Interplay of Spin and Orbital Angular Momentum in the Proton

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We derive the consequences of the Myhrer-Thomas explanation of the proton spin problem for the distribution of orbital angular momentum on the valence and sea quarks. After QCD evolution, these results are found to be in very good agreement with both recent lattice QCD calculations and the experimental constraints from Hermes and JLab.

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There is no more fundamental question concerning the structure of the nucleon than the distribution of spin and orbital angular momentum over its quarks and gluons [1,2]. Over the 20 years since the European Muon Collaboration (EMC) reported that most of the nucleon spin was not carried as the spin of its quarks and antiquarks [3], there has been tremendous progress in unravelling this mystery. It is now known that the missing spin fraction is of order 2/3 [4,5], rather than 90%, and furthermore, the contribution from polarized gluons is less than 5% (corresponding to $|\Delta G| < 0.3$ [6–11]). It was recently shown by Myhrer and Thomas [12] that the modern spin discrepancy can be rather well explained in terms of standard features of the nonperturbative structure of the nucleon, namely, relativistic motion of the valence quarks [13], the pion cloud required by chiral symmetry [14] and an exchange current contribution associated with the one-gluon-exchange hyperfine interaction [15].

Here, we derive the consequences of the Myhrer-Thomas work for the distribution of orbital angular momentum on the quarks and antiquarks. These results are then tested against the latest measurements of the Generalized Parton Distributions from Hermes and JLab, as well as lattice QCD. We shall see that once the appropriate connection between the quark model and QCD is made at an appropriately low scale, there is a remarkable degree of consistency between all three determinations. This not only gives us considerable confidence in the physical picture provided by Myhrer and Thomas, but it also provides much needed insight into the physical content of the lattice QCD simulations.

The structure of the Letter is that we first track where, in the Myhrer-Thomas picture, the missing spin resides as orbital angular momentum on valence quarks and antiquarks. We then recall that orbital angular momentum is not a renormalization group invariant and argue, following 30 years of similar arguments [16,17], that the model values should be associated with a very low scale. Solving the QCD evolution equations for the up and down quark angular momenta then leads to the remarkable result that the orbital angular momentum of the up and down quarks cross over around 1 GeV^2 so that at the scale of current experiments or lattice QCD simulations L^d (the orbital angular momentum carried by down *and* antidown quarks) is positive and greater than L^u, which tends to be negative.

Consider first the relativistic motion of the valence quarks, described (e.g.) by solving the Dirac equation for a spin up particle in an s state. The lower component of the corresponding spinor has the quark spin predominantly down (i.e., spin down to spin up in the ratio 2/3:1/3) because the corresponding *p*-wave orbital angular momentum is up. Thus, the relativistic correction which lowers the quark spin fraction to about 65%, leads to 35% of the proton spin being carried as valence quark orbital angular momentum. If, for simplicity, we start with an SU(6) wave function, the $u - d$ components are in the ratio $+4/3: -1/3$ – c.f. line 2 of Table I.

As originally derived by Hogaasen and Myhrer [18], the exchange current correction to spin dependent quantities, such as baryon axial charges and magnetic moments, arising from the widely used one-gluon-exchange hyperfine interaction, is dominated by those diagrams involving excitation of a *p*-wave antiquark. The total correction to the spin, which involved the same bag model matrix elements, was found [12] to be $\Sigma \to \Sigma - 0.15$. In this case, the 15% of the proton spin lost to quarks is converted to orbital angular momentum of the *p*-wave antiquark—c.f. line 3 of Table I.

The pion cloud of the nucleon required by chiral symmetry [19–21] leads to a multiplicative correction to the

TABLE I. Distribution of the fraction of the spin of the nucleon as spin and orbital angular momentum of its quarks at the model scale. Successive lines down the table show the result of adding a new effect to all the preceding effects.

	$L^{u+\bar{u}}$	$L^{d+\bar{d}}$	Σ
Nonrelativistic	0	0	1.00
Relativistic	0.46	−0.11	0.65
OGE	0.52	−0.02	0.50
Pion cloud	0.50	0.12	0.38

nucleon spin, $Z - \frac{1}{3}P_{N\pi} + \frac{5}{3}P_{\Delta\pi}$ [22] of order 0.75 to 0.80. For the $N\pi$ Fock component of the nucleon wave function, the angular momentum algebra is identical to that of the lower component of the quark spinor mentioned above. That is, the pion tends to have positive (p-wave) orbital angular momentum, while the N spin is down. From the point of view of a deep inelastic probe, the pion is (predominantly) a quark-antiquark pair, but since they are coupled to spin zero, they contribute nothing to the spin structure function.

The flavor structure of the pion-baryon Fock components needs a little care; for example, the dominant $N\pi$ component is $n\pi^+$, so the pion orbital angular momentum in this case is shared by a u-quark and a $\bar{d}$-antiquark—leading naturally to an excess of $\bar{d}$ quarks in the proton sea [23]. The final distribution of spin and orbital angular momentum, obtained after applying the pionic correction to the relativistic quark model, including the effect of the one-gluon-exchange hyperfine interaction, is shown in the final line of Table I.

The very clear physical picture evident from Table I is that the spin of the proton resides predominantly as orbital angular momentum of the u (and $\bar{u}$) quarks. In contrast, the d (and $\bar{d}$) quarks carry very little orbital angular momentum. The total angular momentum is shared between the u (and $\bar{u}$) quarks, J^u, and the d (and $\bar{d}$) quarks, J^d, in the ratio J^u:$J^d = 0.67$: -0.17.

At first appearance, these results seem to disagree with the indications from lattice QCD [24,25], which suggest that L^d tends to be positive, while L^u is negative. One should observe that these calculations were performed at fairly large quark mass and omit disconnected terms, which may carry significant orbital angular momentum [26] and are certainly needed to account for the U(1) axial anomaly. Nevertheless, the apparent discrepancy is of concern. At this point, we recall the crucial fact that neither the total, nor the orbital angular momentum is renormalization group invariant (RGI) [27]. The lattice QCD values are evaluated at a scale set by the lattice spacing, around 4 GeV^2. On the other hand, we have not identified the scale corresponding to the values derived in our chiral quark model.

This problem has been considered for more than 30 years [16], driven initially by the fact that in a typical, valence-dominated quark model, the fraction of momentum carried by the valence quarks is near 100%, whereas at 4 GeV^2, the experimentally measured fraction is nearer 35%. Given that QCD evolution implies that the momentum carried by valence quarks is a monotonically decreasing function of the scale, the only place to match a quark model to QCD is at a low scale, Q_0. Early studies within the bag model found this scale to be considerably less than 1 GeV [17].

Over the last decade, this idea has been used with remarkable success to describe the data from HERA, over an enormous range of x and Q^2, starting from a valence-dominated set of input parton distributions at a scale of order 0.4 GeV [28]. A similar scale is needed to match parton distributions calculated in various modern quark models to experimental data [29]. Indeed, one may view the choice of starting scale as part of the definition of the model. We note that the comparison between theory and experiment after QCD evolution is not very sensitive to the order of perturbation theory at which one works. However, what does change is the unphysical starting scale. In the present work, we show results at leading order, which also avoids questions of scheme dependence.

The QCD evolution equations for angular momentum in the flavor singlet case were studied by Ji, Tang, and Hoodbhoy [27]. The scheme used corresponds to the choice of a renormalization scheme which preserves chiral symmetry, rather than gauge symmetry [30,31], so that Σ is scale invariant. Explicit solutions for $\Delta G(t)$, $L^{u+d+s}(t)$, and $L^g(t)$ are given in Ref. [27], where $t = \ln(Q^2/\Lambda_{\rm QCD}^2)$ [recall $\alpha_s(Q^2) = 4\pi/[\beta_0 \ln(Q^2/\Lambda_{\rm QCD}^2]$ and $\beta_0 = 11 - 2N_f/3$]. We also need the solution for the nonsinglet case, $L^{u-d} \equiv L^u - L^d$, which is much simpler. Specializing to the case of 3 active flavors ($N_f = 3$), we find

$$L^{u-d}(t) + \frac{\Delta u - \Delta d}{2} = \left(\frac{t}{t_0}\right)^{-(32/9\beta_0)} \left(L^{u-d}(t_0) + \frac{\Delta u - \Delta d}{2}\right). \tag{1}$$

One can also solve for the nonsinglet combination $L^{u+d} - 2L^s$ and hence obtain explicit expressions for L^u and L^d (assuming, as in the Myhrer-Thomas work, that $\Delta s = L^s = 0$ at the model scale),

$$\begin{aligned} L^{u(d)} = & -\frac{\Delta u}{2}\left(-\frac{\Delta d}{2}\right) + 0.06 + \frac{1}{3}\left(\frac{t}{t_0}\right)^{-(50/81)} \\ & \times \left[L^{u+d}(t_0) + \frac{\Sigma}{2} - 0.18\right] + \frac{1}{6}\left(\frac{t}{t_0}\right)^{-(32/81)} \\ & \times \left[L^{u+d}(t_0) \pm 3L^{u-d}(t_0) \pm g_A^{(3)} + \frac{\Sigma}{2}\right]. \end{aligned} \tag{2}$$

We are now in a position to evaluate the total and orbital angular momentum carried by each flavor of quark as a function of Q^2, given some choice of initial conditions. Choosing $N_f = 3$, $\Lambda_{\rm QCD} = 0.24$ GeV, and $Q_0 = 0.4$ GeV, together with the values given in Table I [and $L(t_0) = \Delta G(t_0) = 0$], we find the results shown in Fig. 1. The behavior of J^u and J^d is relatively simple, with the former decreasing fairly rapidly at low Q^2 and the latter increasing. Both settle down to slow variation above 1 GeV^2, with the sum around 60% of the total nucleon spin—the rest being carried as orbital angular momentum and spin by the gluons. A similar result has also been reported in the context of the chiral quark soliton model [32].

While the behavior of $J^{u,d}$ is unremarkable, the corresponding behavior of $L^{u,d}$ is spectacular. L^u is large and L^d

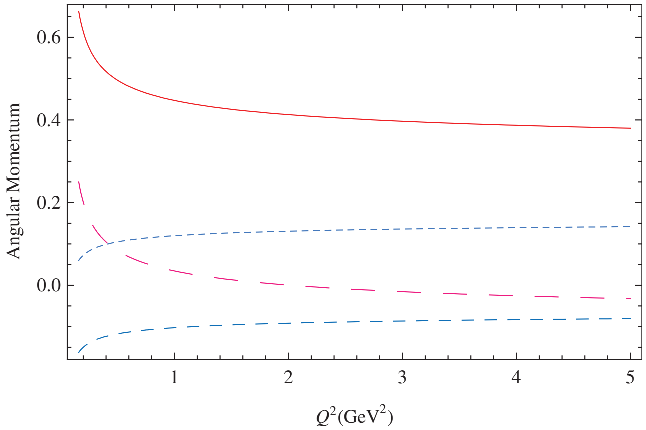

FIG. 1 (color). Evolution of the total angular momentum and the orbital angular momentum of the up and down quarks in the proton—from top to bottom (at 4 GeV^2): J^u (solid line), L^d (smallest dashed line), L^u (largest dashed line), and J^d (middle length dashed line). In this case, it is assumed that the gluons carry no spin or orbital angular momentum at the model scale (0.4 GeV).

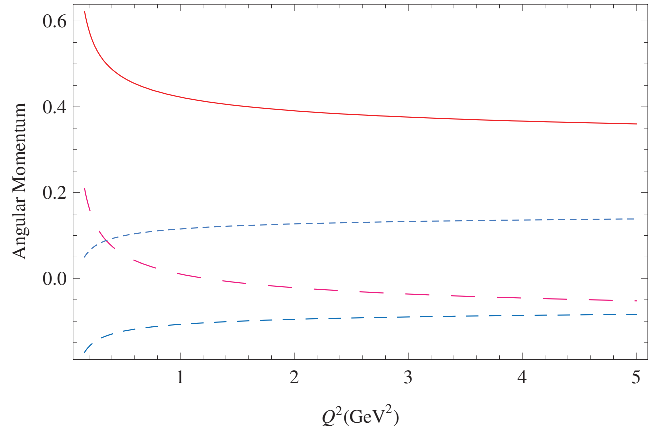

FIG. 2 (color). Evolution of the total angular momentum and the orbital angular momentum of the up and down quarks in the proton—from top to bottom (at 4 GeV^2): J^u (solid line), L^d (smallest dashed line), L^u (largest dashed line), and J^d (middle length dashed line). In this case, it is assumed that the gluons carry 0.1 units of angular momentum at the model scale (0.4 GeV).

small at the model scale but they very rapidly cross and settle down inverted above 1 GeV^2. The reason for this behavior is easily understood because asymptotically L^u and L^d tend to $0.06 - \Delta u/2$ and $0.06 - \Delta d/2$, or -0.36 and $+0.28$, respectively. This is a model independent result and it is simply a matter of how fast QCD evolution takes one from the familiar physics at the model scale to the asymptotic limit.

As we have already noted, the lattice QCD data for the orbital angular momentum carried by the u and d quarks have a number of systematic errors. Disconnected terms are as yet uncalculated, and the data need to be extrapolated over a large range in both pion mass and momentum transfer in order to extract the physical values of J^u and J^d. Nevertheless, for all these cautionary remarks, the results just reported are consistent with the latest lattice results of Hägler *et al.* [24]. For example, they report J^{u+d} in the range 0.25 to 0.29 at the physical pion mass, in comparison with 0.30 in the calculation reported above. They also report $L^{u+d} \sim 0.06$ in comparison with 0.11 in this work. Of course, given the omission of disconnected terms in the lattice simulations, the result for L^{u-d} may be more reliable. The LHPC Collaboration reports $L^{u-d} = -0.124 \pm 0.023$ in Ref. [33] (where the error is obtained by combining errors on L^u and L^d in quadrature), while our present result is -0.16 ± 0.05 [34]. Finally, the qualitative feature that L^d is positive and bigger than L^u is, as we have explained, clearly reproduced in the current work.

Although it is clear that ΔG is too small to give a major correction to the spin sum rule through the axial anomaly [35,36] [e.g., $-N_f \alpha_s \Delta G/(2\pi) \sim 0.05$ for $\Delta G = 0.3$ at $Q^2 = 3$ GeV^2], it can still be nonzero. As just one example of the effect of a small gluon spin fraction at the model scale, in Fig. 2, we show the evolution of the angular momentum on the u and d quarks if ΔG is set to 0.1 at the starting scale (and $L^{u(d)}$ lowered proportionately to preserve the proton spin). While the qualitative behavior is identical, there are nontrivial quantitative changes. In particular, L^u moves down by about 0.04 and J^{u+d} moves down to 0.26 at 4 GeV^2. We note that the nature of the QCD evolution is such that the changes in the values of L^u and L^d at 4 GeV^2 are considerably smaller than at the model scale.

The experimental extraction of information about the quark angular momentum is still in its very early stage of development. One needs to rely on a model to analyze the experimental data, which are still at sufficiently low Q^2 that one cannot be sure that the handbag mechanism really dominates. Nevertheless, the combination of deeply virtual Compton scattering (DVCS) data on the proton from Hermes [37,38] and the neutron from JLab [39] (both at a scale $Q^2 \sim 2$ GeV^2), provides two constraints on J^u and J^d, within the model of Goeke *et al.* [40,41], as shown in Fig. 3. Also shown there is the prediction of the present work [34]. Note that the error bands are the purely experimental (predominantly statistical) errors, and there is, as yet, no information on the possible systematic variation corresponding to a change of model. The exploration of the model dependence is clearly a high priority for future work. Nevertheless, within the present uncertainties, most notably the relatively low Q^2 of the data and the unknown model dependence of the extraction of $J^{u(d)}$, there is a remarkable degree of agreement.

In summary, we have shown that the resolution of the spin crisis proposed by Myhrer and Thomas, which implies that the majority of the spin of the proton resides on u and $\bar{u}$ quarks, after QCD evolution is consistent with current determinations from lattice QCD and experimental data

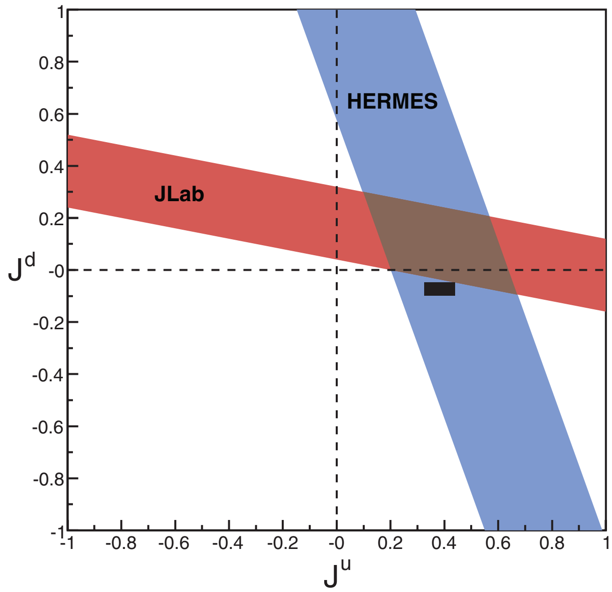

FIG. 3 (color). Comparison between the constraints on the total angular momentum carried by u and d quarks in the proton, derived from experiments on DVCS at Hermes [37,38] and JLab [39] at a scale of order 2 GeV^2, and the model of Myhrer and Thomas (the small dark rectangle) as explained in this work.

on deeply virtual Compton scattering. The effect of QCD evolution in inverting the orbital angular momentum of the u and d quarks in the model was especially important. For the future, we look forward to improvements in both these areas.

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