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Interval Valued Neutrosophic Parameterized Soft Set Theory and its Decision Making

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Abstract – In this work, we present definition of interval valued neutrosophic parameterized (IVNP-)soft set and its operations. Then we define parameter reduction method for IVNP-soft set.We also give an example which shows that they can be successfully applied to problem that contains indeterminacy.

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set (NS) [34], which is the generalization of the classical sets, conventional fuzzy set [40] and intuitionistic fuzzy set [5]. In recent years, neutrosophic sets has been successfully applied to many fields such as;control theory [1],　databases [3,4], clustering [36], medical diagnosis problem [2], decision making problem [25,37], topology [26],and so on.

Presently work on the neutrosophic set theory is progressing rapidly such as; Bhowmik and Pal defined intuitionistic neutrosophic set [9] and intuitionistic neutrosophic relations [10]. Later on Salam, Alblowi [33] introduced another concept called generalized neutrosophic set. Wang et al. [38] proposed another extension of neutrosophic set which is single valued neutrosophic. Also Wang etal. [39] introduced the notion of interval valued neutrosophic set which is an instance of neutrosophic set. It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree.Many applications of neutrosophic theory have been worked by Geogiev [23],Ye

[36,37], Majumdar and Samanta [31], P.D. Liu [41,42] and Broumi and Smarandache [14] and so on.

In 1999 a Russian researcher [32] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [15,,27], on FP-soft sets [20,21], on intuitionistic fuzzy soft set theory [8,17,28], on intuitionistic fuzzy parameterized soft set theory [18], oninterval valued intuitionistic fuzzy soft set [24], on generalized fuzzy soft sets [30,35], on generalized intuitionistic fuzzy soft [6], on possibility intuitionistic fuzzy soft set [7], on intuitionistic neutrosophic soft set [11], on generalized neutrosophic soft set [12], on fuzzy parameterized fuzzy soft set theory [16], on IFP−fuzzy soft set theory [19], on neutrosophic soft set [29]. Recently, Deli [22] introduced the concept of interval valued neutrosophic soft set as a combination of interval neutrosophic set and soft sets.

In this paper our main objective is to introduce the notion of interval valued neutrosophic parameterized soft set which is a generalization of neutrosophic parameterized soft sets [13].The paper is structured as follows. In Section 2, we first recall the necessary background on neutrosophic sets, interval neutrosophic sets and soft sets. In Section 3,we present interval valued neutrosophic parameterized soft set theory and examines their respective properties. In section 4, we present a interval valued neutrosophic parameterized aggregation operator. Section 5, interval valued neutrosophic parameterized decision methods is presented with example. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set, interval valued neutrosophic set and soft set. For more details, the reader could refer to [29, 32, 34, 39].

Definition 2.1. [34] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)} >, x \in U\}$$

where the functions $\mu, \nu, \omega : U \rightarrow]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+. \tag{1}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]−0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such

as in scientific and engineering problems.

Definition 2.2. [39] Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]$.

For two IVNS

$$A_{\text{IVNS}} = \{ <x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] > | \ x \in X \}$$

and

$$B_{\text{IVNS}} = \{ <x, [\mu_B^L(x), \mu_B^U(x)], [\nu_B^L(x), \nu_B^U(x)], [\omega_B^L(x), \omega_B^U(x)] > | \ x \in X \}$$

Then,

1. $A_{\text{IVNS}} \subseteq B_{\text{IVNS}}$ if and only if

$$\mu_A^L(x) \leq \mu_B^L(x), \mu_A^U(x) \leq \mu_B^U(x), \nu_A^L(x) \geq \nu_B^L(x), \omega_A^U(x) \geq \omega_B^U(x), \ \omega_A^L(x) \geq F\omega_B^L(x)$$
$$, \omega_A^U(x) \geq \omega_B^U(x).$$

2. $A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if,

$$\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x) \text{ for any} x \in X.$$

3.   The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

$$A_{IVNS}^o = \{ <x, [\omega_A^L(x), \omega_A^U(x)]>, [1 - \nu_A^U(x), 1 - \nu_A^L(x)], [\mu_A^L(x), \mu_A^U(x)] | \ x \in X \}$$

4.   A∩B = { $<x, [\min(\mu_A^L(x), \mu_B^L(x)), \min(\mu_A^U(x), \mu_B^U(x))], [\max(\nu_A^L(x), \nu_B^L(x)),$
       $\max(\nu_A^U(x), \nu_B^U(x)], [\max(\omega_A^L(x), \omega_B^L(x)), \max(\omega_A^U(x), \omega_B^U(x))] >: x \in X$ }

5.   A∪B = { $<x, [\max(\mu_A^L(x), \mu_B^L(x)), \max(\mu_A^U(x), \mu_B^U(x))], [\min(\nu_A^L(x), \nu_B^L(x)),$
       $\min(\nu_A^U(x), \nu_B^U(x)], [\min(\omega_A^L(x), \omega_B^L(x)), \min(\omega_A^U(x), \omega_B^U(x))] >: x \in X$ }

As an illustration, let us consider the following example.

Example 2.3. Assume that the universe of discourse U= $\{x_1, x_2, x_3, x_4\}$. Then, A is a interval valued neutrosophic set (IVNS) of U such that,

A = {< x_1, [0.1 0.8], [0.2 0.6], [0.8 0.9] >, < x_2, [0.2 0.5], [0.3 0.5], [0.6 0.8]>, < x_3, [0.5 0.8], [0.4 0.5], [0.45 0.6] >, < x_4, [0.1 0.4], [0.1 0.5], [0.4 0.8] >}

Definition 2.4. [32] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P (U).

As an illustration, let us consider the following example.

Example 2.5 . Suppose that U is the set of houses under consideration, say U = $\{h_1, h_2 \ldots, h_8\}$. Let E be the set of some attributes of such houses, say E = $\{e_1, e_2, \ldots, e_4\}$, where $e_1, e_2, \ldots, e_4$ stand for the attributes "beautiful", "costly", "in the green surroundings"", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, moderate houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, saysMr. X, and may be defined like this:

A= E,(K, A) =$\{(e_1, \{h_1, h_2\}), (e_2, \{h_3\}), (e_3, \{h_1, h_2.h_5\}), (e_4, U)\}$.

3. Interval Neutrosophic Parameterized Soft Set Theory

In this section, we define interval neutrosophic parameterized soft set and their operations.

Definition 3.1. Let U be an initial universe, P(U) be the power set of U,E be a set of all parameters and K be an interval valued neutrosophic set over E . Then an interval neutrosophic parameterized soft sets(IVNP-soft sets), denoted byΨ_K, defined as;

$$\Psi_K = \{(< x, [\mu_K^L(x), \mu_K^U(x)], [v_K^L(x), v_K^U(x)], [\omega_K^L(x), \omega_K^U(x)] >, f_K(x)): x \in E\}$$
$where \ \mu_K:E \to [0, 1], v_K:E \to [0, 1], \omega_K:E \to [0, 1]$ and $f_K:E \to P(U)$ such that $f_K(x) = \Phi$ if $\mu_K(x) = 0$, $v_K(x) = 1$ and$\omega_K(x) = 1$.

Here, theμ_K, $v_K$and ω_K called truth-membership function, indeteminacy-membership function and falsity-membership function of (IVNP-soft set), respectively.

Example 3.2. Assume that U= $\{u_1, u_2, u_3\}$ is a universal set and E= $\{x_1, x_2\}$ is a set of parameters. If

K= $\{(<x_1, [0.2, 0.3], [0.3,0.5], [0.4 ,0.5]>) , (<x_2, [0.3 ,0.4], [0.5,0.6], [0.4 ,0.5]>)\}$

and

$$f_K(x_1) = \{u_2 , u_3\}, f_K(x_2) = U$$

then a IVNP-soft set $\Psi_K$is written by

$\Psi_K = \{(<x_1, [0.2, 0.3], [0.3,0.5], [0.4 ,0.5]>, \{u_2, u_3\}), (<x_2, [0.3 ,0.4], [0.5,0.6], [0.4 ,0.5]>, U)\}$

Definition 3.3. Let $\Psi_K \in$IVNP-soft sets. If $f_K(x) = \Phi$, $\mu_K^L(x) = \mu_K^U(x) = 0$, $v_K^L(x) = v_K^U(x) = 0$ and $\omega_K^L(x) = \omega_K^U(x) = 1$ all x $\in$ E. then Ψ_K is called empty IVNP-soft set, denoted by Ψ_Φ.

Definition 3.4.Let $\Psi_K \in$IVNP-soft sets. If $f_K(x) = U, \mu_K^L(x) = \mu_K^U(x) = 1$, $v_K^L(x) = v_K^U(x) = 0$ and $\omega_K^L(x) = \omega_K^U(x) = 0$ all x $\in$ E. Then Ψ_K is called K-universal IVNP-soft set, denoted by$\Psi_{\tilde{K}}$.
If K= E, then the K-universal IVNP-soft set is called universal IVNP-soft set, denoted by $\Psi_{\tilde{E}}$.

Definition 3.5. Ψ_K and Ω_M are two IVNP-soft set. Then, $\Psi_K = \Omega_M$, if and only if $\Psi_K \sqsubseteq \Omega_M$ and $\Omega_M \sqsubseteq \Psi_K$ for all $x \in E$.

Definition 3.6. Ψ_K and Ω_M are two IVNP-soft set. Then, Ψ_K is IVNP-subset of Ω_M, denoted by $\Psi_k \sqsubseteq \Omega_L$ if and only if $\mu_K^L(x) \le \mu_M^L(x)$, $\mu_K^U(x) \le \mu_M^U(x)$, $v_K^L(x) \ge v_M^L(x)$, $v_K^U(x) \ge v_M^U(x)$, $\omega_K^L(x) \ge \omega_M^L(x)$, $\omega_K^U(x) \ge \omega_M^U(x)$ and $f_K(x) \subseteq g_M(x)$ for all $x \in E$.

Definition 3.7. Let $\Psi_K \in$ IVNP-soft set. Then, the complement of Ψ_K, denoted by Ψ_K^c, is defined by

$$\Psi_K^c = \{(<x, [\omega_K^L(x), \omega_K^U(x)], [v_K^L(x), v_K^U(x)], [\mu_K^L(x), \mu_K^U(x)], f_{K^c}(x)): x \in E\}$$

where $f_{K^c}(x) = U \setminus f_K(x)$

Definition 3.8. Let Ψ_K and Ω_M are two IVNP-soft set. Then, union of Ψ_K and Ω_M, denoted by $\Psi_K \sqcup \Omega_M$, is defined by

$$\Psi_K \sqcup \Omega_M = \{(<x, [\max\{\mu_K^L(x), \mu_M^L(x)\}, \max\{\mu_K^U(x), \mu_M^U(x)\}], [\min\{v_K^L(x), v_M^L(x)\},$$
$$\min\{v_K^U(x), v_M^U(x)\}], [\min\{\omega_K^L(x), \omega_M^L(x)\}, \min\{\omega_K^U(x), \omega_M^U(x)\}]>,$$
$$f_{K\cup M}(x)): x \in E\}$$

Where $f_{K\cup M}(x) = f_K(x) \cup f_M(x)$

Definition 3.9. Let Ψ_K and Ω_M are two IVNP-soft set. Then, intersection of Ψ_K and Ω_M, denoted by $\Psi_K \sqcap \Omega_M$, is defined by

$$\Psi_K \sqcap \Omega_M = \{(<x, [\min\{\mu_K^L(x), \mu_M^L(x)\}, \min\{\mu_K^U(x), \mu_M^U(x)\}], [\max\{v_K^L(x), v_M^L(x)\},$$
$$\max\{v_K^U(x), v_M^U(x)\}], [\max\{\omega_K^L(x), \omega_M^L(x)\}, \max\{\omega_K^U(x), \omega_M^U(x)\}]>,$$
$$f_{K\cap M}(x)): x \in E\}$$

where $f_{K\cap M}(x) = f_K(x) \cap f_M(x)$

Example 3.10. Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3\}$. Then,
$\Psi_K = \{(<x_1, [0.1,0.5], [0.4, 0.5], [0.2, 0.3]>, \{u_1, u_2\}),$
$(<x_2, [0.2,0.3], [0.5, 0.7], [0.1, 0.3]>, \{u_2, u_3\})\}$
$\Omega_M = \{(<x_2, [0.1,0.6], [0.2, 0.3], [0.2, 0.4]>, \{u_3, u_4\}),$
$(<x_3, [0.4,0.7], [0.1, 0.2], [0.3, 0.4]>, \{u_3\})\}$

Then

$\Psi_K \sqcup \Omega_M = \{(<x_1, [0.1,0.5], [0.4, 0.5], [0.2, 0.3]>, \{u_1, u_2\}), (<x_2, [0.2,0.6], [0.2, 0.3], [0.1, 0.3]>, \{u_2, u_3, u_4\}), (<x_3, [0.4,0.7], [0.1, 0.2], [0.3, 0.4]>, \{u_3\})\}$
$\Psi_K \sqcap \Omega_M = \{(<x_2, [0.1,0.3], [0.5, 0.7], [0.2, 0.4]>, \{u_3\})\}$.
$\Psi_K^c = \{(<x_1, [0.2,0.3], [0.4, 0.5], [0.1, 0.5]>, \{u_3, u_4\}),$
$(<x_2, [0.1,0.3], [0.5, 0.7], [0.2, 0.3]>, \{u_1, u_4\})\}$

Remark 3.11. $\Psi_K \sqsubseteq \Omega_M$ does not imply that every element of Ψ_K is an element of Ω_M as in the definition of classical subset. For example assume that U=$\{u_1,u_2,u_3,u_4\}$ is a universal set of objects and E=$\{x_1,x_2,x_3\}$ is a set of all parameters, if IVNP-soft sets Ψ_K and Ω_M are defined as

$\Psi_K= \{(<x_1,[0.1,0.3], [\ 0.5,\ 0.5]\ ,[0.2\ ,0.3]>,\{u_1,u_2\}),(<x_2, [0.3,0.4], [\ 0.4,\ 0.5]\ ,$
$[0.3,\ 0.5]>,\{u_2\})\}$
$\Omega_M= \{(<x_1,[0.2,0.6], [\ 0.3,\ 0.4]\ ,[0.1\ ,0.2]>,U),(<x_2, [0.4,0.7], [\ 0.1,\ 0.3]\ ,$
$[0.2,\ 0.3]>,\{u_1,u_4\})\}$

It can be seen that $\Psi_K \sqsubseteq \Omega_M$, but every element of Ψ_K is not an element of Ω_M.

Proposition 3.12. Let $\Psi_K, \Omega_M \in$ IVNP-soft set .Then
  i. $\Psi_K \sqsubseteq \Psi_{\tilde{E}}$
  ii. $\Psi_{\Phi} \sqsubseteq \Psi_K$
  iii. $\Psi_K \sqsubseteq \Psi_K$

Proof. It is clear from Definition 3.3-3.5.

Proposition 3.13. Let Ψ_K, Ω_M and $\Upsilon_N \in$ IVNP-soft set. Then
  i. $\Psi_K = \Omega_M$ and $\Omega_M = \Upsilon_N \Leftrightarrow \Psi_K = \Upsilon_N$
  ii. $\Psi_K \sqsubseteq \Omega_M$ and $\Omega_M \sqsubseteq \Psi_K \Leftrightarrow \Psi_K = \Omega_M$
  iii. $\Psi_K \sqsubseteq \Omega_M$ and $\Omega_M \sqsubseteq \Upsilon_N \Rightarrow \Psi_K \sqsubseteq \Upsilon_N$

Proof. It can be proved by Definition 3.3-3.5

Proposition 3.14 Let $\Psi_K \in$ IVNP-soft set. Then
  i. $(\Psi_K^c)^c = \Psi_K$
  ii. $\Psi_{\Phi}^c = \Psi_{\tilde{E}}$
  iii. $\Psi_E^c = \Psi_{\tilde{\Phi}}$

Proof. It is trial.

Proposition 3.15. Let Ψ_K, Ω_M and $\Upsilon_N \in$ IVNP-soft set. Then
  i. $\Psi_K \sqcup \Psi_K = \Psi_K$
  ii. $\Psi_K \sqcup \Psi_{\tilde{\Phi}} = \Psi_K$
  iii. $\Psi_K \sqcup \Psi_{\tilde{E}} = \Psi_{\tilde{E}}$
  iv. $\Psi_K \sqcup \Omega_M = \Omega_M \sqcup \Psi_K$
  v. $(\Psi_K \sqcup \Omega_M) \sqcup \Upsilon_N = \Psi_K \sqcup (\Omega_M \sqcup \Upsilon_N)$

Proof. It is clear

Proposition 3.16. Let Ψ_K, Ω_M and $\Upsilon_N \in$ IVNP-soft set, Then
  i. $\Psi_K \sqcap \Psi_K = \Psi_K$
  ii. $\Psi_K \sqcap \Psi_{\tilde{\Phi}} = \Psi_{\tilde{\Phi}}$
  iii. $\Psi_K \sqcap \Psi_{\tilde{E}} = \Psi_K$
  iv. $\Psi_K \sqcap \Omega_M = \Omega_M \sqcap \Psi_K$
  v. $(\Psi_K \sqcap \Omega_M) \sqcap \Upsilon_N = \Psi_K \sqcap (\Omega_L \sqcap \Upsilon_N)$

Proof. It is clear

Proposition 3.17. Let Ψ_K, Ω_M and Υ_N ∈IVNP-soft set, Then

i. $\Psi_K \sqcup (\Omega_M \sqcap \Upsilon_N) = (\Psi_K \sqcup \Omega_M) \sqcap (\Psi_K \sqcup \Upsilon_N)$

ii. $\Psi_K \sqcap (\Omega_M \sqcup \Upsilon_N) = (\Psi_K \sqcap \Omega_M) \sqcup (\Psi_K \sqcap \Upsilon_N)$

Proof. It can be proved by definition 3.8 and 3.9

Proposition 3.18. Let Ψ_K, Ω_M ∈IVNP-soft set, Then

i. $(\Psi_K \sqcup \Omega_M)^c = \Psi_K^c \sqcap \Omega_M^c$

ii. $(\Psi_K \sqcap \Omega_M)^c = \Psi_K^c \sqcup \Omega_M^c$

Proof. It is clear.

Definition 3.19. Let Ψ_K, Ω_M ∈IVNP-soft set, Then

i. OR-product of Ψ_K and Ω_M denoted by $\Psi_K \underline{\vee} \Omega_M$, is defined as following

$\Psi_K \underline{\vee} \Omega_M(x, y) = \{ \ \ (<(x,y),([\max\{\mu_K^L(x), \mu_M^L(y)\}, \max\{\mu_K^U(x), \mu_M^U(y)\}],$
$[\min\{\nu_K^L(x), \nu_M^L(y)\}, \min\{\nu_K^U(x), \nu_M^U(y)\}], [\min\{\omega_K^L(x), \omega_M^L(y)\},$
$\min\{\omega_K^U(x), \omega_M^U(y)]) >, \Psi_K \cup \Omega_M(x,y)) : x, y \in E \ \ \}$
where $\Psi_K \cup \Omega_M(x,y) = \Psi_K(x) \cup \Omega_M(y)$

ii. AND-product of Ψ_K and Ω_L denoted by $\Psi_K \overline{\wedge} \Omega_M$ is defined as following

$\Psi_K \overline{\wedge} \Omega_M(x,y) = \{ \ \ (<(x,y), ([\min\{\mu_K^L(x), \mu_M^L(y)\}, \min\{\mu_K^U(x), \mu_M^U(y)\}],$
$[\max\{\nu_K^L(x), \nu_M^L(y)\}, \max\{\nu_K^U(x), \nu_M^U(y)], [\max\{\omega_K^L(x), \omega_M^L(y)\},$
$\max\{\omega_K^U(x), \omega_M^U(y)]) >, \Psi_K \cap \Omega_M(x,y)) : x, y \in E \ \ \}$
where $\Psi_K \cap \Omega_M(x,y) = \Psi_K(x) \cap \Omega_M(y)$

Proposition 3.20. Let Ψ_K, Ω_M and Υ_N ∈IVNP-soft set. Then

i. $\Psi_K \overline{\wedge} \Psi_\Phi = \Psi_\Phi$

ii. $\Psi_K \overline{\wedge} \Omega_M = \Omega_M \overline{\wedge} \Psi_K$

iii. $\Psi_K \underline{\vee} \Omega_M = \Omega_M \underline{\vee} \Psi_K$

iv. $(\Psi_K \overline{\wedge} \Omega_M) \overline{\wedge} \Upsilon_N = \Psi_K \overline{\wedge} (\Omega_M \overline{\wedge} \Upsilon_N)$

v. $(\Psi_K \underline{\vee} \Omega_M) \underline{\vee} \Upsilon_N = \Psi_K \underline{\vee} (\Omega_M \underline{\vee} \Upsilon_N)$

Proof . It can be proved by definition 3.15

4. Parameter Reduction Method

In this section, we have defined a parameter reduction method of an IVNP-soft set, that produce a soft set from an IVNP-soft set. For this, we define level set for IVNP-soft set. This concept presents an adjustable approach to IVNP–soft sets based decision making problems.

Throughout this section we will accept that the parameter set E and the initial universe U are finite sets.

Definition 4.1 Let $\Psi_K \in$ IVNPS. Then for a $s = [s^-, s^+]$, $t = [t^-, t^+]$, $q = [q^-, q^+] \subseteq [0, 1]$, the (s, t, q) –level soft set of Ψ_K is a crisp soft set, denoted by $(\Psi_K; (s, t, q))$, defined by

$$(\Psi_K; (s,t,q)) = \{((x_i, \Psi_K(x_i)) : x_i \in E\}$$

where

$$s^- \leq \mu_K^L(x_i), s^+ \leq \mu_K^U(x_i), t^- \geq v_K^L(x_i), t^+ \geq v_K^U(x_i) \quad \text{and} \quad q^- \geq \omega_K^L(x_i), q^+ \geq \omega_K^U(x_i)$$

Remark In Definition 4.1, s $\subseteq$ [0,1] can be viewed as a given least threshold on degrees of truth-membership, t$\subseteq$[0, 1] can be viewed as a given greatest threshold on degrees of indeterminacy-membership and q $\subseteq$ [0, 1] can be viewed as a given greatest threshold on degrees of falsity-membership. If $s^- \leq \mu_K^L(x_i), s^+ \leq \mu_K^U(x_i), t^- \geq v_K^L(x_i), t^+ \geq v_K^U(x_i)$ and $q^- \geq \omega_K^L(x_i), q^+ \geq \omega_K^U(x_i)$, it shows that the degree of the truth-membership of x with respect to the u is not less than s, and the degree of the indeterminacy-membership of u with respect to the parameter x is not more than t and the degree of the falsity-membership of u with respect to the parameter x is not more than q. In practical applications of IVNP-soft sets, the thresholds s,t,q$\subseteq$[0, 1] is pre-established by decision makers and reflect decision makers requirements on "truth-membership levels", "indeterminacy-membership levels "and "falsity-membership levels".

Definition 4.2 Let$\Psi_K \in$ IVNPS and an $s_{min} = [s_{min}^-, s_{min}^+]$, $t_{min} = [t_{min}^-, t_{min}^+]$, $q_{min} = [q_{min}^-, q_{min}^+] \subseteq [0, 1]$ which is called a threshold of IVNSP-soft set. The level soft set of Ψ_K with respect to $(s_{min}, t_{min}, q_{min})$ is a crisp soft set , denoted by

$$(\Psi_K; (s_{min}, t_{min}, q_{min})),$$

defined by;

$$(\Psi_K; (s_{min}, t_{min}, q_{min})) = \{((x_i, \Psi_K(x_i)) : x_i \in E\}$$

where,

$$s_{min}^- \leq \mu_K^L(x_i), s_{min}^+ \leq \mu_K^U(x_i), t_{min}^- \geq v_K^L(x_i), t_{min}^+ \geq v_K^U(x_i) \quad \text{and} \quad q_{min}^- \geq \Omega_K^L(x_i),$$
$$q_{min}^+ \geq \omega_K^U(x_i)$$
$$s_{min}^- = \inf \{\mu_K^L(x_i) : x_i \in E\}, s_{min}^+ = \inf \{\mu_K^U(x_i) : x_i \in E\}$$
$$t_{min}^- = \inf \{v_K^L(x_i) : x_i \in E\}, t_{min}^+ = \inf \{v_K^U(x_i) : x_i \in E\}$$
$$q_{min}^- = \inf \{\omega_K^L(x_i) : x_i \in E\}, q_{min}^+ = \inf \{\omega_K^U(x_i) : x_i \in E\}$$

The $(s_{min}, t_{min}, q_{min})$ is called the mmm-threshold of the IVNP-soft set Ψ_K . In the following discussions, the mmm-level decision rule will mean using the mmm-threshold and considering the mmm-level soft set in IVNP-soft sets based decision making.

Definition 4.3 Let $\Psi_K \in$ IVNPS and an $s_{mid} = [s_{mid}^-, s_{mid}^+]$, $t_{mid} = [t_{mid}^-, t_{mid}^+]$, $q_{mid} = [q_{mid}^-, q_{mid}^+] \subseteq [0,1]$ which is called a threshould of IVNSP-soft set . The level soft set of Ψ_K with respect to $(s_{mid}, t_{mid}, q_{mid})$ is a crisp soft set, denoted by $(\Psi_K; (s_{mid}, t_{mid}, q_{mid}))$,defined by;

$$(\Psi_K; (s_{mid}, t_{mid}, q_{mid})) = \{((x_i, \Psi_K(x_i)) : x_i \in E\}$$

where,

$$s_{mid}^- \leq \mu_K^L(x_i), s_{mid}^+ \leq \mu_K^U(x_i), t_{mid}^- \geq v_K^L(x_i), t_{mid}^+ \geq v_K^U(x_i) \text{and} \quad q_{mid}^- \geq \omega_K^L(x_i), q_{mid}^+ \geq \omega_K^U(x_i)$$

$$s_{mid}^- = \sum_{x \in E^*} \mu_K^L(x_i) \Big/ |E^*|, \quad s_{mid}^+ = \sum_{x \in E^*} \mu_K^U(x_i) \Big/ |E^*|$$

$$t_{mid}^- = \sum_{x \in E^*} v_K^L(x_i) \Big/ |E^*|, \quad t_{mid}^+ = \sum_{x \in E^*} v_K^U(x_i) \Big/ |E^*|$$

$$q_{mid}^- = \sum_{x \in E^*} \omega_K^L(x_i) \Big/ |E^*|, \quad q_{mid}^+ = \sum_{x \in E^*} \omega_K^U(x_i) \Big/ |E^*|$$

For all $x_i \in E^*$, where if $x_i \in E/E^*$ then $\Psi_K(x_i) = \emptyset$.

The $(s_{mid}, t_{mid}, q_{mid})$ is called the mmm-threshold of the IVNP-soft set Ψ_K. In the following discussions, the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in IVNP-soft sets based decision making.

Definition 4.4 Let $\Psi_K \in$ IVNPS and an $s_{max} = [\, s_{max}^-, s_{max}^+\,]$, $t_{min} = [\, t_{min}^-, t_{min}^+\,]$, $q_{min} = [\, q_{min}^-, q_{min}^+\,] \subseteq [\, 0, 1\,]$ which is called a threshould of IVNSP-soft set . The level soft set of Ψ_K with respect to $(s_{max}, t_{min}, q_{min})$ is a crisp soft set, denoted by $(\, \Psi_K; (s_{max}, t_{min}, q_{min}))$,defined by;

$$(\, \Psi_K; (s_{max}, t_{min}, q_{min})) = \{((x_i, \Psi_K(x_i)) : x_i \in E\}$$

where,

$s_{max}^- \le \mu_K^L(x_i)$,$s_{max}^+ \le \mu_K^U(x_i)$,$t_{min}^- \ge v_K^L(x_i)$,$t_{min}^+ \ge v_K^U(x_i)$ and $q_{min}^- \ge \omega_K^L(x_i)$,$q_{min}^+ \ge \omega_K^U(x_i)$

$$s_{max}^- = \sup\{\mu_K^L(x_i) : x_i \in E\}, \quad s_{max}^+ = \sup\{\, \mu_K^U(x_i) : x_i \in E\}$$
$$t_{min}^- = \inf\{v_K^L(x_i) : x_i \in E\}, \quad t_{min}^+ = \inf\{\, v_K^U(x_i) : x_i \in E\}$$
$$q_{min}^- = \inf\{\omega_K^L(x_i) : x_i \in E\}, \quad q_{min}^+ = \inf\{\, \omega_K^U(x_i) : x_i \in E\}$$

The $(s_{max}, t_{min}, q_{min})$ is called the Mmm-threshold of the IVNP-soft set Ψ_K . In the following discussions, the Mmm-level decision rule will mean using the Mmm-threshold and considering the Mmm-level soft set in IVNP-soft sets based decision making.

Definition 4.5 Let $\Psi_K \in$ IVNPS. The threshold based on median could be expressed as a function

$\text{med}_\psi : A \to [0, 1]^3$, i.e. $s_{med} = [s_{med}^-, s_{med}^+]$, $t_{med} = [t_{med}^-, t_{med}^+]$, $q_{med} = [q_{med}^-, q_{med}^+] \subseteq [0,1]$ for all $\varepsilon \in A$, where for $\forall \varepsilon \in A$, s_{med}^-, s_{med}^+ is the median by ranking the degree of interval truth membership of all alternatives according to order from large to small (or from small to large), namely

$$s_{med}^- = \begin{cases} \mu_K^L\left(x_{\left(\frac{|E|+1}{2}\right)}\right) & \text{if } |E| \text{ is odd} \\[2em] \left(\mu_K^L\left(x_{\left(\frac{|E|}{2}\right)}\right) + \mu_K^L\left(x_{\left(\frac{|E|}{2}+1\right)}\right)\right) \Big/ 2 & \text{if } |E| \text{ is even} \end{cases}$$

$$s_{med}^+ = \begin{cases} \mu_K^U\left(x_{\left(\frac{|E|+1}{2}\right)}\right) & \text{if } |E| \text{ is odd} \\[2em] \left(\mu_K^U\left(x_{\left(\frac{|E|}{2}\right)}\right) + \mu_K^U\left(x_{\left(\frac{|E|}{2}+1\right)}\right)\right) \Big/ 2 & \text{if } |E| \text{ is even} \end{cases}$$

t_{med}^-, t_{med}^+ is the median by ranking the degree of interval indeterminacy membership of all alternatives according to order from large to small (or from small to large), namely

$$t_{med}^- = \begin{cases} \nu_K^L\left(\mathbf{x}_{\left(\frac{|E|+1}{2}\right)}\right) & \textbf{if } |E| \textbf{ is odd} \\ \left(\left(\nu_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}\right)}\right) + \mu_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}+1\right)}\right)\right)\right)\Big/ 2 & \textbf{if } |E| \textbf{ is even} \end{cases}$$

$$t_{med}^+ = \begin{cases} \nu_K^U\left(\mathbf{x}_{\left(\frac{|E|+1}{2}\right)}\right) & \textbf{if } |E| \textbf{ is odd} \\ \left(\left(\nu_K^U\left(\mathbf{x}_{\left(\frac{|E|}{2}\right)}\right) + \nu_K^U\left(\mathbf{x}_{\left(\frac{|E|}{2}+1\right)}\right)\right)\right)\Big/ 2 & \textbf{if } |E| \textbf{ is even} \end{cases}$$

And q_{med}^- ,q_{med}^+ is the median by ranking the interval degree of falsity membership of all alternatives according to order from large to small (or from small to large), namely

$$q_{med}^- = \begin{cases} \omega_K^L\left(\mathbf{x}_{\left(\frac{|E|+1}{2}\right)}\right) & \textbf{if } |E| \textbf{ is odd} \\ \left(\left(\omega_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}\right)}\right) + \omega_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}+1\right)}\right)\right)\right)\Big/ 2 & \textbf{if } |E| \textbf{ is even} \end{cases}$$

$$q_{med}^+ = \begin{cases} \omega_K^L\left(\mathbf{x}_{\left(\frac{|E|+1}{2}\right)}\right) & \textbf{if } |E| \textbf{ is odd} \\ \left(\left(\omega_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}\right)}\right) + \omega_K^L\left(\mathbf{x}_{\left(\frac{|E|}{2}+1\right)}\right)\right)\right)\Big/ 2 & \textbf{if } |E| \textbf{ is even} \end{cases}$$

The $(s_{med}, t_{med}, q_{med})$ is called themed-threshold of the IVNP-soft set Ψ_K . In the following discussions, the Med-level decision rule will mean using the Med-threshold and considering the Med-level soft set in IVNP-soft sets based decision making.

Example 4.6 Ψ_K= {(<x_1 ,[0.1,0.5], [0.4, 0.5] ,[0.2 ,0.3]>,{u_1 ,u_2 }),(<x_2 ,[0.2,0.3], [0.5, 0.7], [0.1, 0.3]>,{u_2 ,u_3 }),(<x_3,[0.1,0.3], [0.1, 0.7] ,[0.3 ,0.4]>,{u_1 ,u_2 ,u_3 })}
Then

$$s_{mid}= [0.13, 0.36], \quad t_{mid}= [0.33, 0.63], \quad q_{mid} = [0.2 \ 0.43]$$
$$s_{min}= [0.1, 0.3], \quad t_{min}= [0.1, 0.5], \quad q_{min} = [0.1,0.3]$$
$$s_{max}= [0.2, 0.5], \quad t_{min}= [0.1, 0.5], q_{min} = [0.1,0.3]$$
$$s_{med}= [0.2, 0.3], \quad t_{med}= [0.5, 0.7], \quad q_{med} = [0.1, 0.3]$$

Theorem 4.7. Let $\Psi_K \in$ IVNP-soft set $(\Psi_K; (s_{mid}, t_{mid}, q_{mid}))$, $(\Psi_K; (s_{min}, t_{min}, q_{min}))$and $(\Psi_K; (s_{max}, t_{min}, q_{min}))$ be the mid-level soft set, max –level soft set and min –level soft set of Ψ_K ,respectively. Then,
1. $(\Psi_K; (s_{max}, t_{min}, q_{min})) \subseteq (\Psi_K; (s_{mid}, t_{mid}, q_{mid}))$
2. $(\Psi_K; (s_{max}, t_{min}, q_{min})) \subseteq (\Psi_K; (s_{mid}, t_{mid}, q_{mid}))$

Proof. Let $\Psi_K \in$ IVNPSS.From definition, definition and definition,it can be seen that
$$s_{min}^- \leq s_{mid}^- \leq s_{max}^-, \quad t_{min}^- \leq t_{mid}^- \leq t_{max}^- \text{ and } q_{min}^- \leq q_{mid}^- \leq q_{max}^-.$$

Thus

i.   for all $x_j \in$ E which providing the inequalities

$s^-_{mid} \leq \mu^L_K(x_i) \leq s^-_{max}, (x_i, \Psi_K(x_i) \notin (\Psi_K; (s_{max}, t_{min}, q_{min}))$.

So, $(\Psi_K; (s_{max}, t_{min}, q_{min})) \subseteq (\Psi_K; (s_{mid}, t_{mid}, q_{mid}))$

ii.   it can be proved similar way

Now, we construct an IVNP –soft sets decision making method by the following algorithm;

Algorithm:

Step 1. Input the IVNP –soft sets -soft set Ψ_K

Step 2. Input a threshold ($s_{mid}, t_{mid}, q_{mid}$) (or ($s_{max}, t_{min}, q_{min}$), ($s_{min}, t_{min}, q_{min}$)) by using mid –level decision rule (or Mmm-level decision rule, mmm-level decision rule) for decision making.

Step 3. Compute mid-level soft set ($\Psi_K; (s_{mid}, t_{mid}, q_{mid})$) (or Mmm-level soft set ($\Psi_K; (s_{max}, t_{min}, q_{min})$) ,mmm –level soft set ($\Psi_K; (s_{min}, t_{min}, q_{min})$), Med –level soft set ($\Psi_K; (s_{min}, t_{min}, q_{min})$)

Step 4. Present the level soft set ($\Psi_K; (s_{mid}, t_{mid}, q_{mid})$) (or the level soft set ($\Psi_K; (s_{max}, t_{min}, q_{min})$), the level soft set ($\Psi_K; (s_{min}, t_{min}, q_{min})$), Med–level soft set ($\Psi_K; (s_{min}, t_{min}, q_{min})$))in tabular form.

Step 5. Compute the choice value c_i of u_i for any $u_i \in$ U ,

Step 6. The optimal decision is to select u_k if $c_k = \max_{u_i \in U} c_i$

Remark If k has more than one value then any one of u_k may be chosen. If there are too many optimal choices in Step 6, we may go back to the second step and change the threshold (or decision rule) such that only one optimal choice remains in the end.

Example 4.8. Assume that a company wants to fill a position. There are 4 candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM) that is from the department of human resources. He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the parameter reduction method, the numbers of candidates are reduced to a suitable one. Assume that the set of candidates U={u_1, u_2, u_3, u_4,u_5, u_6, u_7} which may be characterized by a set of parameters E={x_1, x_2, x_3 ,x_4, x_5, x_6} For i=1,2,3,4,5,6 the parameters i stand for experience, computer knowledge, training, young age, diction and flexible working hours compatible, respectively. Now, we can apply the method as follows:

Step 1. After thinking thoroughly, he/she evaluates the alternative according to choosing parameters and constructs an IVNP-soft set $\Psi_K$as follows

Ψ_K= {<x_1, ([0.6, 0.8], [0.1, 0.2], [0.3, 0.5])>,{u_1,u_4,u_5,u_7}), (<x_2,([0.5, 0.6], [0.3, 0.4], [0.2, 0.3])>,{u_2,u_5,u_6,u_8}),(<x_3, ([0.4, 0.5], [0.3, 0.4], [0.1 ,0.4])>, {u_2,u_3,u_6,u_7}), (<x_4, ([0.1, 0.2], [0.4, 0.8], [0.4 ,0.5])>, {u_1,u_4,u_5,u_8}}, (<x_5, ([0.4, 0.5], [0.2, 0.4], [0.3, 0.6])>, {u_3,u_6,u_7})},(<x_6, ([0.7, 0. 7], [0.1, 0.3], [0.1, 0.5])>, {u_2,u_4,u_8}})}

Step 2. Then, we have

$$s_{mid}, = [0.45, 0.655], t_{mid} = [0.23, 0.41], q_{mid} = [0.23, 0.46]$$
$$s_{max}, = [0.7, 0.8], t_{min} = [0.1, 0.2], q_{min} = [0.1, 0.3]$$

$$s_{min}, = [0.1, 0.2], \quad t_{min} = [0.1, 0.2], q_{min} = [0.1, 0.3]$$
$$s_{med}, = [0.25, 0.35], \quad t_{med} = [0.35, 0.6], q_{med} = [0.25, 0.45]$$

Step 3. Thus, the $(s_{med}, t_{med}, q_{med})$-level soft set of Ψ_K is (after the necessary calculations, they can be seen that $(s_{max}, t_{min}, q_{min})$-level soft set, $(s_{min}, t_{min}, q_{min})$-level soft set, and $(s_{mid}, t_{mid}, q_{mid})$- level soft set of Ψ_K are not suitable for decision making in this problem.)
$(\Psi_K; (s_{med}, t_{med}, q_{med})) = \{(<\underline{x_2},([0.5, 0.6], [0.3, 0.4], [0.2, 0.3])>, \{u_2, u_5, u_6, u_8\}),$

$(<x_3, ([0.4, 0.5], [0.3, 0.4], [0.1, 0.4])>, \{u_2, u_3, u_6, u_7\})\}$

Step 4. Tabular form of $(\Psi_K; (s_{med}, t_{med}, q_{med}))$ is

u	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
x_2	0	1	0	0	1	1	0	1
x_3	0	1	1	0	0	1	1	0

Step 5. Then, we have the choice value c_i for i = 1, 2, 3,…,8
$$c_1 = 0, c_2 = 2, c_3 = 1, c_4 = 0, c_5 = 1, c_6 = 2, c_7 = 1 \text{ and } c_8 = 1$$

Step 6. So, the optimal decision is u_2 or u_6
Note that this decision making method can be applied for group decision making easily with help of the definition 3.19.

5. Conclusions

In this work, we have introduced the concept of interval valued neutrosophic parameterized soft set and studied some of its properties. The complement, union and intersection operations have been defined on the interval valued neutrosophic parameterized soft set. The definition of parameter reduction method is introduced with application of this operation in decision making problems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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