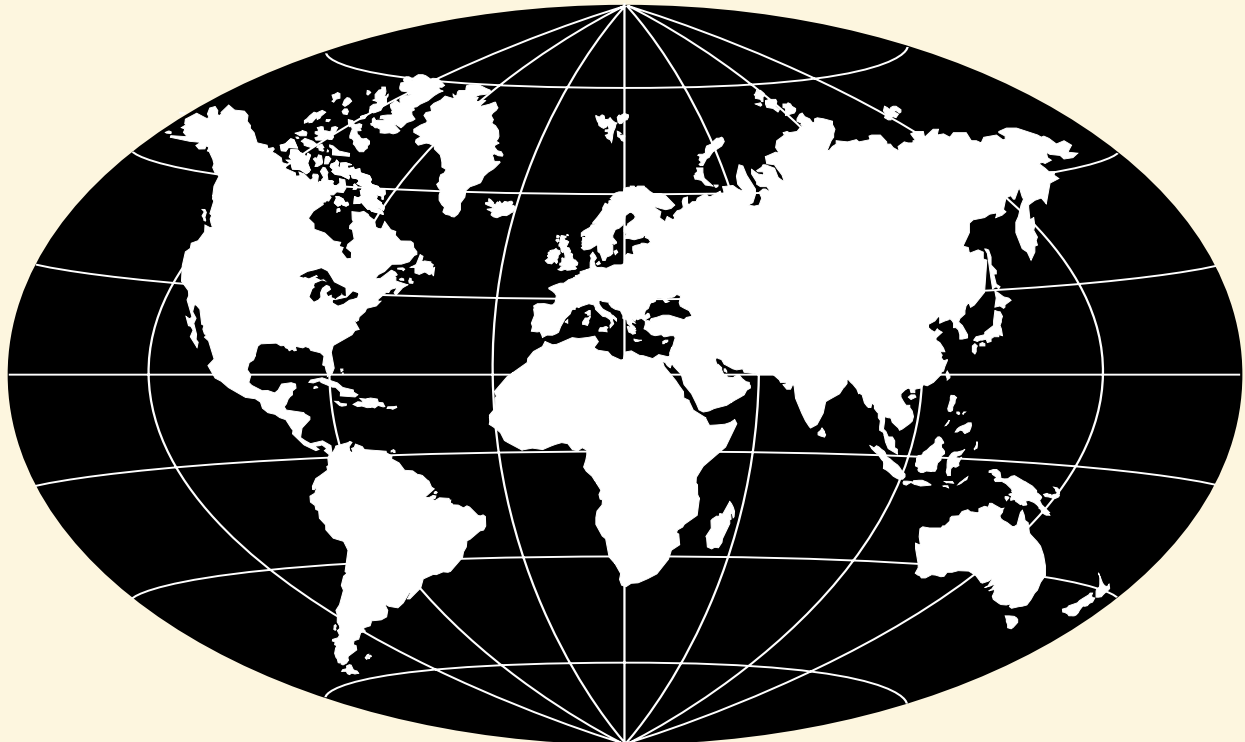**CIMMYT**

Sustainable

Maize and Wheat

Systems for the Poor

Interpolation Techniques for

Climate Variables

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Abstract: This paper examines statistical approaches for interpolating climatic data over large regions, providing a brief introduction to interpolation techniques for climate variables of use in agricultural research, as well as general recommendations for future research to assess interpolation techniques. Three approaches—1) inverse distance weighted averaging (IDWA), 2) thin plate smoothing splines and 3) co-kriging—were evaluated for a 20,000 km^2 square area covering the state of Jalisco, Mexico. Taking into account valued error prediction, data assumptions, and computational simplicity, we recommend use of thin-plate smoothing splines for interpolating climate variables.

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Summary

Understanding spatial variation in climatic conditions is key to many agricultural and natural resource management activities. However, the most common source of climatic data is meteorological stations, which provide data only for single locations. This paper examines statistical approaches for interpolating climatic data over large regions, providing a brief introduction to interpolation techniques for climate variables of use in agricultural research, as well as general recommendations for future research to assess interpolation techniques. Three approaches—1) inverse distance weighted averaging (IDWA), 2) thin plate smoothing splines and 3) co-kriging— were evaluated for a 20,000 km^2 square area covering the state of Jalisco, Mexico. Monthly mean data were generated for 200 meteorological stations and a digital elevation model (DEM) based on 1 km^2 grid cells was used. Due to low correlation coefficients between the prediction variable (precipitation) and the co-variable (elevation), interpolation using co-kriging was carried out for only four months. Validation of the surfaces using two independent sets of test data showed no difference among the three techniques for predicting precipitation. For maximum temperature, splining performed best. IDWA does not provide an error surface and therefore splining and co-kriging were preferred. However, the rigid prerequisites of co-kriging regarding the statistical properties of the data used (e.g., normal distribution, non-stationarity), along with its computational demands, may put this approach at a disadvantage. Taking into account error prediction, data assumptions, and computational simplicity, we recommend use of thin-plate smoothing splines for interpolating climate variables.

Acknowledgments

Interpolation Acronyms and Terminology

CIMMYT	International Maize and Wheat Improvement Center.
DEM	Digital elevation model; a digital description of a terrain in the shape of data and algorithms.
ERIC	Extractor Rápido de Información Climatológica.
GCV	Generalized cross validation. A measure of the predictive error of the fitted surface which is calculated by removing each data point, one by one, and calculating the square of the difference between each removed data point from a surface fitted to all the other points.
IDWA	Inverse distance weighted averaging.
IMTA	Instituto Mexicano de Tecnología del Agua.
INIFAP	Mexican National Institute of Forestry, Agriculture, and Livestock Research (Instituto Nacional de Investigaciones Forestales y Agropecuarias).
Interpolation	The procedure of estimating the value of properties at unsampled sites within an area covered by sampled points, using the values of properties from those points.

Interpolation Techniques for Climate Variables

Introduction

Geographic information systems (GIS) and modeling are becoming powerful tools in agricultural research and natural resource management. Spatially distributed estimates of environmental variables are increasingly required for use in GIS and models (Collins and Bolstad 1996). This usually implies that the quality of agricultural research depends more and more on methods to deal with crop and soil variability, weather generators (computer applications that produce simulated weather data using climate profiles), and spatial interpolation—the estimation of the value of properties at unsampled sites within an area covered by sampled points, using the data from those points (Bouman et al. 1996). Especially in developing countries, there is a need for accurate and inexpensive quantitative approaches to spatial data acquisition and interpolation (Mallawaarachchi et al. 1996).

Most data for environmental variables (soil properties, weather) are collected from point sources. The spatial array of these data may enable a more precise estimation of the value of properties at unsampled sites than simple averaging between sampled points. The value of a property between data points can be interpolated by fitting a suitable model to account for the expected variation.

A key issue is the choice of interpolation approach for a given set of input data (Burrough and McDonnell 1998). This is especially true for areas such as mountainous regions, where data collection is sparse and measurements for given variables may differ significantly even at relatively reduced spatial scales

(Collins and Bolstad 1996). Burrough and McDonnell (1998) state that when data are abundant most interpolation techniques give similar results. When data are sparse, the underlying assumptions about the variation among sampled points may differ and the choice of interpolation method and parameters may become critical.

With the increasing number of applications for environmental data, there is also a growing concern about accuracy and precision. Results of spatial interpolation contain a certain degree of error, and this error is sometimes measurable. Understanding the accuracy of spatial interpolation techniques is a first step toward identifying sources of error and qualifying results based on sound statistical judgments.

Interpolation Techniques

One of the most simple techniques is interpolation by drawing boundaries—for example Thiessen (or Dirichlet) polygons, which are drawn according to the distribution of the sampled data points, with one polygon per data point and the data point located in the center of the polygon (Fig. 1). This technique, also referred to as the "nearest neighbor" method, predicts the attributes of unsampled points based on those of the nearest sampled point and is best for qualitative (nominal) data, where other interpolation methods are not applicable. Another example is the use of nearest available weather station data, in absence of other local data (Burrough and McDonnell 1998). In contrast to this discrete method, all other methods embody a

model of continuous spatial change of data, which can be described by a smooth, mathematically delineated surface.

Methods that produce smooth surfaces include various approaches that may combine regression analyses and distance-based weighted averages. As explained in more detail below, a key difference among these approaches is the criteria used to weight values in relation to distance. Criteria may include simple distance relations (e.g., inverse distance methods), minimization of variance (e.g., kriging and co-kriging), minimization of curvature, and enforcement of smoothness criteria (splining). On the basis of how weights are chosen, methods are "deterministic" or "stochastic." Stochastic methods use statistical criteria to determine weight factors. Examples of each include:

- *Deterministic techniques:* Thiessen polygons, inverse distance weighted averaging.
- *Stochastic techniques*: polynomial regression, trend surface analysis, and (co)kriging.

Thiessen polygons : ?? is closest to 141 , therefore ??=141 mm.
Inverse distance weighted averaging: The value for ?? is calculated by weighting the values of all 5 points by the inverse of their distance squared to point ??. After interpolation, ?? = 126 mm. The number of neighbors taken into account is a choice in this interpolation procedure.
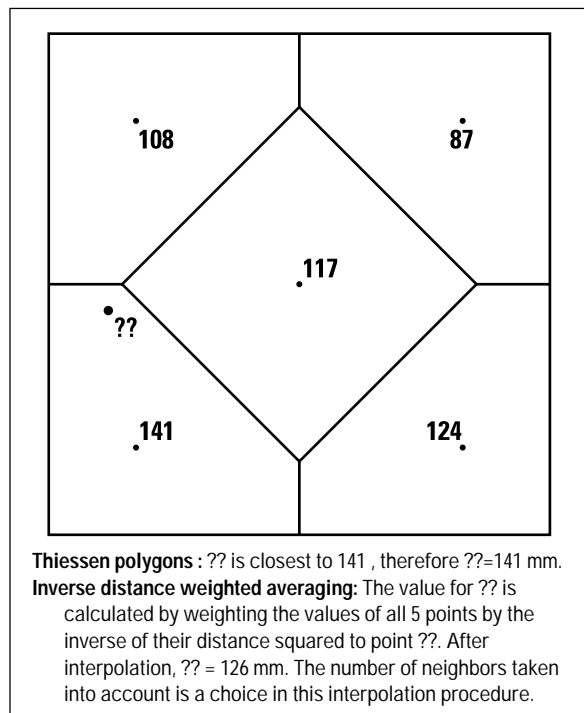
Figure 1. An example of interpolation using Thiessen polygons and inverse distance weighted averaging to predict precipitation.

Interpolation techniques can be "exact" or "inexact." The former term is used in the case of an interpolation method that, for an attribute at a given, unsampled point, assigns a value identical to a measured value from a sampled point. All other interpolation methods are described as "inexact." Statistics for the differences between measured and predicted values at data points are often used to assess the performance of inexact interpolators.

Interpolation methods can also be described as "global" or "local." Global techniques (e.g. inverse distance weighted averaging, IDWA) fit a model through the prediction variable over all points in the study area. Typically, global techniques do not accommodate local features well and are most often used for modeling long-range variations. Local techniques, such as splining, estimate values for an unsampled point from a specific number of neighboring points. Consequently, local anomalies can be accommodated without affecting the value of interpolation at other points on the surface (Burrough 1986). Splining, for example, can be described as deterministic with a local stochastic component (Burrough and McDonnell 1998; Fig. 1).

For soil data, popular methods include kriging, co-kriging, and trend surface analysis (McBratney and Webster 1983; Yates and Warrick 1987; Stein et al. 1988a, 1989a,1989b). In climatology, IDWA, splining, polynomial regression, trend surface analysis, kriging, and co-kriging are common approaches (Collins and Bolstad 1996; Hutchinson and Corbett 1995; Phillips et al. 1992; Hutchinson 1991; Tabios and Salas 1985). For temperature interpolations, methods often allow for an effect of the adiabatic lapse rate (decrease in tempera-ture with elevation) (e.g. Jones 1996). An overview and comparison of interpolation techniques, their assump-tions, and their limitations is presented in Table 1.

In the following section, three interpolation techniques commonly used in interpolating climate data—IDWA, splining and (co)kriging—are described in more detail.

Inverse distance weighted averaging—IDWA is a deterministic estimation method whereby values at unsampled points are determined by a linear combination of values at known sampled points. Weighting of nearby points is strictly a function of distance—no other criteria are considered. This approach combines ideas of proximity, such as Thiessen polygons, with a gradual change of the trend surface. The assumption is that values closer to the unsampled location are more representative of the value to be estimated than values from samples further away. Weights change according to the linear distance of the samples from the unsampled point; in other words, nearby observations have a heavier weight. The spatial arrangement of the samples does not affect the weights. This approach has been applied extensively in the mining industry, because of its ease of use (Collins and Bolstad 1996). Distance-based weighting methods have been used to interpolate climatic data (Legates and Willmont 1990; Stallings et

Table 1. A comparison of interpolation techniques.

Method	Deterministic/ stochastic	Local/global	Transitions (abrupt/gradual)	Exact interpolator	Limitations of the procedure	Best for	Computing load	Output data structure	Assumptions of interpolation model
Classification	Deterministic 'soft' information	Global	Abrupt if used alone	No	Delineation of areas and classes may be subjective. Error assesment limited to within-class standard derivations.	Quick assessments when data are sparse. Removing systematic differences before continuous interpolation from data points.	Small	Classified polygons	Homogeneity within boundaries
Trend surfaces	Essentially deterministic (empirical)	Global	Gradual	No	Physical meaning of trend may be unclear. Outliers and edge effects may distort surface. Error assessment limited to goodness of fit	Quick assessment and removal of spatial trends	Small	Continuous, gridded surface	Phenomeno-logical explanation of trend, normally distributed data
Regression models	Essentially deterministic (empirical-statistical)	Global with local refinements	Gradual if inputs have gradual variation	No	Results depend on the fit of the regression model and the quality and detail of the input data surfaces. Error assessment possible if input errors are known.	Simple numerical modeling of expensive data when better methods are not available or budgets are limited	Small	Polygons or continous, gridded surface	Phenomenological explanation of regression model
Thiessen polygons (proximal mapping)	Deterministic	Local	Abrupt	Yes	No errors assessment, only one data point per polygon. Tessellation pattern depends on distribution of data.	Nominal data from point observations	Small	Polygons or gridded surface	Best local predictor is nearest data point
Pycnophylatic interpolation	Deterministic	Local	Gradual	No, but conserves volumes	Data inputs are counts or densities	Transforming step-wise patterns of population counts to continous surfaces	Small-moderate	Gridded surface or contours	Continous, smooth variation is better than ad hoc areas
Linear interpolation	Deterministic	Local	Gradual	Yes	No error assessments	Interpolating from point data when data densities are high, as in converting gridded data from one project to another	Small	Gridded surface	Data densities are so large that linear approximation is no problem
Moving averages and inverse distance weighting	Deterministic	Local	Gradual	Not with regular smoothing window, but can be forced	No error assessments. Results depend on size of search window and choice of weighting parameter. Poor choice of window can give artifacts when used with high data densities such as digitized contours	Quick interpolation from sparse data on regular grid or irregularly spaced samples	Small	Gridded surface	Underlying surface is smooth
Thin plate splines	Deterministic with local stochasatic component	Local	Gradual	Yes, within smoothing limits	Goodness of fit possible, but within the assumption that the fitted surface is perfectly smooth.	Quick interpolation (univariate or multivariate) of digital elevation data and related attributes to create DEMs from moderately detailed data.	Small	Gridded surface, contour lines	Underlying surface is smooth everywhere
Kriging	Stochastic	Local with global variograms. Local with local variograms when stratified. Local with global trends	Gradual	Yes	Error assessment depends on variogram and distribution of data points and size of interpolated blocks. Requires care when modeling spatial correlation structures.	When data are sufficient to compute variograms, kriging provides a good interpolator for sparse data. Binary and nominal data can be interpolated with indicator kriging. Soft information can also be incorporated as trends or stratification. Multivariate data can be interpolated with co-kriging.	Moderate	Gridded surface	Interpolated surface is smooth. Statistical stationarity and the intrinsic hyphotesis.
Conditional simulation	Stochastic	Local with global variograms. Local with local variograms when stratified. Local with global trends.	Irregular	No	Understanding of underlying stochastic process and models is necessary.	Provides an excellent estimate of the range of possible values of an attribute at unsampled locations that are necessary for Monte Carlo analysis of numerical models, also for error assessments that do not depend on distribution of the data but on local values.	Moderate-heavy	Gridded surfaces	Statistical stationarity and the intrinsic hypothesis

Source: Based on Burrough and McDonnell 1998.

al. 1992). The choice of power parameter (exponential degree) in IDWA can significantly affect the interpolation results. At higher powers, IDWA approaches the nearest neighbor interpolation method, in which the interpolated value simply takes on the value of the closest sample point. IDWA interpolators are of the form:

$$\hat{y}(x) = \Sigma \lambda_i\, y(x_i)$$

where:

$\lambda_i =$ the weights for the individual locations.

$y(x_i) =$ the variables evaluated in the observation locations.

The sum of the weights is equal to 1. Weights are assigned proportional to the inverse of the distance between the sampled and prediction point. So the larger the distance between sampled point and prediction point, the smaller the weight given to the value at the sampled point.

Splining—This is a deterministic, locally stochastic interpolation technique that represents two dimensional curves on three dimensional surfaces (Eckstein 1989; Hutchinson and Gessler 1994). Splining may be thought of as the mathematical equivalent of fitting a long flexible ruler to a series of data points. Like its physical counterpart, the mathematical spline function is constrained at defined points.

The polynomial functions fitted through the sampled points are of degree m or less. A term r denotes the constraints on the spline. Therefore:

- When $r = 0$, there are no constraints on the function.
- When $r = 1$, the only constraint is that the function is continuous.
- When $r = m+1$, constraints depend on the degree m.

For example, if $m = 1$ there are two constraints ($r = 2$):

- The function has to be continuous.

- The first derivative of the function has to be continuous at each point.

For $m = 2$, the second derivative must also be continuous at each point. And so on for $m = 3$ and more. Normally a spline with $m = 1$ is called a "linear spline", a spline with $m = 2$ is called a "quadratic spline," and a spline with $m = 3$ is called a "cubic spline". Rarely, the term "bicubic" is used for the three-dimensional situation where surfaces instead of lines need to be interpolated (Burrough and McDonnell 1998).

Thin plate smoothing splines—Splining can be used for exact interpolation or for "smoothing." Smoothing splines attempt to recover a spatially coherent—i.e., consistent—signal and remove the noise (Hutchinson and Gessler 1994). Thin plate smoothing splines, formerly known as "laplacian smoothing splines," were developed principally by Wahba and Wendelberger (1980) and Wahba (1990). Applications in climatology have been implemented by Hutchinson (1991), Hutchinson (1995), and Hutchinson and Corbett (1995). Hutchinson (1991) presents a model for partial thin plate smoothing splines with two independent spline variables:

$$q_i = f(x_i,\, y_i) + \sum_{j=1}^{p} \beta_i \psi_{ij} + \varepsilon_i \ (i = 1,......,n)$$

where:

$f(x_i, y_i) =$ unknown smooth function

$\beta_j =$ set of unknown parameters

$x_i, y_i, \psi_{ij} =$ independent variables

$\varepsilon_i =$ independent random errors with zero mean and variance $d_i \sigma^2$

$d_i =$ known weights

The smoothing function f and the parameters β_j are estimated by minimizing:

$$\sum_{i=1}^{n} \left[\left(q_i - f(x_i, y_i - \sum_{j=1}^{p} \beta_j\, \psi_{ij}) / d_i \right]^2 + \lambda\, J_m(f) \right.$$

where:

$$J_m(f) = \text{a measure of the smoothness of } f \text{ defined in terms of } m^{\text{th}} \text{ order derivates of } f$$

$$\lambda = \text{a positive number called the smoothing parameter}$$

The solution to this partial thin plate spline becomes an ordinary thin plate spline, when there is no parametric sub-model (i.e.; when p=0).

The smoothing parameter λ is calculated by minimizing the generalized cross validation function (GCV). This technique is considered relatively robust, since the method of minimizing of the GCV directly addresses the predictive accuracy and is less dependent on the veracity of the underlying statistical model (Hutchinson 1995).

Co-kriging and fitting variogram models—Named after its first practitioner, the south-African mining engineer Krige (1951), kriging is a stochastic technique similar to IDWA, in that it uses a linear combination of weights at known points to estimate the value at an unknown point. The general formula for kriging was developed by Matheron (1970). The most commonly applied form of kriging uses a "semi-variogram"—a measure of spatial correlation between pairs of points describing the variance over a distance or lag *h.* Weights change according to the spatial arrangement of the samples. The linear combination of weights are of the form:

$$\sum \lambda_i\, y_i$$

where:

$$y_i = \text{the variables evaluated in the observation locations}$$

$$\lambda_i = \text{the kriging weights}$$

Kriging also provides a measure of the error or uncertainty of the estimated surface.

The semi-variogram and model fitting—The semi-variogram is an essential step for determining the spatial variation in the sampled variable. It provides useful information for interpolation, sampling density, determining spatial patterns, and spatial simulation. The semi-variogram is of the form:

$$\gamma(h) = \tfrac{1}{2}E(y(x) - y(x+h))^2$$

where:

$$\gamma(h) = \text{semi-variogram, dependent on lag or distance } h$$

$$(x, x+h) = \text{pair of points with distance vector } h$$

$$y(x) = \text{regionalized variable } y \text{ at point x}$$

$$y(x)-y(x+h) = \text{difference of the variable at two points separated by } h$$

$$E = \text{mathematical expectation}$$

Two assumptions need to be met to apply kriging: stationarity and isotropy. Stationarity for spatial correlation (necessary for kriging and co-kriging) is based on the assumption that the variables are stationary. When there is stationarity, $\gamma(h)$ does not depend on x, where x is the point location and h is the distance between the points. So the semi-variogram depends only on the distance between the measurements and not on the location of the measurements. Unfortunately, there are often problems of non-stationarity in real-world datasets (Collins and Bollstad 1996; Burrough 1986). Stein et al. (1991a) propose several equations to deal with this issue. In other cases the study area may be stratified into more homogeneous units before co-kriging (Goovaerts 1997); e.g., using soil maps (Stein et al. 1988b).

When there is isotropy for spatial correlation, then $\gamma(h)$ depends only on h. So the semi-variogram depends only on the magnitude of *h* and not on its direction. For example, it is highly likely that the amount of groundwater increases when approaching a

river. In this case there is anisotropy, because the semi-variogram will depend on the direction of *h*. Usually, stationarity is also necessary for the expectation $\mathrm{E}y(x)$, to ensure that the expectation doesn't depend on x and is constant.

From the semi-variogram (Fig 2.), various properties of the data are determined: the sill (A), the range (r), the nugget (C_0), the sill/nugget ratio, and the ratio of the square sum of deviance to the total sum of squares (SSD/SST). The nugget is the intercept of the semi-variogram with the vertical axis. It is the non-spatial variability of the variable and is determined when *h* approaches 0. The nugget effect can be caused by variability at very short distances for which no pairs of observations are available, sampling inaccuracy, or inaccuracy in the instruments used for measurement. In an ideal case (e.g., where there is no measurement error), the nugget value is zero. The range of the semi-variogram is the distance *h* beyond which the variance no longer shows spatial dependence. At *h,* the sill value is reached. Observations separated by a distance larger than the range are spatially independent observations. To obtain an indication of the part of the semi-variogram that shows spatial dependence, the sill:nugget ratio can be determined. If this ratio is close to 1, then most of the variability is non-spatial.
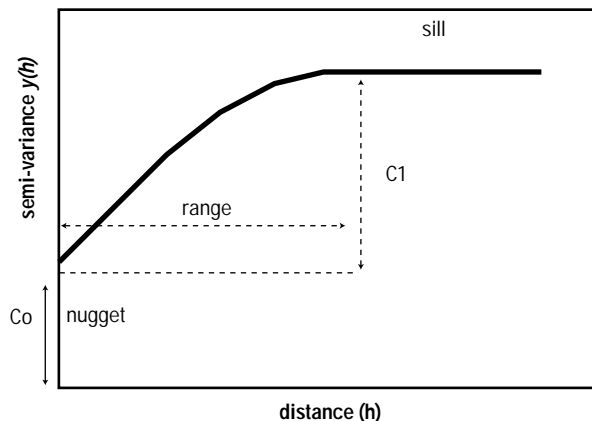
Figure 2. An example of a semi-variogram with range, nugget, and sill.

Normally a "variogram" model is fitted through the empirical semi-variogram values for the distance classes or lag classes. The variogram properties—the sill, range and nugget—can provide insights on which model will fit best (Cressie 1993; Burrough and McDonnell 1998). The most common models are the linear model, the spherical model, the exponential model, and the Gaussian model (Fig. 3). When the nugget variance is important but not large and there is a clear range and sill, a curve known as the spherical model often fits the variogram well.

Spherical model:

$$\gamma(\mathrm{h}){=}\mathrm{C}_0{+}\mathrm{A}*\left(\tfrac{3}{2}\left(\tfrac{\mathrm{h}}{\mathrm{r}}\right){-}\tfrac{1}{2}\left(\tfrac{1}{2}\right)^3\right) \quad \text{for} \quad \mathrm{h}\in 0,\mathrm{r}]$$

$$= \mathrm{C}_0{+}\mathrm{A} \quad \text{for} \quad \mathrm{h}>\mathrm{r}$$

(where = *r* is the range, *h* is lag or distance, and C_0+A is the sill)

If there is a clear nugget and sill but only a gradual approach to the range, the exponential model is often preferred.

Exponential model:

$$\gamma(\mathrm{h}){=}\mathrm{C}_0{+}\mathrm{A}*(1{-}\mathrm{e}^{\frac{\mathrm{h}}{\mathrm{r}}}) \quad \text{for} \quad \mathrm{h}{>}0$$

If the variation is very smooth and the nugget variance is very small compared to the spatially random variation, then the variogram can often best be fitted by a curve having an inflection such as the Gaussian model:

$$\gamma(\mathrm{h}){=}\mathrm{C}_0{+}\mathrm{A}*(1{-}\mathrm{e}^{-(\frac{\mathrm{h}}{\mathrm{r}})^2}) \quad \text{for} \quad \mathrm{h}{>}0$$

All these models are known as "transitive" variograms, because the spatial correlation structure varies with the distance *h.* Non-transitive variograms have no sill
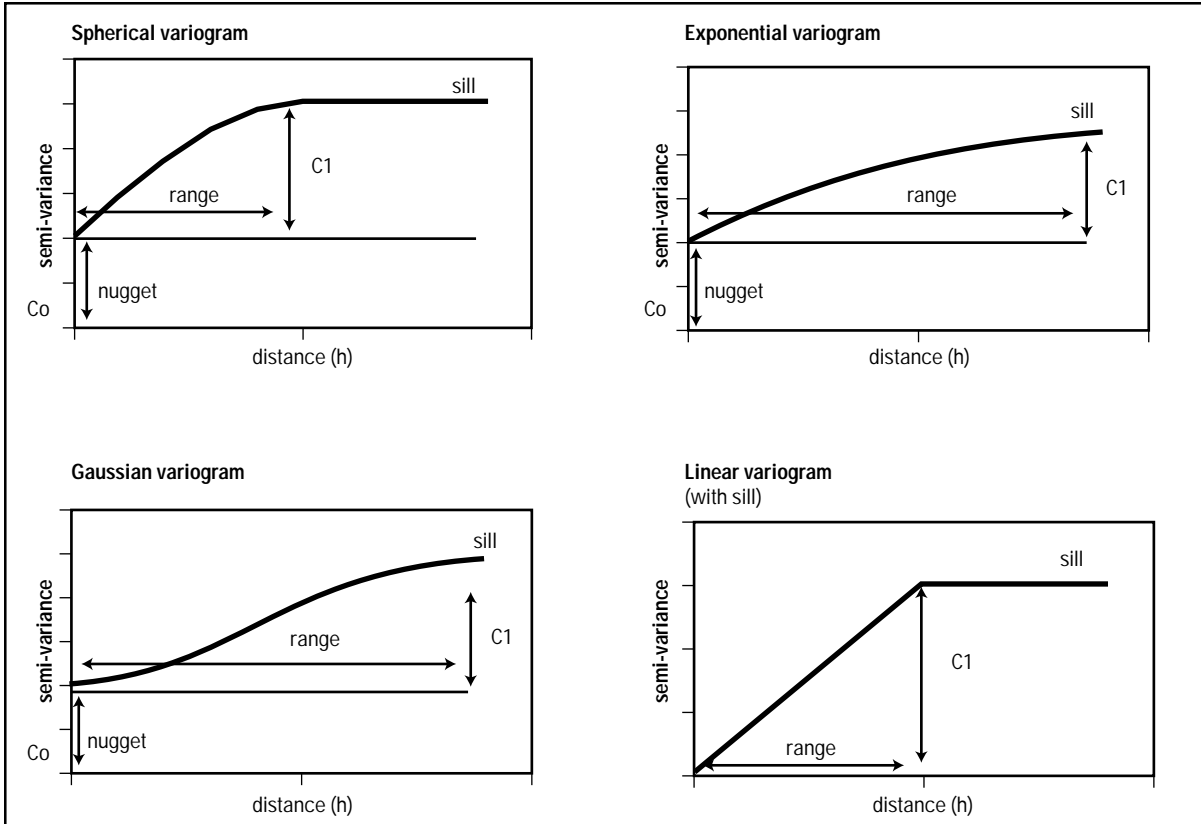
Figure 3. Examples of most commonly used variogram models a) spherical, b) exponential, c) Gaussian, and d) linear.

within the sampled area and may be represented by the linear model:

$$\gamma(h) = Co + bh$$

However, linear models with sill also exist and are in the form of:

$$\gamma(h) = A * \frac{h}{r} \quad \text{for} \quad h \in (o,r]$$

The ratio of the square sum of deviance (SSD) to the total sum of squares (SST) indicates which model best fits the semi-variogram. If the model fits the semi-variogram well, the SSD/SST ratio is low; otherwise, SSD/SST will approach 1. To test for anisotropy, the semi-variogram needs to be determined in a different direction than h. To ensure isotropy, the semi-variogram model should be unaffected by the direction in which h is taken.

Co-kriging—Co-kriging is a form of kriging that uses additional covariates, usually more intensely sampled than the prediction variable, to assist in prediction. Co-kriging is most effective when the covariate is highly correlated with the prediction variable. To apply co-kriging one needs to model the relationship between the prediction variable and a co-variable. This is done by fitting a model through the cross-variogram. Estimation of the cross-variogram is carried out similarly to estimation of the semi-variogram:

$$y_{1,2}(h) = \tfrac{1}{2} E \left((y_1(x) - y_1(x+h))(y_2(x) - y_2(x+h)) \right)$$

High cross-variogram values correspond to a low covariance between pairs of observations as a function of the distance h. When interpolating with co-kriging, the variogram models have to fit the "linear

model of co-regionalization" as described by Journel and Huijbregts (1978) and Goulard and Voltz (1992). (See Annex 1 for a description of the model.) To have positive definiteness, the semi-variograms and the cross-variogram have to obey the following relationship:

$$y_{1,2}(h) \leq \sqrt{y_1(h)y_2(h)}$$

This relationship should hold for all *h*.

The actual fitting of a variogram model is an interactive process that requires considerable judgment and skill (Burrough and McDonnell 1998).

Reviewing Interpolation Techniques

Early reviews of interpolation techniques (Lam 1983; Ripley 1981) often provided little information on their efficacy and did not evaluate them quantitatively. Recent studies, however, have focused on efficacy and quantitative criteria, through comparisons using datasets (Stein et al. 1989a; Stein et al. 1989b; Hutchinson and Gessler 1994; Laslett 1994; Collins and Bolstad 1996). Collins and Bolstad (1996) compared eight spatial interpolators across two regions for two temperature variables (maximum and minimum) at three temporal scales. They found that several variable characteristics (range, variance, correlation with other variables) can influence the choice of a spatial interpolation technique. Spatial scale and relative spatial density and distribution of sampling stations can also be determinant factors. MacEachren and Davidson (1987) concluded that data measurement accuracy, data density, data distribution and spatial variability have the greatest influence on the accuracy of interpolation. Burrough and McDonnell (1998) concluded that most interpolation techniques give similar results when data are abundant. For sparse data the underlying assumptions about the variation among sampled points differ and, therefore, the choice of interpolation method and parameters becomes critical.

The most common debate regards the choice of kriging or co-kriging as opposed to splining (Dubrule 1983; Hutchinson 1989; Hutchinson 1991; Stein and Corsten 1991; Hutchinson and Gessler 1994; Laslett 1994). Kriging has the disadvantage of high computational requirements (Burrough and McDonnell 1998). Modeling tools to overcome some of the problems include those developed by Pannatier (1996). However, the success of kriging depends upon the validity of assumptions about the statistical nature of variation. Several studies conclude that the best quantitative and accurate results are obtained by kriging (Dubrule 1983; Burrough and McDonnell 1998; Stein and Corsten 1991; Laslett 1994). Cristobal-Acevedo (1993) evaluated thin splines, inverse distance weighting, and kriging for soil parameters. His conclusion was that thin splines were the less exact of the three. Collins and Bolstad (1996) confirm what has been said before: splining has the disadvantage of providing no error estimates and of masking uncertainty. Also, it performs much better when dense, regularly-spaced data are available; it is not recommended for irregular spaced data. Martinez Cob and Faci Gonzalez (1994) compared co-kriging to kriging for evapotranspiration and rainfall. Predictions with co-kriging were not as good for evaporation but better for precipitation. However, prediction error was less with co-kriging in both cases.

The debate does not end there. For example, Hutchinson and Gessler (1994) pointed out that most of the aforementioned comparisons of interpolation methods did not examine high-order splines and that data smoothing in splining is achieved in a statistically rigorous fashion by minimizing the generalized cross validation (GCV). Thus, thin plate smooth splining *does* provide a measure of spatial accuracy (Wahba and Wendelberger 1980; Hutchinson 1995).

There appears to be no simple answer regarding choice of an appropriate spatial interpolator. Method performance depends on the variable under study, the spatial configuration of the data, and the underlying

assumptions of the methods. Therefore a method is "best" only for specific situations (Isaaks and Srivastava 1989).

A Case Study for Jalisco, Mexico

The GIS/Modeling Lab of the CIMMYT Natural Resources Group (NRG) is interfacing GIS and crop simulation models to address temporal and spatial issues simultaneously. A GIS is used to store the large volumes of spatial data that serve as inputs to the crop models. Interfacing crop models with a GIS requires detailed spatial climate information. Interpolated climate surfaces are used to create grid-cell-size climate files for use in crop modeling. Prior to the creation of climate surfaces, we evaluated different interpolation techniques—including inverse distance weighting averaging (IDWA), thin plate smoothing splines, and co-kriging—for climate variables for 20,000 km^2 roughly covering the state of Jalisco in northwest Mexico. While splining and co-kriging have been described as formally similar (Dubrule 1983; Watson 1984), this study aimed to evaluate practical use of related techniques and software.

Material and methods—Regarding *software*, the ArcView spatial analyst (ESRI 1998) was used for inverse distance weighting interpolation. For thin plate smoothing splines, the ANUSPLIN 3.2 multi-module package (Hutchinson 1997) was used. The first module or program (either SPLINAA or SPLINA[1]) is used to fit different partial thin plate smoothing spline functions for more independent variables. Inputs to the module are a point data file and a covariate grid. The program yields several output files:

- A large residual file which is used to check for data errors.
- An optimization parameter file containing parameters used to calculate the optimum smoothing parameter(s).

- A file containing the coefficients defining the fitted surfaces that are used to calculate values of the surfaces by LAPPNT and LAPGRD.
- A file that contains a list of data and fitted values with Bayesian standard error estimates (useful for detecting data errors).
- A file that contains an error covariance matrix of fitted surface coefficients. This is used by ERRPNT and ERRGRD to calculate standard error estimates for the fitted surfaces.

The program LAPGRD produces the prediction variable surface grid. It uses the surface coefficients file from the SPLINAA program and the co-variable grid, in this case the DEM. The program ERRGRD calculates the error grid, which depicts the standard predictive error.

For co-kriging, the packages SPATANAL and CROSS (Staritsky and Stein 1993), WLSFIT (Heuvelink 1992), and GSTAT (Pebesma 1997) were used. The SPATANAL and CROSS programs were used to create semi-variograms and cross-variograms respectively from ASCII input data files. The WLSFIT program was used to get an initial model fit to the semi-variogram and cross-variogram. GSTAT was used to improve the model. GSTAT produces a prediction surface grid and a prediction variance grid. A grid of the prediction error can be produced from the prediction variance grid using the map calculation procedure in the ArcView Spatial analyst.

Data—The following sources were consulted:

- Digital elevation model (DEM): 1 km^2 (USGS 1997).
- Daily precipitation and temperature data from 1940 to 1990, Instituto Mexicano de Tecnología del Agua (IMTA; 868 stations/20,000 km^2).

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1 This depends on the type of variable to be predicted. The SPLINAA program uses year to year monthly variances to weigh sampled points and is more suitable for precipitation, the SPLINA program uses month to month variance to weigh sampling points and is more suitable for temperature.

- Monthly precipitation data from 1940 to 1996, Instituto Nacional de Investigaciones Forestales y Agropecuarias (INIFAP; 100 stations/20,000 km²).

In this study, daily precipitation and temperature (maximum and minimum) data were extracted from the Extractor Rápido de Información Climatológica (ERIC, IMTA 1996). We selected a square (106° W; -101° W; 18° N; 23° N) that covered the state of Jalisco, northwest Mexico, encompassing approximately 20,000 km² (Fig. 4). A subset of station data from 1965-1990 was "cleaned up" using the Pascal program and the following criteria:

- If more then 10 days were missing from a month, the month was discarded.
- If more then 2 months were missing from a year, the year was discarded.
- If fewer than 19 or 16 years were available for a station, the station was discarded.

Data for monthly precipitation from 180 stations were provided by INIFAP. There were 70 data points with station numbers identical to some in ERIC (IMTA 1996). The coordinates from these station numbers were compared and, in a few cases, were different. INIFAP had verified the locations for Jalisco stations using a geographic positioning system, so the INIFAP coordinates were used instead of those from ERIC, wherever there were differences of more than 10 km (Table 2). For the other states in the selected area, we used ERIC data. In four cases stations had identical

Table 2. Stations for which geographic coordinates were changed to INIFAP values.

Station NR.	Name	ERIC latitude	ERIC longitude	INIFAP latitude	INIFAP longitude
14089	La Vega, Teuchitlan	20.58	-103.75	20.595	- 103.844
14073	Ixtlahuacan del Rio	20.87	-103.33	20.863	-103.241
14043	Ejutla, Ejutla	19.97	-104.03	19.90	-104.167
14006	Ajojucar, Teocaltiche	21.42	-102.40	21.568	-102.435

coordinates (Table 3), and the second station was removed from the dataset that was to be used for interpolation.

Table 3. Station numbers with identical geographic coordinates (stations in bold were kept for interpolation).

Station NR.	Latitude	Longitude
16164	**19.42**	**102.07**
16165	19.42	102.07
16072	**19.57**	**102.58**
16073	19.57	102.58
18002	**21.05**	**104.48**
18040	21.05	104.48

Daily data were used to calculate the monthly means per year and consequently the station means using SAS 6.12 (SAS Institute 1997). The monthly means by station yielded the following files:

- Monthly precipitation based on 19 years or more for 194 stations.
- Monthly precipitation based on 16 years or more for 316 stations.
- Monthly mean maximum temperature based on 19 years for 140 stations.
- Monthly mean minimum temperature based on 19 years for 175 stations.

Validation sets—To evaluate whether splining or co-kriging was best for interpolating climate variables for the selected area, we determined the precision of prediction of each using test sets. These sets contain randomly selected data points from the available observations. They are not used for prediction nor variogram estimation, so it is possible to compare predicted points with independent observations. In this study two test sets were used.

First five smaller, almost equal sub-areas were defined (Fig. 4). For precipitation, 10 stations were randomly selected from each. These 50 points were divided into two sets. Each dataset had 25 validation points and 169 interpolation points. The benefit of working with
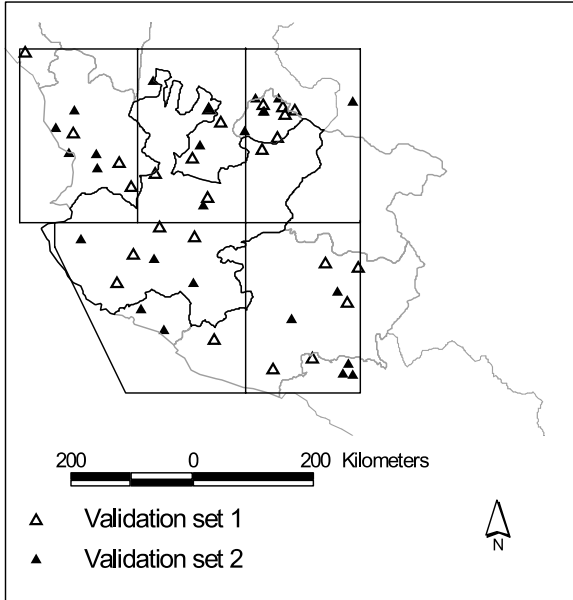
Figure 4. Validation selection areas and two validation sets of 25 points each for precipitation.

two datasets of 169 points each is that all 194 points are used for analysis and interpolation, but the validation stations are still independent of the dataset.

The interpolation techniques were tested as well for maximum temperature. Because only 140 stations were available, only 6 validation points were randomly selected from each square. Therefore, interpolation for maximum temperature was executed using 125 points and 15 points were kept independent as a validation set.

Exploratory data analysis and co-kriging requirements— An exploratory data analysis was conducted prior to interpolation to consider the need for transformation of precipitation data, the characteristics of the dataset to be used, and the correlation coefficients between the prediction variable and the co-variable "elevation." Log transformation is commonly applied to give precipitation data a more normal distribution. However, back-transforming the precipitation values can be problematic because exponentiation tends to exaggerate any interpolation-related error (Goovaerts 1997).

The two precipitation datasets were compared to see if the dataset from 194 stations (19 years or more) had greater precision than that from 320 stations (16 years or more). This was done by comparing the nugget effects of the variograms. As an indication of measurement accuracy, if the nugget of the large dataset is larger than the nugget of the small dataset, then the large dataset is probably less accurate. For each variogram, the number of lags and the lag distance were kept at 20 and 0.2 respectively. The model type fitted through the variogram was also the same for each dataset. This allowed a relatively unbiased comparison of the two nugget values, because the nugget difference is independent of model, number of lags, and lag distance. Variogram fitting was done with the WLSFIT program (Heuvelink 1992). The nugget difference can be calculated as:

(nugget of the 320 station dataset) – (nugget of the 194 station dataset).

Thus, the relative nugget difference can be presented as:

$$\left(\frac{\text{nugget320} - \text{nugget194}}{\text{nugget320}} \right) * 100\%$$

Results—In the exploratory data analysis, precipitation data for all months showed an asymmetric distribution. The difference between the non-transformed surface and the transformed surface was high only in areas without stations. In most areas, the difference was smaller than the prediction error. We therefore decided not to transform the precipitation data for interpolation. The temperature data did not show an asymmetric distribution, so it was not necessary to test transformation (De Beurs 1998).

The relative nugget difference of the large precipitation dataset (320 stations, 16 years of data) was compared to that for the small dataset (194 stations, 19 years of data). For every month except July and November, the relative nugget difference was less then 30% (Annex 2)

and, in two cases, the nugget value was smaller for the small dataset. Because the difference in accuracy between the two datasets was not large, the small dataset of monthly means based on more than 19 years was used.

Co-kriging works best when there is a high absolute correlation between the co-variable and the prediction variable. In general, during the dry season precipitation shows a positive correlation with altitude, whereas during the wet season there is a negative correlation. The correlation between each variable to be interpolated (precipitation and maximum temperature) and the co-variable (elevation) were determined. For the selected area, April, May, August and September had acceptable correlation coefficients between precipitation and elevation (Table 4). May to

Table 4. Correlation coefficients between prediction variables: precipitation (P), maximum temperature (Tmax), and the co-variable (elevation).

Month	Correlation P*Elevation	Correlation Tmax*Elevation
January	-0.26	-0.82
February	0.26	-0.80
March	0.20	-0.71
April	0.68	-0.63
May	0.59	-0.63
June	-0.02	-0.74
July	-0.36	-0.84
August	-0.52	-0.84
September	-0.59	-0.84
October	-0.39	-0.85
November	-0.37	-0.85
December	-0.39	-0.84

October had the highest precipitation values. The lack of a correlation between precipitation and elevation for June may be because it rains everywhere, making co-kriging difficult for that month. There is little precipitation in the other months.

Maximum temperature showed a greater absolute correlation with elevation, so the interpolation methods were evaluated for the same months (April, May, August and September). April and May had the lowest and August and September the highest correlation coefficients.

Semi-variogram fitting for the co-kriging technique— Variograms were made and models fitted to them. For months with a negative correlation, cross-variogram values were also negative. To fit a rough model with the WLSFIT program (Heuvelink 1992), it was necessary to make the correlation values positive, because WLSFIT does not accept negative correlations. This first round of model fitting was used to obtain an initial impression. The final model was then fitted using GSTAT (Pebesma 1997). Linear models of co-regionalization were determined only for the months April, May, August and September (Table 5 and 6). A linear model of co-regionalization occurs when the variogram and the cross-variogram are given the same basic structures and the co-regionalization matrices are positive semi-definite (Annex 1). For precipitation the other months had correlation coefficients that were too low for

Table 5. Variogram and cross-variogram values for the linear model of co-regionalization for precipitation.

		Semi-variogram				Cross –variogram			
Month	Variable	Model	Nugget	Sill	Range	Model	Nugget	Sill	Range
April	Precip.	Exponential	3.20	46.3	2.10	Exponential	17.8	3720	2.10
	Elevation	Exponential	5050	565000	2.10				
May	Precip.	Exponential	38.4	256	2.10	Exponential	0	8680	2.10
	Elevation	Exponential	5050	565000	2.10				
August	Precip.	Gaussian	1110	43400	5.65	Gaussian	2330	-198000	5.65
	Elevation	Gaussian	62300	2990000	5.65				
September	Precip.	Gaussian	1260	64300	7.00	Gaussian	1560	-365000	7.00
	Elevation	Gaussian	63500	4400000	7.00				

Table 6. Variograms and cross-variograms for the linear model of co-regionalization for maximum temperature.

		Semi-variogram				Cross-variogram			
Month	Variable	Model	Nugget	Sill	Range	Model	Nugget	Sill	Range
April	Tmax	Exponential	1.40	9.69	0.60	Exponential	22.4	-1310	0.60
	Elevation	Exponential	360	303000	0.60				
May	Tmax	Exponential	1.10	9.81	0.60	Exponential	20.3	-1280	0.60
	Elevation	Exponential	380	303000	0.60				
August	Tmax	Spherical	0.926	10.7	1.50	Spherical	-11.2	-1640	1.50
	Elevation	Spherical	2810	309000	1.50				
September	Tmax	Spherical	1.13	10.1	1.50	Spherical	-20.2	-1580	1.50
	Elevation	Spherical	2810	309000	1.50				

satisfactory co-kriging. The final ASCII surfaces interpolated at 30 arc seconds were created with GSTAT.

Surface characteristics and surface validation— Splining and co-kriging technique results were truncated to zero to avoid unrealistic, negative precipitation values. Interpolated monthly precipitation surfaces are displayed for April, May, August, and September in Annex 3. Surfaces were also created with IDWA, splining, and co-kriging (not shown). The IDWA surfaces show clear "bubbles" around the actual station points. Visually, the co-kriging surfaces follow the IDWA surfaces very well. The splined surfaces are similar to the DEM surface but appear more precise.

Basic characteristics of the DEM, monthly precipitation, and temperature surfaces created through IDWA, co-kriging, and splining are presented in Annex 4. Maximum elevation as reported in the stations is 2,361 m. Maximum elevation from the DEM was 4,019 m—much higher than the elevation of the highest station. Therefore precipitation and maximum temperature were estimated at elevations higher than elevations of the stations. It is not possible to validate these values because there are no measured values for such high elevations. However, the extreme values of the interpolated surfaces can be evaluated. For precipitation, it is difficult to know whether values at high elevations were reasonable estimates, because there is no generic association with elevation as

occurs with temperature. The maximum value of the splined surfaces was smaller than the maximum measured value from the station.

Measured precipitation data have a distribution that is skewed to the right. A frequency distribution of precipitation after interpolation (Fig. 5) provides another means of comparing the effects of interpolation methods. The interpolated surfaces were clipped to the area of Jalisco to avoid side effects. Depending on the month, splining and co-kriging produced contrasting distributions. In May, splining indicated that 77% or more grid cells had less than 30 mm precipitation, whereas co-kriging allocated 70% of cells to this precipitation range. For September, co-kriging showed over 28% of the cells had from 138 to 161 mm precipitation, whereas splining assigned 24.5% of the cells to this precipitation class. In both cases co-kriging gave a wider precipitation range. The frequency of the co-variable "elevation" within Jalisco is not normally distributed either (Fig. 6). Considering that there was a positive correlation between precipitation and altitude in May and a negative correlation in September, splining seemed to follow the distribution of elevation more than co-kriging. However, in the absence of more extensive validation data, it is not possible to state that one method was superior to the other based on resulting frequency distributions.

Usually, temperature decreases 5 to 6°C per 1,000-m increase in elevation, depending on relative humidity and starting temperature (Monteith and Unsworth
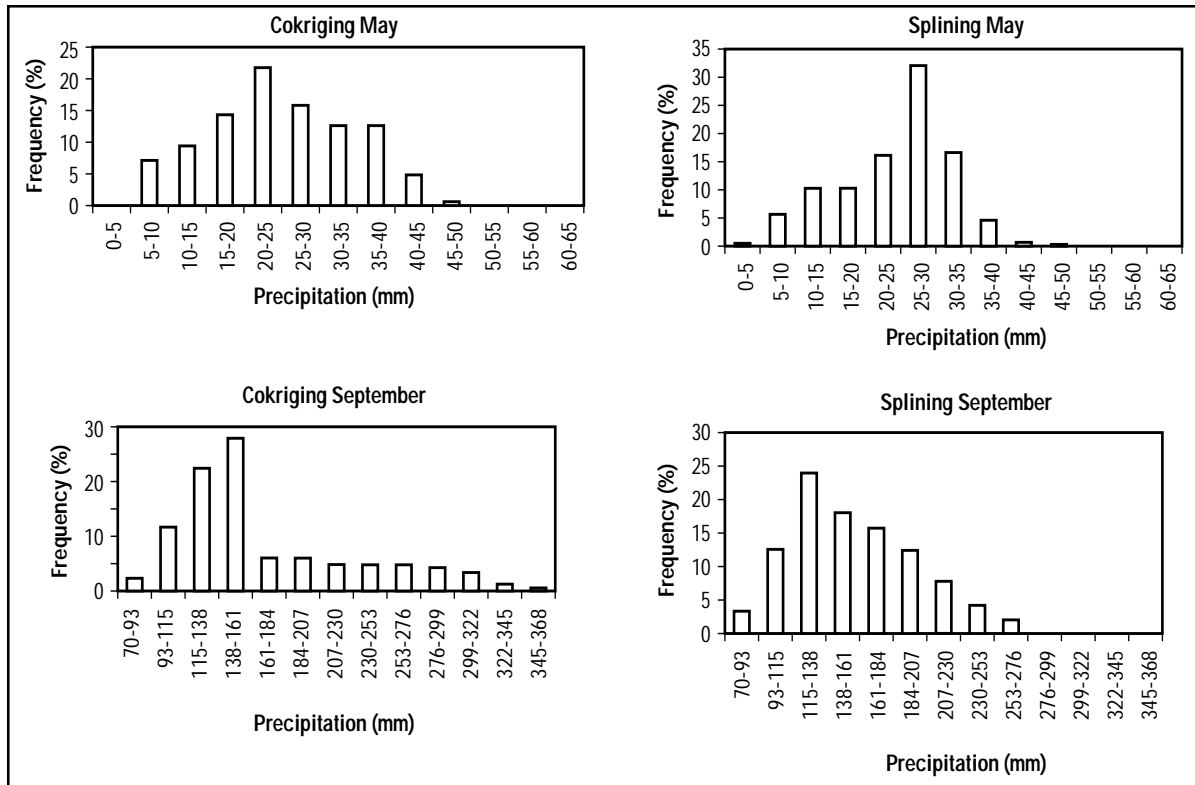
Figure 5. Frequency distribution of precipitation values after splining and co-kriging for two months.
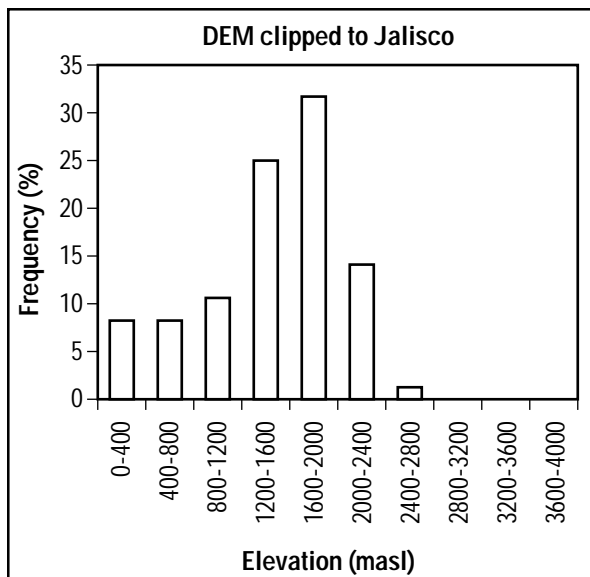

Figure 6. Frequency distribution of elevation, the co-variable for interpolation in this study, for Jalisco.

1990; see also Linacre and Hobbs 1977). The difference between the maximum elevation of the stations and the maximum elevation in the DEM was 1,658 m. Thus, the estimated maximum temperature should be approximately 11 degrees below values measured from the stations. This can be seen for the minimum value of the spline-interpolated values, which were about 9-10 degrees below measured values (Annex 4). The range of the co-krige and IDWA interpolated values was almost the same as that for the measured data. Therefore, at higher elevations splining appeared to predict the maximum temperature better than co-kriging.

The surfaces were validated using the two independent test sets. For *precipitation*, the IDWA appeared to perform better than the other techniques (Table 7), but the difference was not significant (statistical analysis not shown). There was little difference between splining and co-kriging, but we could apply the latter only for the four months when

there was a high correlation with the co-variable. The predictions for August and September using the second interpolation set were less accurate than those obtained using the first set.

Validation showed that splining performed better for all months for *maximum temperature* (Table 8). There was no difference between co-kriging and IDWA predictions (statistical analysis not shown).

Prediction uncertainty (GCV)— Prediction uncertainty or 'error' surfaces were produced with the splining and co-kriging techniques. Annex 5 shows this for precipitation. The prediction error from splining was more constant across months. The co-kriging error surfaces showed greater variability spatially and between months.

Conclusions for the Study Area

IDWA gave the best results for *precipitation*, though its superiority was not significant over results obtained through the other methods. There was no gain from using elevation as a co-variable to interpolate precipitation. Distance to sea was another co-variable checked. However, the correlation was local and not always present (De Beurs 1998). Other co-variables were not readily available.

For maximum *temperature* there was a higher correlation with elevation and interpolation improved when this co-variable was used. Interpolation of maximum temperature was better handled by splining than by co-kriging or IDWA.

Table 7. Validation statistics for four monthly precipitation surfaces.

Precipitation	Mean absolute difference (mm)				Relative difference (%)			
	April	May	August	September	April	May	August	September
Validation 1								
IDWA	2.0	5.4	23.9	25.9	31	23	12	17
Splining	2.5	5.5	35.9	31.3	38	23	18	20
Co-kriging	2.0	5.5	33.6	32.5	31	23	17	21
Validation 2								
IDWA	1.9	6.6	41.2	41.7	41	31	20	24
Splining	2.2	6.1	55.1	47.3	46	28	26	27
Co-kriging	1.7	6.4	39.1	40.9	37	30	19	23

Table 8. Validation statistics for four maximum temperature surfaces.

Tmax	Mean absolute difference (ºC)				Relative difference (%)			
	April	May	August	September	April	May	August	September
IDWA	2.7	2.5	2.0	1.9	8.6	7.6	7.0	6.7
Splining	1.6	1.4	1.2	1.1	5.0	4.5	4.2	3.9
Co-kriging	2.6	2.3	1.8	1.9	8.4	7.3	6.6	6.5

Conclusions and Recommendations for Further Work

Conclusions of this work apply to this case study only, but several general recommendations can be made for future case studies:

• Splining and co-kriging should be preferred over the IDWA technique, because the former provide prediction uncertainty or "error" surfaces that describe the spatial quality of the prediction surfaces. Co-kriging was possible for only four months for precipitation in the study area, due to the data prerequisites for this technique. Spline interpolation was preferred over co-kriging because it is faster and easier to use, as also noted in other studies (e.g., Hutchinson and Gessler 1994).

• For all techniques interpolation can be improved by using more stations.

• For splining and co-kriging, interpolation can be improved by using more independent co-variables that are strongly correlated with the prediction variable.

- Preferably, all surfaces for one environmental variable should be produced using only one technique.
- Interested readers might wish to evaluate kriging with external drift, where the trend is modeled as a linear function of smoothly varying secondary (external) variables, or regression kriging, which looks very much like co-kriging with more variables. In regression kriging there is no need to estimate the cross-variogram of each co-variable individually—all co-variables are incorporated into one factor.

Taking into account error prediction, data assumptions, and computational simplicity, we would recommend use of thin-plate smoothing splines for interpolating climate variables.

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Yates, S.R., and A.W. Warrick. 1987. Estimating soil water content using Cokriging. *Soil Science Society of America Journal* 51:23-30.

Annex 1.
Description of Applying the Linear Model of Co-regionalization[2]

The linear model of co-regionalization is a model that ensures that estimates derived from co-kriging have positive or zero variance. For example there is the following model:

$$\Gamma(h) = \begin{pmatrix} \gamma_{11}(h) & \gamma_{12}(h) \\ \gamma_{21}(h) & \gamma_{22}(h) \end{pmatrix} = \begin{pmatrix} b^0_{11} & b^0_{12} \\ b^0_{21} & b^0_{22} \end{pmatrix} * g_0(h) + \begin{pmatrix} b^0_{11} & b^0_{12} \\ b^0_{21} & b^0_{22} \end{pmatrix} * g_1(h)$$

where:

$\Gamma(h)$ = the semi-variogram matrix, and

$g_1(h)$ = l^{th} basic variogram model in the linear model of co-regionalization.

So the basic variogram models are the same for every variogram, or cross-variogram. In this case, $g_0(h)$ is the nugget model and $g_1(h)$ is the sill model.

For a linear model of co-regionalization, all of the co-regionalization matrices (B_1) should be positive definite. A symmetric matrix is positive semi-definite if its determinants and all its principal minor determinants are non-negative. If Nv = 2, as in the example or with the precipitation data:

$b^1_{11} \geq 0$ and b^1_{22}

$b^1_{11}b^1_{22} - b^1_{12}b^1_{12} \geq 0 \rightarrow b^1_{12} \leq \sqrt{b^1_{11}b^1_{22}}$

Thus, when fitting the basic structure $g_1(h)$ in the linear model of co-regionalization, these four general rules should be considered:

$b^1_{ij} \neq 0 \rightarrow b^1_{ii} \neq 0$ and $b^1_{jj} \neq 0$

$b^1_{ii} = 0 \rightarrow b^1_{ij} = 0 \ \forall j$

b^1_{ij} may be equal to zero

$b^1_{ii} \neq 0$ and $b^1_{jj} \neq 0 \rightarrow b^1_{ij} = 0$ or $b^1_{ij} \neq 0$.

To fit a linear model of co-regionalization:

- Take the smallest set of semi-variogram models $g_1(h)$ that captures the major features of all N_v.
- Estimate the sill and the slope of the semi-variogram models $g_1(h)$ while taking care that the co-regionalization matrices are positive definite.
- Evaluate the "goodness" of fit of all models. When a compromise is necessary, then the priority lies in fitting a model to the variogram of the variable to be predicted, as opposed to the variogram of the co-variable or cross-variogram.

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[2] As described by Goulard and Voltz, 1992.

Annex 2.
Dataset Comparison
for Precipitation

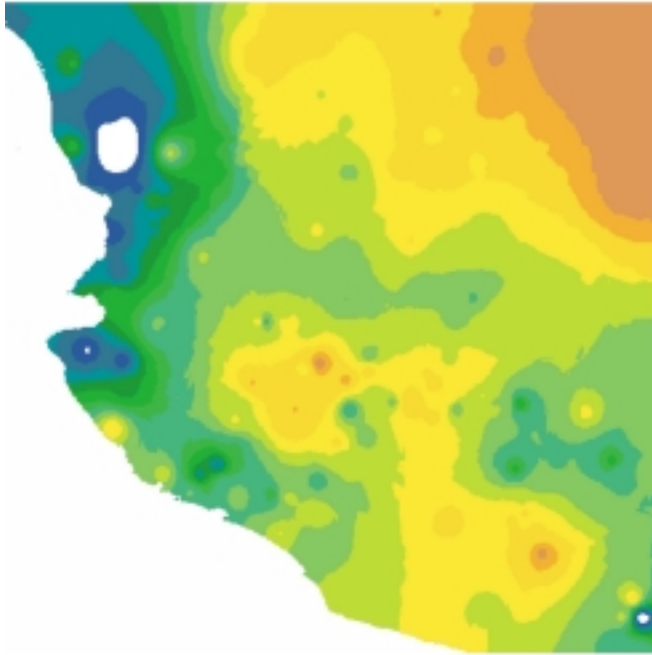**Comparison between the dataset with 320 points and the dataset with 194 points.**

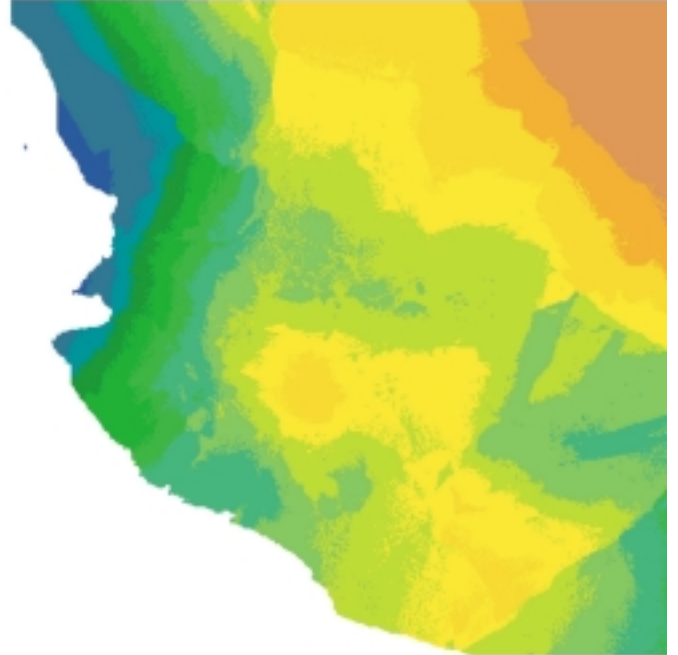|	Variable	Model	Nugget	Sill	Range	ssd/sst	nugget diff.[1]	rel. nugget diff. (%)[2]
198	January	Exponential	0.0254	0.0724	1.36	0.086	0.0071	21.8
320	January	Exponential	0.0325	0.0785	4.06	0.025		
198	February	Spherical	0.0099	0.0269	1.07	0.155	0.00213	17.8
320	February	Spherical	0.0120	0.0183	1.10	0.236		
198	March	Gaussian	0.0226	0.0678	9.47	0.487	-0.0019	-9.2
320	March	Gaussian	0.0207	0.0041	2.65	0.631		
198	April	Gaussian	0.0175	0.192	9.47	0.073	- 0.0014	- 8.7
320	April	Gaussian	0.0161	0.243	9.47	0.039		
198	May	Spherical	0.0786	3.41	9.47	0.055	0.0174	-0.3
320	May	Spherical	0.0594	0.322	9.47	0.066		
198	June	Spherical	0.434	5.40	5.33	0.019	0.121	21.8
320	June	Spherical	0.555	5.09	6.09	0.014		
198	July	Spherical	0.993	8.94	6.00	0.022	0.452	36.7
320	July	Spherical	1.23	6.47	6.00	0.034		
198	August	Spherical	0.778	13.7	7.00	0.063	0.292	27.3
320	August	Spherical	1.07	10.3	7.00	0.094		
198	September	Gaussian	1.85	20.5	3.58	0.061	0.02	1.1
320	September	Gaussian	1.87	95.3	9.47	0.050		
198	October	Gaussian	0.475	4.98	5.98	0.017	-0.052	-12.3
320	October	Gaussian	0.423	10.2	8.95	0.025		
198	November	Exponential	0.0129	0.170	1.38	0.038	0.0162	55.7
320	November	Exponential	0.0291	0.428	7.00	0.037		
198	December	Spherical	0.0058	0.179	9.47	0.019	0.0015	20.5
320	December	Spherical	0.0073	0.139	9.47	0.075		

1. Nugget difference = nugget of the big dataset - nugget of the small dataset.

2. Relative nugget difference is: $\left(\dfrac{nugget320 - nugget194}{nugget320} \right) * 100$

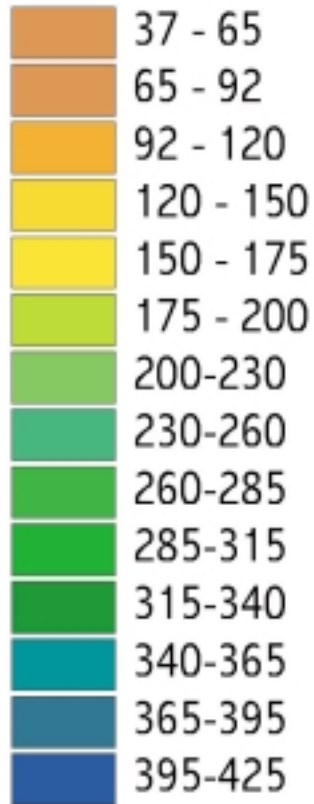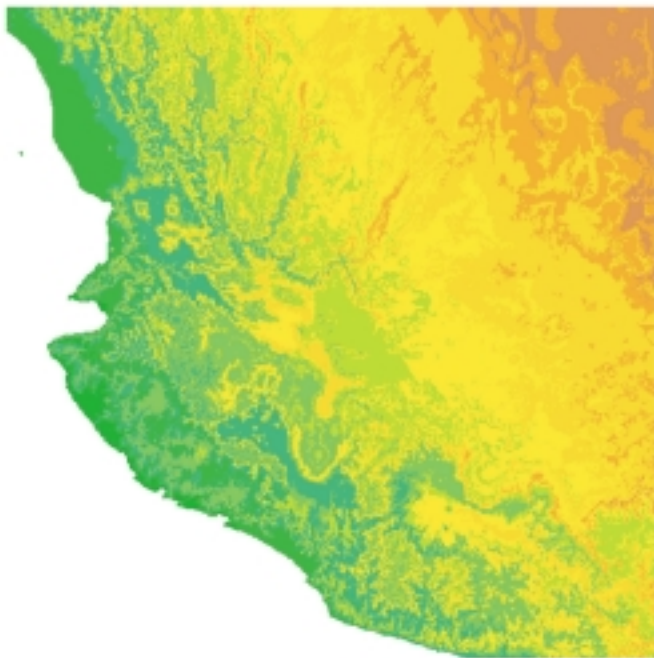# Annex 3. Interpolated Monthly Precipitation Surfaces from IDWA, Splining, and Co-kriging for April and August

IDW interpolation August

Cokrige interpolation August

Spline interpolation August

N

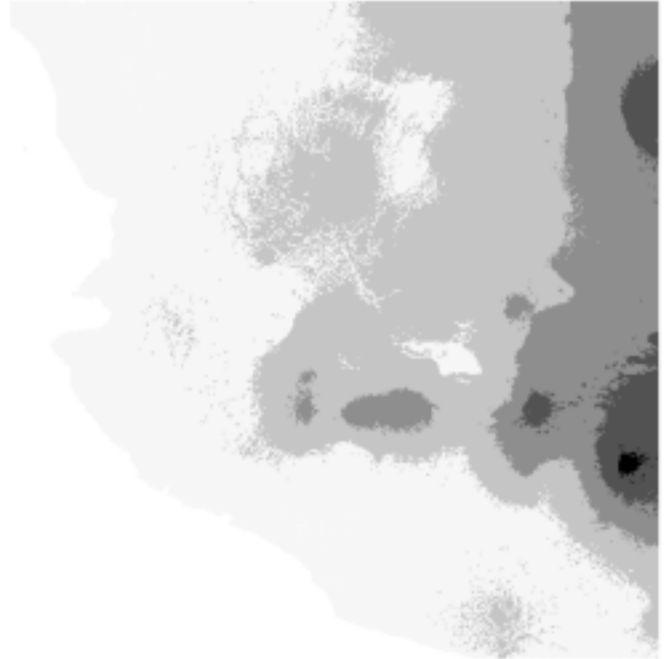|	
	37 - 65
	65 - 92
	92 - 120
	120 - 150
	150 - 175
	175 - 200
	200-230
	230-260
	260-285
	285-315
	315-340
	340-365
	365-395
	395-425

0   200   400   Kilometers

IDW interpolation April

Cokrige interpolation April

Spline interpolation April

Precipitation (mm)

0 - 5
5 - 10
10 - 15
15 - 20
20 - 25

0   200   400   Kilometers

N
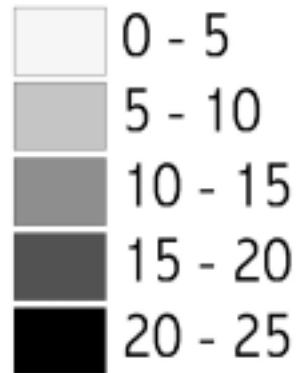
Station values and DEM surface values for elevation

Elevation	Measured (m)	DEM (m)
Minimum	27	1
Maximum	2361	4019
Mean	1396.56	1455.321

Measured values and interpolated surface values for precipitation.

April	Measured (mm)	IDW (mm)	Spline (mm)	Co-krige (mm)
Minimum	0	0.0	-1.4	-1.5
Maximum	20.3	24.9	15.6	20.4
Mean	5.9	5.2	284.7	6.2

May	Measured (mm)	IDW (mm)	Spline (mm)	Co-krige (mm)
Minimum	3.8	3.7	-11	1.3
Maximum	60.8	75.1	57.1	57.1
Mean	27.2	22.6	24.0	24.9

August	Measured (mm)	IDW (mm)	Spline (mm)	Co-krige (mm)
Minimum	76.8	47.0	55.7	38.0
Maximum	426.8	495.7	317.3	423.1
Mean	197.6	216.7	175.4	198.3

September	Measured (mm)	IDW (mm)	Spline (mm)	Co-krige (mm)
Minimum	95.8	44.0	28.9	44.5
Maximum	429.6	472.4	280.0	382.8
Mean	166.4	186.7	143.6	164.4

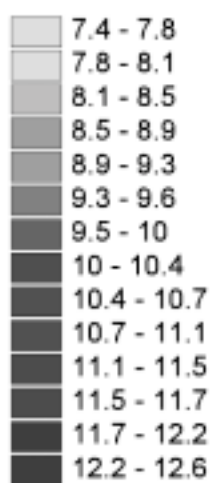**Measured values and interpolated surfaces for maximum temperature (ºC).**

April	Measured ºC	IDWA ºC	Spline ºC	Co-krige ºC
Minimum	25.4	24.7	14.9	27.5
Maximum	40.1	40.9	40.0	38.8
Mean	31.9	32.0	31.0	32.0

May	Measured ºC	IDWA ºC	Spline ºC	Co-krige ºC
Minimum	26.9	25.4	15.7	28.0
Maximum	41.2	41.2	41.2	40.1
Mean	32.9	33.0	31.9	33.2

August	Measured ºC	IDWA ºC	Spline ºC	Co-krige ºC
Minimum	21.3	20.9	12.3	22.6
Maximum	35.2	35.5	36.0	34.7
Mean	28.3	29.4	28.1	28.8

September	Measured ºC	IDWA ºC	Spline ºC	Co-krige ºC
Minimum	21.3	21.3	12.1	22.5
Maximum	35.5	35.1	35.6	35.4
Mean	28.2	29.1	27.8	28.7

Annex 5. Prediction Error Surfaces for Precipitation Interpolated by Splining and Co-kriging
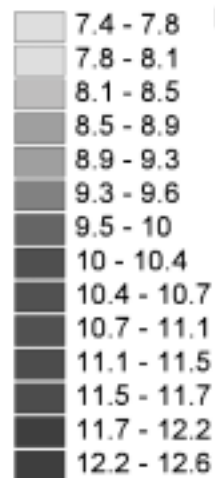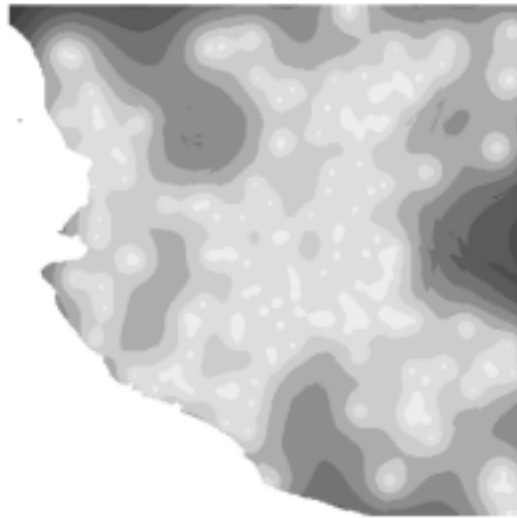
Spline error April

Spline error May

Error (mm)

	7.4 - 7.8
	7.8 - 8.1
	8.1 - 8.5
	8.5 - 8.9
	8.9 - 9.3
	9.3 - 9.6
	9.5 - 10
	10 - 10.4
	10.4 - 10.7
	10.7 - 11.1
	11.1 - 11.5
	11.5 - 11.7
	11.7 - 12.2
	12.2 - 12.6
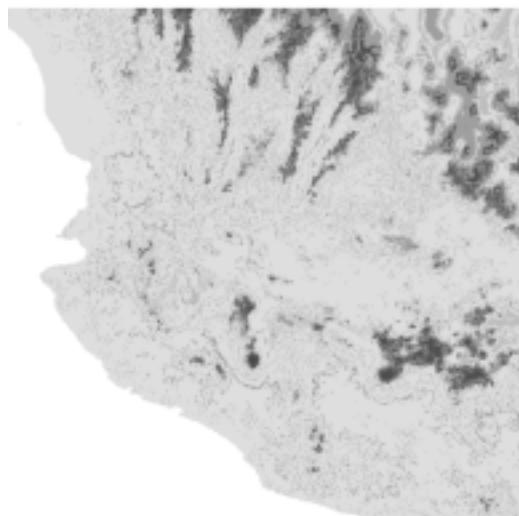
Spline error August

Spline error September

Error (mm)

	7.4 - 7.8
	7.8 - 8.1
	8.1 - 8.5
	8.5 - 8.9
	8.9 - 9.3
	9.3 - 9.6
	9.5 - 10
	10 - 10.4
	10.4 - 10.7
	10.7 - 11.1
	11.1 - 11.5
	11.5 - 11.7
	11.7 - 12.2
	12.2 - 12.6

0   200   400 Kilometers

N

Cokrige error surface April

Error (mm)

2.0 - 2.3	6.9 - 7.4
2.3 - 2.5	7.4 - 7.9
2.5 - 2.8	7.9 - 8.4
2.8 - 3.0	8.4 - 8.9
3.0 - 3.2	8.9 - 9.4
3.2 - 3.5	9.4 - 9.8
3.5 - 3.7	9.8 - 10.3
3.7 - 4.0	10.3 - 10.8
4.0 - 4.2	10.8 - 11.3
4.2 - 4.4	11.3 - 11.8
4.4 - 4.7	11.8 - 12.3
4.7 - 4.9	12.3 - 12.8
4.9 - 5.2	12.8 - 13.3
5.2 - 5.4	13.3 - 13.7
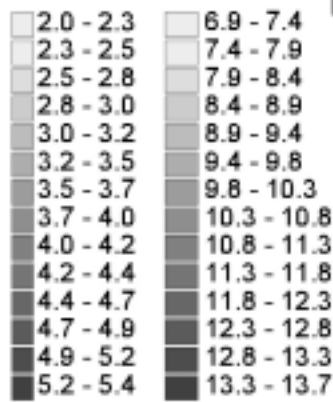
Cokrige error surface May
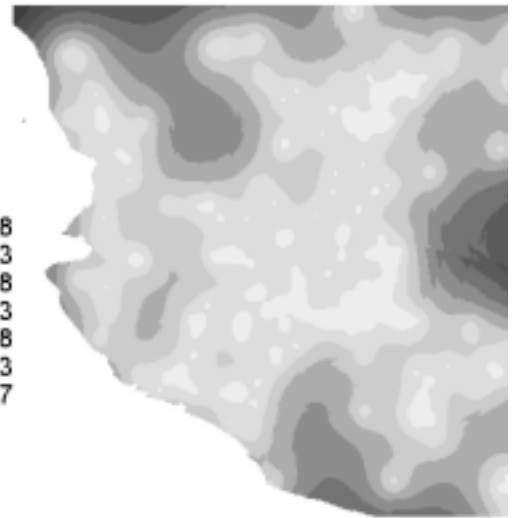
Cokrige error surface August

Error (mm)

33.5 - 34.6	36.5 - 37.5
34.6 - 35.7	37.5 - 38.6
35.7 - 36.9	38.6 - 39.7
36.9 - 38.0	39.7 - 40.8
38.0 - 39.1	40.8 - 41.8
39.1 - 40.2	41.8 - 42.9
40.2 - 41.3	42.9 - 44.0
41.3 - 42.5	44.0 - 45.0
42.5 - 43.6	45.0 - 46.1
43.6 - 44.7	46.1 - 47.2
44.7 - 45.8	47.2 - 48.3
45.8 - 46.9	48.3 - 49.3
46.9 - 48.0	49.3 - 50.4
48.0 - 49.2	50.4 - 51.5
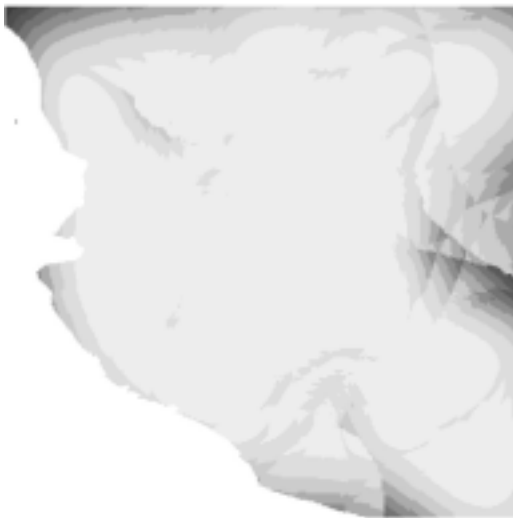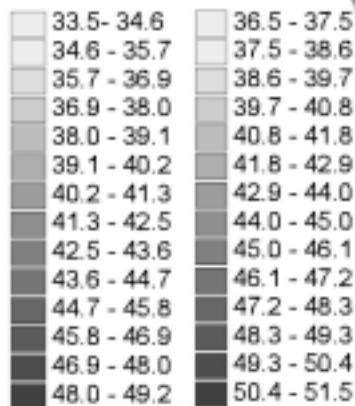
Cokrige error surface September

0   200   400 Kilometers

N

Interpolation Techniques for Climate Variables

NRG-GIS 99-01