Center for Economic Studies, University of Munich, 80539 Munich, Germany

CENTER FOR ECONOMIC STUDIES

INTERTEMPORAL POPULATION ETHICS: CRITICAL-LEVEL UTILITARIAN PRINCIPLES

Charles Blackorby
Walter Bossert
David Donaldson

Working Paper No. 86

UNIVERSITY OF MUNICH

CES

Working Paper Series

CES Working Paper Series

INTERTEMPORAL POPULATION ETHICS: CRITICAL-LEVEL UTILITARIAN PRINCIPLES

Charles Blackorby
Walter Bossert
David Donaldson

Working Paper No. 86

1995

Center for Economic Studies
University of Munich
Schackstr. 4
80539 Munich
Germany
Telephone & Telefax:
++49-89-2180-3112

The results reported in this paper were previously circulated in Blackorby, Bossert, and Donaldson (1993). Earlier versions were presented at the University of British Columbia, the University of California at Davis, the University of Karlsruhe, the University of Namur, the University of Ulm, the University of Waterloo, the University of Western Ontario, the University of Windsor, the 1993 Canadian Economic Theory Conference in Vancouver, the 1993 Latin American Meeting of the Econometric Society in Tucumán, and the Second Meeting of the Society for Social Choice and Welfare in Rochester, 1994. We are indebted to John Broome, Peter Hammond, Rosa Matzkin, Marc Nerlove, John Roemer, John Weymark, seminar participants, three referees and Guy Laroque for comments and suggestions. Financial support through a grant from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

CES Working Paper No. 86
July 1995

INTERTEMPORAL POPULATION ETHICS: CRITICAL-LEVEL UTILITARIAN PRINCIPLES

Abstract

This paper considers the problem of social evaluation in a model where population size, individual lifetime utilities, lengths of life, and birth dates vary across states. We investigate principles for social evaluation in an intertemporal framework and show that history must matter to some extent if they are to be ethically acceptable. Using an axiom called *independence of the utilities of the dead,* we provide a characterization of Critical-Level Generalized Utilitarian rules. As a by-product of our analysis, we show that social discounting is ruled out in an intertemporal welfarist environment. A simple population-planning example is also discussed.

Charles Blackorby
Department of Economics
University of British Columbia
Vancouver, BC V6T 1Z1
Canada

1. Introduction

The evaluation of public policies often requires the comparison of possible states of the world with different populations. For example, the design of social security systems, resource use, and birth-control policies all involve different populations and population sizes. These variable-population issues arise naturally in an intertemporal context. Consequently, fundamental ethical issues must be addressed in order to deal with them.

Despite the importance and relevance of these issues, the theoretical foundations of social evaluation with variable populations have received little attention in the literature. This paper introduces a model that provides a theoretical framework for the requisite ethical investigation.

Welfarist population ethics is concerned with principles that use information about the well-being (utilities) of the individuals who are alive in alternative states of affairs to make social evaluations.[1] In most cases, these principles are employed 'timelessly', based on individual lifetime utilities, but without taking into account intertemporal aspects;[2] sometimes, however, utilities experienced in a single period are used.[3] We adopt the standard normalization that individual *neutrality* is represented by a lifetime-utility level of zero.[4] An individual life is worth living if and only if lifetime utility is positive (see Section 2 for a discussion).

The most commonly used welfarist principles are *Classical Utilitarianism* and *Average Utilitarianism* whose value functions are total utility and average utility. Other principles have been proposed as well,[5] and one family that concerns us specifically is *Critical-Level Utilitarianism* and its generalized versions (Blackorby and Donaldson [1984]). These principles contain a parameter—the critical level of utility. The value function for Critical-Level Utilitarianism is obtained by subtracting the critical level from the lifetime utility of each individual alive in a state and adding the resulting numbers.

The critical level is easily interpreted: if the addition of another individual to an existing population does not affect its members' utilities, the change is a social improvement

[1] See Blackorby and Donaldson [1984] and Sen [1991]. Dasgupta [1988] presents a principle that does not produce orderings of alternative states. In this paper, we ignore the well-being of sentient non-humans, but refer readers who are interested in the ethics of animal exploitation to Blackorby and Donaldson [1992b].

[2] See Blackorby and Donaldson [1984, 1991] and Bossert [1990a,b].

[3] See Broome [1992a,b,c], Cowen [1989] and Hurka [1983].

[4] Any number will do, but zero is standard in the population literature. See Broome [1992a,b,c], Dasgupta [1988], Parfit [1976, 1982, 1984], and Sikora [1978].

[5] See Hurka [1983] and Ng [1986].

if the added person's well-being exceeds the critical level and a social deterioration if the added utility is below the critical level. Sikora [1978] argues that the critical level should be zero and uses an axiom he calls the *Pareto plus principle* which requires it, but Blackorby and Donaldson [1984] argue that it should be positive. Parfit [1982] suggests an interval of critical levels: an addition to the population, *ceteris paribus*, is a social improvement if and only if the added individual's utility exceeds the highest critical level, and it is a social deterioration if and only if the added utility is less than the lowest critical level, resulting in an incomplete ranking of social states (see Blackorby, Bossert, and Donaldson [1995b]).

A principle implies the *repugnant conclusion* (Parfit [1976, 1982, 1984]) if any state in which each member of the population enjoys a life above neutrality is declared inferior to a state in which each member of a sufficiently large population lives a life that is above, but arbitrarily close to, neutrality. Such principles may recommend creating a large population even if the result is mass poverty. Classical Utilitarianism is an example of such a rule. Imagine a social change in which a million babies are born, live barely happy lives that last a single minute, and die.[6] Assuming that no one else is affected, in utility terms, by the addition of the babies, Classical Utilitarianism declares the change good. This results in a lower average utility. The change can be repeated any number of times, moving average utility toward the level of well-being of the babies, and each change counts as a social improvement. It is this property that leads to the repugnant conclusion. Principles such as Average (Generalized) Utilitarianism and Critical-Level (Generalized) Utilitarianism with a positive critical level avoid the repugnant conclusion.

Because the Pareto plus principle forces the critical level to zero, it can only be imposed if one is willing to accept the repugnant conclusion. Although it is possible to provide plausible arguments in favor of the Pareto plus principle, any view (such as our own) which considers the repugnant conclusion ethically unacceptable requires its rejection.

We provide a characterization of Critical-Level Generalized Utilitarian rules in an intertemporal framework. Our approach is welfarist in the sense that our assumptions (in particular, the *strong Pareto* principle) imply that the individual *lifetime utilities* achieved in possible states of the world are the only determinants of the social ranking. We propose an axiom called *independence of the utilities of the dead.* It allows an evaluator in the present to ignore the utilities of individuals whose lives are over in the states to be compared; lifetime utilities of individuals that continue into the present in at least one of these states *are* permitted to influence the social ranking.[7] This axiom permits history to matter to some extent. This is desirable because rules that are based only on present and

[6] This example was suggested by Amartya Sen.

[7] In a different framework, Hammond [1988] uses a similar axiom called *independence of long dead ancestors*. Suitably reformulated, it is sufficient to prove the results of the present paper.

future utilities lead to the repugnant conclusion if a plausible intertemporal consistency requirement is satisfied (Blackorby, Bossert, and Donaldson [1995a]).

History-dependent principles are able to distinguish between an individual dying just before the period in which the evaluation is made and an individual not being born at all. Independence of the utilities of the dead allows this important distinction to be made, thereby permitting ethically attractive social decision rules.

The *only* rules that satisfy independence of the utilities of the dead and our additional (standard) fixed-population assumptions are the Critical-Level Generalized Utilitarian rules. Because these principles escape the repugnant conclusion when the critical level is positive, this provides a strong argument in their favour.

The strong Pareto condition rules out social discounting of future utilities. Because *Pareto indifference* (which is implied by strong Pareto) is an essential component of welfarism,[8] it follows that, in our model, social discounting is not welfarist. There is a good reason for discounting the well-being of future generations if their existence is uncertain, of course, but that consideration does not apply to our model (in which all outcomes are certain).

2. An Intertemporal Framework

In this section, we present a general model for intertemporal social evaluation with a variable population. Lifetime utilities, birth dates and lengths of life vary across states. Lifetime utilities represent individual well-being and may depend on such 'prudential values' as accomplishment, health, autonomy, liberty, understanding, pleasure, and personal relations in addition to consumption (Griffin [1986]).

The set of positive (nonnegative) integers is denoted by $\mathcal{Z}_{++}$ ($\mathcal{Z}_{+}$). For $n \in \mathcal{Z}_{++}$, $\mathcal{R}^n$ is Euclidean n-space; the superscript is omitted in $\mathcal{R}^1$.

$X = \{x, y, \ldots\}$ is the set of social states of affairs, $N = \mathbf{N}(x)$ is the set of individuals alive in state x, and $n = \mathbf{n}(x)$ is the number alive in x. If person i is alive in x, $s^i = S^i(x)$ is the period just before he or she is born and $l^i = L^i(x)$ is his or her lifetime. Given this, person i is alive in periods $(s^i + 1, \ldots, s^i + l^i)$. No person can live more than $L \in \mathcal{Z}_{++}$ periods.

The lifetime utility enjoyed by individual i in state x, $i \in \mathbf{N}(x)$, is $u^i = U^i(x)$ and $U^i(x) > U^i(y)$ means that person i's life in state x is better than in state y, all

[8] In social-choice theory, welfarism is a consequence of three axioms: *Pareto indifference, unlimited domain,* and *independence of irrelevant alternatives.* For a discussion, see Blackorby and Donaldson [1984] and Blackorby, Donaldson, and Weymark [1984].

things considered.[9] Lifetime utilities are normalized so that a lifetime-utility level of zero ($U^i(x) = 0$) represents neutrality. For an individual, a life, taken as a whole, is worth living if and only if lifetime utility is above neutrality. A fully informed individual whose lifetime-utility level is negative prefers not to have any of his or her experiences (that is, to be permanently unconscious). Because people who do not exist cannot have preferences, it does not make sense to say that an individual who is not alive gains by existing with a positive utility level. In this paper, we do not assume that the existence of a person at neutrality is socially equivalent to nonexistence. We note, as well, that killing a person who is above neutrality reduces his or her lifetime utility; it does not change population size.

We assume that lifetime utilities are numerically measurable and interpersonally fully comparable in order to allow for the largest possible class of social orderings.[10]

The set of individuals alive in state x, together with birth dates, lifetimes and lifetime utilities, can be written as an *alternative*

$$A = \Big\{N, \big\{(s^i, l^i, u^i)\big\}_{i\in N}\Big\} = \mathbf{A}(x) = \Big\{\mathbf{N}(x), \big\{(S^i(x), L^i(x), U^i(x))\big\}_{i\in \mathbf{N}(x)}\Big\}. \tag{2.1}$$

The set $\mathcal{A}$ of admissible alternatives consists of all alternatives A in which (i) N is finite, and (ii) $s^i \in \mathcal{Z}_+$, $l^i \in \{1, \ldots, L\}$, $u^i \in \mathcal{R}$. In addition, $\mathcal{A}$ includes the null alternative $A_\emptyset$, the alternative in which no one is alive. Because of our welfarist approach, we assume that a single ordering of $\mathcal{A}$ is sufficient to order X given $\mathbf{A}(x)$ for all x in X. The general definition of $\mathcal{A}$ corresponds to an unlimited domain assumption commonly used in social-choice theory.

3. Social Evaluation With Lifetime Utilities

To describe welfarist social evaluations in an intertemporal framework, the following notation is used: $\{\overline{A}, \widehat{A}, \ldots\}$ are alternatives, $\{\overline{N}, \widehat{N}, \ldots\}$ are the corresponding sets of named individuals alive in the states, $\{\bar{n}, \hat{n}, \ldots\}$ are the numbers of people alive, and $\bar{u}^i$ and $\hat{u}^j$ are the lifetime utilities of $i \in \overline{N}$, $j \in \widehat{N}$. In addition, R denotes the social ordering of the alternatives in $\mathcal{A}$, and $\overline{A}\ R\ \widehat{A}$ means that $\overline{A}$ is socially at least as good as $\widehat{A}$. The strict preference and indifference relations corresponding to R are P and I.[11]

We assume that R satisfies the strong Pareto condition.

Strong Pareto: *For all $\overline{A}, \widehat{A} \in \mathcal{A}$ with $\overline{N} = \widehat{N} = N$:*

(i) if, for all $i \in N$, $\bar{u}^i = \hat{u}^i$, then $\overline{A}$ and $\widehat{A}$ are socially indifferent ($\overline{A}\ I\ \widehat{A}$);

(ii) if, for all $i \in N$, $\bar{u}^i \geq \hat{u}^i$ with at least one strict inequality, then $\overline{A}$ is socially preferred to $\widehat{A}$ ($\overline{A}\ P\ \widehat{A}$).

Strong Pareto implies Pareto indifference ((i) in the definition), a condition that plays a critical role in this paper. It ensures that social evaluations are based on individuals' assessments of their own lives—lifetime utilities.

Strong Pareto is an axiom that applies to fixed populations only. To link alternatives with populations of the same size but with different individuals alive in each, we require the ordering R to treat people anonymously, paying no attention to their identities.

Anonymity: *For all $\overline{A}, \widehat{A} \in \mathcal{A}$ with $\bar{n} = \hat{n}$, if there exists a bijection $\pi : \overline{N} \longmapsto \widehat{N}$ such that $\bar{s}^{\bar{\imath}} = \hat{s}^{\pi(\bar{\imath})}$, $\bar{l}^{\bar{\imath}} = \hat{l}^{\pi(\bar{\imath})}$, and $\bar{u}^{\bar{\imath}} = \hat{u}^{\pi(\bar{\imath})}$ for all $\bar{\imath} \in \overline{N}$, then $\overline{A}$ and $\widehat{A}$ are socially indifferent ($\overline{A}\ I\ \widehat{A}$).*

In addition, we assume that the fixed-population orderings implied by R are continuous.

Continuity: *For all $\widetilde{N} \neq \emptyset$, the restriction of the ordering R to $\{A \in \mathcal{A} \mid N = \widetilde{N}\}$ is continuous in lifetime utilities.*

Strong Pareto, anonymity, and continuity allow us to find a value function for the ordering R. For any $\overline{A}$ and $\widehat{A}$ in $\mathcal{A}$, there exist alternatives $\overline{A}'$ and $\widehat{A}'$ in $\mathcal{A}$ with $\overline{N}' = \overline{N}$ and $\widehat{N}' = \widehat{N}$ such that each person has the same lifetime utility and length of life but is born in period 1 ($s^i = 0$). Pareto indifference implies $\overline{A}\, I\, \overline{A}'$ and $\widehat{A}\, I\, \widehat{A}'$. It follows that the social ranking of $\overline{A}$ and $\widehat{A}$ is the same as the social ranking of $\overline{A}'$ and $\widehat{A}'$. Thus, for social evaluation, we can confine attention to alternatives in which everyone is born in period one. Strong Pareto also implies that the ordering R is independent of lengths of life. It follows that R is fully welfarist; it depends on lifetime utilities only.

For alternatives with a given number of people $n \in \mathcal{Z}_{++}$, we can represent the fixed-population orderings by the continuous, increasing, and symmetric social-evaluation functions $\{W^n : \mathcal{R}^n \longmapsto \mathcal{R} \mid n \in \mathcal{Z}_{++}\}$. The *representative utility* v is defined to be that level of lifetime utility which, if enjoyed by everyone, is socially indifferent to the actual utility vector ($\{u^i\}_{i\in N}$). Thus,

$$W^n(v\mathbf{1}_n) = W^n(\{u^i\}_{i\in N}), \tag{3.1}$$

[9] Lifetime utilities are experienced by individuals who are able to rank states which involve different lifetimes. Although it is not necessary for our analysis, it is possible to regard lifetime utilities as functions of the utilities experienced in each period of life. Thus, for person i, we can define a set of *lifetime* utility functions $\{U^i_{l^i} : \mathcal{R}^{l^i} \longmapsto \mathcal{R} \mid 1 \leq l^i \leq L\}$, so that $u^i = U^i_{l^i}(u^i_{s^i+1}, \ldots, u^i_{s^i+l^i})$ where u^i_t is person i's utility in period t. This formulation does not require individual utility functions to be separable—per-period utilities may depend on present, past, and anticipated experiences and consumption.

[10] Other information assumptions could be used but would restrict the class of possible orderings. For discussions of information assumptions in social-choice theory, see Blackorby, Donaldson, and Weymark [1984], Bossert [1991], d'Aspremont and Gevers [1977], Hammond [1979], Roberts [1980a,b], and Sen [1974, 1977].

[11] P is defined by $\overline{A}\ P\ \widehat{A} \longleftrightarrow \overline{A}\ R\ \widehat{A}$ *and not* $\widehat{A}\ R\ \overline{A}$; I is defined by $\overline{A}\ I\ \widehat{A} \longleftrightarrow \overline{A}\ R\ \widehat{A}$ *and* $\widehat{A}\ R\ \overline{A}$.

where $\mathbf{1_n} = (1, \ldots, 1) \in \mathcal{R}^n$. Given continuity and increasingness, (3.1) can be solved uniquely for v, and we write

$$v = \Upsilon^n(\{u^i\}_{i \in N}). \tag{3.2}$$

The function $\Upsilon^n : \mathcal{R}^n \longmapsto \mathcal{R}$ is continuous and ordinally equivalent to W^n for all $n \in \mathcal{Z}_{++}$.

The family of functions $\{\Upsilon^1, \Upsilon^2, \ldots\}$ (or $\{W^1, W^2, \ldots\}$) is not sufficient to rank states with different population sizes. By Theorem 2.1 of Blackorby and Donaldson [1984], there exists a *social value function* $W : \mathcal{Z}_{++} \times \mathcal{R} \longmapsto \mathcal{R}$, increasing in its second argument, such that x is socially at least as good as y if and only if

$$W\left(\mathbf{n}(x), \Upsilon^{\mathbf{n}(x)}\left(\{U^i(x)\}_{i \in \mathbf{N}(x)}\right)\right) \geq W\left(\mathbf{n}(y), \Upsilon^{\mathbf{n}(y)}\left(\{U^i(y)\}_{i \in \mathbf{N}(y)}\right)\right) \tag{3.3}$$

whenever $\mathbf{n}(x)$ and $\mathbf{n}(y)$ are positive. Therefore R can be represented by W; that is, for all $\overline{A}$ and $\widehat{A}$ in $\mathcal{A} \setminus \{A_\emptyset\}$,

$$\overline{A} \, R \, \widehat{A} \longleftrightarrow W(\bar{n}, \bar{v}) \geq W(\hat{n}, \hat{v}), \tag{3.4}$$

where $\bar{n}$ ($\hat{n}$) is population size in $\overline{A}$ ($\widehat{A}$) and $\bar{v}$ ($\hat{v}$) is representative utility in $\overline{A}$ ($\widehat{A}$).

For any $A \in \mathcal{A} \setminus \{A_\emptyset\}$, consider another alternative with one additional person in which the utilities of those alive in both states are the same, person for person.[12] The *critical level* c is that lifetime-utility level which, if enjoyed by the additional person, results in social indifference. That is, for any $A \in \mathcal{A} \setminus \{A_\emptyset\}$,

$$W\left(n+1, \Upsilon^{n+1}(\{u^i\}_{i \in N}, c)\right) = W\left(n, \Upsilon^n(\{u^i\}_{i \in N})\right). \tag{3.5}$$

If a $c \in \mathcal{R}$ exists such that (3.5) is satisfied, the solution is unique because Υ^{n+1} is increasing. Therefore, we write

$$c = C^n(\{u^i\}_{i \in N}), \tag{3.6}$$

where $\{C^1, C^2, \ldots\}$ is a family of *critical-level functions* for the principle in question. Because we assume strong Pareto, critical levels—if they exist—cannot depend on the birth date of the added individual.

We *assume* that there exists a critical level $c_\emptyset$ for the null alternative $A_\emptyset$; that is, $A_\emptyset$ is indifferent to the alternative where one person is alive with lifetime utility $c_\emptyset$. Again, because of strong Pareto, $c_\emptyset$ is independent of the birth date of this person. This is a very weak assumption. Given continuity, nonexistence of $c_\emptyset$ would imply either that a society with one person alive is always better than a society with nobody alive, no matter how miserable this one individual might be, or that a one-person society is always worse than the situation represented by $A_\emptyset$, no matter how well off the single person is. Those nonsensical consequences are ruled out by assuming the existence of a critical level for the null alternative. It should be noted that the existence of critical levels for alternatives other than $A_\emptyset$ is not assumed for our results—existence of those critical levels follows from the existence of $c_\emptyset$ and our axioms.

For each social state and each person alive in it, the evaluator knows the date at which the individual is born. Should the social value function depend on that information? If it does not, social discounting is ruled out. If it does, discounting of future lifetime utilities is permitted. The first of these positions is a consequence of Pareto indifference. For any alternative $A \in \mathcal{A}$, consider another alternative which is identical except that each person's life begins in another period. Because individual lifetime utilities are the same, Pareto indifference forces social indifference.[13]

A possible objection to this argument is the claim that any individual's birth date is fixed. Although it is true that a person cannot be born at a completely arbitrary time, because the duration of pregnancy is not fixed, his or her birth date may vary over several months.[14] This possibility is sufficient, given Pareto indifference, to rule out discounting.

It is possible, however, to modify our framework by assuming fixed birth dates. In this case, a strengthening of anonymity that rules out the relevance of birth dates in social evaluations could be used, together with a domain assumption that guarantees, for each alternative A, the existence of another alternative such that the number of people alive is equal to the number of people in A, individual lifetime utilities are the same, and birth dates are arbitrary. Because the exposition of our results is simplified in the unlimited domain framework, we have chosen to use it in this paper.

4. Population Principles

The most commonly used principles are extensions of fixed-population (Generalized) Utilitarian rules to societies with a variable population. Fixed-population Utilitarianism is described by the representative-utility functions

$$v = \Upsilon^n(\{u^i\}_{i \in N}) = \frac{1}{n} \sum_{i \in N} u^i, \tag{4.1}$$

average utility. Utilitarianism can be generalized by writing

$$v = \Upsilon^n(\{u^i\}_{i \in N}) = g^{-1}\left(\frac{1}{n} \sum_{i \in N} g(u^i)\right) \tag{4.2}$$

12 This hypothetical situation is examined to shed light on different ethical judgments. It is important to realize that utilities are assumed to capture *all* aspects of individual well-being, including the 'externalities' of friendship, family life, crowding, crime, and so on.

13 Cowen [1992] and Hammond [1988] use Pareto indifference to argue against social discounting.

14 Similarly, although it is unlikely, the date of conception may vary because spermatozoa may survive for some time.

where $g : \mathcal{R} \longmapsto \mathcal{R}$ is increasing and continuous. Without loss of generality, g can be chosen so that $g(0) = 0$. If g is (strictly) concave, the resulting rule represents (strict) inequality aversion.

Classical Utilitarianism ranks states according to the total utilities of those alive in them, and

$$W(n, v) \overset{\circ}{=} nv = \sum_{i \in N} u^i \tag{4.3}$$

where $\overset{\circ}{=}$ means 'is ordinally equivalent to'. *Classical Generalized Utilitarianism* uses transformed utilities, with

$$W(n, v) \overset{\circ}{=} ng(v) = \sum_{i \in N} g(u^i). \tag{4.4}$$

Average Utilitarianism uses average utilities to compare states, and *Average Generalized Utilitarianism* uses representative utilities. In both cases,

$$W(n, v) \overset{\circ}{=} v. \tag{4.5}$$

Blackorby and Donaldson [1984, 1991] and Hurka [1982] provide discussions of Average Utilitarianism, and generalized averaging rules are considered in Blackorby and Donaldson [1984] and Bossert [1990a,b].

In an atemporal model, Blackorby and Donaldson [1984] propose two families of principles based on Utilitarianism and Generalized Utilitarianism. In the former case, *Critical-Level Utilitarianism*, the value function is

$$W(n, v) \overset{\circ}{=} n\big[v - \alpha\big] = \sum_{i \in N} \big[u^i - \alpha\big], \tag{4.6}$$

where $\alpha \in \mathcal{R}$ is the critical utility level. Critical-Level Utilitarianism is a different principle for each value of α.[15] *Critical-Level Generalized Utilitarianism* uses transformed utilities, and

$$W(n, v) \overset{\circ}{=} n\big[g(v) - g(\alpha)\big] = \sum_{i \in N} \big[g(u^i) - g(\alpha)\big]. \tag{4.7}$$

When α is positive, these principles avoid the repugnant conclusion (Parfit [1976, 1982, 1984]) which is implied by Classical Utilitarianism and its generalized counterpart. Principles that imply the repugnant conclusion declare any state with a representative utility above neutrality inferior to another state with a representative utility above but arbitrarily close to neutrality and a suitably large population. Although critical-level principles allow representative utilities just above the critical level with a large enough population size to substitute for higher representative utilities and smaller populations, this should not prove to be ethically unattractive as long as the critical level is chosen so that new lives above it are regarded as desirable. As Sen [1991] remarks, "a scenario in which more people enjoy a utility level... [above α]... must be seen as a better outcome". In addition, for representative utilities above α, increasing α 'sharpens' the tradeoff between representative utility and population size (see Blackorby and Donaldson [1984, 1991, 1992a] for discussions).

[15] It is possible to normalize utilities so that neutrality is represented by a utility level of $-\alpha$. In that case, the value function is the sum of normalized utilities; the social ordering is, of course, unchanged. This makes it clear that the standard normalization of zero for neutrality is essential in order to write the Classical Utilitarian value function as (4.3).

5. History-Dependent Population Principles

When population decisions are made in period t, all feasible states have a common history, but the lifetime utilities of some individuals are not fixed. If person i is alive in period $t-1$, then there may be states, available to decision makers at time t, in which person i's life is extended to period t and beyond, and other states in which i dies at the end of period $t-1$. If decisions in period t are made on the basis of present and future per-period utilities only, any per-period principle that is consistent with the overall ordering R *must* produce the repugnant conclusion. This is the case because the only principles satisfying such a consistency condition and our other axioms are the Generalized Classical Utilitarian rules—see Blackorby, Bossert, and Donaldson [1995a] for details.

The source of the problem is that, if the past is ignored in ranking states of the world in period t, it is impossible to distinguish between a person dying just before period t and a person not being born in period t. Therefore, in order to avoid these difficulties, history has to matter to some extent. On the other hand, one might not want to take into account the utilities of individuals whose lives are over in all states of the world to be compared.

This can be accomplished by identifying, for each alternative $A \in \mathcal{A}$, the set of individuals who do not live beyond period $t-1$,

$$D_t(A) = \big\{i \in N \mid s^i + l^i < t\big\}. \tag{5.1}$$

These individuals are the *dead* in period t. In addition, we identify the set of individuals who are born before period t, which is

$$B_t(A) = \big\{i \in N \mid s^i + 1 < t\big\}. \tag{5.2}$$

Suppose that, in any two alternatives, the same individuals are born and die before period t and have the same birth dates, lengths of life, and lifetime utilities. *Independence of the utilities of the dead* requires that, if these individuals are removed from the alternatives in question, the social ranking is unchanged. The axiom that uses this construction is

Independence of the Utilities of the Dead: *For all $\overline{A}$, $\widehat{A} \in \mathcal{A}$ and all $t \in \mathcal{Z}_{++}$, if*

$$B_t(\overline{A}) = D_t(\overline{A}) = D_t(\widehat{A}) = B_t(\widehat{A}) =: \Delta_t, \tag{5.3}$$

$(\bar{s}^i, \bar{l}^i, \bar{u}^i) = (\hat{s}^i, \hat{l}^i, \hat{u}^i)$ for all $i \in \Delta_t$, and the alternatives $\overline{A}_t$ and $\widehat{A}_t$ are constructed by removing all individuals in Δ_t from $\overline{A}$ and $\widehat{A}$, then

$$\overline{A}\, R\, \widehat{A} \longleftrightarrow \overline{A}_t\, R\, \widehat{A}_t \quad \text{and} \quad \widehat{A}\, R\, \overline{A} \longleftrightarrow \widehat{A}_t\, R\, \overline{A}_t. \tag{5.4}$$

Independence of the utilities of the dead is a very weak axiom. It only applies to the comparison of alternatives where all individuals who are born before period t die before period t, and have the same birth dates, lifetimes, and lifetime utilities in both alternatives. Our domain assumption ensures that this axiom is sufficient to prove our results.

When independence of the utilities of the dead is combined with strong Pareto, anonymity and continuity, additive separability of the fixed-population functions $\{\Upsilon^n\}$ and $\{W^n\}$ results. We consider $\mathcal{A}^n$, the subset of $\mathcal{A}$ in which each alternative has n people alive.

Lemma 1: *Strong Pareto, Anonymity, Continuity, and Independence of the Utilities of the Dead imply that, for all $n \geq 3$, Υ^n is additively separable and can be written as*

$$\Upsilon^n(\{u^i\}_{i \in N}) = g^{-1}\left(\frac{1}{n}\sum_{i \in N} g(u^i)\right) \tag{5.5}$$

where $g : \mathcal{R} \longmapsto \mathcal{R}$ is continuous and increasing and $g(0) = 0$.

Proof: For $n \geq 3$, consider any $\overline{A}$, $\widehat{A} \in \mathcal{A}^n$ with $\overline{N} = \widehat{N} = \{1, \ldots, n\} =: N$ in which $(\bar{s}^i, \bar{l}^i, \bar{u}^i) = (\hat{s}^i, \hat{l}^i, \hat{u}^i)$ for all $i \in N^*$ where $\emptyset \neq N^* \subset N$. Because L (the maximal possible lifetime) and n are finite, there exists $\widetilde{L} \in \mathcal{Z}_{++}$ such that $\bar{s}^i + \bar{l}^i = \hat{s}^i + \hat{l}^i < \widetilde{L}$ for all $i \in N$.

Now consider the alternatives $\overline{A}'$ and $\widehat{A}'$ which differ from $\overline{A}$ and $\widehat{A}$ in that everyone in $N \setminus N^*$ is born in period $\widetilde{L} + 1$ ($\bar{s}'^i = \hat{s}'^i = \widetilde{L}$ for all $i \in N \setminus N^*$). Then

$$B_{\widetilde{L}+1}(\overline{A}') = D_{\widetilde{L}+1}(\overline{A}') = D_{\widetilde{L}+1}(\widehat{A}') = B_{\widetilde{L}+1}(\widehat{A}') = \Delta_{\widetilde{L}+1} = N^*. \tag{5.6}$$

By Pareto Indifference, $\overline{A}'$ and $\overline{A}$ are socially indifferent and $\widehat{A}'$ and $\widehat{A}$ are socially indifferent, and, therefore,

$$\overline{A}\, R\, \widehat{A} \longleftrightarrow \overline{A}'\, R\, \widehat{A}' \quad \text{and} \quad \widehat{A}\, R\, \overline{A} \longleftrightarrow \widehat{A}'\, R\, \overline{A}'. \tag{5.7}$$

Removing all individuals in N^* from $\overline{A}'$ and $\widehat{A}'$ results in the alternatives $\overline{A}''$ and $\widehat{A}''$ which are ranked in the same way as $\overline{A}'$ and $\widehat{A}'$ by Independence of the Utilities of the Dead. We then consider the alternatives $\overline{A}'''$ and $\widehat{A}'''$ which are identical to $\overline{A}''$ and $\widehat{A}''$ except that each person's date of birth is the same as in $\overline{A}$ and $\widehat{A}$ ($\bar{s}'''^i = \hat{s}'''^i = \bar{s}^i = \hat{s}^i$ for all $i \in N$). Pareto Indifference implies that these alternatives are indifferent to $\overline{A}''$ and $\widehat{A}''$ respectively, and we get

$$\overline{A}\, R\, \widehat{A} \longleftrightarrow \overline{A}'''\, R\, \widehat{A}''' \quad \text{and} \quad \widehat{A}\, R\, \overline{A} \longleftrightarrow \widehat{A}'''\, R\, \overline{A}''' \tag{5.8}$$

which is equivalent to

$$\Upsilon^n(\bar{u}^1, \ldots, \bar{u}^n) \geq \Upsilon^n(\hat{u}^1, \ldots, \hat{u}^n) \longleftrightarrow \Upsilon^m\left(\{\bar{u}^i\}_{i \in N \setminus N^*}\right) \geq \Upsilon^m\left(\{\hat{u}^i\}_{i \in N \setminus N^*}\right) \tag{5.9}$$

and

$$\Upsilon^n(\hat{u}^1, \ldots, \hat{u}^n) \geq \Upsilon^n(\bar{u}^1, \ldots, \bar{u}^n) \longleftrightarrow \Upsilon^m\left(\{\hat{u}^i\}_{i \in N \setminus N^*}\right) \geq \Upsilon^m\left(\{\bar{u}^i\}_{i \in N \setminus N^*}\right), \tag{5.10}$$

where $m := |N \setminus N^*|$. This implies that, in Υ^n, $N \setminus N^*$ is separable from its complement and, therefore, that every subset of variables is separable from its complement. Gorman's [1968] theorem on overlapping separable sets (see also Blackorby, Primont, and Russell [1978])[16] implies additive separability, which results in

$$\Upsilon^n(u^1, \ldots, u^n) = F^n\left(\sum_{i \in N} g_n^i(u^i)\right) \tag{5.11}$$

where F^n and each g_n^i are continuous and increasing. Symmetry of Υ^n implies that each g_n^i can be chosen to be independent of i, and, writing $g_n = g_n^i$, we can choose $g_n(0) = 0$ by a normalization. (5.9) and (5.10) imply that g_n, and, therefore, F^n, can be chosen to be independent of n. Because $\Upsilon^n(u, \ldots, u) = u$ for all $u \in \mathcal{R}$,

$$\Upsilon^n(u^1, \ldots, u^n) = g^{-1}\left(\frac{1}{n}\sum_{i \in N} g(u^i)\right) \tag{5.12}$$

where $g := g_n$ for all n. Anonymity implies (5.5). ■

An interesting fixed-population result follows from Lemma 1. Independence of the utilities of the dead and our other axioms imply that fixed-population orderings must be consistent with additively separable social-evaluation functions.[17]

Theorem 1 establishes that critical levels must exist for all A in $\mathcal{A}$ and, in addition, must be equal to $c_\emptyset$ (the critical level for the null alternative, the existence of which we assume) for all A in $\mathcal{A}$. Note that this implies, in particular, that critical levels cannot depend on the size of the existing population. We call the common critical level α, and show that Critical-Level Generalized Utilitarianism is the only family of population principles satisfying independence of the utilities of the dead and our other axioms.

16 The version of Gorman's theorem that is relevant here is stated in the appendix.

17 See also d'Aspremont and Gevers [1977] for a separability axiom with respect to 'unconcerned individuals'.

Theorem 1: *The ordering R satisfies Strong Pareto, Anonymity, Continuity, and Independence of the Utilities of the Dead if and only if, for all $\overline{A}$, $\widehat{A} \in \mathcal{A} \setminus \{A_\emptyset\}$,*

$$\overline{A}\, R\, \widehat{A} \longleftrightarrow \sum_{i\in\overline{N}} \left[g(\bar{u}^i) - g(\alpha)\right] \geq \sum_{i\in\widehat{N}} \left[g(\hat{u}^i) - g(\alpha)\right], \tag{5.13}$$

where $g : \mathcal{R} \longmapsto \mathcal{R}$ is continuous and increasing, $g(0) = 0$ and $\alpha \in \mathcal{R}$, and if $\overline{A}$ or $\widehat{A}$ is $A_\emptyset$, the appropriate sum in (5.13) is replaced with zero.

Proof: To argue necessity, we first establish the existence of the critical-level functions $\{C^n \mid n \in \mathcal{Z}_{++}\}$, and show that, for all $n \in \mathcal{Z}_{++}$ and all $(u^1,\ldots,u^n) \in \mathcal{R}^n$,

$$C^n(u^1,\ldots,u^n) = c_\emptyset =: \alpha. \tag{5.14}$$

To do this, consider any $A \in \mathcal{A}$ in which $N = \{1,\ldots,n\}$. Let $\widetilde{L} \in \mathcal{Z}_{++}$ be such that $s^i + l^i < \widetilde{L}$ for all $i \in N$, and create the alternative $A' \in \mathcal{A}$ by adding an $n+1^{\text{st}}$ person born in period $\widetilde{L}+1$ ($s^{n+1} = \widetilde{L}$) with lifetime utility $u^{n+1} = \alpha$. In period $\widetilde{L}+1$,

$$B_{\widetilde{L}+1}(A') = D_{\widetilde{L}+1}(A') = D_{\widetilde{L}+1}(A) = B_{\widetilde{L}+1}(A) = \Delta_{\widetilde{L}+1} = \{1,\ldots,n\}, \tag{5.15}$$

the original n individuals. When these people are removed from A' and A, the alternatives A'' and $A_\emptyset$ result. Only person $n+1$ is alive in A'', and $A_\emptyset$ is the null alternative. Independence of the Utilities of the Dead implies that

$$A'\, R\, A \longleftrightarrow A''\, R\, A_\emptyset \quad\text{and}\quad A\, R\, A' \longleftrightarrow A_\emptyset\, R\, A''. \tag{5.16}$$

Because the only person alive in A'' has a lifetime utility of $\alpha = c_\emptyset$, we have $A''\, I\, A_\emptyset$, and therefore, (5.16) implies $A'\, I\, A$. This means that α is the critical level for A; that is, C^n exists for all $n \in \mathcal{Z}_{++}$ and (5.14) is satisfied for all $n \in \mathcal{Z}_{++}$ and all $(u^1,\ldots,u^n) \in \mathcal{R}^n$. Anonymity and Strong Pareto imply that, for any alternative $A \in \mathcal{A} \setminus \{A_\emptyset\}$,

$$C^n(\{u^i\}_{i\in N}) = \alpha. \tag{5.17}$$

Now consider any two alternatives $\overline{A}$ and $\widehat{A}$ in $\mathcal{A}\setminus\{A_\emptyset\}$ such that $\bar{n} \geq 3$ and, without loss of generality, $\bar{n} \geq \hat{n}$. The alternative $\widehat{A}'$ with $|\widehat{N}'| = \bar{n}$ is created by adding $(\bar{n} - \hat{n})$ individuals to $\widehat{A}$, each with a lifetime utility of α. Because α is the critical utility level,

$$\widehat{A}'\, I\, \widehat{A}, \tag{5.18}$$

and

$$\overline{A}\, R\, \widehat{A} \longleftrightarrow \overline{A}\, R\, \widehat{A}' \quad\text{and}\quad \widehat{A}\, R\, \overline{A} \longleftrightarrow \widehat{A}'\, R\, \overline{A}. \tag{5.19}$$

Applying Lemma 1, we have

$$\begin{aligned} \overline{A}\, R\, \widehat{A} &\longleftrightarrow \overline{A}\, R\, \widehat{A}' \\ &\longleftrightarrow g^{-1}\left(\frac{1}{\bar{n}}\sum_{i\in\overline{N}} g(\bar{u}^i)\right) \geq g^{-1}\left(\frac{1}{\bar{n}}\left[\sum_{i\in\widehat{N}} g(\hat{u}^i) + (\bar{n}-\hat{n})g(\alpha)\right]\right) \\ &\longleftrightarrow \sum_{i\in\overline{N}} g(\bar{u}^i) \geq \sum_{i\in\widehat{N}} g(\hat{u}^i) + (\bar{n}-\hat{n})g(\alpha) \\ &\longleftrightarrow \sum_{i\in\overline{N}} \left[g(\bar{u}^i) - g(\alpha)\right] \geq \sum_{i\in\widehat{N}} \left[g(\hat{u}^i) - g(\alpha)\right] \end{aligned} \tag{5.20}$$

and

$$\widehat{A}\, R\, \overline{A} \longleftrightarrow \sum_{i\in\widehat{N}} \left[g(\hat{u}^i) - g(\alpha)\right] \geq \sum_{i\in\overline{N}} \left[g(\bar{u}^i) - g(\alpha)\right], \tag{5.21}$$

which implies (5.13).

Lemma 1 applies only to alternatives with three or more people, but it is easily extended to $n = 2, 1$. For any alternative A in $\mathcal{A}^2$ ($n = 2$), the addition of a third person with a lifetime utility equal to the critical level α transforms A into $A' \in \mathcal{A}^3$ with social indifference. It follows that the results of Lemma 1 must apply to the case $n = 2$, and, trivially, to $n = 1$ as well. Because $c_\emptyset = \alpha$, the result extends to $A_\emptyset$.

Sufficiency is easily checked. ■

Theorem 1 permits us to describe all the principles that order X, the set of social states. For any two states x and y in X with associated alternatives $\mathbf{A}(x)$ and $\mathbf{A}(y)$, x is socially at least as good as y if and only if

$$\sum_{i\in\mathbf{N}(x)} \left[g(U^i(x)) - g(\alpha)\right] \geq \sum_{i\in\mathbf{N}(y)} \left[g(U^i(y)) - g(\alpha)\right]. \tag{5.22}$$

If $\mathbf{N}(x)$ or $\mathbf{N}(y)$ is empty, the appropriate sum is replaced with zero. If the fixed-population principles are utilitarian, then the function g must be linear, and (5.22) becomes

$$\sum_{i\in\mathbf{N}(x)} \left[U^i(x) - \alpha\right] \geq \sum_{i\in\mathbf{N}(y)} \left[U^i(y) - \alpha\right], \tag{5.23}$$

the Critical-Level Utilitarian family.

The Critical-Level Generalized Utilitarian principles induce orderings from the vantage point of period t. Define the ordering R_t by

$$\overline{A}\, R_t\, \widehat{A} \longleftrightarrow \sum_{i\in\overline{N}\setminus\left[D_t(\bar{A})\cap D_t(\hat{A})\right]} \left[g(\bar{u}^i) - g(\alpha)\right] \geq \sum_{i\in\widehat{N}\setminus\left[D_t(\bar{A})\cap D_t(\hat{A})\right]} \left[g(\hat{u}^i) - g(\alpha)\right]. \tag{5.24}$$

For all $\overline{A}$ and $\widehat{A}$ with a common history, R_t and R are the same. Note that the past *does* matter, because the lifetime utility of everybody who is alive at t in $\overline{A}$ or $\widehat{A}$ is taken into account.

The repugnant conclusion occurs with these principles when $\alpha \leq 0$, but not when $\alpha > 0$ because

$$W(n,v) \stackrel{\circ}{=} \sum_{i \in N} \left[g(u^i) - g(\alpha)\right] = n\left[g(v) - g(\alpha)\right] \tag{5.25}$$

is positive and increasing in n when $v > \alpha$, and negative and decreasing in n when $v < \alpha$. Therefore, any state with representative utility below α is socially inferior to any state with v above α, no matter what the (non-zero) population sizes are. This result is summarized in

Corollary 1: *The ordering R satisfies Strong Pareto, Anonymity, Continuity, and Independence of the Utilities of the Dead, and avoids the Repugnant Conclusion if and only if R is represented by a member of the Critical-Level Generalized Utilitarian family (5.22) and the critical level of lifetime utility, α, is greater than zero.*

It is possible to argue in favor of setting the critical level equal to zero (the Pareto-Plus principle). Theorem 1 implies that, in conjunction with our axioms, Classical Generalized Utilitarianism results. However, as Corollary 1 indicates, this position requires acceptance of the repugnant conclusion. Because we consider the repugnant conclusion ethically unattractive, we recommend a positive value for α.

There are, of course, principles such as Average Utilitarianism that do not imply the repugnant conclusion and cannot be written as (5.22). The reason that such principles do not appear in Corollary 1 is that they do not satisfy independence of the utilities of the dead—the inclusion or exclusion of the utilities of Cleopatra and Socrates in averages can make a difference to social rankings in the present.

As long as the critical lifetime-utility level α is not equal to zero, Critical-Level Generalized Utilitarianism distinguishes between killing someone and preventing his or her existence. With a positive α, this principle recommends that individuals whose lifetime-utility levels would be below α not be born, other things equal. This means that some lives that are above neutrality should be prevented if the lifetime utilities experienced are low. Once an individual has been born, however, all positive utility levels have value, and strong Pareto ensures that reducing an individual's lifetime utility in any way (without changing the utilities of others) is socially undesirable.

Another effect of a positive α is that weight is given to individual lives. For example, suppose that states x and y differ for persons 1 and 2 only. In x, person 1 enjoys a lifetime utility of 100, and person 2 is not alive. In y, both people are alive with lifetime utilities of 50 each. Classical Utilitarianism ($\alpha = 0$) declares x and y to be socially indifferent, but

Critical-Level Utilitarianism with $\alpha > 0$ prefers x to y: weight is given to the fact that *one* person experiences the lifetime utility of 100.

Hammond [1988] investigates a contingent-states model and uses an axiom that is similar to independence of the utilities of the dead. He suggests that a particular interpretation of Classical Utilitarianism avoids the repugnant conclusion if parents care about the well-being of their children.

6. An Application

Suppose resources can be allocated to a birth-control program and that Critical-Level Utilitarianism is employed to determine the optimal expenditure level. There are two periods and a single resource. In each period, the total amount available of the resource is ω. In period one, the population (which, for simplicity, we treat as a continuous variable in this section) is fixed at $\bar{n} > 0$. Expenditures on birth control in the first period affect the population size in the second period. Without any birth control expenditures, the population in period two is $\bar{n}$, and to reduce the population in period two to $n \in (0, \bar{n})$, the expenditure needed is $\gamma(\bar{n} - n)$ where $\gamma \geq 0$ is a parameter.

Individuals live for one period, all individuals in the society have the same increasing and *strongly concave*[18] utility function, and person i's utility is $U(z^i)$ where z^i is his or her consumption. Note that, because the utility function is the same for each person and strongly concave, Critical-Level Utilitarianism requires consumption to be equal within each generation. Therefore, in period one each person consumes z, and in period two each person consumes ω/n. We assume that ω is large enough so that each person can enjoy a utility level above the critical level α.

The budget constraint for period one is

$$\bar{n}z + \gamma(\bar{n} - n) \leq \omega. \tag{6.1}$$

Using Critical-Level Utilitarianism with a positive critical level α, we obtain the maximization problem

$$\max_{(z,n)} \left\{ \bar{n}\left[U(z) - \alpha\right] + n\left[U\left(\frac{\omega}{n}\right) - \alpha\right] \right\} \text{ subject to (6.1).} \tag{6.2}$$

We assume that α is large enough so that the optimal period-two population for the case $\gamma = 0$ is less than $\bar{n}$; our assumptions imply that it is positive.[19] Given this, (6.2) has a unique interior solution $(\overset{*}{z}, \overset{*}{n})$ for all values of γ in an interval $[0, \bar{\gamma})$ with $\bar{\gamma} > 0$. Because

[18] See Diewert, Avriel, and Zang [1981].

[19] When $\gamma = 0$, Critical-Level Utilitarianism requires that the optimal period-two population solve the *pure population problem* subject to the constraint $n \leq \bar{n}$. Given our assumptions, the optimal population is positive (Blackorby and Donaldson [1984]).

U is strongly concave, so is the objective function in (6.2), and therefore the first-order conditions are sufficient for a solution. Using standard comparative-statics techniques, we obtain the results

$$\frac{\partial \overset{*}{n}}{\partial \gamma} > 0, \qquad \frac{\partial \overset{*}{z}}{\partial \alpha} < 0, \quad \text{and} \quad \frac{\partial \overset{*}{n}}{\partial \alpha} < 0. \tag{6.3}$$

The response of $\overset{*}{z}$ to a change in γ cannot be signed.

If the cost of birth control increases (represented by an increase in γ), the optimal amount spent on birth control in period one falls, thereby increasing the population in period two. This, in turn, leads to a decrease in per-capita consumption in period two. There are two opposite influences on consumption in period one which are analogous to income and substitution effects, and the overall effect cannot be signed in general.

If the critical level α increases, it is better to have a smaller population (given that resources are fixed) and, therefore, the amount spent on birth control in period one increases. The amount of the resource available for consumption in period one decreases as a consequence.

7. Conclusion

Blackorby and Donaldson [1984] derive the Critical-Level Generalized Utilitarian principles from an axiom that produces additive separability of the fixed-population social-evaluation functions—the *population substitution principle*—and an axiom that requires the critical levels to be independent of existing utilities—the *critical-level population principle*. In an intertemporal setting, independence of the utilities of the dead is sufficient to substitute for both of these axioms. This leads to an alternative characterization of Critical-Level Generalized Utilitarianism.

The critical level of lifetime utility is an important ethical parameter. Broome [1992a,b] argues that it must be small enough so that lives that are well worth living are not prevented and large enough to avoid approximating the repugnant conclusion.[20] Blackorby, Bossert, and Donaldson [1995b] propose that an interval of critical levels could be chosen, with one state declared to be better than another if and only if it is better, according to Critical-Level Generalized Utilitarianism, for all critical levels in the interval. This is consistent with a suggestion of Parfit [1982], and it results in an incomplete ranking of social states.

We suggest, in closing, that the Critical-Level Generalized Utilitarian principles may prove to be useful in a wide variety of circumstances. They provide a way to formulate the tradeoff between the well-being of present and future generations when population sizes differ. And their greatest strength is they can handle problems where population size is endogenous, such as population control programs, health expenditures on pre-natal care, and foreign aid or development programs that have population-growth consequences.

[20] Population issues are also discussed in Broome [1991, 1992c].

APPENDIX: Gorman's Overlapping Theorem

Let $F : \mathcal{R}^n \longmapsto \mathcal{R}$ be a continuous and increasing function. Let $I = \{1, \ldots, n\}$ be the index set for the arguments of F. Let $I^r \subseteq I$ be such that $\emptyset \neq I^r \neq I$. Furthermore, let X^r be the subvector of $X \in \mathcal{R}^n$ corresponding to I^r.

For a continuous and increasing function $F : \mathcal{R}^n \longmapsto \mathcal{R}$, the set of variables I^r is *(strictly) separable in* F *from its complement* if and only if there exist continuous and increasing functions $F^r : \mathcal{R}^{|I^r|} \longmapsto \mathcal{R}$ and $F^0 : A^r \times \mathcal{R}^{|I^c|} \longmapsto \mathcal{R}$ such that

$$F(X) = F^0(F^r(X^r), X^c)$$

for all $X \in \mathcal{R}^n$, where $I^c := I \setminus I^r$ and $A^r := \{Y \in \mathcal{R} \mid \exists X^r \in \mathcal{R}^{|I^r|} \ni Y = F^r(X^r)\}$.

The following is the version of the *overlapping theorem* which is relevant for our purposes (Gorman [1968] actually proves a stronger result):

Theorem: *If I^r and I^s are nonempty and (strictly) separable in F from their respective complements in I, and $I^r \cap I^s$, $I^r \setminus I^s$, $I^s \setminus I^r$ are nonempty, then there exist continuous and increasing functions $F^1 : \mathcal{R}^{|I^1|} \longmapsto \mathcal{R}$, $F^2 : \mathcal{R}^{|I^2|} \longmapsto \mathcal{R}$, $F^3 : \mathcal{R}^{|I^3|} \longmapsto \mathcal{R}$, and $F^0 : A^{r,s} \times \mathcal{R}^{|I^c|} \longmapsto \mathcal{R}$ such that*

$$F(X) = F^0(F^1(X^1) + F^2(X^2) + F^3(X^3), X^c)$$

for all $X \in \mathcal{R}^n$, where $I^1 := I^r \setminus I^s$, $I^2 := I^r \cap I^s$, $I^3 := I^s \setminus I^r$, $I^c := I \setminus (I^r \cup I^s)$, and $A^{r,s} := \{Y \in \mathcal{R} \mid \exists X^1 \in \mathcal{R}^{|I^1|}, X^2 \in \mathcal{R}^{|I^2|}, X^3 \in \mathcal{R}^{|I^3|} \ni Y = F^1(X^1) + F^2(X^2) + F^3(X^3)\}$.

REFERENCES

Blackorby, C., W. Bossert, and D. Donaldson (1993): "Intertemporal Population Ethics: A Welfarist Approach," University of British Columbia Discussion Paper 93-13.

Blackorby, C., W. Bossert, and D. Donaldson (1995a): "Intertemporally Consistent Population Ethics: Classical Utilitarian Principles," in *Social Choice Re-Examined*, ed. by K. Arrow, A. Sen, and K. Suzumura.

Blackorby, C., W. Bossert, and D. Donaldson (1995b): "Quasi-Orderings and Population Ethics," *Social Choice and Welfare*, forthcoming.

Blackorby, C. and D. Donaldson (1984): "Social Criteria for Evaluating Populatfon Change," *Journal of Public Economics*, 25, 13–33.

Blackorby, C. and D. Donaldson (1991): "Normative Population Theory: A Comment," *Social Choice and Welfare*, 8, 261–267.

Blackorby, C. and D. Donaldson (1992a): "The Value of Living: A Comment," *Recherches Economiques de Louvain*, 58, 143–145.

Blackorby, C., and D. Donaldson (1992b): "Pigs and Guinea Pigs: A Note on the Ethics of Animal Exploitation," *Economic Journal*, 102, 1345–1369.

Blackorby, C., D. Donaldson, and J. Weymark (1984): "Social Choice with Interpersonal Utility Comparisons: A Diagrammatic Introduction," *International Economic Review*, 25, 327–356.

Blackorby, C., D. Primont, and R.R. Russell (1978): *Duality, Separability and Functional Structure: Theory and Economic Applications.* Amsterdam/New York: North-Holland/American Elsevier.

Bossert, W. (1990a): "Maximin Welfare Orderings with Variable Population Size," *Social Choice and Welfare*, 7, 39–45.

Bossert, W. (1990b): "Social Evaluation with Variable Population Size: An Alternative Concept," *Mathematical Social Sciences*, 19, 143–158.

Bossert, W. (1991): "On Intra- and Interpersonal Utility Comparisons," *Social Choice and Welfare*, 8, 207–219.

Broome, J. (1991): *Weighing Goods.* Oxford: Basil Blackwell.

Broome, J. (1992a): "The Value of Living," *Recherches Economiques de Louvain*, 58, 125–142.

Broome, J. (1992b): "Reply to Blackorby and Donaldson, and Drèze," *Recherches Economiques de Louvain*, 58, 167–171.

Broome, J. (1992c): *Counting the Cost of Global Warming.* Cambridge: White Horse.

Cowen, T. (1989): "Normative Population Theory," *Social Choice and Welfare*, 6, 33–43.

Cowen, T. (1992): "Consequentialism Implies a Zero Rate of Intergenerational Discount," in *Justice Between Age Groups and Generations*, ed. by P. Laslett and J. Fishkin. New Haven/London: Yale University Press.

Dasgupta, P. (1988): "Lives and Well-Being," *Social Choice and Welfare*, 5, 103–126.

d'Aspremont, C., and L. Gevers (1977): "Equity and the Informational Basis of Collective Choice," *Review of Economic Studies*, 44, 199–209.

Diewert, W.E., M. Avriel, and I. Zang (1981): "Nine Kinds of Quasiconcavity and Concavity," *Journal of Economic Theory*, 25, 397–420.

Gorman, W.M. (1968): "The Structure of Utility Functions," *Review of Economic Studies*, 32, 369–390.

Griffin, J. (1986): *Well-Being: Its Meaning, Measurement, and Moral Importance.* Oxford: Clarendon.

Hammond, P. (1979): "Equity in Two-Person Situations: Some Consequences," *Econometrica*, 47, 1127–1136.

Hammond, P. (1988): "Consequentialist Demographic Norms and Parenting Rights," *Social Choice and Welfare*, 5, 127–146.

Hurka, T.M. (1982): "Average Utilitarianism," *Analysis*, 42, 65–69.

Hurka, T.M. (1983): "Value and Population Size," *Ethics*, 93, 496–507.

Ng, Y.-K. (1986): "Social Criteria for Evaluating Population Change: An Alternative to the Blackorby-Donaldson Criterion," *Journal of Public Economics*, 29, 375–381.

Parfit, D. (1976): "On Doing the Best for Our Children," in *Ethics and Populations*, ed. by M. Bayles. Cambridge: Schenkman.

Parfit, D. (1982): "Future Generations, Further Problems," *Philosophy and Public Affairs*, 11, 113–172.

Parfit, D. (1984): *Reasons and Persons.* Oxford/New York: Oxford University Press.

Roberts, K.W.S. (1980a): "Possibility Theorems with Interpersonally Comparable Welfare Levels," *Review of Economic Studies*, 47, 409–420.

Roberts, K.W.S. (1980b): "Interpersonal Comparability and Social Choice Theory," *Review of Economic Studies*, 47, 421–439.

Sen, A. (1974): "Informational Bâses of Alternative Welfare Approaches: Aggregation and Income Distribution," *Journal of Public Economics*, 3, 387–403.

Sen, A. (1977): "On Weights and Measures: Informational Constraints in Social Welfare Analysis," *Econometrica*, 45, 1539–1572.

Sen, A. (1991): "Welfare Economics and Population Ethics," presented at the Nobel Jubilee Symposium on Population Development and Welfare, December 5–7, 1991, mimeo, Harvard University.

Sikora, R.I. (1978): "Is it Wrong to Prevent the Existence of Future Generations?," in *Obligations to Future Generations*, ed. by R.I. Sikora and B. Barry. Philadelphia: Temple.

CES Working Paper Series

---

12 Marc Nerlove, Assaf Razin, Efraim Sadka and Robert K. von Weizsäcker, Comprehensive Income Taxation, Investments in Human and Physical Capital, and Productivity, January 1992

13 Tapan Biswas, Efficiency and Consistency in Group Decisions, March 1992

14 Kai A. Konrad and Kjell Erik Lommerud, Relative Standing Comparisons, Risk Taking and Safety Regulations, June 1992

15 Michael Burda and Michael Funke, Trade Unions, Wages and Structural Adjustment in the New German States, June 1992

16 Dominique Demougin and Hans-Werner Sinn, Privatization, Risk-Taking and the Communist Firm, June 1992

17 John Piggott and John Whalley, Economic Impacts of Carbon Reduction Schemes: Some General Equilibrium Estimates from a Simple Global Model, June 1992

18 Yaffa Machnes and Adi Schnytzer, Why hasn't the Collective Farm Disappeared?, August 1992

19 Harris Schlesinger, Changes in Background Risk and Risk Taking Behavior, August 1992

20 Roger H. Gordon, Do Publicly Traded Corporations Act in the Public Interest?, August 1992

21 Roger H. Gordon, Privatization: Notes on the Macroeconomic Consequences, August 1992

22 Neil A. Doherty and Harris Schlesinger, Insurance Markets with Noisy Loss Distributions, August 1992

23 Roger H. Gordon, Fiscal Policy during the Transition in Eastern Europe, September 1992

24 Giancarlo Gandolfo and Pier Carlo Padoan, The Dynamics of Capital Liberalization: A Macroeconometric Analysis, September 1992

25 Roger H. Gordon and Joosung Jun, Taxes and the Form of Ownership of Foreign Corporate Equity, October 1992

26 Gaute Torsvik and Trond E. Olsen, Irreversible Investments, Uncertainty, and the Ramsey Policy, October 1992

27 Robert S. Chirinko, Business Fixed Investment Spending: A Critical Survey of Modeling Strategies, Empirical Results, and Policy Implications, November 1992

28 Kai A. Konrad and Kjell Erik Lommerud, Non-Cooperative Families, November 1992

29 Michael Funke and Dirk Willenbockel, Die Auswirkungen des "Standortsicherungsgesetzes" auf die Kapitalakkumulation - Wirtschaftstheoretische Anmerkungen zu einer wirtschaftspolitischen Diskussion, January 1993

30 Michelle White, Corporate Bankruptcy as a Filtering Device, February 1993

31 Thomas Mayer, In Defence of Serious Economics: A Review of Terence Hutchison; Changing Aims in Economics, April 1993

32 Thomas Mayer, How Much do Micro-Foundations Matter?, April 1993

33 Christian Thimann and Marcel Thum, Investing in the East: Waiting and Learning, April 1993

34 Jonas Agell and Kjell Erik Lommerud, Egalitarianism and Growth, April 1993

35 Peter Kuhn, The Economics of Relative Rewards: Pattern Bargaining, May 1993

36 Thomas Mayer, Indexed Bonds and Heterogeneous Agents, May 1993

37 Trond E. Olsen and Gaute Torsvik, Intertemporal Common Agency and Organizational Design: How much Decentralization?, May 1993

38 Henry Tulkens and Philippe vanden Eeckaut, Non-Parametric Efficiency, Progress and Regress Measures for Panel Data: Methodological Aspects, May 1993

39 Hans-Werner Sinn, How Much Europe? - Subsidiarity, Centralization and Fiscal Competition, July 1993

40 Harald Uhlig, Transition and Financial Collapse, July 1993

41 Jim Malley and Thomas Moutos, Unemployment and Consumption: The Case of Motor-Vehicles, July 1993

42 John McMillan, Autonomy and Incentives in Chinese State Enterprises, August 1993

43 Murray C. Kemp and Henry Y. Wan, Jr., Lumpsum Compensation in a Context of Incomplete Markets, August 1993

44 Robert A. Hart and Thomas Moutos, Quasi-Permanent Employment and the Comparative Theory of Coalitional and Neoclassical Firms, September 1993

45 Mark Gradstein and Moshe Justman, Education, Inequality, and Growth: A Public Choice Perspective, September 1993

46 John McMillan, Why Does Japan Resist Foreign Market-Opening Pressure?, September 1993

47 Peter J. Hammond, History as a Widespread Externality in Some Arrow-Debreu Market Games, October 1993

48 Michelle J. White, The Costs of Corporate Bankruptcy: A U.S.-European Comparison, October 1993

49 Gerlinde Sinn and Hans-Werner Sinn, Participation, Capitalization and Privatization, Report on Bolivia's Current Political Privatization Debate, October 1993

50 Peter J. Hammond, Financial Distortions to the Incentives of Managers, Owners and Workers, November 1993

51 Hans-Werner Sinn, Eine neue Tarifpolitik (A New Union Policy), November 1993

52 Michael Funke, Stephen Hall and Martin Sola, Rational Bubbles During Poland's Hyperinflation: Implications and Empirical Evidence, December 1993

53 Jürgen Eichberger and Ian R. Harper, The General Equilibrium Foundations of Modern Finance Theory: An Exposition, December 1993

54 Jürgen Eichberger, Bayesian Learning in Repeated Normal Form Games, December 1993

55 Robert S. Chirinko, Non-Convexities, Labor Hoarding, Technology Shocks, and Procyclical Productivity: A Structural Econometric Approach, January 1994

56 A. Lans Bovenberg and Frederick van der Ploeg, Consequences of Environmental Tax Reform for Involuntary Unemployment and Welfare, February 1994

57 Jeremy Edwards and Michael Keen, Tax Competition and Leviathan, March 1994

58 Clive Bell and Gerhard Clemenz, The Desire for Land: Strategic Lending with Adverse Selection, April 1994

59 Ronald W. Jones and Michihiro Ohyama, Technology Choice, Overtaking and Comparative Advantage, May 1994

60 Eric L. Jones, Culture and its Relationship to Economic Change, May 1994

61 John M. Hartwick, Sustainability and Constant Consumption Paths in Open Economies with Exhaustible Resources, June 1994

62 Jürg Niehans, Adam Smith and the Welfare Cost of Optimism, June 1994

63 Tönu Puu, The Chaotic Monopolist, August 1994

64 Tönu Puu, The Chaotic Duopolists, August 1994

65 Hans-Werner Sinn, A Theory of the Welfare State, August 1994

66 Martin Beckmann, Optimal Gambling Strategies, September 1994

67 Hans-Werner Sinn, Schlingerkurs - Lohnpolitik und Investitionsförderung in den neuen Bundesländern, September 1994

68 Karlhans Sauernheimer and Jerome L. Stein, The Real Exchange Rates of Germany, September 1994

69 Giancarlo Gandolfo, Pier Carlo Padoan, Giuseppe De Arcangelis and Clifford R. Wymer, The Italian Continuous Time Model: Results of the Nonlinear Estimation, October 1994

70 Tommy Staahl Gabrielsen and Lars Sørgard, Vertical Restraints and Interbrand Competition, October 1994

71 Julia Darby and Jim Malley, Fiscal Policy and Consumption: New Evidence from the United States, October 1994

72 Maria E. Maher, Transaction Cost Economics and Contractual Relations, November 1994

73 Margaret E. Slade and Henry Thille, Hotelling Confronts CAPM: A Test of the Theory of Exhaustible Resources, November 1994

74 Lawrence H. Goulder, Environmental Taxation and the "Double Dividend": A Reader's Guide, November 1994

75 Geir B. Asheim, The Weitzman Foundation of NNP with Non-constant Interest Rates, December 1994

76 Roger Guesnerie, The Genealogy of Modern Theoretical Public Economics: From First Best to Second Best, December 1994

77 Trond E. Olsen and Gaute Torsvik, Limited Intertemporal Commitment and Job Design, December 1994

78

79 Richard J. Arnott and Ralph M. Braid, A Filtering Model with Steady-State Housing, April 1995

80 Vesa Kanniainen, Price Uncertainty and Investment Behavior of Corporate Management under Risk Aversion and Preference for Prudence, April 1995

81 George Bittlingmayer, Industry Investment and Regulation, April 1995

82 Richard A. Musgrave, Public Finance and Finanzwissenschaft Traditions Compared, April 1995

83 Christine Sauer and Joachim Scheide, Money, Interest Rate Spreads, and Economic Activity, May 1995

84 Jay Pil Choi, Preemptive R&D, Rent Dissipation and the "Leverage Theory", May 1995

85 Stergios Skaperdas and Constantinos Syropoulos, Competing for Claims to Property, July 1995

86 Charles Blackorby and Walter Bossert and David Donaldson, Intertemporal Population Ethics: Critical-Level Utilitarian Principles, July 1995