Intracavity electromagnetically induced transparency

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The effect of intracavity electromagnetically induced transparency (EIT) on the properties of optical resonators and active laser devices is discussed theoretically. Pronounced frequency pulling and cavity-linewidth narrowing are predicted. The EIT effect can be used to reduce classical and quantum-phase noise of the beat note of an optical oscillator substantially. Fundamental limits of this stabilization mechanism as well as its potential application to high-resolution spectroscopy are discussed. © 1998 Optical Society of America

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Electromagnetically induced transparency[1] (EIT) is a dense-media analog of dark resonances,[2] which occur in three-level Λ systems driven by coherent optical fields. In recent years a number of potential applications of EIT have been described. These include, in particular, enhancement of nonlinear optical processes,[3] high-resolution spectroscopy, and optical magnetometry.[4,5]

In this Letter we describe theoretically the effect of an intracavity-induced transparency. When a dense ensemble of coherently prepared Λ atoms is placed inside an optical resonator, the resonator response is drastically modified, resulting in frequency pulling[6] and a substantial narrowing of spectral features. This effect can be used for frequency-difference stabilization of lasers[7] or other two-mode light sources such as broadband parametric oscillators. Here intracavity EIT results in locking of the beat note to the resonance frequency of a two-photon transition between metastable atomic levels and causes a substantial reduction of quantum and classical noise in the beat signal. Possible applications of intracavity EIT include sensitivity intracavity spectroscopy, novel frequency standards, and optical magnetometry.

The profound effects of intracavity EIT are due to large dispersion close to the point of almost-vanishing absorption,[8] which can easily exceed the empty-cavity dispersion in the case of an optically thick Λ medium. To illustrate the locking and narrowing mechanism let us consider a ring cavity containing a cell of length l with a linear dispersive medium. The medium response is characterized by the real (χ') and the imaginary (χ'') parts of the susceptibility, for which we assume $\chi' = \beta(\nu - \nu_0)$ and constant χ'' for frequencies ν that are sufficiently close to some resonance frequency ν_0. β and χ'' are proportional to the atomic density N/V. The cavity-response function, i.e., the ratio of circulating to input intensity, is given by[9]

$$S(\nu) = \frac{I_{\mathrm{circ}}}{I_{\mathrm{in}}} = \frac{t^2}{1 + r^2\kappa^2 - 2r\kappa \cos[\Phi(\nu)]}, \tag{1}$$

where t and r are the transmissivity and the reflectivity of the input coupler ($t^2 + r^2 = 1$), $\Phi(\nu) = \nu L/c + \kappa l \chi'/2 \approx \nu L/c + kl\beta(\nu - \nu_0)/2$ is the total phase shift, and $\kappa = \exp(-kl\chi'')$ describes the medium absorption per round trip, L. On inspection of the round-trip phase shift one finds that the resonance frequency of the combined cavity + medium system $[\Phi(\nu_r) = 2m\pi]$ is governed by a pulling equation:

$$\nu_r = \frac{1}{1+\eta}\,\nu_c + \frac{\eta}{1+\eta}\,\nu_0 . \tag{2}$$

Here $\eta = (ck/2)(l/L)\beta$ defines a frequency-locking or -stabilization coefficient and ν_c is the resonance frequency of the empty cavity. Similarly, by expanding the cosine in Eq. (1) around ν_r, one also finds that the width of cavity resonances $\Delta\nu$ is changed by the intracavity medium:

$$\frac{\Delta\nu}{C} = \frac{1 - r\kappa}{\sqrt{\kappa}\,(1-r)}\,\frac{1}{1+\eta}, \tag{3}$$

where C is the empty-cavity linewidth. The first factor describes an enhancement of the effective cavity + medium width owing to additional losses, and the second one describes the reduction owing to the linear dispersion. When EIT is established in an intracavity medium, the absorption can be negligible ($\chi'' \to 0$), whereas the dispersion is large, resulting in substantial line narrowing.

To quantify this conclusion we consider the response of the typical Λ system [Fig. 1(a)] driven by a strong laser field of Rabi frequency Ω_2 to the weak test field Ω_1. The corresponding linear susceptibility near the two-photon resonance[10] is

$$\chi' = \xi\,\frac{\gamma_1(\nu - \nu_0)}{{\Omega_2}^2}, \qquad \chi'' = \xi\,\frac{\gamma_1\gamma_0}{{\Omega_2}^2}. \tag{4}$$

Here $\xi = (3/4\pi^2)(N\lambda^3/V)$; $\nu_0 = \nu_2 - \omega_{b_1b_2}$, where ν_2 is the drive frequency; and $\omega_{b_1b_2}$ is the frequency of the $b_1 \to b_2$ transition. In a situation typical for EIT, i.e., when the lower levels are metastable, γ_0 can be very small compared with γ_1, and thus the absorption can be made small ($\kappa \approx 1$) even for a large density–length product in the atomic-vapor cell. Under these conditions, the phase shifts are large even for a γ_1, small

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detuning, resulting in a large stabilization coefficient. The ultimate limit of stabilization can be obtained by imposition of the condition that the residual absorption losses in the cell should not exceed the empty-cavity losses. One finds that for the maximum stabilization coefficient

$$\eta \le C/2\gamma_0 . \tag{5}$$

We note that for a long-lived ground-state coherence the ratio C/γ_0 can become very large. In this case the effective resonance frequency of the cavity coincides with $\nu_2 - \omega_{b_1 b_2}$ and the cavity width can be reduced by several orders of magnitude, whereas the photon losses are practically unaffected. The above conclusion is illustrated in Fig. 1(b), in which the cavity transmission function is shown for different atomic densities.

It is instructive to estimate the lower limit to the cavity linewidth. For a good cavity and the maximum stabilization coefficient, as in inequality (5), we find that $\Delta\nu \to 4\gamma_0$, i.e., a linewidth that can be orders of magnitude smaller than both the empty-cavity linewidth and the single-atom transparency window (width of Λ resonance). In the strong-field limit the latter is power broadened and scales as Ω_2. Hence the effect of power broadening on the combined cavity + atom system is completely eliminated here in the high-density regime.

Let us now discuss the effect of the Λ medium on the phase-difference noise of two optical modes that are independently oscillating inside the cavity. A three-level intracavity medium displaying EIT can be used to lock the beat note to the resonance frequency of the two-photon transition $\omega_{b_1 b_2}$. In particular, we focus here on the spectral properties of a two-mode laser. We emphasize, however, that the two-mode laser serves only as a generic example. Alternatively, one can consider locking the beat note of two independent single-mode lasers or of a broadband nondegenerate parametric oscillator. The two-mode lasers considered here are especially convenient when frequency differences are to be measured, since the beat note of the two modes can be intrinsically narrow, provided that the optical paths are similar. The evolution of the coherent amplitudes a_1 and a_2 of the oscillating (laser) fields can be described by stochastic c-number equations ($n = 1, 2$):

$$\dot{a}_n = -\left(\frac{C}{2} + i\Delta_n^c\right)a_n + \frac{A_n}{2}\, a_n + i g_n N \sigma_n + F_n(t) . \tag{6}$$

Here $A_{1,2}$ are the effective gain coefficients for the two modes, which have the generic structure $A_{1,2} = \alpha_{1,2} - \beta_{1,2}|a_{1,2}|^2 - \tilde{\beta}_{1,2}|a_{2,1}|^2$. The linear gain coefficients α_n as well as the self-saturation and cross-saturation coefficients, β_n and $\tilde{\beta}_n$, respectively, depend on the specific laser model.[9] The exact form of the saturation coefficients is unimportant for the present discussion as long as $\tilde{\beta}_1 \tilde{\beta}_2 < \beta_1 \beta_2$. F_n are noise operators associated with the gain processes. The correlation function of the operators is given by[9] $\langle F_n(t)^* F_n(t')\rangle = C\,\delta(t - t')$. $g_{1,2}$ describe the coupling to the Λ medium. $\Delta_{1,2}^c = \nu_{1,2}^c - \nu_{1,2}$, where $\nu_{1,2}$ are the actual lasing frequencies of the two modes and $\nu_{1,2}^c$ are the corresponding eigenfrequencies of the empty cavity. Absorption, dispersion, and noise properties of the Λ atomic system are also described by a set of c-number Langevin equations for the polarizations σ_i.[10] Below we restrict ourselves to a symmetric configuration and assume equal gain, cavity losses, coupling constants, etc.

One can study the semiclassical behavior of the laser modes by disregarding all noise contributions and eliminating the atomic variables. Laser equations (6) have a solution with equal amplitudes of the modes $a_1 = a_2$. In this case of equal strength of both fields we have, close to the resonance, $\chi_{1,2}' = \xi_\gamma(\Delta_{2,1} - \Delta_{1,2})/(2\Omega^2)$ and $\chi_{1,2}'' = \xi\gamma\gamma_0/(2\Omega^2)$ with $\gamma = \gamma_1 = \gamma_2$, $\Omega = g a_1 = g a_2$, and $\Delta_{1,2} = \omega_{ab_{1,2}} - \nu_{1,2}$.

It is convenient to write the field equations in terms of square amplitudes (photon numbers) $n_{1,2}$ and phases $\phi_{1,2}$. The steady-state solution of the laser phase equations immediately leads to the frequency-pulling equation

$$\nu_1 - \nu_2 = \frac{1}{1+\eta}(\nu_1^c - \nu_2^c) + \frac{\bar{\eta}}{1+\eta}\,\omega_{b_2 b_1} , \tag{7}$$

with $\bar{\eta} = \eta/2$. For $\eta \gg 1$ the beat-note frequency is locked to $\omega_{b_1 b_2}$, i.e., to the Λ-resonance frequency. The additional absorption in the cavity increases the effective decay rate for the laser modes by $\bar{\eta}\gamma_0$. This absorption is, however, unimportant, provided that inequality (5) is fulfilled.

Let us now turn to the phase-noise properties of the two-mode laser. The noise contribution owing to the interaction with the Λ medium is negligible.[10] We model the effect of technical noise by a fluctuation of the spacing between the cavity-resonance frequencies $\delta\omega_c = \omega_1^c - \omega_2^c$. $\delta\omega_c$ is assumed to obey a linear stochastic equation with a Markovian noise force and a phenomenological damping rate γ_c, $\delta\dot{\omega}_c = -\gamma_c \delta\omega_c + F_c$, with $\langle F_c(t) F_c(t')\rangle = \gamma_c \langle \delta\omega_c^2\rangle \delta(t - t')$, where $\langle \delta\omega_c^2\rangle$ characterizes the strength of technical fluctuations. The stochastic equations can be solved by linearization, and the beat-note phase-noise spectrum can be calculated. In the low- (fluctuation-) frequency regime ($\omega \ll \Omega$) we find that

$$S(\omega) = \frac{1}{(1+\bar{\eta})^2}\,\frac{2\gamma_c}{\omega^2 + \gamma_c^2}\,\langle \delta\omega_c^2\rangle + \frac{2C^2}{(1+\bar{\eta})^2}\,\frac{\hbar\nu}{P_{\text{out}}} \cdot \tag{8}$$

The two terms in Eq. (8) represent the influence of technical fluctuations and the beat-note phase diffusion of the laser (Shawlow–Townes) linewidth.[9] Thus,

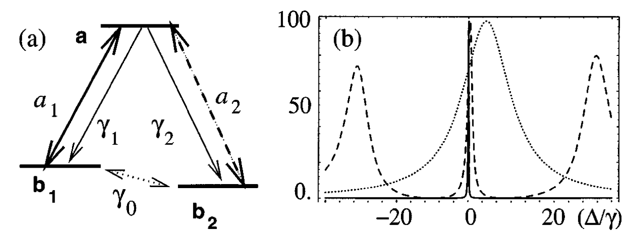

Fig. 1. Generic Λ system for EIT. The frequencies of two fields are close to the resonant frequencies of transitions $a \to b_1$ and $a \to b_2$. $b_{1,2}$ are metastable states. (b) Cavity response as a function of test-field frequency for different values of atomic density. The dotted, dashed, and solid curves correspond to $\eta = 0$, $\eta = 10$, and $\eta = 100$, respectively. The parameters are $\Omega_2 = 10\gamma$, $r = 0.98$, and $\nu_c - \nu_0 = 5\gamma$.

owing to the intracavity medium the (Markovian) technical as well as the quantum fluctuations are reduced by a factor of $1/(1 + \bar{\eta})^2$. Slow technical fluctuations, such as temperature drifts of the cavity resonances, are reduced by only $1/(1 + \bar{\eta})$ [see Eq. (7)]. Note that for strong stabilization ($\bar{\eta} = C/\gamma_0 \ll 1$) the phase diffusion is proportional to γ_0^2 instead of C^2. Such a suppression of quantum-phase noise in the laser beat note[11] is a consequence of the cavity line-narrowing effect described above.

We note here that in addition to the increased intracavity losses there exists another important limitation on the maximal η value that is due to the dynamic instabilities that often arise in different stabilization schemes.[12,13] For the present system the frequency-pulling regime described above is stable as long as $\bar{\eta} < 2\gamma/\gamma_0$ for a homogeneously broadened system and $\bar{\eta} < 2\Delta_D/\gamma_0$ for a medium that is Doppler broadened (one photon Doppler width Δ_D).

It is interesting to consider a particular example of the beat-note laser stabilization. A variety of gas and dye lasers as well as certain types of extended-cavity diode laser can operate on two modes (possibly of orthogonal polarizations) with frequency separation of the order of a few gigahertz. In this case the frequency difference of the modes can be locked to the transition between hyperfine components of alkali atomic vapors. The natural linewidth of such two-photon transitions can be made as low as 10–100 Hz by use of buffer-gas or wall-coating techniques. Taking the empty-cavity width of $\sim 10^7$ Hz, we find that atomic densities that correspond to the stabilization factor $\eta > 10^5$ can be used without affecting the output power of the laser. The frequency locking of can be achieved in an alkali-vapor cell by use of transitions of the D absorption lines at moderate atomic densities of $\sim 10^{12}$ cm^{-3} and laser intensities above optical saturation. Depending on the initial degree of technical-noise correlation, the resulting beat-note linewidths can be in or below the millihertz region.

The potentially interesting feature of the present approach is that it allows one to combine strong locking of two laser modes and narrow linewidths with intense laser fields. It was already demonstrated[5] that dispersive effects in a dense coherent medium can be used to reduce power broadening of two-photon resonances significantly and thus can lead to a potentially attractive regime of laser spectroscopy in which narrow resonances coexist with strong fields. An interferometric measurement in a dispersive medium typically leads to several narrow interferometric fringes. In practice it is therefore often difficult to distinguish and determine the position of the central fringe. This determination is no longer a problem if intracavity EIT is used, since in the regime of strong frequency pulling the beat note automatically locks to the two-photon resonance, whereas the effective width is equivalent to the width of the interferometric fringes. These features make various applications of the technique proposed above for improvement of atomic-frequency standards and optical magnetometers feasible.

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