eqs. 11–14. The current-value Hamiltonian associated to this problem is

$$\mathcal{L} \equiv u(x_h + z_h) + \lambda_k \left[a \cdot \mathcal{F}(k, g, m_h) - x_h - x_f - w \cdot (m_h + m_f) \right] +$$
$$+ \lambda_b \left[rb + x_f + p_m m_f - z_h - p_g g \right] - \lambda_s \left[m_h + m_f \right],$$

where $\{\lambda_k, \lambda_b, \lambda_s\}$ are the dynamic multipliers associated to the state variables $\{k, b, s\}$. Maximizing $\mathcal{L}$ with respect to the control variables $\{x_h, x_f, z_h, m_h, m_f, g\}$ we obtain

$$\lambda_k = \lambda_b = u'(x_h + z_h), \tag{21}$$

$$\lambda_s = \lambda_k \cdot (a\mathcal{F}_{m_h} - w), \tag{22}$$

$$\lambda_s = \lambda_b p_m - \lambda_k w = \lambda_k \cdot (p_m - w), \tag{23}$$

$$\lambda_b p_g = \lambda_k a\mathcal{F}_g. \tag{24}$$

The co-state equations for $\{k, b, s\}$ read

$$\rho\lambda_k - \dot{\lambda}_k = \lambda_k a\mathcal{F}_k, \tag{25}$$

$$\rho\lambda_b - \dot{\lambda}_b = \lambda_b r, \tag{26}$$

$$\rho\lambda_s - \dot{\lambda}_s = 0, \tag{27}$$

and the transversality conditions require

$$\lim_{t\to\infty} \lambda_k(t)\, k(t)\, e^{-\rho t} = \lim_{t\to\infty} \lambda_b(t)\, b(t)\, e^{-\rho t} = \lim_{t\to\infty} \lambda_s(t)\, s(t)\, e^{-\rho t} = 0. \tag{28}$$

Notice that Eqs. 21–23 and 25–26 imply the following no-arbitrage conditions:

$$a +_k = r, \quad a\mathcal{F}_{m_h} = p_m, \quad a\mathcal{F}_g = p_g, \tag{29}$$

$$\dot{p}_m - \dot{w} = (p_m - w) \cdot r, \tag{30}$$

where Eq. 29 establishes the equality between prices and marginal productivities of the inputs in final production, and Eq. 30 is Hotelling's rule. Also, combining constraints (12)–(14), we obtain

$$\dot{k} + \dot{b} = a\mathcal{F}(k, g, m_h) - c - wm + rb + p_m m_f - p_g g. \tag{31}$$

Substituting Eq. 31 in Eq. 15, augmented net investment equal

$$\text{ANI} = a\mathcal{F}(k, g, m_h) - c - wm + rb + p_m m_f - p_g g + (p_m - w) \cdot \dot{s} + Q. \tag{32}$$

Time-differentiating Eq. 32 we have

$$\dot{\text{ANI}} = \dot{a}\mathcal{F}(k, g, m_h) + a\mathcal{F}_k\dot{k} + a\mathcal{F}_g\dot{g} + a\mathcal{F}_{m_h}\dot{m}_h - \dot{c} +$$
$$- \dot{w}m - w\dot{m} + \dot{r}b + r\dot{b} + \dot{p}_m m_f + p_m\dot{m}_f - \dot{p}_g g - p_g\dot{g} +$$
$$+ (\dot{p}_m - \dot{w}) \cdot \dot{s} + (p_m - w) \cdot \ddot{s} + \dot{Q}. \tag{33}$$

Substituting Eq. 29 to eliminate marginal productivities, the Hotelling rule (30) to eliminate $(\dot{p}_m - \dot{w})$, and using Eq. 11 to substitute $\ddot{s} = -(\dot{m}_h + \dot{m}_f)$,

we obtain

$$\text{A\dot{N}I} = \left[r\dot{k} + r\dot{b} + r\left(p_m - w\right) \cdot \dot{s}\right] - \dot{c} + \dot{Q} + \dot{a}\mathcal{F}\left(k, g, m_h\right)$$
$$- \dot{w}m + \dot{r}b + \dot{p}_m m_f - \dot{p}_g g,$$
$$\text{A\dot{N}I} = \left[r\dot{k} + r\dot{b} + r\left(p_m - w\right) \cdot \dot{s}\right] - \dot{c} + \dot{Q} + q,$$

where we have used result (16) to obtain the last expression. By definition (15), the term in square brackets equals $r\text{ANI} - rQ$, implying

$$\text{A\dot{N}I} = r\text{ANI} - \dot{c} + \dot{Q} - rQ + q.$$

By definition (9), the total time-derivative of the time premium is $\dot{Q} = rQ - q$. As a consequence, the above expression reduces to

$$\text{A\dot{N}I} = r\text{ANI} - \dot{c}. \tag{34}$$

Integrating Eq. 31 between over the interval (t, T), we have

$$\text{ANI}\left(t\right) = \int_t^T \dot{c}\left(v\right) e^{-\int_t^v r(v')dv'} dv + \text{ANI}\left(T\right) \cdot e^{-\int_t^T r(v)dv}. \tag{35}$$

Notice that, using definition (15) and the Hotelling rule (30), the last term in Eq. 35 can be written as

$$\text{ANI}\left(T\right) \cdot e^{-\int_t^T r(v)dv} = \left[\dot{k}\left(T\right) + \dot{b}\left(T\right) + Q\left(T\right)\right] \cdot e^{-\int_t^T r(v)dv}$$
$$+ \left(p_m\left(t\right) - w\left(t\right)\right) \cdot \dot{s}\left(T\right). \tag{36}$$

Using the co-state equations (25)–(27), the transversality conditions (28) imply

$$\lim_{T \to \infty} k\left(T\right) e^{-\int_t^T r(v)dv} = \lim_{T \to \infty} b\left(T\right) e^{-\int_t^T r(v)dv} = 0 \text{ and } \lim_{T \to \infty} s\left(T\right) = 0.$$

As a consequence,

$$\lim_{T \to \infty} \dot{k}\left(T\right) e^{-\int_t^T r(v)dv} = \lim_{T \to \infty} \dot{b}\left(T\right) e^{-\int_t^T r(v)dv} = \lim_{T \to \infty} \dot{s}\left(T\right) = 0. \tag{37}$$

Moreover, $\dot{Q} = rQ - q$ (which is well defined only if $q = 0$ because $Q = 0$ otherwise) implies

$$\lim_{T \to \infty} Q\left(T\right) e^{-\int_t^T r(v)dv} = 0. \tag{38}$$

Results (37) and (38) imply that taking the limit as $T \to \infty$ in Eq. 36, we have

$$\lim_{T \to \infty} \text{ANI}\left(T\right) \cdot e^{-\int_t^T r(v)dv} = 0.$$

As a consequence, taking the limit as $T \to \infty$ in Eq. 35, we obtain $\text{ANI}\left(t\right) = \int_t^T \dot{c}\left(v\right) e^{-\int_t^v r(v')dv'} dv$, as stated in Proposition 3. The fact that $\text{ANI}\left(t\right) < 0$ implies unsustainability follows by analogy with Proposition 2.   □

Derivation of Eq. 17 By definitions (9) and (16), the value of time is given by

$$Q(t) = \int_t^\infty \frac{\dot{a}(v)}{a(v)} \cdot a(v) \mathcal{F}(k(v), g(v), m_h(v)) \cdot e^{-\bar{r}(v-t)} +$$

$$+ \int_t^\infty \dot{p}_m(v) m_f(v) \cdot e^{-\bar{r}(v-t)} dv - \int_t^\infty \dot{w}(v) m(v) \cdot e^{-\bar{r}(v-t)} dv +$$

$$- \int_t^\infty \frac{\dot{p}_g(v)}{p_g(v)} \cdot p_g(v) g(v) \cdot e^{-\bar{r}(v-t)} dv +$$

$$+ \int_t^\infty \dot{r}(v) b(v) \cdot e^{-\bar{r}(v-t)} dv. \tag{39}$$

Considering the first line in Eq. 39, we substitute $\dot{a}(v)/a(v) \approx \gamma_a$ inside the integral and, defining the average future growth rate of output as $\gamma_x \equiv \frac{1}{v-t} \cdot \int_t^v \dot{x}(\tau)/x(\tau) d\tau$, obtain

$$Q_1 = \gamma_a \cdot \int_t^\infty a(v) \mathcal{F}(k(v), g(v), m_h(v)) \cdot e^{-\bar{r}(v-t)} dv = \gamma_a \cdot x(t) \cdot \int_t^\infty e^{-(\bar{r}-\gamma_x)(v-t)} dv.$$

Substituting the approximation based on the Keynes-Ramsey rule, $\rho \approx \bar{r} - \gamma_x$ with $\rho > 0$ constant, direct integration yields $Q_1 = \gamma_a x(t)/\rho$. Exploiting $m = m_h + m_f$, the second line of Eq. 39 can be re-written as

$$Q_2 - Q_4 = \int_t^\infty \left[\dot{p}_m(v) - \dot{w}(v)\right] m_f(v) \cdot e^{-\bar{r}(v-t)} dv$$

$$- \int_t^\infty \frac{\dot{w}(v)}{w(v)} w(v) m_h(v) \cdot e^{-\bar{r}(v-t)} dv$$

where we can substitute $\dot{w}(v)/w(v) \approx \gamma_w$ and the Hotelling rule $\left[\dot{p}_m(v) - \dot{w}(v)\right] = \bar{r} \cdot \left[p_m(v) - w(v)\right]$ to obtain

$$Q_2 - Q_4 = \bar{r} \cdot \int_t^\infty \left[p_m(v) - w(v)\right] \cdot m_f(v) \cdot e^{-\bar{r}(v-t)} dv$$

$$- \gamma_w \int_t^\infty w(v) m_h(v) \cdot e^{-\bar{r}(v-t)} dv.$$

Further substitute $w(v) \approx w(t) e^{\gamma_w(v-t)}$, $m_h(v) \approx m_h(t) e^{\gamma_{m_h}(v-t)}$ and $p_m(v) - w(v) = \left[p_m(t) - w(t)\right] \cdot e^{\bar{r}(v-t)}$ yields

$$Q_2 - Q_4 = \bar{r} \cdot (p_m(t) - w(t)) \cdot \int_t^\infty m_f(v) dv - \frac{\gamma_w}{\bar{r} - \gamma_w - \gamma_{m_h}} \cdot w(t) m_h(t).$$

Considering the third line in Eq. 39, substituting $\dot{p}_g(v)/p_g(v) \approx \gamma_{p_g}$, $p_g(v) \approx p_g(t) e^{\gamma_{p_g}(v-t)}$ and $g(v) \approx g(t) e^{\gamma_g(v-t)}$ we obtain

$$Q_3(t) = \frac{\gamma_{p_g}}{\bar{r} - \gamma_{p_g} - \gamma_g} \cdot p_g(t) g(t).$$

Finally, the assumption $r(v) \approx \bar{r}$ implies $\dot{r}(v) \approx 0$ so that the last line in Eq. 39 is equal to zero. Notice that the integrals yielding Q_3 and Q_4 are bounded

provided that $\bar{r} > \gamma_w + \gamma_{m_h}$ and $\bar{r} > \gamma_{p_g} + \gamma_g$. Both these inequalities can be shown to hold necessarily along an optimal path in order to fulfill the various transversality conditions associated to the state variables. In particular, if $\bar{r} < \gamma_{p_g} + \gamma_g$, the integral yielding $Q_3(t)$ becomes unbounded and does not fulfill intertemporal solvency with the rest of the world: the sequence of future trade deficits explodes at a rate that exceeds the interest on foreign debt.

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