(the last integral has been taken from a table[11]). Therefore, E_0 behaves like $(\ln H)^2$ for large values of H. A more careful, tedious, but straightforward study of (3), with the use of majorizations and minorizations, gives the following more precise result for the asymptotic behavior of E_0:

$$E_0 = mc^2 + (\alpha/4\pi)mc^2\{[\ln(2e\hbar H/m^2c^3) - C - \tfrac{3}{2}]^2 + A + \cdots\}, \quad (5)$$

where $C=0.577$ is Euler's constant, and where A is a numerical constant for which we have only found bounds: $-6<A<7$.

One readily sees from (3) that even for tremendous values of H (the characteristic field $m^2c^3/e\hbar$ being 4.4×10^{13} G), the radiative correction to E_0 remains of relative order α. In particular, E_0 certainly does not vanish at $H=(4\pi/\alpha)(m^2c^3/e\hbar)=7.6\times10^{16}$ G, a field value for which (1) is not valid. Some doubts about the limits of validity of the anomalous magnetic moment concept have actually been raised by the authors of Ref. 2 themselves.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, edited by A. Jeffrey (Academic Press Inc., New York, 1965).

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Interpretation of a Unified Theory of Gravitation and Symmetry Breaking*

DAVID PEAK AND AKIRA INOMATA
Department of Physics, State University of New York, Albany, New York 12203
(Received 15 July 1969)

The formalism of Moen and Moffat is interpreted as a Yang-Mills theory set in a space-time generally endowed with curvature and torsion.

In a recent paper,[1] Moen and Moffat describe the possibility of a generalized definition of "parallel" transport of a vector nonet [an element of the tensor representation of the combined group of space-time and $U(3)$ transformations] resulting in (a) a connection between space-time and internal symmetries without reference to a "supergroup" and (b) unitary symmetry breaking induced by the presence of a zero-mass boson (to first approximation). We show that it is possible to interpret the formalism in this work as an extended Yang-Mills theory. From this point of view we see that a total symmetry group is already "embedded" in the theory, and that the character of the background space-time is sufficient to break the internal symmetry.

To see how it may be possible to make the aforementioned interpretation, we first review some aspects of a local gauge theory set in a curved background. At the outset there is, presumably, a matter field which displays a unitary symmetry characterized by[2]

$$\psi'(x)=S^{-1}(x)\psi(x). \quad (1)$$

The entities generically designated S are taken to be matrix representations of elements of a group of internal transformations, and are by assumption functions of the space-time coordinates of the event point at which the transformation is made. The internal degrees of freedom of the ψ field are thus adjustable at all other points of space-time, in keeping with the requirements of a local picture of interaction. To ensure the invariance of the dynamical structure of this system, it is necessary to introduce auxiliary field operators B_μ that couple universally with the various ψ components, and which transform under local internal group action as

$$B'_\mu=S^{-1}(B_\mu S-\nabla_\mu S). \quad (2)$$

Here ∇_μ denotes the relevant space-time covariant derivative with respect to the μth coordinate.

In a sense, the B_μ fields are like components of an affine connection[3]; as a consequence, we may define a totally covariant derivative operator expressed symbolically as

$$D_\mu=\nabla_\mu+B_\mu. \quad (3)$$

D_μ commutes with both space-time and internal transformations, and serves to establish a meaning for a parallel transport of fields with mixed indices. In terms of the vector nonets mentioned in I, the operation of D_μ provides, for example,

$$D_\nu A^{\sigma i}=\nabla_\nu A^{\sigma i}+B_\nu{}^i{}_j A^{\sigma j}=\partial_\nu A^{\sigma i}+\left\{\begin{matrix}\sigma\\ \mu\nu\end{matrix}\right\}A^{\mu i}+B_\nu{}^i{}_j A^{\sigma j}, \quad (4)$$

where Greek indices refer to space-time structure, Latin indices to internal.

Now, the covariant derivative defined in I is just such an operator, that is, it measures the effect of the total variation of fields. As expressed in that work, the

* Supported in part by the National Science Foundation.

[1] I. O. Moen and J. W. Moffat, Phys. Rev. **179**, 1233 (1969); herein this paper shall be referred to as I.

[2] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

[3] See, e.g., J. L. Anderson, *Principles of Relativity Physics* (Academic Press Inc., New York, 1967), p. 44.

covariant derivative of a contravariant vector nonet is

$$A^{\sigma i}{}_{;\nu}=\partial_\nu A^{\sigma i}+h^i{}_{jk}\Gamma_{\mu\nu}{}^{\sigma j}A^{\mu k}, \tag{5}$$

which the $h^i{}_{jk}$ given in terms of the conventional $f^i{}_{jk}$ and $d^i{}_{jk}$ of $U(3)$ symmetry[4] as

$$h^i{}_{jk}=(1-\alpha)f^i{}_{jk}+\alpha d^i{}_{jk}. \tag{6}$$

The right-hand side of Eq. (5) is obviously

$$\partial_\nu A^{\sigma i}+h^i{}_{0k}\Gamma_{\mu\nu}{}^{\sigma 0}A^{\mu k}+h^i{}_{ak}\Gamma_{\mu\nu}{}^{\sigma a}A_\mu{}^k, \tag{7}$$

where the sum on a is 1–8. Consideration of the transformation law

$$\Gamma'_{\mu\nu}{}^{\lambda i}=\frac{\partial x'^\lambda}{\partial x^\alpha}\frac{\partial x^\beta}{\partial x'^\mu}\frac{\partial x^\gamma}{\partial x'^\nu}\Gamma_{\beta\gamma}{}^{\alpha i}+\frac{\partial^2 x'^\lambda}{\partial x^\alpha\partial x^\beta}\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\delta^i{}_0$$

shows that, under change of coordinates, only the unitary scalar component of $\Gamma_{\mu\nu}{}^{\lambda i}$ transforms as a connection while the remaining internal components transform as space-time tensors. Hence, it is plausible to interpret (5) as (4) by allowing the identifications

$$A^{\sigma i}{}_{;\nu}\to D_\nu A^{\sigma i},$$

$$h^i{}_{0k}\Gamma_{\mu\nu}{}^{\sigma 0}\;(=\delta^i{}_k\beta\Gamma_{\mu\nu}{}^{\sigma 0})\to\delta^i{}_k\left\{\begin{matrix}\sigma\\ \mu\nu\end{matrix}\right\},$$

and

$$h^i{}_{ak}\Gamma_{\mu\nu}{}^{\sigma a}\to\delta^\sigma{}_\mu B_\nu{}^i{}_k.$$

In fact, the second replacement is already given in I [Eq. (59)]. After interpretation, assuming as in I that internal transformations may be made path-independent, we are always able to select an internal basis such that the third term of (7) is zero.[5] Consequently, the total divergence of a vector density nonet $\mathfrak{V}^{\mu i}$,

$$D_\mu\mathfrak{V}^{\mu i}=\partial_\mu\mathfrak{V}^{\mu i}+2\left\{\begin{matrix}\sigma\\ [\mu\sigma]\end{matrix}\right\}\mathfrak{V}^{\mu i}+B_\mu{}^i{}_j\mathfrak{V}^{\mu j}, \tag{8}$$

becomes

$$D_\mu\mathfrak{V}^{\mu i}=\partial_\mu\mathfrak{V}^{\mu i}+2\left\{\begin{matrix}\sigma\\ [\mu\sigma]\end{matrix}\right\}\mathfrak{V}^{\mu i}.$$

As a result, the conservation law

$$D_\mu\mathfrak{V}^{\mu i}=0 \tag{9}$$

yields

$$\dot{F}^i(t)=\int\partial_\mu\mathfrak{V}^{\mu i}d^3x=-2\int\left\{\begin{matrix}\sigma\\ [\mu\sigma]\end{matrix}\right\}\mathfrak{V}^{\mu i}d^3x. \tag{10}$$

[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[5] H. G. Loos, J. Math. Phys. **8**, 2114 (1967).

The symmetry of the F-spin operators is broken, even in the case of zero Yang-Mills fields, by the unconventional space-time structure available in our hypotheses. The right-hand side of (10) vanishes, we note, both in the event of a torsion-free space-time and when the torsion present is completely antisymmetric.

Let us examine, in the light of our interpretation, statements (a) and (b) given initially. The assumptions in I appear tacitly to include a supergroup, namely, the direct product of space-time and internal groups. One then sees a trivial combination of the two sets of symmetries, a situation manifested in the vanishing of the Yang-Mills fields. On the other hand, symmetry breaking is still feasible as a result of the assumed torsion. The torsion acts as an independent field which couples to the current $\mathfrak{V}^{\mu i}$ to break the unitary symmetry, but the unambiguous identification of a particle with this field is problematic.[6]

A slight modification in the unitary transformation laws given in I provides a nontrivial local gauge picture, replete with symmetry breaking even in the ordinary Minkowski background. If we let the vector nonets transform internally as

$$\bar{\delta}A^{\sigma i}(x)=i\epsilon^j(x)L_{jk}{}^iA^{\sigma k}(x), \tag{11}$$

the variation of $A^{\sigma i}{}_{;\nu}$ gives a "connectionlike" law for $h^i{}_{jk}\Gamma_{\mu\nu}{}^{\sigma j}$

$$\bar{\delta}(h^i{}_{jk}\Gamma_{\mu\nu}{}^{\sigma j})=i\epsilon^n(L_{nm}{}^ih^m{}_{jk}\Gamma_{\mu\nu}{}^{\sigma j}-L_{nk}{}^mh^i{}_{jm}\Gamma_{\mu\nu}{}^{\sigma j})-i(\partial_\nu\epsilon^j)L_{kj}{}^i\delta^\sigma{}_\mu. \tag{12}$$

Since the parameters $\epsilon^j(x)$ are taken as scalar-valued functions of space-time, (12) is the statement in I language of the infinitesimal version of (2). With the wider generality, (8) and (9) imply

$$\dot{F}^i(t)=-2\int\left\{\begin{matrix}\sigma\\ [\mu\sigma]\end{matrix}\right\}\mathfrak{V}^{\mu i}d^3x-\int B_\mu{}^i{}_j\mathfrak{V}^{\mu j}d^3x, \tag{13}$$

which indicates that the coupling of the Yang-Mills field to the current density alone is sufficient to break the symmetry. In the usual theory[2] massless spin-1 bosons are associated with the B_μ fields; these can be held responsible for the breaking (13). The prototype (and as yet singular) example is, as mentioned in I, that of the electromagnetic potentials A_μ.

[6] See, e.g., R. Finkelstein, J. Math. Phys. **1**, 440 (1960).