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Interval Valued T-Spherical Fuzzy Information Aggregation Based on Dombi t-Norm and Dombi t-Conorm for Multi-Attribute Decision Making Problems

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Abstract: Multi-attribute decision-making (MADM) is commonly used to investigate fuzzy information effectively. However, selecting the best alternative information is not always symmetric because the alternatives do not have complete information, so asymmetric information is often involved. Expressing the information under uncertainty using closed subintervals of [0, 1] is beneficial and effective instead of using crisp numbers from [0, 1]. The goal of this paper is to enhance the notion of Dombi aggregation operators (DAOs) by introducing the DAOs in the interval-valued T-spherical fuzzy (IVTSF) environment where the uncertain and ambiguous information is described with the help of membership grade (MG), abstinence grade (AG), non-membership grade (NMG), and refusal grade (RG) using closed sub-intervals of [0, 1]. One of the key benefits of the proposed work is that in the environment of information loss is reduced to a negligible limit. We proposed concepts of IVTSF Dombi weighted averaging (IVTSFDWA) and IVTSF Dombi weighted geometric (IVTSFDWG) operators. The diversity of the IVTSF DAOs is proved and the influences of the parameters, associated with DAOs, on the ranking results are observed in a MADM problem where it is discussed how a decision can be made when there is asymmetric information about alternatives.

Keywords: Dombi operations; T-spherical fuzzy set; interval-valued T-spherical fuzzy set; Dombi aggregation operators; multi-attribute decision making

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1. Introduction

Almost every event of the world, as if it were a rule, attracts some degree of uncertainty and imprecision; the events that involve human opinion are not an exception to this rule. To eradicate these imprecisions and inaccuracies from the data influenced by human opinion, the notion of the fuzzy sets (FSs) and MG of elements and objects were proposed by Zadeh [1]. In addition, Atanassov [2] made his contribution by associating elements and objects with NMG along with an MG. The concept of intuitionistic FS (IFS) is also a part of his contribution. According to Atanassov's IFS, only those duplets of information are allowed where the sum of NMG and MG lies in $[\mathbf{0}, \mathbf{1}]$, consequently restricting the assigning of MG and NMG up to a certain extent and hence removing our flexibility of choice of MG and NMG. On the other hand, Yager [3] introduced the concept of the Pythagorean fuzzy set (PyFS), which allows only those duplets where the sum of squares of the MG and NMG lies in $[\mathbf{0}, \mathbf{1}]$. Later on, q-rung ortho pair FS (QROFS) [4], a further generalization of PyFS, was introduced, which permits all those duplets for which the sum of qth power lies in $[\mathbf{0}, \mathbf{1}]$.

The realization that human opinion might also contain some degree of refusal and abstinence, the absence of which can result in some loss of information. Due to this fact, Cuong [5] introduced the concept of Picture FS (PFS), which associates human opinion with an additional grade, namely an abstinence grade (AG), along with NMG and MG, and permits all such triplets whose sum lies in $[\mathbf{0}, \mathbf{1}]$. Cuong's concept of PFS was further generalized to the notion of the spherical fuzzy set (SFS) and T-SFS by Mahmood et al. [6] where the range for assigning the MGs, AGs, and NMGs is increased to a maximal level to express uncertain information.

Information or data influenced by uncertainty and imprecision has garnered an extensive discussion in MADM, where optimum alternatives are selected. For MADM problems, the aggregation of information is the main tool. There are several aggregation operators (AOs), that are based on various t-norms and t-conorms. Among these AOs, the discussion of arithmetic AOs has a rich background, as such AOs have been discussed in various frameworks, for example, intuitionistic fuzzy arithmetic AOs [7,8], fuzzy number intuitionistic fuzzy arithmetic AOs [9], intuitionistic trapezoidal fuzzy arithmetic AOs [10], and intuitionistic fuzzy hybrid arithmetic AOs [11]. Einstein AOs based on Einstein t-norms have also been investigated by various authors as Einstein interactive AOs of TSFSs are investigated by [12], Einstein geometric AOs of IFSs are investigated in [13], Einstein AOs of QROFSs are investigated by Riaz et al. [14] and some Einstein t-norm based-Choquet AOs are discussed in [15]. Fuzzy mathematics also exploit the concept of Hamacher t-norms, which resulted in the development of Hamacher AOs and their utilization in MADM problems, and has been investigated by various authors. Entropy-based Hamacher AOs for IFSs are developed in [16] while a study of the TSF Hamacher AOs for analyzing the performance of the search and rescue robots is investigated by Ullah et al. [17] Jana et al. [18] studied the applicability of the picture fuzzy Hamacher AOs for the evaluation of the enterprise performance and entropy-based Pythagorean fuzzy Hamacher AOs are developed by Wang et al. [19].

Dombi t-norms (DTN) and Dombi t-conorms (DTCN) [20] are also studied widely among several other t-norms and t-conorms, which lead to the development of DAOs. The notion of DAOs in the framework of IFSs and that of DAOs in the Pythagorean fuzzy environment have been introduced by Seikh and Mandal [21] and Jana et al. [22], respectively. Jana et al. [23] have also studied problems influenced by DAOs of QROFSs. Since real-life problems cannot be described more accurately with the help of duplets of IFSs, PyFSs, and QROFSs, DAOs of PFSs have been introduced by Jana et al. [24]. DAOs in the environment of bipolar fuzzy sets have been set up in [25], whereas DAOs in a neutrosophic environment have been introduced by Shi and Ye [26]. The notion of DAOs for hesitant fuzzy sets and linguistic cubic sets has been developed by He [27] and Lu and Ye [28], respectively. The applications of DAOs in performance evaluation are investigated by He [29]. Lie et al. [30] proposed Dombi t-norm-based Bonferroni mean operators for MADM purposes. Dombi t-norm-based Heronian mean operators in picture fuzzy settings is investigated by Zhang et al. [31]. Khan et al. [32] proposed Dombi t-norm-based power Bonferroni mean operators in a neutrosophic environment for MADM purposes. Li et al. [33] developed some methods for MADM problems based on Dombi hamy mean operators in intuitionistic fuzzy settings. Wei et al. [34] introduces the notion of DAOs associated with Bonferroni mean operators in a 2-touple linguistic neutrosophic environment.

The notion of interval-valued T-spherical fuzzy sets Dombi t-norm and t-conorms are very closely related to the notion of symmetry. Based on symmetry, we can talk about the mixture of both theories. Our earlier discussion has revealed the limited scope of the operators developed in intuitionistic, Pythagorean, and q-rung orthopair fuzzy environments, and has also underlined the possibility of information loss to some extent. The DAOs of IFSs [21], PyFSs [22], and QROFSs [23] express two aspects of human opinion and lead to information loss as the AG and the RG remain undiscussed. Moreover, the DAOs of PFSs [24] face the issue of applicability and can be applied to a specific type of data only, due to the strict restraints on these fuzzy frameworks. Ullah et al. [35] suggested that expressing the uncertain information using an interval instead of a crisp number reduces information loss by introducing the framework of IVTSFSs. The frame of IVTSFSs has two main advantages; first, it describes the AG and RG along with the MG and NMG. Second, it reduces information loss by modeling the human opinion in terms of intervals instead of crisp numbers. Keeping the limited nature of the frame of IFSs, PyFSs, QROFSs, PFSs, SFSs, and their DAOs, this paper aims to develop the DAOs in the environment of IVTSFSs. The main contributions of the presented research are as follows:

1. The interval-valued TSF DAOs discuss four aspects of uncertain information, which is comparatively better than the DAOs of IFSs, PyFSs, and QROFSs, where only two aspects of the uncertain information are discussed.
2. The proposed DAOs allow the description of the uncertain information in the form of closed sub-intervals $[0,\ 1]$, which reduces information loss as. This is already proved in [35] by Ullah et al.
3. The proposed DAOs are more improved than the DAOs of the PFSs and SFSs by providing a limitless range for assigning the degrees of membership, non-membership, refusal, and abstinence.
4. A thorough investigation into the limitations of the previously defined AOs is observed in Section 6, where it is shown that the DAOs of IVTSFSs can not only be applied to the information provided using the MG, AG, NMG, and RG, but can also be applied to the problems involving IFSs, PyFSs, QROFSs, PFSs, and SFSs and proved to be the most generalized form of DAOs.

The structure of this paper is as follows. In Section 1, we discussed some history of fuzzy frameworks and aggregation operators. In Section 2, some basic terms are studied related to IVTSFSs and TSFSs. In Section 3, Dombi operations in the IVTSF environment are investigated. Sections 4 and 5 are based on IVTSFDWA and IVTSFDWG operators, respectively. In Section 6, some special cases of the proposed aggregation operators are studied. In Section 7, we investigated the applicability of the DAOs of IVTSFSs in MADM problems illustrated by examples. Section 8 is based on a comparative study while Section 9 summarizes the article.

2. Preliminaries

In this section, we recall some basic but necessary definitions and discuss the research gap for the proposed work. Throughout the paper, X denotes a non-empty set, and $\dot{\check{s}}, \underset{\cdot}{i}, \underset{,}{d}$, and r denote the MG, NMG, AG, and RG, respectively.

The notion of TSFS was proposed by Mahmood et al. [6] by describing the uncertain information using an MG, NMG, AG, and RG with a most favorable condition compared to the notions of IFSs, PyFSs, QROFSs, and PFSs.

Definition 1. [6] *A TSFS on X, based on an MG denoted by $\dot{\check{s}}$, NMG $\underset{,}{d}$, AG $\underset{\cdot}{i}$, and RG r, is of the form:*

$$\text{Ꝑ} = \left\{ \begin{array}{c} \left(x, \left(\dot{\check{s}}, \underset{\cdot}{i}, \underset{,}{d}\right)\right). \dot{\check{s}} : X \to [0,\ 1],\ \underset{\cdot}{i} : X \to [0,\ 1]\ \textit{and}\ \underset{,}{d} : X \to [0,\ 1]\ \forall\ x \in X \\ 0 \leq (\dot{\check{s}}^{\mathfrak{q}}(x) + \underset{\cdot}{i}^{\mathfrak{q}}(x) + \underset{\cdot}{i}^{\mathfrak{q}}(x) \leq 1,\ \mathfrak{q} \in \mathbb{Z}^{+} \\ r(x) = \sqrt[\mathfrak{q}]{1 - \left(\dot{\check{s}}^{\mathfrak{q}}(x) + \underset{\cdot}{i}^{\mathfrak{q}}(x) + \underset{,}{d}^{\mathfrak{q}}(x)\right)} \end{array} \right\} \quad (1)$$

The triplet $\left(\dot{\check{s}}\ (x), \underset{\cdot}{i}(x), \underset{,}{d}(x)\right)$ is known as a TSF number (TSFN).

In 2019, Ullah et al. [35] gave the idea of IVTSFS by generalizing the notion of TSFS by expressing each MG, NMG, AG, and RG using a closed subinterval of $[0,\ 1]$ instead of crisp values from it. To see an example of how interval-valued frameworks effectively give a result with less information loss compare to non-interval valued frameworks, one is referred to Section 3 of Ullah et al. [35] Below we define the IVTSFS.

Definition 2. [35] *An IVTSFS on X, based on an MG denoted by* $\dot{\text{s}} = \left[\dot{\text{s}}^l(x),\ \dot{\text{s}}^u(x)\right]$, *NMG* $\text{d} = \left[\text{d}^l(x),\ \text{d}^u(x)\right]$, *AG* $\text{i} = \left[\text{i}^l(x),\ \text{i}^u(x)\right]$ *and RG* $r = \left[r^l(x), r^u(x)\right]$, *is of the form.*

$$\text{P} = \left\{ \begin{array}{c} \left(x, (\dot{\text{s}}, \text{i}, \text{d})\right).\ \dot{\text{s}},\ \text{i}, \text{d}.X \to \text{some closed subinterval of } [0,\ 1]\ \forall\ x \in X, \\ 0 \le (\dot{\text{s}}^{u\mathfrak{q}}(x) + \text{i}^{u\mathfrak{q}}(x) + \text{d}^{u\mathfrak{q}}(x) \le 1,\ \mathfrak{q} \in \mathbb{Z}^+ \\ r(x) = \left[r^l(x), r^u(x)\right] = \left(\begin{array}{c} \left[\left(1 - (\dot{\text{s}}^u)^{\mathfrak{q}}(x) - (\text{i}^u)^{\mathfrak{q}}(x) + (\text{d}^u)^{\mathfrak{q}}\right)^{\frac{1}{q}}\right], \\ \left[\left(1 - (\dot{\text{s}}^l)^{\mathfrak{q}}(x) - (\text{i}^l)^{\mathfrak{q}}(x) + (\text{d}^l)^{\mathfrak{q}}\right)^{\frac{1}{q}}\right] \end{array} \right) \end{array} \right\} \tag{2}$$

The triplet $\left(\dot{\text{s}}\ (x), \text{i}(x), \text{d}(x)\right) = \left(\left[\dot{\text{s}}^l,\ \dot{\text{s}}^u\right], \left[\text{i}^l,\ \text{i}^u\right], \left[\text{d}^l,\ \text{d}^u\right]\right)$ is known as an IVTSF number (IVTSFN).

Remark. *The following conditions, when applied to Definition 2, reduce the frame of an IVTSFS to a reduced fuzzy framework of:*

1. *TSFS: if* $\dot{\text{s}}^l = \dot{\text{s}}^u, \text{i}^l = \text{i}^u$ *and* $\text{d}^l = \text{d}^u$
2. *IVSFS: if* $q = 2$.
3. *SFS: if* $q = 2$ *and* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u$ *and* $\text{d}^l = \text{d}^u$.
4. *IVPFS: if* $q = 1$.
5. *PFS: if* $q = 1$ *and* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u$ *and* $\text{d}^l = \text{d}^u$.
6. *IVQROPFS: if* $\text{i}^l = \text{i}^u = 0$.
7. *QROPFS: if* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u$ *and* $\text{d}^l = \text{d}^u$.
8. *IVPyFS: if* $q = 2$ *and* $\text{i}^l = \text{i}^u = 0$.
9. *PyFS: if* $q = 2$ *and* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u = 0$ *and* $\text{d}^l = \text{d}^u$.
10. *IVIFS: if* $q = 1$ *and* $\text{i}^l = \text{i}^u = 0$.
11. *IFS: if* $q = 1$ *and* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u = 0$ *and* $\text{d}^l = \text{d}^u$.
12. *IVFS: if* $q = 1$ *and* $\text{i}^l = \text{i}^u = \text{d}^l = \text{d}^u = 0$.
13. *FS: if* $q = 1$ *and* $\dot{\text{s}}^l = \dot{\text{s}}^u,\ \text{i}^l = \text{i}^u = 0 = \text{d}^l = \text{d}^u$.

Ullah et al. [35] also describe a ranking tool for the ranking of two or more IVTSFNs. We use the following score functions for the ranking of IVTSFNs.

Definition 3. [35] *For an IVTSFN* $\text{P} = \left(\dot{\text{s}}(\text{x}),\ \text{i}(\text{x}),\ \text{d}(\text{x})\right) = \left(\left[\dot{\text{s}}^l,\ \dot{\text{s}}^u\right], \left[\text{i}^l,\ \text{i}^u\right], \left[\text{d}^l,\ \text{d}^u\right]\right)$, *the score function is defined as:*

$$SC(\text{P}) = \frac{\left(\dot{\text{s}}^l\right)^q \left(1 - \left(\text{i}^l\right)^q - \left(\text{d}^l\right)^q\right) + \left(\dot{\text{s}}^u\right)^q \left((1 - (\text{i}^u)^n - (\text{d}^u)^q)\right)}{3},\ SC() \in [0,\ 1] \tag{3}$$

To aggregate fuzzy information, there are several AOs in different fuzzy frameworks based on t-norms and t-conorms. Among several t-norms and t-conorms, the notion of DTN and DTCN [20] is one of the widely used tools to form aggregation formulas to handle uncertain information. The conception of DTN and DTCN is given in Definition 4 below.

Definition 4. [20] *For* $f,\ g \in \mathbb{R}$. *The DTN and DTCN are of the following form:*

$$Dom^{tn}(f, g) = \frac{1}{1 + \left\{\left(\frac{1-f}{f}\right) + \left(\frac{1-g}{g}\right)\right\}^{\frac{1}{}}} \tag{4}$$

$$Dom^{tcn}(f,g) = 1 - \frac{1}{1 + \left\{ \left(\frac{f}{1-f} \right) + \left(\frac{g}{1-g} \right) \right\}^{1}} \tag{5}$$

Based on the notion of DTN and DTCN, DAOs are being investigated in several fuzzy frameworks for MADM purposes. So far, DAOs are defined in the layouts of IFSs [2], PyFSs [3], QROFSs [4], PFSs [5], and SFSs [6]. All of the said DAOs in different fuzzy frameworks have somehow failed to have discussed some scenarios. The DAOs of IFSs, PyFSs, and QROFSs discussed only two aspects of uncertain information where the DAOs of the IFSs and PyFSs have some applicability issues in assigning their MG and NMG as well. Similarly, the DAOs of PFSs and SFSs also have applicability issues whenever it comes to assign them the MG, AG, and the NMG. More specifically, the layouts of PFSs and SFSs allow the MG, AG, and NMG from a certain range with very little independency. Due to these facts, the DAOs of IFSs, PyFSs, QROFSs, PFSs, and SFSs also have some limitations when it comes to their applicability. Therefore, in this paper, we aim to develop the Dombi operations in the IVTSF environment and hence DAOs for MADM purposes.

In the next section, we aim to propose Dombi operational laws for IVTSFNs and investigate their basic characteristics.

3. Dombi Operations on IVTSFSs

In this section, we introduce four basic operations for IVTSFNs [35] followed by a supportive example. Some unproven results are also given, which can be proved straightforwardly.

Definition 5. *Let* $\wp = \left(\left[\dot{s}^{l}, \dot{s}^{u} \right], \left[\underset{\cdot}{i}^{l}, \underset{\cdot}{i}^{u} \right], \left[\underset{\cdot}{d}^{l}, \underset{\cdot}{d}^{u} \right] \right)$, $\wp_1 = \left(\left[\dot{s}_1^{l}, \dot{s}_1^{u} \right], \left[\underset{\cdot}{i}_1^{l}, \underset{\cdot}{i}_1^{u} \right], \left[\underset{\cdot}{d}_1^{l}, \underset{\cdot}{d}_1^{u} \right] \right)$ *and* $\wp_2 = \left(\left[\dot{s}_2^{l}, \dot{s}_2^{u} \right], \left[\underset{\cdot}{i}_2^{l}, \underset{\cdot}{i}_2^{u} \right], \left[\underset{\cdot}{d}_2^{l}, \underset{\cdot}{d}_2^{u} \right] \right)$ *be three IVTSFNs and* $\lambda > 0$*. Then IVTSF Dombi operations are given by:*

1 $$\wp_1 \oplus \wp_2 = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\dot{s}_1^{l\mathfrak{q}}}{1-\dot{s}_1^{l\mathfrak{q}}} \right) + \left(\frac{\dot{s}_2^{l\mathfrak{q}}}{1-\dot{s}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\dot{s}_1^{u\mathfrak{q}}}{1-\dot{s}_1^{u\mathfrak{q}}} \right) + \left(\frac{\dot{s}_2^{u\mathfrak{q}}}{1-\dot{s}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\underset{\cdot}{i}_1^{l\mathfrak{q}}}{\underset{\cdot}{i}_1^{l\mathfrak{q}}} \right) + \left(\frac{1-\underset{\cdot}{i}_2^{l\mathfrak{q}}}{\underset{\cdot}{i}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\underset{\cdot}{i}_1^{u\mathfrak{q}}}{\underset{\cdot}{i}_1^{u\mathfrak{q}}} \right) + \left(\frac{1-\underset{\cdot}{i}_2^{u\mathfrak{q}}}{\underset{\cdot}{i}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\underset{\cdot}{d}_1^{l\mathfrak{q}}}{\underset{\cdot}{d}_1^{l\mathfrak{q}}} \right) + \left(\frac{1-\underset{\cdot}{d}_2^{l\mathfrak{q}}}{\underset{\cdot}{d}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\underset{\cdot}{d}_1^{u\mathfrak{q}}}{\underset{\cdot}{d}_1^{u\mathfrak{q}}} \right) + \left(\frac{1-\underset{\cdot}{d}_2^{u\mathfrak{q}}}{\underset{\cdot}{d}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right] \end{array} \right)$$

2 $$\wp_1 \otimes \wp_2 = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\dot{s}_1^{l\mathfrak{q}}}{\dot{s}_1^{l\mathfrak{q}}} \right) + \left(\frac{1-\underset{\cdot}{i}_2^{l\mathfrak{q}}}{\dot{s}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{ \left(\frac{1-\dot{s}_1^{u\mathfrak{q}}}{\dot{s}_1^{u\mathfrak{q}}} \right) + \left(\frac{1-\dot{s}_2^{u\mathfrak{q}}}{\dot{s}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right], \\ \left[\sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\underset{\cdot}{i}_1^{l\mathfrak{q}}}{1-\underset{\cdot}{i}_1^{l\mathfrak{q}}} \right) + \left(\frac{\underset{\cdot}{i}_2^{l\mathfrak{q}}}{1-\underset{\cdot}{i}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\underset{\cdot}{i}_1^{u\mathfrak{q}}}{1-\underset{\cdot}{i}_1^{u\mathfrak{q}}} \right) + \left(\frac{\underset{\cdot}{i}_2^{u\mathfrak{q}}}{1-\underset{\cdot}{i}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right], \\ \left[\sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\underset{\cdot}{d}_1^{l\mathfrak{q}}}{1-\underset{\cdot}{d}_1^{l\mathfrak{q}}} \right) + \left(\frac{\underset{\cdot}{d}_2^{l\mathfrak{q}}}{1-\underset{\cdot}{d}_2^{l\mathfrak{q}}} \right) \right\}^{1}}}, \sqrt[\mathfrak{q}]{1 - \frac{1}{1+\left\{ \left(\frac{\underset{\cdot}{d}_1^{u\mathfrak{q}}}{1-\underset{\cdot}{d}_1^{u\mathfrak{q}}} \right) + \left(\frac{\underset{\cdot}{d}_2^{u\mathfrak{q}}}{1-\underset{\cdot}{d}_2^{u\mathfrak{q}}} \right) \right\}^{1}}} \right] \end{array} \right),$$

$$3\ \lambda^{\mathfrak{p}} = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\lambda\left(\frac{1-\dot{s}^{l\mathfrak{q}}}{\dot{s}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\lambda\left(\frac{1-\dot{s}^{u\mathfrak{q}}}{\dot{s}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\dot{i}^{l\mathfrak{q}}}{1-\dot{i}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\dot{i}^{u\mathfrak{q}}}{1-\dot{i}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right] \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\d{d}^{l\mathfrak{q}}}{1-\d{d}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\d{d}^{u\mathfrak{q}}}{1-\d{d}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right] \end{array} \right)$$

$$4\ \mathfrak{p}^{\lambda} = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\lambda\left(\frac{1-\dot{s}^{l\mathfrak{q}}}{\dot{s}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\lambda\left(\frac{1-\dot{s}^{u\mathfrak{q}}}{\dot{s}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\dot{i}^{l\mathfrak{q}}}{1-\dot{i}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\dot{i}^{u\mathfrak{q}}}{1-\dot{i}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right] \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\d{d}^{l\mathfrak{q}}}{1-\d{d}^{l\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}},\ \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\lambda\left(\frac{\d{d}^{u\mathfrak{q}}}{1-\d{d}^{u\mathfrak{q}}}\right)\right\}^{\frac{1}{}}}} \right] \end{array} \right)$$

The following example illustrates the applicability of the above-defined operations.

Example 1. *Let* $\mathfrak{p} = ([0.65, 0.83],\ [0.4, 0.54],\ [0.26, 0.41])$, $\mathfrak{p}_1 = ([0.43, 0.53],\ [0.29, 0.43],\ [0.39, 0.47])$ *and* $\mathfrak{p}_2 = ([0.77, 0.89],\ [0.34, 0.49],\ [0.59, 0.68])$ *be three IVTSFNs for* $\mathfrak{q} = 4$ *and* $\lambda = 2$ *and* $= 1$. *Then*

$$\mathfrak{p}_1 \oplus \mathfrak{p}_2 = ([0.092, 0.16],\ [0.0012, 0.0055],\ [0.005, 0.01])$$

$$\mathfrak{p}_1 \otimes \mathfrak{p}_2 = ([0.008, 0.019], [0.0051, 0.022], [0.035, 0.061])$$

$$2\mathfrak{p} = ([0.076, 0.161], [0.0032, 0.0111], [0, 0.004])$$

$$\mathfrak{p}^2 = ([0.026, 0.078], [0.0125, 0.0392], [0.002, 0.014])$$

Theorem 1. *Let* $\mathfrak{p} = \left(\left[\dot{s}^{l},\ \dot{s}^{u}\right],\ \left[\dot{i}^{l},\ \dot{i}^{u}\right], \left[\d{d}^{l},\ \d{d}^{u}\right]\right)$, $\mathfrak{p}_1 = \left(\left[\dot{s}^{l}_1, \dot{s}^{u}_1\right],\ \left[\dot{i}^{l}_1, \dot{i}^{u}_1\right],\ \left[\d{d}^{l}_1, \d{d}^{u}_1\right]\right)$ *and* $\mathfrak{p}_2 = \left(\left[\dot{s}^{l}_2, \dot{s}^{u}_2\right],\ \left[\dot{i}^{l}_2, \dot{i}^{u}_2\right],\ \left[\d{d}^{l}_2, \d{d}^{u}_2\right]\right)$ *be three IVTSFNs and* $\lambda \geq 0$. *Then*

1. $\mathfrak{p}_1 \oplus \mathfrak{p}_2 = \mathfrak{p}_2 \oplus \mathfrak{p}_1$
2. $\mathfrak{p}_1 \otimes \mathfrak{p}_2 = \mathfrak{p}_2 \otimes \mathfrak{p}_1$
3. $\lambda(\mathfrak{p}_1 \oplus \mathfrak{p}_2) = \lambda\mathfrak{p}_1 \oplus \lambda\mathfrak{p}_2$
4. $(\lambda_1 \oplus \lambda_2)\mathfrak{p} = \lambda_1\mathfrak{p} \oplus \lambda_2\mathfrak{p}$
5. $(\mathfrak{p}_1 \otimes \mathfrak{p}_2)^{\lambda} = \mathfrak{p}_1^{\lambda} \otimes \mathfrak{p}_2^{\lambda}$
6. $\mathfrak{p}^{\lambda_1} \otimes \mathfrak{p}^{\lambda_2} = \mathfrak{p}^{\lambda_1+\lambda_2}$

Proof. Trivial. □

4. Interval Valued TSF Dombi Arithmetic Aggregation Operator

The goal of this section is to develop and investigate the applicability of IVTSFDWA operators. The conceptions of IVTSFDWA, IVTSF Dombi ordered weighted averaging (IVTSFDOWA), and IVTSF Dombi hybrid averaging (IVTSFDHA) operators are developed followed by a numerical example, and their basic properties of aggregation are discussed. It is to be noted that throughout this paper, by $\psi = (\psi_1, \psi_2, \psi_3 \ldots \psi_n)^T$ we mean the weight vector of IVTSFNs $\text{Ƥ}_j (j = 1, 2, 3 \ldots n)$ with $\psi_j > 0$ and $\sum\limits_{j=1}^{n} \psi_j = 1$.

In the next example, we propose the definition of the IVTSFDWA operator followed by a relevant theorem and its proof via induction.

Definition 6. *Let* $\text{Ƥ}_j = \left(\left[\dot{\check{s}}^l_j, \dot{\check{s}}^u_j\right], \left[\underset{\cdot}{i}^l_j, \underset{\cdot}{i}^u_j\right], \left[\underset{\varsigma}{d}^l_j, \underset{\varsigma}{d}^u_j\right] \right)$ $(j = 1, 2, 3 \ldots n)$ *be IVTSFNs. Then IVTSFDWA operator is defined as:*

$$IVTSFDWA(\text{Ƥ}_1, \text{Ƥ}_2 \ldots . \text{Ƥ}_n) = \overset{n}{\underset{j=1}{\oplus}} \left(\psi_j \text{Ƥ}_j \right) \tag{6}$$

Hence, we get the accompanying theorem that follows the Dombi operations on IVTSFNs.

Theorem 2. *Let* $\text{Ƥ}_j = \left(\left[\dot{\check{s}}^l_j, \dot{\check{s}}^u_j\right], \left[\underset{\cdot}{i}^l_j, \underset{\cdot}{i}^u_j\right], \left[\underset{\varsigma}{d}^l_j, \underset{\varsigma}{d}^u_j\right] \right)$ $(j = 1, 2, 3 \ldots n)$ *be IVTSFNs. Then aggregated value of using the IVTSFDWA operator is also an IVTSFN and*

$$IVTSFDWA(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3 \ldots \text{Ƥ}_n) = \left(\begin{array}{c} \left[\sqrt[\text{ɋ}]{1 - \frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{\dot{\check{s}}^{l\,\text{ɋ}}_j}{1-\dot{\check{s}}^{l\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{1 - \frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{\dot{\check{s}}^{u\,\text{ɋ}}_j}{1-\dot{\check{s}}^{u\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-\underset{\cdot}{i}^{l\,\text{ɋ}}_j}{\underset{\cdot}{i}^{l\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-\underset{\cdot}{i}^{u\,\text{ɋ}}_j}{\underset{\cdot}{i}^{u\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-\underset{\varsigma}{d}^{l\,\text{ɋ}}_j}{\underset{\varsigma}{d}^{l\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-\underset{\varsigma}{d}^{u\,\text{ɋ}}_j}{\underset{\varsigma}{d}^{u\,\text{ɋ}}_j} \right) \right\}^{\underline{1}}}} \right] \end{array} \right) \tag{7}$$

Proof. We prove the Theorem 2 by induction as follows.

When $n = 2$. Then,

$$IVTSFDWA(\text{Ƥ}_1, \text{Ƥ}_2) = \text{Ƥ}_1 \oplus \text{Ƥ}_2 = \psi_1 \left(\left[\dot{\check{s}}^l_1, \dot{\check{s}}^u_1\right], \left[\underset{\cdot}{i}^l_1, \underset{\cdot}{i}^u_1\right], \left[\underset{\varsigma}{d}^l_1, \underset{\varsigma}{d}^u_1\right] \right) \oplus \psi_2 \left(\left[\dot{\check{s}}^l_2, \dot{\check{s}}^u_2\right], \left[\underset{\cdot}{i}^l_2, \underset{\cdot}{i}^u_2\right], \left[\underset{\varsigma}{d}^l_2, \underset{\varsigma}{d}^u_2\right] \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[\text{ɋ}]{1 - \frac{1}{1+\left\{ \psi_1 \left(\frac{\dot{\check{s}}^{l\,\text{ɋ}}_1}{1-\dot{\check{s}}^{l\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{1 - \frac{1}{1+\left\{ \psi_1 \left(\frac{\dot{\check{s}}^{u\,\text{ɋ}}_1}{1-\dot{\check{s}}^{u\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \psi_1 \left(\frac{1-\underset{\cdot}{i}^{l\,\text{ɋ}}_1}{\underset{\cdot}{i}^{l\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \psi_1 \left(\frac{1-\underset{\cdot}{i}^{u\,\text{ɋ}}_1}{\underset{\cdot}{i}^{u\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}} \right] \\ \left[\sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \psi_1 \left(\frac{1-\underset{\varsigma}{d}^{l\,\text{ɋ}}_1}{\underset{\varsigma}{d}^{l\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}}, \sqrt[\text{ɋ}]{\frac{1}{1+\left\{ \psi_1 \left(\frac{1-\underset{\varsigma}{d}^{u\,\text{ɋ}}_1}{\underset{\varsigma}{d}^{u\,\text{ɋ}}_1} \right) \right\}^{\underline{1}}}} \right] \end{array} \right)$$

$$
\oplus\left(\begin{array}{c}
\left[\sqrt[q]{1-\frac{1}{1+\left\{\psi_2\left(\frac{\dot{s}^{lq}_2}{1-\dot{s}^{lq}_2}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{1-\frac{1}{1+\left\{\psi_2\left(\frac{\dot{s}^{uq}_2}{1-\dot{s}^{uq}_2}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\psi_2\left(\frac{1-\dot{i}^{lq}_2}{\dot{i}^{lq}_2}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\psi_2\left(\frac{1-\dot{i}^{uq}_2}{\dot{i}^{uq}_2}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\psi_2\left(\frac{1-\dot{d}^{lq}_2}{\dot{d}^{lq}_2}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\psi_2\left(\frac{1-\dot{d}^{uq}_2}{\dot{d}^{uq}_2}\right)\right\}^{\underline{1}}}}\right]
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{\dot{s}^{lq}_j}{1-\dot{s}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{\dot{s}^{uq}_j}{1-\dot{s}^{uq}_j}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{1-\dot{i}^{lq}_j}{\dot{i}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{1-\dot{i}^{uq}_j}{\dot{i}^{uq}_j}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{1-\dot{d}^{lq}_j}{\dot{d}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{2}\psi_j\left(\frac{1-\dot{d}^{uq}_j}{\dot{d}^{uq}_j}\right)\right\}^{\underline{1}}}}\right]
\end{array}\right)
$$

Hence the result is valid for $n = 2$.
Let the result be valid for $n = k$. Then we have

$$
IVTSFDWA(\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3 \ldots \mathrm{P}_k) = \left(\begin{array}{c}
\left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{\dot{s}^{lq}_j}{1-\dot{s}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{\dot{s}^{uq}_j}{1-\dot{s}^{uq}_j}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{i}^{lq}_j}{\dot{i}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{i}^{uq}_j}{\dot{i}^{uq}_j}\right)\right\}^{\underline{1}}}}\right],\\
\left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{d}^{lq}_j}{\dot{d}^{lq}_j}\right)\right\}^{\underline{1}}}},\ \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{d}^{uq}_j}{\dot{d}^{uq}_j}\right)\right\}^{\underline{1}}}}\right]
\end{array}\right)
$$

Now for $n = k+1$, then

$$IVTSFDWA(\text{P}_1, \text{P}_2, \text{P}_3 \ldots \text{P}_k, \text{P}_{k+1}) = \left(\begin{array}{c} \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{\dot{s}^{l\mathfrak{q}}_{j}}{1-\dot{s}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{\dot{s}^{u\mathfrak{q}}_{j}}{1-\dot{s}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{i}^{l\mathfrak{q}}_{j}}{\dot{i}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{i}^{u\mathfrak{q}}_{j}}{\dot{i}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{d}^{l\mathfrak{q}}_{j}}{\dot{d}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k}\psi_j\left(\frac{1-\dot{d}^{uj}_{j}}{\dot{d}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right] \end{array} \right) \oplus \\ \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{\dot{s}^{l\mathfrak{q}}_{k+1}}{1-\dot{s}^{l\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{\dot{s}^{u\mathfrak{q}}_{k+1}}{1-\dot{s}^{u\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{i}^{l\mathfrak{q}}_{k+1}}{\dot{i}^{l\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{i}^{u\mathfrak{q}}_{k+1}}{\dot{i}^{u\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{d}^{l\mathfrak{q}}_{k+1}}{\dot{d}^{l\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{d}^{u\mathfrak{q}}_{k+1}}{\dot{d}^{u\mathfrak{q}}_{k+1}}\right)\right\}^{\underline{1}}}} \right] \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{\dot{s}^{l\mathfrak{q}}_{j}}{1-\dot{s}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{\dot{s}^{u\mathfrak{q}}_{j}}{1-\dot{s}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{s}^{l\mathfrak{q}}_{j}}{\dot{s}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{i}^{u\mathfrak{q}}_{j}}{\dot{i}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right] \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{d}^{l\mathfrak{q}}_{j}}{\dot{d}^{l\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{k+1}\psi_j\left(\frac{1-\dot{d}^{u\mathfrak{q}}_{j}}{\dot{d}^{u\mathfrak{q}}_{j}}\right)\right\}^{\underline{1}}}} \right] \end{array} \right)$$

Thus, the result is true for $n = k+1$.
Therefore, we conclude that the result holds for any n. □

The following example illustrate the applicability of the IVTSFDWA operator.

Example 2. *Let* $P_1 = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41])$, $P_2 = ([0.43, 0.53], [0.29, 0.43], [0.39, 0.47])$, $P_3 = ([0.77, 0.89], [0.34, 0.49], [0.59, 0.68])$ *and* $P_4 = ([0.71, 0.81], [0.56, 0.65], [0.64, 0.65])$ *be four IVTSFNs for* $q = 4$ *and also let* $= 1$ *and* $\psi = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$. *Then*

$$IVTSFDWA(P_1, P_2, P_3, P_4) = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[4]{1 - \frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{0.65^4}{1-0.65^4}\right)^1 + 0.25\left(\frac{0.43^4}{1-0.43^4}\right)^1 + \\ 0.41\left(\frac{0.77^4}{1-0.77^4}\right)^1 + 0.19\left(\frac{0.71^4}{1-0.71^4}\right)^1 \end{array} \right\}^{\frac{1}{1}}}}, \\ \sqrt[4]{1 - \frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{0.83^4}{1-0.83^4}\right)^1 + 0.25\left(\frac{0.53^4}{1-0.53^4}\right)^1 + \\ 0.41\left(\frac{0.89^4}{1-0.89^4}\right)^1 + 0.19\left(\frac{0.81^4}{1-0.81^4}\right)^1 \end{array} \right\}^{\frac{1}{1}}}} \end{array} \right] \\ \left[\begin{array}{c} \sqrt[4]{\frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{1-0.4^4}{0.4^4}\right)^1 + 0.25\left(\frac{1-0.29^4}{0.29^4}\right)^1 + \\ 0.41\left(\frac{1-0.34^4}{0.34^4}\right)^1 + 0.19\left(\frac{1-0.56^4}{0.56^4}\right)^1 \end{array} \right\}^{\frac{1}{1}}}}, \\ \sqrt[4]{\frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{1-0.54^4}{0.54^4}\right)^1 + 0.25\left(\frac{1-0.43^4}{0.43^4}\right)^1 + \\ 0.41\left(\frac{1-0.49^4}{0.49^4}\right)^1 + 0.19\left(\frac{1-0.65^4}{0.65^4}\right)^1 \end{array} \right\}^{\frac{1}{1}}}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[4]{\frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{1-0.26^4}{0.26^4}\right)^1 + 0.25\left(\frac{1-0.39^4}{0.39^4}\right)^1 + \\ 0.41\left(\frac{1-0.59^4}{0.59^4}\right)^1 + 0.19\left(\frac{1-0.64^4}{0.64^4}\right)^1 \end{array} \right\}^{\frac{1}{1}}}}, \\ \sqrt[4]{\frac{1}{1+\left\{ \begin{array}{c} 0.15\left(\frac{1-0.41^4}{0.41^4}\right)^1 + 0.25\left(\frac{1-0.47^4}{0.47^4}\right)^1 + \\ 0.41\left(\frac{1-0.68^4}{0.68^4}\right)^1 + 0.19\left(\frac{1-0.65^4}{0.65^4}\right)^1 \end{array} \right\}^{\frac{1}{2}}}} \end{array} \right] \end{array} \right)$$

$$IVTSFDWA(P_1, P_2, P_3, P_4) = ([0.705, 0.84], [0.3412, 0.4907], [0.38, 0.523])$$

Next, we prove some properties of the IVTSFDWA as follows.

Theorem 3. *(Idempotency property) If* $P_j = \left(\left[\dot{s}^l_j, \dot{s}^u_j\right], \left[i^l_j, i^u_j\right], \left[d^l_j, d^u_j\right]\right)$ $(j = 1, 2, 3, \ldots n)$ *be some identical IVTSFNs. i.e.,* $P_j = P$ *for all* j, *then* $IVTSFDWA(P_1, P_2, P_3, \ldots P_n) = P$

Proof. Since $P_j = \left(\left[\dot{s}^l_j, \dot{s}^u_j\right], \left[i^l_j, i^u_j\right], \left[d^l_j, d^u_j\right]\right) = P(j = 1, 2, 3, \ldots n)$

Then, we have:

$$IVTSFDWA(P_1, P_2, P_3 \ldots P_n,) = \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{l\,q}_j}{1-\dot{s}^{l\,q}_j}\right)\right\}^{\frac{1}{}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{u\,q}_j}{1-\dot{s}^{u\,q}_j}\right)\right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-i^{l\,q}_j}{i^{l\,q}_j}\right)\right\}^{\frac{1}{}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-i^{u\,q}_j}{i^{u\,q}_j}\right)\right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-d^{l\,q}_j}{d^{l\,q}_j}\right)\right\}^{\frac{1}{}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-d^{u\,q}_j}{d^{u\,q}_j}\right)\right\}^{\frac{1}{}}}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\dot{s}^{l\,q}_{j}}{1-\dot{s}^{l\,q}_{j}}\right)\right\}^{\frac{1}{q}}}},\ \sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\dot{s}^{u\,q}_{j}}{1-\dot{s}^{u\,q}_{j}}\right)\right\}^{\frac{1}{q}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-i^{l\,q}_{j}}{i^{l\,q}_{j}}\right)\right\}^{\frac{1}{q}}}},\ \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-i^{u\,q}_{j}}{i^{u\,q}_{j}}\right)\right\}^{\frac{1}{q}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-d^{l\,q}_{j}}{d^{l\,q}_{j}}\right)\right\}^{\frac{1}{q}}}},\ \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-d^{u\,q}_{j}}{d^{u\,q}_{j}}\right)\right\}^{\frac{1}{q}}}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\frac{\dot{s}^{l\,q}}{1-\dot{s}^{l\,q}}}},\ \sqrt[q]{1 - \frac{1}{1+\frac{\dot{s}^{u\,q}}{1-\dot{s}^{u\,q}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\frac{1-i^{l\,q}}{i^{l\,q}}}},\ \sqrt[q]{\frac{1}{1+\frac{1-i^{u\,q}}{i^{u\,q}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\frac{1-d^{l\,q}}{d^{l\,q}}}},\ \sqrt[q]{\frac{1}{1+\frac{1-d^{u\,q}}{d^{u\,q}}}} \right] \end{array} \right)$$

$$= \left(\left[\dot{s}^{l},\ \dot{s}^{u}\right], \left[i^{l},\ i^{u}\right], \left[d^{l},\ d^{u}\right] \right) = \wp$$

Thus $IVTSFDWA(\wp_1, \wp_2, \wp_3, \ldots \wp_n) = \wp$. □

Theorem 4. *(Boundedness property) Let* $\wp_j = \left(\left[\dot{s}^{l}_{j}, \dot{s}^{u}_{j}\right], \left[i^{l}_{j}, i^{u}_{j}\right], \left[d^{l}_{j}, d^{u}_{j}\right] \right)$ $(j = 1, 2, 3, \ldots n)$ *be a few IVTSFNs. Let* $\wp^- = min(\wp_1, \wp_2, \wp_3, \ldots \wp_n)$ *and* $\wp^+ = max(\wp_1, \wp_2, \wp_3, \ldots \wp_n)$*. Then* $\wp \le IVTSFDWA(\wp_1, \wp_2, \wp_3, \ldots \wp_n) \le \wp^+$.

Proof. Let $\wp_j = \left(\left[\dot{s}^{l}_{j}, \dot{s}^{u}_{j}\right], \left[i^{l}_{j}, i^{u}_{j}\right], \left[d^{l}_{j}, d^{u}_{j}\right] \right)$ $(j = 1, 2, 3, \ldots n)$ be IVTSFNs and $\wp^- = min(\wp_1, \wp_2, \wp_3, \ldots \wp_n) = \left(\left[\dot{s}^{l-}_{j}, \dot{s}^{u-}_{j}\right], \left[i^{l-}_{j}, i^{u-}_{j}\right], \left[d^{l-}_{j}, d^{u-}_{j}\right] \right)$ and $\wp^+ = max(\wp_1, \wp_2, \wp_3, \ldots \wp_n) = \left(\left[\dot{s}^{l+}_{j}, \dot{s}^{u+}_{j}\right], \left[i^{l+}_{j}, i^{u+}_{j}\right], \left[d^{l+}_{j}, d^{u+}_{j}\right] \right)$.

We have $\dot{s}^- = min\left\{ \left[\dot{s}^{l}_{j}, \dot{s}^{u}_{j}\right] \right\}$, $i^- = max\left\{ \left[i^{l}_{j}, i^{u}_{j}\right] \right\}$, $d^- = max\left\{ \left[d^{l}_{j}, d^{u}_{j}\right] \right\}$, $\dot{s}^+ = max\left\{ \left[\dot{s}^{l}_{j}, \dot{s}^{u}_{j}\right] \right\}$, $i^+ = min\left\{ \left[i^{l}_{j}, i^{u}_{j}\right] \right\}$, $d^+ = min\left\{ \left[d^{l}_{j}, d^{u}_{j}\right] \right\}$.

Now consider,

$$\sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{-l\,q}}{1-\dot{s}^{l-q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{l\,q}}{1-\dot{s}^{l\,q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{+l\,q}}{1-\dot{s}^{+l\,q}}\right)\right\}^{\frac{1}{q}}}}$$

$$\sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{-u\,q}}{1-\dot{s}^{-u\,q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{u\,q}}{1-\dot{s}^{u\,q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{\dot{s}^{+u\,q}}{1-\dot{s}^{+u\,q}}\right)\right\}^{\frac{1}{q}}}}$$

$$\sqrt[q]{\frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-i^{+l\,q}}{i^{+l\,q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{\frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-i^{l\,q}}{i^{l\,q}}\right)\right\}^{\frac{1}{q}}}} \le \sqrt[q]{\frac{1}{1 + \left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-i^{-l\,q}}{i^{+l\,q}}\right)\right\}^{\frac{1}{q}}}}$$

$$\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{i}}^{+u\mathfrak{q}}}{\dot{\text{i}}^{+u\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{i}}^{u\mathfrak{q}}}{\dot{\text{i}}^{u\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{i}}^{-u\mathfrak{q}}}{\dot{\text{i}}^{+u\mathfrak{q}}}\right)\right\}^{\underline{1}}}}$$

$$\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{+l\mathfrak{q}}}{\dot{\text{d}}^{+l\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{l\mathfrak{q}}}{\dot{\text{d}}^{l\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{-l\mathfrak{q}}}{\dot{\text{d}}^{-l\mathfrak{q}}}\right)\right\}^{\underline{1}}}}$$

$$\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{+u\mathfrak{q}}}{\dot{\text{d}}^{+u\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{u\mathfrak{q}}}{\dot{\text{d}}^{u\mathfrak{q}}}\right)\right\}^{\underline{1}}}} \leq \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{-u\mathfrak{q}}}{\dot{\text{d}}^{-u\mathfrak{q}}}\right)\right\}^{\underline{1}}}}$$

Therefore, $\mathcal{P}^{-} \leq IVTSFDWA(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots \mathcal{P}_n) \leq \mathcal{P}^{+}$. □

Theorem 5. *(Monotonicity Property) Let* $\mathcal{P}_{\text{j}}$ *and* $\mathcal{P}'_{\text{j}}(\text{j} = 1, 2, 3, \ldots n)$ *be IVTSFNs such that* $\mathcal{P}_{\text{j}} \leq \mathcal{P}'_{\text{j}}$ *for all* j*. Then* $IVTSFDWA = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots \mathcal{P}_n) \leq IVTSFDWA = \left(\mathcal{P}'_1, \mathcal{P}'_2, \mathcal{P}'_3, \ldots \mathcal{P}'_n\right)$.

Proof. Trivial. □

In the definition, we introduce the IVTSFDOWA operator keeping in mind the significance of the ordered position of the information as follows.

Definition 7. *Let* $\mathcal{P}_{\text{j}} = \left(\left[\dot{\check{s}}_{\text{j}}^{l}, \dot{\check{s}}_{\text{j}}^{u}\right], \left[\dot{\text{i}}_{\text{j}}^{l}, \dot{\text{i}}_{\text{j}}^{u}\right], \left[\dot{\text{d}}_{\text{j}}^{l}, \dot{\text{d}}_{\text{j}}^{u}\right]\right)$ $(\text{j} = 1, 2, 3, \ldots n)$ *be an IVTSFNs. An IVTSFDOWA operator is a function* $IVTSFDOWA.\mathcal{P}^{n} \to \mathcal{P}$ *defined as:*

$$IVTSFDOWA(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots \mathcal{P}_n) = \mathop{\oplus}_{\text{j}=1}^{n}\left(\psi_{\text{j}}\mathcal{P}_{\sigma(\text{j})}\right)$$

where $(\sigma(1), \sigma(2), \sigma(3), \ldots \sigma(n))$ *are the permutation of* $(\text{j} = 1, 2, 3, \ldots n)$*, for which* $\mathcal{P}_{\sigma(\text{j}=1)} \geq \mathcal{P}_{\sigma(\text{j})}$ *for all* $\text{j} = 1, 2, 3, \ldots n$.

Based on Dombi operations on IVTSFNs, we have the following result.

Theorem 6. *Let* $\mathcal{P}_{\text{j}} = \left(\left[\dot{\check{s}}_{\text{j}}^{l}, \dot{\check{s}}_{\text{j}}^{u}\right], \left[\dot{\text{i}}_{\text{j}}^{l}, \dot{\text{i}}_{\text{j}}^{u}\right], \left[\dot{\text{d}}_{\text{j}}^{l}, \dot{\text{d}}_{\text{j}}^{u}\right]\right)$ $(\text{j} = 1, 2, 3 \ldots n)$ *be IVTSFNs. Then aggregated value using the IVTSFDOWA operator is also an IVTSFN given by:*

$$IVTSFDOWA(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots \mathcal{P}_n) = \mathop{\oplus}_{\text{j}=1}^{n}\left(\psi_{\text{j}}\mathcal{P}_{\sigma(\text{j})}\right) = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{\dot{\check{s}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}{1-\dot{\check{s}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{\dot{\check{s}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}{1-\dot{\check{s}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}\right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{i}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}{\dot{\text{i}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\psi_{\text{j}}\left(\frac{1-\dot{\text{i}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}{\dot{\text{i}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}\right], \\ \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\Psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}{\dot{\text{d}}^{l\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n}\Psi_{\text{j}}\left(\frac{1-\dot{\text{d}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}{\dot{\text{d}}^{u\,\mathfrak{q}}_{\sigma(\text{j})}}\right)\right\}^{\underline{1}}}}\right] \end{array}\right) \quad (8)$$

where $(\sigma(1), \sigma(2), \sigma(3), \ldots \sigma(n))$ *are the permutation of* $(\text{j} = 1, 2, 3, \ldots n)$*, for which* $\mathcal{P}_{\sigma(\text{j}=1)} \geq \mathcal{P}_{\sigma(\text{j})}$ *for all* $\text{j} = 1, 2, 3, \ldots n$.

The following example illustrates the applicability of the IVTSFDOWA operator.

Example 3. *Let* $\text{P}_1 = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41])$, $\text{P}_2 = ([0.43, 0.53], [0.29, 0.43], [0.39, 0.47])$ *and* $\text{P}_3 = ([0.77, 0.89], [0.34, 0.49], [0.59, 0.68])$ *and* $\text{P}_4 = ([0.71, 0.81], [0.56, 0.65], [0.64, 0.65])$ *be four IVTSFNs for* $q = 4$ *and also let* $= 1$ *and* $\psi = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$ *denote the aggregation associated weight vector.*

First, we compute the scores using Definition 3 as follows.

$$T(\text{P}_1) = 0.254,\ T(\text{P}_2) = 0.224,\ T(\text{P}_3) = 0.121,\ T(\text{P}_4) = 0.192$$

Based on scores, we have $\text{P}_{\sigma(1)} = \text{P}_1$, $\text{P}_{\sigma(2)} = \text{P}_2$, $\text{P}_{\sigma(3)} = \text{P}_4$, $\text{P}_{\sigma(4)} = \text{P}_3$. So, using the IVTSFOWA operator, we have:

$$TSFDOWA(\text{P}_1, \text{P}_2, \text{P}_4,\ \text{P}_3) = ([0.685, 0.817],\ [0.3724, 0.5227],\ [0.35, 0.502])$$

Theorem 7. *(Idempotency Property) If* $\text{P}_j (j = 1, 2, 3, \ldots n)$ *are all the same, i.e.,* $\text{P}_j = \text{P}$ *for all* j. *Then*

$$IVTSFDOWA(\text{P}_1, \text{P}_2, \ldots \text{P}) = \text{P}$$

Theorem 8. *(Boundedness Property) Let* $\text{P}_j = \left(\left[\dot{s}_j^l, \dot{s}_j^u \right],\ \left[i_j^l, i_j^u \right],\ \left[d_j^l, d_j^u \right] \right)$ $(j = 1, 2, 3, \ldots n)$ *be a few IVTSFNs and let* $\text{P}^- = min(\text{P}_1, \text{P}_2, \text{P}_3, \ldots \text{P}_n)$ *and* $\text{P}^+ = max(\text{P}_1, \text{P}_2, \text{P}_3, \ldots \text{P}_n)$.

Then, $\text{P} \leq IVTSFDOWA(\text{P}_1, \text{P}_2, \text{P}_3, \ldots \text{P}_n) \leq \text{P}^+$.

Theorem 9. *(Monotonicity Property) Let* P_j *and* $\text{P}'_j (j = 1, 2, 3, \ldots n)$ *be two collections of IVTSFNs such that* $\text{P}_j \leq \text{P}'_j$ *for all* j. *Then* $IV\ TSFDOWA = (\text{P}_1, \text{P}_2, \text{P}_3, \ldots \text{P}_n) \leq\ IVTSFDWA = \left(\text{P}'_1, \text{P}'_2, \text{P}'_3, \ldots \text{P}'_n \right)$.

Theorem 10. *(Commutativity Property) Let* P_j *and* $\text{P}'_j (j = 1, 2, \ldots n)$ *be two collections of IVTSFNs. Then*

$$IVTSFDOWA(\text{P}_1, \text{P}_2, \ldots \text{P}_n) = IVTSFDOWA(\text{P}_1, \text{P}_2, \ldots \text{P}_n)$$

where $\text{P}_j (1, 2, \ldots n)$ *is any permutation of* $\text{P}_j (j = 1, 2, \ldots n)$.

Now we propose the concept of IVTSF Dombi hybrid averaging (IVTSFDHA) operators. The motivation behind this is we need hybrid operators whenever we need to weigh the ordered position as well as the argument itself.

Definition 9. *An IVTSFDHA operator is a function* $IV\ TSFDHA.\text{P}^n \to \text{P}$, *defined as:*

$$IVTSFDHA(\text{P}_1, \text{P}_2, \ldots \text{P}_n) = \overset{n}{\underset{j=1}{\oplus}} \left(\psi_j \dot{\text{P}}_{\sigma(j)} \right) 1 = \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{\dot{s}^{l\,q}_{\sigma(j)}}{1 - \dot{s}^{l\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}},\ \sqrt[q]{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{\dot{s}^{u\,q}_{\sigma(j)}}{1 - \dot{s}^{u\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{1 - i^{l\,q}_{\sigma(j)}}{i^{l\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}},\ \sqrt[q]{\frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{1 - i^{u\,q}_{\sigma(j)}}{i^{u\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{1 - d^{l\,q}_{\sigma(j)}}{d^{l\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}},\ \sqrt[q]{\frac{1}{1 + \left\{ \sum_{j=1}^n \psi_j \left(\frac{1 - d^{u\,q}_{\sigma(j)}}{d^{u\,q}_{\sigma(j)}} \right) \right\}^{\frac{1}{}}}} \right] \end{array} \right) \quad (9)$$

where $\dot{\mathrm{P}}_{\sigma(\mathrm{j})}$ *is the jth biggest weighted IVTSF values* $\dot{\mathrm{P}}_{\mathrm{j}}$ $\left(\dot{\mathrm{P}}_{\mathrm{j}} = n\,\Psi_{\mathrm{j}}\mathrm{P}_{\mathrm{j}}, \mathrm{j} = 1, 2, \ldots n\right)$, *and* $\Psi = (\Psi_1, \Psi_2, \ldots \Psi_n)^T$ *be the weight vector of* $\dot{\mathrm{P}}_{\mathrm{j}}$ *with* $\sum_{\mathrm{j}=1}^{n} \Psi_{\mathrm{j}} = 1$. *Here n denotes the balancing coefficient.*

When $\Psi = \left(\frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)$, IVTSFDWA and IVTSFDOWA operator becomes special cases of the IVTSFDHA operator.

The following example illustrates the applicability of the IVTSFDOWA operator.

Example 4. *Let* $\mathrm{P}_1 = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41])$, $\mathrm{P}_2 = ([0.43, 0.53], [0.29, 0.43], [0.39, 0.47])$ *and* $\mathrm{P}_3 = ([0.77, 0.89], [0.34, 0.49], [0.59, 0.68])$ *and* $\mathrm{P}_4 = ([0.71, 0.81], [0.56, 0.65], [0.64, 0.65])$ *be four IVTSFNs for* $\mathfrak{q} = 4$ *and also let* $= 1$ *and* $\psi = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$ *denote the aggregation associated weight vector and* $\Psi_{\mathrm{j}} = (0.1,\ 0.19,\ 0.41,\ 0.3)^T$. *Then first we compute* $\dot{\mathrm{P}}_{\mathrm{j}} = n\,\Psi_{\mathrm{j}}\mathrm{P}_{\mathrm{j}}$ *as follows.*

$$\dot{\mathrm{P}}_1 = n\,\Psi_1\mathrm{P}_1 = ((4)(0.1)([0.65, 0.83],\ [0.4, 0.54],\ [0.26, 0.41])) = ([0.020, 0.066], [0.015, 0.047], [0.003, 0.017])$$

$$\dot{\mathrm{P}}_2 = n\,\Psi_2\mathrm{P}_2 = ((4)(0.19)([0.43, 0.53],\ [0.29, 0.43],\ [0.39, 0.47])) = ([0.007, 0.015], [0.002, 0.011], [0.008, 0.016])$$

$$\dot{\mathrm{P}}_3 = n\,\Psi_3\mathrm{P}_3 = ((4)(0.41)([0.77, 0.89],\ [0.34, 0.49],\ [0.59, 0.68])) = ([0.118, 0.184], [0.002, 0.009], [0.019, 0.036])$$

$$\dot{\mathrm{P}}_4 = n\,\Psi_4\mathrm{P}_4 = ((4)(0.3)([0.71, 0.81],\ [0.56, 0.65],\ [0.64, 0.65])) = ([0.073, 0.119], [0.021, 0.038], [0.036, 0.038])$$

$$T\left(\dot{\mathrm{P}}_1\right) = 0.0000065$$

$$T\left(\dot{\mathrm{P}}_2\right) = 0.000000019$$

$$T\left(\dot{\mathrm{P}}_3\right) = 0.00044$$

$$T\left(\dot{\mathrm{P}}_4\right) = 0.000076$$

Based on scores, we have $\dot{\mathrm{P}}_{\sigma(1)} = \dot{\mathrm{P}}_3$, $\dot{\mathrm{P}}_{\sigma(2)} = \dot{\mathrm{P}}_4$, $\dot{\mathrm{P}}_{\sigma(3)} = \dot{\mathrm{P}}_1$, $\dot{\mathrm{P}}_{\sigma(4)} = \dot{\mathrm{P}}_2$. So, using IVTSFOWA operator, we have

$$IVTSFDHA\left(\dot{\mathrm{P}}_3, \dot{\mathrm{P}}_4, \dot{\mathrm{P}}_1,\ \dot{\mathrm{P}}_2\right)$$

$$IVTSFDHA\left(\dot{\mathrm{P}}_3, \dot{\mathrm{P}}_4, \dot{\mathrm{P}}_1,\ \dot{\mathrm{P}}_2\right) = ([0.078, 0.123],\ [0.0026, 0.0129],\ [0.004, 0.019])$$

5. Interval Valued T-Spherical Fuzzy Dombi Geometric Aggregation Operators

In this section, we propose IVTSFDWG, IVTSF Dombi ordered weighted geometric (IVTSFDOWG) and IVTSF Dombi hybrid geometric (IVTSFDHG) operators using the Dombi operational laws of the IVTSFNs. The fitness of the proposed DAOs is also investigated using the induction method.

Definition 10. *Let* $\mathrm{P}_{\mathrm{j}} = \left(\left[\check{\mathrm{s}}^l_{\mathrm{j}}, \check{\mathrm{s}}^u_{\mathrm{j}}\right],\ \left[\mathrm{i}^l_{\mathrm{j}}, \mathrm{i}^u_{\mathrm{j}}\right],\ \left[\mathrm{d}^l_{\mathrm{j}}, \mathrm{d}^u_{\mathrm{j}}\right]\right)$ $(\mathrm{j} = 1, 2, 3 \ldots n)$ *be IVTSFNs. Then IVTSFDWG operator is defined as:*

$$IVTSFDWG(\mathrm{P}_1, \mathrm{P}_2 \ldots .\mathrm{P}_n) = \bigoplus_{\mathrm{j}=1}^{n} (\mathrm{P}_{\mathrm{j}})^{\psi_{\mathrm{j}}} \tag{10}$$

Theorem 11. *Let* $\text{Ƥ}_{\text{j}} = \left(\left[\dot{\check{s}}^{l}_{\text{j}}, \dot{\check{s}}^{u}_{\text{j}}\right], \left[\dot{\text{i}}^{l}_{\text{j}}, \dot{\text{i}}^{u}_{\text{j}}\right], \left[\text{ɖ}^{l}_{\text{j}}, \text{ɖ}^{u}_{\text{j}}\right]\right)$ $(\text{j} = 1, 2, 3 \ldots n)$ *be IVTSFNs. Then aggregated value using the IVTSFDWG operator is also an IVTSFN and*

$$IVTSFDWG(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3 \ldots \text{Ƥ}_n) = \left(\begin{array}{c} \left[\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{1-\dot{\check{s}}^{l}{}^{\mathfrak{q}}_{\text{j}}}{\dot{\check{s}}^{l}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}, \sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{1-\dot{\check{s}}^{u}{}^{\mathfrak{q}}_{\text{j}}}{\dot{\check{s}}^{u}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}\right], \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{\dot{\text{i}}^{l}{}^{\mathfrak{q}}_{\text{j}}}{1-\dot{\text{i}}^{l}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{\dot{\text{i}}^{u}{}^{\mathfrak{q}}_{\text{j}}}{1-\dot{\text{i}}^{u}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}\right] \\ \left[\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{\text{ɖ}^{l}{}^{\mathfrak{q}}_{\text{j}}}{1-\text{ɖ}^{l}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}, \sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{\text{j}=1}^{n} \psi_{\text{j}} \left(\frac{\text{ɖ}^{u}{}^{\mathfrak{q}}_{\text{j}}}{1-\text{ɖ}^{u}{}^{\mathfrak{q}}_{\text{j}}}\right)\right\}^{\frac{1}{}}}}\right] \end{array}\right) \quad (11)$$

Proof. Similar to Theorem 2. □

Theorem 12. *(Idempotency Property) If* $\text{Ƥ}_{\text{j}} (\text{j} = 1, 2, 3, \ldots n)$ *are all the same, i.e.,* $\text{Ƥ}_{\text{j}} = \text{Ƥ}$ *for all* j*. Then* $IVTSFDWG(\text{Ƥ}_1, \text{Ƥ}_2, \ldots \text{Ƥ}) = \text{Ƥ}$.

The proof of this theorem is similar to the proof of Theorem 3. However, an example is provided below to justify the theorem.

Example 5. *Let* $\text{Ƥ}_1 = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41]) = \text{Ƥ}_2 = \text{Ƥ}_3 = \text{Ƥ}_4$ *be four IVTSFNs for* $\mathfrak{q} = 4$ *and also let* $= 1$ *and* $\psi = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$. *Then*

$$IVTSFDWG(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3, \text{Ƥ}_4) = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41]) = \text{Ƥ}_1$$

Theorem 13. *(Boundedness Property) Let* $\text{Ƥ}_{\text{j}} = \left(\left[\dot{\check{s}}^{l}_{\text{j}}, \dot{\check{s}}^{u}_{\text{j}}\right], \left[\dot{\text{i}}^{l}_{\text{j}}, \dot{\text{i}}^{u}_{\text{j}}\right], \left[\text{ɖ}^{l}_{\text{j}}, \text{ɖ}^{u}_{\text{j}}\right]\right)$ $(\text{j} = 1, 2, 3, \ldots n)$ *be a few IVTSFNs and let* $\text{Ƥ}^- = min(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3, \ldots \text{Ƥ}_n)$ *and* $\text{Ƥ}^+ = max(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3, \ldots \text{Ƥ}_n)$ *denote the minimum and maximum of all IVTSFNs. Then*

$$\text{Ƥ}^- \le IVTSFDWG(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3, \ldots \text{Ƥ}_n) \le \text{Ƥ}^+$$

The proof of this theorem is similar to the proof of Theorem 4. However, an example is provided below to justify the theorem.

Example 6. *Let* $\text{Ƥ}_1 = ([0.65, 0.83], [0.4, 0.54], [0.26, 0.41])$, $\text{Ƥ}_2 = ([0.43, 0.53], [0.29, 0.43], [0.39, 0.47])$ *and* $\text{Ƥ}_3 = ([0.77, 0.89], [0.61, 0.67], [0.19, 0.27])$ *and* $\text{Ƥ}_4 = ([0.71, 0.85], [0.56, 0.65], [0.21, 0.29])$ *be four IVTSFNs for* $\mathfrak{q} = 4$ *and also let* $= 1$ *and* $\psi = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$ *denote the aggregation associated weight vector.*

First, we compute the scores using Definition 3 as follows.

$$T(\text{Ƥ}_1) = 0.254,\ T(\text{Ƥ}_2) = 0.224,\ T(\text{Ƥ}_3) = 0.121,\ T(\text{Ƥ}_4) = 0.192$$

From the score value analysis, it is clear that $\text{Ƥ}^- = \text{Ƥ}_3$ and $\text{Ƥ}^+ = \text{Ƥ}_1$.
Now, the aggregated value of Ƥ_1, Ƥ_2, Ƥ_3, and Ƥ_4 is given as follows:

$$IVTSFDWG(\text{Ƥ}_1, \text{Ƥ}_2, \text{Ƥ}_3, \text{Ƥ}_4) = ([0.561,\ 0.687], [0.5392,\ 0.6155], [0.294,\ 0.378])$$

Hence, by considering the definition of subsethood from [5], we have

$$Ꝑ^- = Ꝑ_3 \leq IVTSFDWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, Ꝑ_4) \leq Ꝑ^+ = Ꝑ_1$$

Theorem 14. *(Monotonicity Property) Let* $Ꝑ_ȷ$ *and* $Ꝑ'_ȷ (ȷ = 1,2,3,\ldots n)$ *be two collections of IVTSFNs such that* $Ꝑ_ȷ \leq Ꝑ'_ȷ$ *for all* ȷ. *Then* $IVTSFDWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, \ldots Ꝑ_n) \leq IVTSFDWG(Ꝑ'_1, Ꝑ'_2, Ꝑ'_3, \ldots Ꝑ'_n)$.

Likewise Example 5, an example can be constructed for the support of Theorem 14.

Definition 11. *Let* $Ꝑ_ȷ = \left(\left[ṡ^l_ȷ, ṡ^u_ȷ\right], \left[i^l_ȷ, i^u_ȷ\right], \left[ḑ^l_ȷ, ḑ^u_ȷ\right]\right)$ $(ȷ = 1,2,3,\ldots n)$ *be an IVTSFNs. Then interval-valued TSF Dombi ordered WG (IVTSFDOWG) operator is a function* $IVTSFDOWG.Ꝑ^n \to Ꝑ$ *defined as:*

$$IVTSFDOWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, \ldots Ꝑ_n) = \bigoplus_{ȷ=1}^{n} \left(Ꝑ_{\sigma(ȷ)}\right)^{\psi_ȷ} \quad (12)$$

where $(\sigma(1), \sigma(2), \sigma(3), \ldots \sigma(n))$ *are the permutation of* $(ȷ = 1,2,3,\ldots n)$, *for which* $Ꝑ_{\sigma(ȷ-1)} \geq Ꝑ_{\sigma(ȷ)}$ *for all* $ȷ = 1,2,3,\ldots n$.

Theorem 15. *Let* $Ꝑ_ȷ = \left(\left[ṡ^l_ȷ, ṡ^u_ȷ\right], \left[i^l_ȷ, i^u_ȷ\right], \left[ḑ^l_ȷ, ḑ^u_ȷ\right]\right)$ $(ȷ = 1,2,3,\ldots n)$ *be IVTSFNs. Then IVTSFDOWG operator is a function* $IVTSFDOWG.Ꝑ^n \to Ꝑ$ *defined as:*

$$IVTSFDOWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, \ldots Ꝑ_n) = \bigotimes_{j=1}^{n} \left(Ꝑ_{\sigma(j)}\right)^{\psi_j} = \left(\begin{array}{c} \left[\sqrt[ꝗ]{\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{1-ṡ^{lꝗ}_{\sigma(j)}}{ṡ^{lꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}}, \sqrt[ꝗ]{\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{1-ṡ^{uꝗ}_{\sigma(j)}}{ṡ^{uꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}} \right], \\ \left[\sqrt[ꝗ]{1-\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{i^{lꝗ}_{\sigma(j)}}{1-i^{lꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}}, \sqrt[ꝗ]{1-\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{i^{uꝗ}_{\sigma(j)}}{1-i^{uꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}} \right], \\ \left[\sqrt[ꝗ]{1-\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{ḑ^{lꝗ}_{\sigma(j)}}{1-ḑ^{lꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}}, \sqrt[ꝗ]{1-\dfrac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\dfrac{ḑ^{uꝗ}_{\sigma(j)}}{1-ḑ^{uꝗ}_{\sigma(j)}}\right)\right\}^{\frac{1}{ꝗ}}}} \right] \end{array} \right) \quad (13)$$

where $(\sigma(1), \sigma(2), \sigma(3), \ldots \sigma(n))$ *are the permutation of* $(ȷ = 1,2,3,\ldots n)$, *for which* $Ꝑ_{\sigma(ȷ-1)} \geq Ꝑ_{\sigma(ȷ)}$ *for all* $ȷ = (1,2,3,\ldots n)$.

Theorem 16. *(Idempotency Property) If* $Ꝑ_ȷ (ȷ = 1,2,3,\ldots n)$ *are all the same i.e.,* $Ꝑ_ȷ = Ꝑ$ *for all* ȷ. *Then*

$$IVTSFDOWG(Ꝑ_1, Ꝑ_2, \ldots Ꝑ) = Ꝑ$$

Theorem 17. *(Boundedness Property) Let* $Ꝑ_ȷ = \left(\left[ṡ^l_ȷ, ṡ^u_ȷ\right], \left[i^l_ȷ, i^u_ȷ\right], \left[ḑ^l_ȷ, ḑ^u_ȷ\right]\right)$ $(ȷ = 1,2,3,\ldots n)$ *be few IVTSFNs and let* $Ꝑ^- = min Ꝑ_ȷ$ *and* $Ꝑ^+ = max Ꝑ_ȷ$. *Then*

$$Ꝑ^- \leq IVTSFDOWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, \ldots Ꝑ_n) \leq Ꝑ^+.$$

Theorem 18. *(Monotonicity Property) Let* $Ꝑ_ȷ = \left(\left[ṡ^l_ȷ, ṡ^u_ȷ\right], \left[i^l_ȷ, i^u_ȷ\right], \left[ḑ^l_ȷ, ḑ^u_ȷ\right]\right)$ *and* $Ꝑ'_ȷ (ȷ = 1,2,3,\ldots n)$ *be two collections of IVTSFNs,* $Ꝑ_ȷ \leq Ꝑ'_ȷ$ *for all* ȷ. *Then* $IVTSFDOWG(Ꝑ_1, Ꝑ_2, Ꝑ_3, \ldots Ꝑ_n) \leq IVTSFDOWG (Ꝑ'_1, Ꝑ'_2, Ꝑ'_3, \ldots Ꝑ'_n)$.

Theorem 19. *(Commutativity Property) Let* $\mathcal{P}_j$ *and* $\mathcal{P}'_j(j = 1, 2, , \ldots n)$ *be two sets of IVTSFNs, then* $IVTSFDOWG(\mathcal{P}_1, \mathcal{P}_2, \ldots \mathcal{P}_n) = IVTSFDOWG(\mathcal{P}'_1, \mathcal{P}'_2, \ldots \mathcal{P}'_n)$ *where* $\mathcal{P}_j(1, 2, \ldots n)$ *is any permutation of* $\mathcal{P}_j(j = 1, 2, \ldots n)$*.*

To deal with the case when the argument and the ordered position of the information both are significant, we introduce the notion of hybrid geometric DAO as follows.

Definition 12. *An IVTSFDHG operator is a function* $IVTSFDHG : \mathcal{P}^n \to \mathcal{P}$*, defined as:*

$$IVTSFDHG(\mathcal{P}_1, \mathcal{P}_2, \ldots \mathcal{P}_n) = \underset{j=1}{\overset{n}{\otimes}}\left(\dot{\mathcal{P}}_{\sigma(j)}\right)^{\psi_j} = \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{s}^{l\,q}_{\sigma(j)}}{\dot{s}^{l\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{s}^{u\,q}_{\sigma(j)}}{\dot{s}^{u\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{i}^{l\,q}_{\sigma(j)}}{1-\dot{i}^{l\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{i}^{u\,q}_{\sigma(j)}}{1-\dot{i}^{u\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{d}^{l\,q}_{\sigma(j)}}{1-\dot{d}^{l\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{d}^{u\,q}_{\sigma(j)}}{1-\dot{d}^{u\,q}_{\sigma(j)}}\right)\right\}^{\underline{1}}}} \right] \end{array} \right) \quad (14)$$

where $\dot{\mathcal{P}}_{\sigma(j)}$ *is the jth biggest weighted interval valued TSF values* $\dot{\mathcal{P}}_j$ $\left(\dot{\mathcal{P}}_j = n\,\Psi_j \dot{\mathcal{P}}_j, j = 1, 2, \ldots n\right)$*, and* $\Psi_j = (\Psi_1, \Psi_2, \ldots \Psi_n)^T$ *be the weight vector of* $\dot{\mathcal{P}}_j$ *with* $\sum_{j=1}^{n}\Psi_j = 1$*. Here n denotes the balancing coefficient.*

When $\Psi_j = \left(\frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)$, IVTSFDWG and IVTSFDOWG operator become special cases of the IVTSFDHG operator.

6. Special Cases of Interval Valued TSF Dombi Operators

In this section, we use a few conditions on the IVTSFDWA and IVTSFDWG operators to show their generalization over the previously developed DAOs.

Consider the IVTSFDWA operator as follows.

$$IVTSFDWA(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3 \ldots \mathcal{P}_n) = \left(\begin{array}{c} \left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}^{l\,q}_{j}}{1-\dot{s}^{l\,q}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}^{u\,q}_{j}}{1-\dot{s}^{u\,q}_{j}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{i}^{l\,q}_{j}}{\dot{i}^{l\,q}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{i}^{u\,q}_{j}}{\dot{i}^{u\,q}_{j}}\right)\right\}^{\underline{1}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{d}^{l\,q}_{j}}{\dot{d}^{l\,q}_{j}}\right)\right\}^{\underline{1}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\dot{d}^{u\,q}_{j}}{\dot{d}^{u\,q}_{j}}\right)\right\}^{\underline{1}}}} \right] \end{array} \right) \quad (15)$$

1. By placing $\dot{s}^{l}=\dot{s}^{u}=\dot{s}$, $\underset{\cdot}{i}^{l}=\underset{\cdot}{i}^{u}=\underset{\cdot}{i}$ and $\underset{\varsigma}{d}^{l}=\underset{\varsigma}{d}^{u}=\underset{\varsigma}{d}$, the IVTSFDWA operator reduces to T-spherical fuzzy DAOs given as:

$$TSFDWA(\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\ldots\mathcal{P}_n)=\left(\sqrt[\mathfrak{q}]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{\mathfrak{q}}}{1-\dot{s}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{\mathfrak{q}}}{\underset{\cdot}{i}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt[\mathfrak{q}]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{\mathfrak{q}}}{\underset{\varsigma}{d}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}}\right) \tag{16}$$

2. By placing $\mathfrak{q}=2$ reduces the IVTSFDWA operator to interval-valued spherical fuzzy DAOs given as:

$$IVSFDWA(\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\ldots\mathcal{P}_n)=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{l\mathfrak{q}}}{1-\dot{s}_j^{l\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{u\mathfrak{q}}}{1-\dot{s}_j^{u\mathfrak{q}}}\right)\right\}^{\underline{1}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{l\mathfrak{q}}}{\underset{\cdot}{i}_j^{l\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{u\mathfrak{q}}}{\underset{\cdot}{i}_j^{u\mathfrak{q}}}\right)\right\}^{\underline{1}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{l\mathfrak{q}}}{\underset{\varsigma}{d}_j^{l\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{u\mathfrak{q}}}{\underset{\varsigma}{d}_j^{u\mathfrak{q}}}\right)\right\}^{\underline{1}}}}\right]\end{array}\right) \tag{17}$$

3. By placing $\mathfrak{q}=2$, $\dot{s}^{l}=\dot{s}^{u}=\dot{s}$, $\underset{\cdot}{i}^{l}=\underset{\cdot}{i}^{u}=\underset{\cdot}{i}$ and $\underset{\varsigma}{d}^{l}=\underset{\varsigma}{d}^{u}=\underset{\varsigma}{d}$ reduces the IVSFDWA operator to spherical fuzzy DAOs given as:

$$SFDWA(\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\ldots\mathcal{P}_n)=\left(\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{\mathfrak{q}}}{1-\dot{s}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{\mathfrak{q}}}{\underset{\cdot}{i}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{\mathfrak{q}}}{\underset{\varsigma}{d}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}}\right) \tag{18}$$

4. By placing $\mathfrak{q}=1$ reduces the IVTSFDWA operator to interval valued picture fuzzy DAOs given as:

$$IV\mathcal{P}FDWA(\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\ldots\mathcal{P}_n)=\left(\begin{array}{c}\left[1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{l}}{1-\dot{s}_j^{l}}\right)\right\}^{\underline{1}}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{u}}{1-\dot{s}_j^{u}}\right)\right\}^{\underline{1}}}\right],\\ \left[\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{l}}{\underset{\cdot}{i}_j^{l}}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{u}}{\underset{\cdot}{i}_j^{u}}\right)\right\}^{\underline{1}}}\right],\\ \left[\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{l}}{\underset{\varsigma}{d}_j^{l}}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{u}}{\underset{\varsigma}{d}_j^{u}}\right)\right\}^{\underline{1}}}\right]\end{array}\right) \tag{19}$$

5. By placing $\mathfrak{q}=1$, $\dot{s}^{l}=\dot{s}^{u}=\dot{s}$, $\underset{\cdot}{i}^{l}=\underset{\cdot}{i}^{u}=\underset{\cdot}{i}$ and $\underset{\varsigma}{d}^{l}=\underset{\varsigma}{d}^{u}=\underset{\varsigma}{d}$ reduces the IVPFDWA operator to picture fuzzy DAOs given as:

$$PFDWA(\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\ldots\mathcal{P}_n)=\left(1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{s}_j^{\mathfrak{q}}}{1-\dot{s}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\cdot}{i}_j^{\mathfrak{q}}}{\underset{\cdot}{i}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\underset{\varsigma}{d}_j^{\mathfrak{q}}}{\underset{\varsigma}{d}_j^{\mathfrak{q}}}\right)\right\}^{\underline{1}}}\right) \tag{20}$$

6. By neglecting, the AG reduces the IVTSFDWA operator to interval-valued q-rung orthopair fuzzy DAOs given as

$$IVQROFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(\begin{array}{c}\left[\sqrt[\text{ɋ}]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{l\,\text{ɋ}}}{1-\dot{\check{s}}_j^{l\,\text{ɋ}}}\right)\right\}^{\underline{1}}}},\sqrt[\text{ɋ}]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{u\,\text{ɋ}}}{1-\dot{\check{s}}_j^{u\,\text{ɋ}}}\right)\right\}^{\underline{1}}}}\right],\\ \left[\sqrt[\text{ɋ}]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{l\,\text{ɋ}}}{\text{ḑ}_j^{l\,\text{ɋ}}}\right)\right\}^{\underline{1}}}},\sqrt[\text{ɋ}]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{u\,\text{ɋ}}}{\text{ḑ}_j^{u\,\text{ɋ}}}\right)\right\}^{\underline{1}}}}\right]\end{array}\right) \tag{21}$$

7. By placing $\dot{\check{s}}^l=\dot{\check{s}}^u=\dot{\check{s}}$, $\text{ị}^l=\text{ị}^u=\text{ị}=0$ and $\text{ḑ}^l=\text{ḑ}^u=\text{ḑ}$ reduces the IVPFDWA operator to q-rung orthopair fuzzy DAOs given as:

$$QROFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(\sqrt[\text{ɋ}]{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{\text{ɋ}}}{1-\dot{\check{s}}_j^{\text{ɋ}}}\right)\right\}^{\underline{1}}}},\sqrt[\text{ɋ}]{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{\text{ɋ}}}{\text{ḑ}_j^{\text{ɋ}}}\right)\right\}^{\underline{1}}}}\right) \tag{22}$$

8. By placing $q=2$, and neglecting the AG reduces the IVTSFDWA operator to interval-valued Pythagorean fuzzy DAOs given as:

$$IVPyFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{l\,2}}{1-\dot{\check{s}}_j^{l\,2}}\right)\right\}^{\underline{1}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{u\,2}}{1-\dot{\check{s}}_j^{u\,2}}\right)\right\}^{\underline{1}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{l\,2}}{\text{ḑ}_j^{l\,2}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{u\,2}}{\text{ḑ}_j^{u\,2}}\right)\right\}^{\underline{1}}}}\right]\end{array}\right) \tag{23}$$

9. By placing $q=2$, and $\dot{\check{s}}^l=\dot{\check{s}}^u=\dot{\check{s}}$, $\text{ị}^l=\text{ị}^u=\text{ị}=0$ and $\text{ḑ}^l=\text{ḑ}^u=\text{ḑ}$ reduces the IVPFDWA operator to Pythagorean fuzzy DAOs given as:

$$PyFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(\sqrt{1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{2}}{1-\dot{\check{s}}_j^{2}}\right)\right\}^{\underline{1}}}},\sqrt{\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{2}}{\text{ḑ}_j^{2}}\right)\right\}^{\underline{1}}}}\right) \tag{24}$$

10. By placing $q=1$, and neglecting the abstinence degree reduces the IVTSFDWA operator to interval-valued Pythagorean fuzzy DAOs given as:

$$IVIFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(\begin{array}{c}\left[1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{l}}{1-\dot{\check{s}}_j^{l}}\right)\right\}^{\underline{1}}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j^{u}}{1-\dot{\check{s}}_j^{u}}\right)\right\}^{\underline{1}}}\right],\\ \left[\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{l}}{\text{ḑ}_j^{l}}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j^{u}}{\text{ḑ}_j^{u}}\right)\right\}^{\underline{1}}}\right]\end{array}\right) \tag{25}$$

11. By placing $q=1$, and $\dot{\check{s}}^l=\dot{\check{s}}^u=\dot{\check{s}}$, $\text{ị}^l=\text{ị}^u=\text{ị}=0$ and $\text{ḑ}^l=\text{ḑ}^u=\text{ḑ}$ reduces the IVPFDWA operator to intuitionistic fuzzy DAOs given as:

$$IFDWA(\text{Ƥ}_1,\text{Ƥ}_2,\text{Ƥ}_3\ldots\text{Ƥ}_n)=\left(1-\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{\dot{\check{s}}_j}{1-\dot{\check{s}}_j}\right)\right\}^{\underline{1}}},\frac{1}{1+\left\{\sum_{j=1}^{n}\psi_j\left(\frac{1-\text{ḑ}_j}{\text{ḑ}_j}\right)\right\}^{\underline{1}}}\right) \tag{26}$$

7. Applications in Multi-Attribute Decision Making

In this section, we investigate the applications of the DAOs of IVTSFNs in the MADM problem. We also analyzed the influence of the parameters q and on the ranking outputs.

In MADM problems, we aim to choose an optimum alternative among a finite set of alternatives, keeping in view some attributes based on the opinion of the decision-makers. Let us suppose that the finite set of alternatives be denoted by A_k (k is finite) and that of attributes be denoted by G_{j}(j is finite). The information about the shortlisted alternatives based on the attributes is given in the form of closed subintervals of $[0, 1]$ in the frame of IVTSFSs. Let the decision matrix containing the information provided by the decision-maker be denoted by $D_{k\times j} = (T)_{k\times j}$ whose elements are in the form of triplets having the MGs, AGs, and NMGs in the form of closed subintervals of the $[0, 1]$. The weight vector of the attributes in our algorithm is denoted by $\psi = (\psi_1, \psi_2, \psi_3 \ldots \psi_n)^T$ with conditions stated above in Section 4. For better understanding, briefly described steps of the MADM algorithm are given as follows.

7.1. MADM Algorithm Based on IVTSF Dombi Operators

This subsection is about the steps of the MADM algorithm in the environment of IVTSFSs. These steps are given as follows.

Step 1 is about taking information from decision makers about the finite set of alternatives observing the attributes. The information provided by the decision-maker is having the form of IVTSFNs.

Step 2 is about to investigate the decision matrix obtained in Step 1 for the possible value of q for which every triplet lies in the frame of IVTSFSs. The least value of q is adapted, which will be used in the aggregation process in Step 3.

Step 3 is about examining the attribute and checking if there is any cost-type attribute. If yes, we normalize the decision matrix by using the definition of the complement of IVTSFN [35]. After all the attributes become of a benefit type, we start the process of aggregation by using IVTSFDWA and IVTSFDWG operators given as follows.

$$IVTSFDWA(\text{P}_1, \text{P}_2, \text{P}_3 \ldots \text{P}_n) = \left(\begin{array}{c} \left[\sqrt[q]{1 - \frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{\dot{\check{s}}^{l\,q}_{j}}{1-\dot{\check{s}}^{l\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{\dot{\check{s}}^{u\,q}_{j}}{1-\dot{\check{s}}^{u\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}} \right], \\ \left[\sqrt[q]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-i^{l\,q}_{j}}{i^{l\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}}, \sqrt[q]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-i^{u\,q}_{j}}{i^{u\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}} \right] \\ \left[\sqrt[q]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-d^{l\,q}_{j}}{d^{l\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}}, \sqrt[q]{\frac{1}{1+\left\{ \sum_{j=1}^{n} \psi_j \left(\frac{1-d^{u\,j}_{j}}{d^{u\,q}_{j}} \right) \right\}^{\frac{1}{\,}}}} \right] \end{array} \right)$$

$$IVTSFDWG(\text{ᵽ}_1, \text{ᵽ}_2, \text{ᵽ}_3 \ldots \text{ᵽ}_n) = \begin{pmatrix} \left[\sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-\dot{s}_j^{l\,q}}{\dot{s}_j^{l\,q}}\right)\right\}^{\frac{1}{q}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{1-\dot{s}_j^{u\,q}}{\dot{s}_j^{u\,q}}\right)\right\}^{\frac{1}{q}}}} \right], \\ \left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\text{ị}_j^{l\,q}}{1-\text{ị}_j^{l\,q}}\right)\right\}^{\frac{1}{q}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\text{ị}_j^{u\,q}}{1-\text{ị}_j^{u\,q}}\right)\right\}^{\frac{1}{q}}}} \right], \\ \left[\sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\text{ḑ}_j^{l\,q}}{1-\text{ḑ}_j^{l\,q}}\right)\right\}^{\frac{1}{q}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{j=1}^{n} \psi_j \left(\frac{\text{ḑ}_j^{u\,q}}{1-\text{ḑ}_j^{u\,q}}\right)\right\}^{\frac{1}{q}}}} \right] \end{pmatrix}$$

Step 4 is about examining the aggregated information based on score function of IVTSFNs proposed in Definition 3. We compute the score of all aggregated information for the ranking of alternatives.
Step 5 is about the ranking of alternatives for the best outcome.

To demonstrate the MADM described above, we give an example as follows.

Example 7. *Technology commercialization is one of the problems where MADM approaches become essential to get some useful results with more accuracy. In [24], authors studied an example based on technology commercialization in which the selection of various software packages is examined based on an aggregation of the information under uncertainty. We follow the same problem but with interval-valued TSF information to get optimum results. Suppose four software packages need to assess under four attributes using some interval-valued TSF information. The attributes based on which the four software packages denoted by* $A_{1\leq k\leq 4}$ *shall be examined include* G_1*. Innovation of the technology,* G_2*. Market Potential,* G_3*. Human resources and financial development,* G_4*. Future perspectives of the packages.* $\psi_j = (0.15,\ 0.25,\ 0.41,\ 0.19)^T$ *is adapted is a weight vector of the attributes under observation. After initial screening, information about the packages from the decision-makers is gathered in the decision matrix given in Step 1 below.*

Step 1. Gathering of information from the decision-makers in the form of IVTSFNs about the software packages.

Step 2. Upon investigation, it is noted that all the information given in Table 1 represents IVTSFNs for $q = 4$, i.e., for every $\left(\left[\dot{s}^l, \dot{s}^u\right], \left[\text{ị}^l, \text{ị}^u\right], \left[\text{ḑ}^l, \text{ḑ}^u\right]\right)$ the $0 \leq \left[\dot{s}^{q\,l}, \dot{s}^{q\,u}\right] + \left[\text{ị}^{q\,l}, \text{ị}^{q\,u}\right] + \left[\text{ḑ}^{q\,l}, \text{ḑ}^{q\,u}\right] \leq 1$ as given in Table 2.

Table 1. Decision Matrix based on IVTSF information.

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{A}_1$	([0.65, 0.83], [0.4, 0.54], [0.26, 0.41])	([0.43, 0.53], [0.29, 0.43], [0.39, 0.47])	([0.77, 0.89], [0.34, 0.49], [0.59, 0.68])	([0.71, 0.81], [0.56, 0.65], [0.64, 0.65])
$\mathcal{A}_2$	([0.59, 0.71], [0.58, 0.65], [0.53, 0.59])	([0.29, 0.39], [0.68, 0.77], [0.79, 0.84])	([0.37, 0.48], [0.72, 0.77], [0.63, 0.74])	([0.11, 0.16], [0.29, 0.39], [0.69, 0.77])
$\mathcal{A}_3$	([0.45, 0.58], [0.55, 0.69], [0.31, 0.45])	([0.59, 0.79], [0.55, 0.56], [0.21, 0.34])	([0.31, 0.39], [0.11, 0.14], [0.55, 0.62])	([0.69, 0.85], [0.41, 0.49], [0.59, 0.73])
$\mathcal{A}_4$	([0.85, 0.94], [0.43, 0.54], [0.29, 0.35])	([0.39, 0.51], [0.29, 0.43], [0.61, 0.69])	([0.11, 0.25], [0.55, 0.72], [0.69, 0.83])	([0.79, 0.85], [0.55, 0.7], [0.41, 0.54])

Table 2. $\left(0 \leq \left[\dot{s}^{q_l}, \dot{s}^{q_u}\right] + \left[i^{q_l}, i^{j_u}\right] + \left[d^{q_l}, d^{q_u}\right] \leq 1,\ q = 4\right)$.

0.588	0.1619	0.8989	0.7875
0.554	0.8725	0.7045	0.3753
0.381	0.5012	0.1713	0.8636
0.881	0.3285	0.7472	0.8471

Step 3. This step involves the aggregation of the information offered by decision-makers in Table 1. Both IVTSFDWA and IVTSFDWG operators are utilized to aggregate the information and the findings are given in Table 3.

Table 3. Aggregated data using IVTSFDWA and IVTSFDWG operators.

	IVTSFDWA Operator	*IVTSFDWG Operator*
A_1	$([0.705, 0.85], [0.3412, 0.4907], [0.38, 0.523])$	$([0.561, 0.684], [0.4196, 0.5349], [0.551, 0.617])$
A_2	$([0.141, 0.522], [0.4242, 0.55], [0.639, 0.724])$	$([0.137, 0.239], [0.6628, 0.7294], [0.695, 0.771])$
A_3	$([0.55, 0.73], [0.1373, 0.1747], [0.285, 0.444])$	$([0.426, 0.473], [0.4576, 0.5262], [0.505, 0.606])$
A_4	$([0.689, 0.811], [0.3834, 0.5443], [0.422, 0.515])$	$([0.155, 0.309], [0.5015, 0.6596], [0.615, 0.75])$

Step 4. The aggregation information obtained in Step 3 is subjected to the score function defined in Equation (3) for ranking purposes.

Step 5. Based on the scores obtained in Table 4, the alternatives are ranked, and the ranking results are given in Table 5 as follows.

Table 4. Scores of aggregated information.

Scores	*IVTSFDWA Operator*	*IVTSFDWG Operator*
A_1	0.224	0.085
A_2	0.024	0.0005
A_3	0.121	0.023
A_4	0.192	0.002

Table 5. Ranking results.

	Ranking Analysis
IVTSFPWA operator	$A_4 > A_1 > A_3 > A_2$
IVTSFPWG operator	$A_4 > A_1 > A_3 > A_2$

The results portrayed in Table 5 are further described geometrically in the following Figure 1.

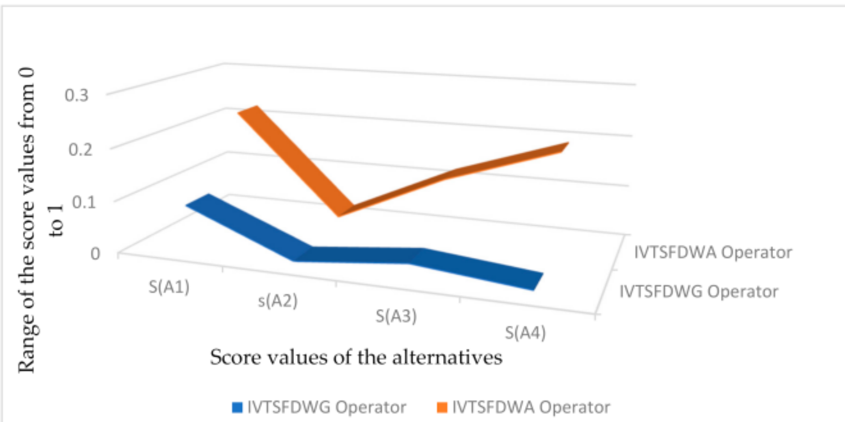

Figure 1. Graphical representation of the score values of alternatives given in Table 5.

The ranking results portrayed in Table 5 and Figure 1, clearly indicate that A_4 i.e., *Creating of Employment and development of technology* is the best technology enterprise according to the results obtained using IVTSFDWA operators while A_2 i.e., the *Market potential* is declared as optimal technology enterprise using IVTSFDWG operators. The ranking results here are not the same in the case of IVTSFDWA and IVTSFDWG operators and the selection operator is up to the decision-maker. Given Ullah et al. [17], in our next study, we aim to investigate the ranking results for various values of q and .

7.2. Impact of q and $\Re$ on Ranking Results

In interval-valued TSF information, usually, the least value q is taken in the aggregation process. However, it has been observed that for larger q the ranking results may vary. Similarly, the condition is also relaxed and variation may result in the variation in ranking results.

First, we observe the variation in q for various values of q. Upon varying the value of q, it is observed that the ranking remains the same and unlike Ullah et al. [17] no significant changes occur in the ranking pattern.

Now we observe the effect on the ranking results. Upon varying the value , it is observed that there is no significant change occur in the case of IVTSFDWA operators. However, for $= 2$ and higher values, the ranking results of IVTSFDWG operators become changed and a new ranking pattern is portrayed in Table 6 below.

Table 6. Effect of on ranking results.

	Ranking Analysis	**Value of $\Re$**
IVTSFDWA operator	$A_4 > A_1 > A_3 > A_2$	For all
IVTSFDWG operator	$A_4 > A_1 > A_3 > A_2$	$= 1$
IVTSFDWG operator	$A_3 > A_1 > A_4 > A_2$	$= 2$ and above

This analysis shows that decision results may alter with varying values of the parameters. However, stability is what we needed in results. As suggested by Ullah et al. [17], the value of the variable parameters should be adopted after there change in the results no longer occurs. Overall, we have three possible cases in the selection of these variable parameters based on the choice of the decision-makers. First, the least possible values should have been opted for these parameters. Second, the stable values of the variable parameters should have opted where the stability is a stage, after which changes in the decision results do not occur. Last, a decision-maker could choose any value as long as their conditions are satisfied. However, for best outcomes, the second one is the best approach as suggested by Ullah et al. [17].

8. Comparative Study

In this section, we aim to compare the aggregated results obtained using IVTSF DAOs with the aggregated results obtained using averaging and geometric aggregation operators of IVTSFSs proposed by Ullah et al. [35] We also show that the previously existing DAOs developed in other fuzzy frameworks cannot be applied to the information discussed in the IVTSF environment. For this purpose, first, we applied the averaging and geometric aggregation operators of IVTSFSs [35] on the data provided in Table 1, which shows the compatibility of the IVTSF framework. Secondly, we listed the DAOs that are developed so far and concluded that their layouts are limited and cannot handle the presented data. We observed that DAOs proposed by Seikh and Mandal and Jana et al. [21,22] deal with two types of human opinion and provide very little independence and hence were unable to handle the given data. We also observed that the DAOs developed by Jana et al. [23] provide independence in the selection of MG, NMG, etc, but can only deal with two types of human opinion and hence are unable to deal with the presented data of Table 1. We also analyzed that the recently developed Hamacher aggregation operators of TSFSs [17] and DAOs of PFSs [24] are also unable to deal with the presented data due to their limited nature. The limited nature of these discussed aggregation operators is already shown in Section 6 comprehensively. A brief survey of the aggregated results of this paper with other papers is given in Table 7 below.

Table 7. Comparative Study.

Operator	Environment	Results
IVTSFDWA operator (Current Work)	IVTSFSs	$A_4 > A_1 > A_3 > A_2$
IVTSFDWG operator (Current Work)	IVTSFSs	$A_4 > A_1 > A_3 > A_2$
IVTSFWA operator Ullah et al. [35]	IVTSFSs	$A_1 > A_3 > A_4 > A_2$
IVTSFWG operator Ullah et al. [35]	IVTSFSs	$A_1 > A_3 > A_4 > A_2$
TSFHWA operator Ullah et al. [17]	TSFSs	Failed
TSFHWG operator Ullah et al. [17]	TSFSs	Failed
PFDWA operator Jana et al. [24]	PFSs	Failed
PFDWG operator Jana et al. [24]	PFSs	Failed
QROFDWA operator Jana et al. [23]	QROFSs	Failed
QROFDWG operator Jana et al. [23]	QROFSs	Failed
PyFDWA operator Jana et al. [22]	PyFSs	Failed
PyFDWG operator Jana et al. [22]	PyFSs	Failed
IFDWA operator Seikh and Mandal [21]	IFSs	Failed
IFDWG operator Seikh and Mandal [21]	IFSs	Failed

From Table 7, it is evident that the DAOs of IFSs, PyFSs, QROFSs, and PFSs failed to deal with IVTSF information where MG, NMG, and AG are defined in terms of closed subintervals instead of crisp numbers. The DAOs discussed in the [21–24] can only be applicable when we have uncertain information in the form of crisp numbers from $[0, 1]$ instead of closed subintervals of $[0, 1]$. Furthermore, another reason for the failure of the work discussed in [21–23] is that such frameworks can apply to the information where only two aspects of the human opinion are considered and hence such DAOs cannot be applied to aggregate the given information due to their limited framework. On the opposite side, the several remarks presented in Section 6 shows that the proposed IVTSF DAOs can be applied to the problems discussed in [21–24]. Even the aggregation operators of TSFSs [17] are unable to handle the IVTSF information as shown in Table 7 where the Hamacher aggregation operators of TSFSs fail to deal with given information. Furthermore, if we observe the results obtained using the averaging and geometric aggregation operators of IVTSFSs [35] with the results of this paper, then it becomes clear that both types of operators are applicable to handle this type of data. However, the choice of selection of the type of aggregation operator is up to the decision-makers. All this leads us to the following points as advantages of the proposed work.

1. IVTSFSDWA and IVTSFDWG operator generalizes the Dombi AOs of IFSs, PyFSs, QROFSs, and PFSs.
2. The IVTSFDWA and IVTSFDWG operators can be applied to the problems where information is provided in terms of IFNs, PyFNs, QROFNs, and PFSs; however, the converse is not true.

9. Conclusions

In this manuscript, the idea of DAOs is discussed in the IVTSF environment by keeping the flexible nature of IVTSFS in mind. An IVTSFS can portray uncertain information using an MG, an NMG, an AG, and an RG using a closed subinterval of $[0, 1]$. The main contribution and key results of the paper are discussed as follows.

1. We proposed Dombi operations based on DTN and DTCN.
2. We investigated the basic features of the Dombi operations.

3. We proposed DAOs including IVTSFDWA and IVTSFDWG operators followed by their basic properties and examples. The fitness of these aggregation operators is also checked using the induction method.
4. An analysis of the consequences of the proposed work is explored where the flexible nature of the DAOs of IVTSFSs is discussed.
5. A MADM algorithm based on the DAOs of IVTSFSs is proposed and is illustrated by a numerical example to see the applicability of the proposed work.
6. A comparative analysis of the currently proposed and previously developed Dombi aggregation operators is also discussed. From the comparative study, it is evident that the given DAOs in the frame of IVTSFSs gave results with no information loss compared to previously defined DAOs.

In the near future, we aim to extend our work to the environment of complex SFSs and TSFSs [36,37], and interval-valued complex TSFS. The work can be extended to complex spherical fuzzy soft sets [38] to investigate the applications. The applications of the current study can also be analyzed in performance measurement systems [39], secure multi-server authentication [40], wireless sensor networks [41], evaluation processes [42], and neural networks [43]. We also aim to utilize the DAOs of IVTSFSs in TOPSIS and other MADM methods.

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