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Intracavity single resonance optical parametric oscillator (I.S.R.O.)

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Résumé. — La théorie de l'oscillateur intracavité des ondes optiques paramétriques ayant une seule onde résonnante est étudiée en supposant que l'onde pompée et l'onde paramétrique résonnante sont des modes gaussiens monomodes des résonateurs du laser et de l'oscillateur. L'onde paramétrique non résonnante est trouvée par résolution de l'équation parabolique. Les équations de mouvement obtenues sont illustrées par des nouveaux termes intégrés que nous avons mis en évidence. Nous avons montré que pour un laser et un cristal donnés deux valeurs optimales du paramètre de focalisation existent : la première détermine le seuil de puissance du laser pompé pour avoir la génération paramétrique, et la deuxième détermine la puissance de sortie maximale de l'onde paramétrique non résonnante. Nous avons montré qu'en régime dégénéré cette puissance maximale était égale à celle du laser de pompe, dans le cas où ce laser fonctionne tout seul (en l'absence de l'oscillateur paramétrique) avec des miroirs ayant un coefficient de transmission optimale, et que pour avoir le rendement maximal ($\eta = 100\,\%$), la puissance du laser de pompe doit être deux fois plus grande que celle du seuil. Les résultats numériques sont calculés pour un oscillateur optique paramétrique comprenant un cristal non linéaire de $Ba_2NaNb_5O_{15}$ et de longueur $l = 4$ mm, placé dans la cavité d'un laser à YAG continu. En régime dégénéré, la puissance de l'onde non résonnante est de 0,35 W si la puissance totale dans la cavité est de 35 W en l'absence du cristal paramétrique.

Abstract. — The theory of the intracavity single resonance optical parametric oscillator has been studied assuming that the pump- and resonant parametric waves were Gaussian eigenmodes of laser and parametric oscillator resonators. The non-resonant parametric wave has been found by solving a parabolic equation. The resulting equations of motion are parametrized in new terms of integrals that we have discovered. With a given laser and a non-linear crystal, two optimal values for the focusing parameter can be found : the first defines the threshold operation regime of the intracavity single resonance optical parametric oscillator, the other defines the maximum output power of the non-resonant parametric wave. In the latter case, we have demonstrated that in the degeneracy operation regime, this maximum output power of non-resonant wave is equal to the maximum pump laser power when the pump laser works alone (that is without the parametric oscillator) with mirrors having optimal transmission coefficients and when the laser power is twice the threshold value of the pump power. Numerical results have been obtained for an intracavity single resonance oscillator based on a c.w.-YAG-Nd^{3+}-laser and a non-linear $Ba_2NaNb_5O_{15}$ crystal of length 4 mm. In the degeneracy operation regime, the power of the non-resonant wave is 0.35 W when the total laser power in the laser cavity is 35 W, in the absence of the optical parametric oscillator.

1. Introduction.

Optical parametric oscillators (O.P.O.) [1, 2] are coherent light sources, operating in the near to middle I.R. region. Having high efficiency, high output power and yielding high quality spectra, O.P.O. are applied in various fields of research, for example, in the study of the structure of atoms and molecules, biochemistry, isotope separation, etc.

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In the Double Resonance optical parametric Oscillator (D.R.O.) [1] the two parametric waves are resonant in the same resonator. For this process, the required pump power is relatively low, but the quality of the spectra is poor as a result of « cluster » effects and mode competition [3]. Furthermore as a consequence of the back coupling process [4] the efficiency cannot be more than 50 %.

To overcome these drawbacks, Single-Resonance optical parametric Oscillators (S.R.O.) [2] have been developed. In these devices only one of two parametric waves is at resonance, and the other propagates freely through the crystal, so that the wavelength can be tuned continuously and the converting efficiency can be 100 % [5]. However a high threshold is necessary.

In a previous paper [6] we have shown that the threshold power of S.R.O. can be reduced by the optimal focusing method, so that there is a possibility of operating extracavity c.w.-S.R.O.

To achieve a pump power many times that of the threshold power needed to reach the stable operation regime, Intracavity Double Resonance optical parametric Oscillator (I.D.R.O.) have been developed [7, 8, 9]. Oshman and Harris [7] studied the operation regime of I.D.R.O. and assumed that the laser medium has a homogeneous gain transition and the interacting waves are plane uniform waves. Experiments on I.D.R.O. have been done by Amann and Yarborough [8] and recently by Volosov [9].

Taking into account the advantages of both mentioned methods (S.R.O., I.D.R.O.), we propose, in this paper, to study the Intracavity Single Resonance optical parametric Oscillator (I.S.R.O.) pumped by a Gaussian laser beam and to treat the focusing condition for which the output power of Non-Resonant Parametric Wave (N.R.P.W.) is maximized.

Figure 1 shows the structure of the I.S.R.O. in the case of three waves mixing collinearly. The pump laser resonator M_1 M_2 is of length L_p, the O.P.O.'s crystal length is equal to the length of O.P.O.'s resonator. The centre of minimum cross-section of the laser beam is made to coincide with the centre of crystal. The two mirrors M_3, M_4 are transparent for the non-resonant wave (idler) and they reflect the resonant parametric wave (signal) 100 %.

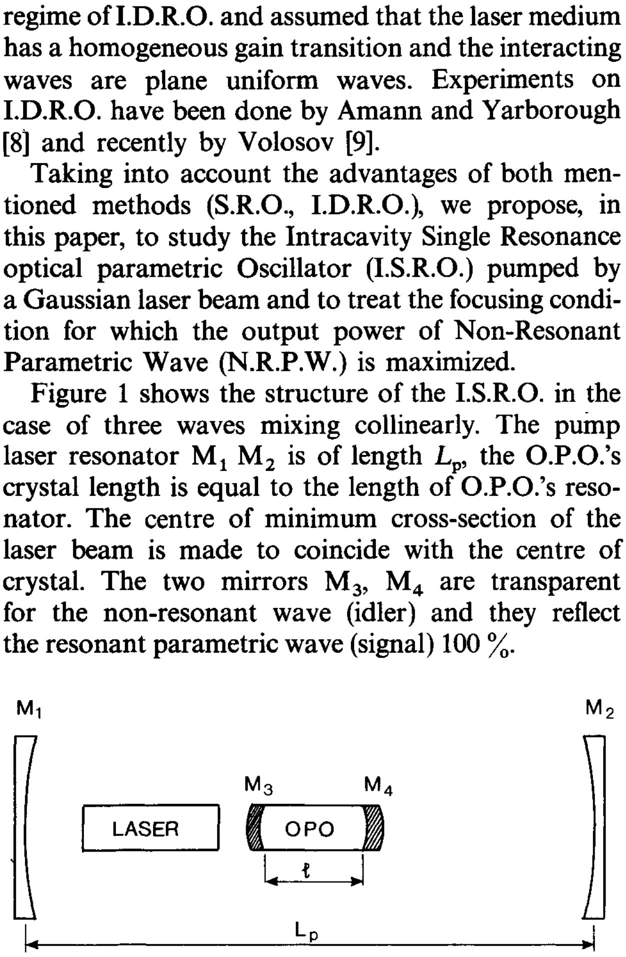

Fig. 1. — Internal optical parametric oscillator.

To obtain strong parametric interaction, the following phase relation between three waves is chosen [10]

$$\phi_p - \phi_s - \phi_i = \frac{\pi}{2} \tag{1}$$

where ϕ_j (j = p, s, i) are the phases of pump-, signal- and idler waves respectively.

In this work, the derivation method is similar to that used by Oshman and Harris [7], and is based upon the equations of motion obtained originally by Lamb [11]. The electric field is expanded in terms of unloaded cavity modes, and the field equations are spatially averaged over the cavity, assuming approximate orthogonality of modes [7].

2. Equation of motion.

2.1 SPATIAL MODE OF THE INTERACTING WAVES. — We shall label all quantities referring to the pump mode, signal and idler with the subscripts p, s, i respectively. The frequencies of these modes are ω_p, ω_s and ω_i satisfying the relation $\omega_p = \omega_i + \omega_s$ for energy conservation. The total electric field may be expressed as

$$\mathbf{E}(r, t) = \sum_{j=i,s,p} E_j(t)\, \mathbf{e}_j\, e^{j(\omega_j t + \phi_j)}\, U_j(r) \tag{2}$$

where $\mathbf{e}_j$ is the polarization unit vector, $U_j(r)$ is the spatial mode and $E_j(t)$ is the amplitude of the electric field. All our calculations are made with the assumption that $E_i \ll E_s$ and $E_i \ll E_p$ in the non-linear medium.

In S.R.O. devices, only the pump- and signal waves are at resonance, and if their TEMoo modes alones are at resonance, we can write $U_j(r)$ as a sum of right and left travelling Gaussian beams, i.e.

$$U_j(r) = u_j(r) + u_j^*(r) \tag{3}$$

with j = s, p.

We shall consider the case of the interaction type I where the walk-off angle is ρ. Boyd and Kleinman [12] have shown that this is equivalent to type II interaction. If the parametric crystal is centred at the origin of a Cartesian system (X, Y, Z) centred in the laser cavity with the Z-axis along the longitudinal cavity axis then, since the signal is at resonance, the one-way travelling signal beam wave is given by [12, 13]

$$u_s(r) = \frac{\exp(jk_s Z)}{1 + j\tau_s} \exp\left\{-\frac{X^2 + Y^2}{W_{0s}^2(1 + j\tau_s)}\right\} \tag{4}$$

$$\tau_s = \frac{2Z}{b_s}. \tag{5}$$

Here W_{0s} is the Gaussian beam waist of the signal and $b_s (= k_s W_{0s}^2)$ is the confocal parameter.

As the pump wave is at resonance in the laser cavity, the one-way travelling resonance pump wave has the form [12, 13]

$$u_p(r) = \frac{\exp(jk_p Z)}{1 + j\tau_p} \exp\left\{-\frac{(X - \rho Z)^2 + Y^2}{W_{0p}^2(1 + j\tau_p)}\right\}. \tag{6}$$

Here we have a difference between D.R.O. and S.R.O. : in the case of D.R.O., the idler beam wave is also at resonance, then its structure has been given in Gaussian form [12]. In S.R.O., the amplitude of the non-resonant idler beam wave has not previously been given, it can be determined, using equations (2), (4) and (6), from the following parabolic equation [14, 15] with certain approximations [16, 17] :

$$\left(\frac{\partial}{\partial z} - \frac{j}{2k_i}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right) E_i(X, Y, Z, t) = j\gamma_i E_p E_s^*(X, Y, Z, t)\, e^{-j\Delta k Z} \tag{7}$$

by means of the Green's function

$$G_i(X - X', Y - Y', Z - Z') = -\frac{jk_i}{2\pi(Z - Z')} \times$$
$$\times \exp\left\{\frac{jk_i}{2(Z - Z')}[(X - X')^2 + (Y - Y')^2]\right\}. \quad (8)$$

The coupling constant γ_i in equation (7) is of the form

$$\gamma_i = \frac{4\pi\omega_i d_{eff}^2}{cn_i}. \quad (9)$$

The effective non-linear coefficient is d_{eff}, given by

$$2 d_{eff} = \chi, \quad (10)$$

where χ is the non-linear susceptibility.

The phase-mismatching is Δk, given by

$$\Delta k = k_p - (k_i + k_s) \quad (11)$$

and the wave vectors of interacting waves are given by

$$k_j = \frac{n_j \omega_j}{c}. \quad (12)$$

The solution of the equation (7) is

$$E_i(X, Y, Z, t) = j\gamma_i \int_{-l/2}^{Z} dZ' \int_{-\infty}^{+\infty} dX' dY' \times$$
$$\times G_i(X - X', Y - Y', Z - Z') E_p E_s(X', Y', Z', t)$$
$$\times e^{-j\Delta k Z'}. \quad (13)$$

After integration and elimination of all high frequency components, equation (13) becomes

$$E_i(X, Y, Z, t) = j\gamma_i e^{jk_i Z} E_p(t) E_s^*(t) \times$$
$$\times \int_{-l/2}^{Z} dZ' \frac{e^{j\Delta k Z'} . e^{j(\omega_i t + \phi_i)}}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'} \exp\{\ \}_1 \quad (14)$$

where

$$\mu = \frac{k_s}{k_p} \quad (15)$$

is the degeneracy parameter, and

$$\{\ \}_1 = -\frac{j(1-\mu)}{W_{0p}^2(\tau - \tau')}(X^2 + Y^2) \times$$
$$\times \left(-1 + \frac{1 + \tau'^2}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'}\right)$$
$$+ \frac{2\rho Z'(1 + j\tau')X'}{W_{0p}^2\left(1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'\right)}$$
$$- \frac{\rho^2 Z'^2}{W_{0p}^2(1 + j\tau')} + \frac{\rho^2 Z'^2}{W_{0p}^2(1 + j\tau')^2 a^*} \quad (16)$$

and

$$a^* = \frac{1}{1 + j\tau'} + \frac{\mu}{1 - j\tau'} - \frac{j(1-\tau)}{\tau - \tau'}. \quad (17)$$

Equation (14) was obtained assuming $b_s = b_p = k_p W_{0p}^2 = k_s W_{0s}^2$ (the non-linear interaction is maximal when the confocal parameters of the pump- and the signal waves are identical (see [6]), and using the specified field approximation [18], which, as is well known, does not consider the reverse action of the excited wave on the exciting waves).

Relation (14) defines the field of the non-resonant idler wave, its spatial distribution is (its spatial mode)

$$U_i(r) = u_i(r) = j\exp(jk_i Z) \times$$
$$\times \int_{-l/2}^{Z} dZ' \frac{e^{j\Delta k Z'}}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'} \exp\{\ \}_1 \quad (18)$$

and its time-dependent part is

$$E_i(t) = \gamma_i E_p(t) E_s^*(t). \quad (19)$$

So, we can conclude that the spatial mode of the non-resonant idler wave does not have a Gaussian distribution.

2.2 Field and phase equations. — According to Lamb [11], the equations of motion are given by

$$(\omega_j + \dot{\phi}_j - \nu_j) E_j = -\frac{1}{2}\left(\frac{\omega_j}{\varepsilon_j}\right) C_j \quad (20)$$

$$\dot{E}_j + \frac{1}{2}\left(\frac{\omega_j}{Q_j}\right) E_j = -\frac{1}{2}\left(\frac{\omega_j}{\varepsilon_j}\right) S_j \quad (20')$$

where ν_j is the unloaded cavity mode frequency, Q_j is the cavity Q for the j^{th} mode, ε_j is the dielectric constant for the polarized medium of interest. The driving terms are defined by the spatially averaged polarization $\overline{P}_j(t)$

$$\overline{P}_j(t) = \frac{\int dr^3\, U_j(r)\, \mathbf{e}_j\, \mathbf{P}(r, t)}{\int dr^3\, U_j^2(r)} =$$
$$= C_j(t)\cos[\omega_j t + \phi_j(t)] +$$
$$+ S_j(t)\sin[\omega_j t + \phi_j(t)] \quad (21)$$

$j = $ p, s

where the integration extends over all space. If the j mode is confined within a cavity of length l_j, we have

$$\int dr^3\, U_j^2(r) = \pi W_{0j}^2 l_j \quad (22)$$

assuming that rapidly oscillating terms $\exp(\pm 2 k_j Z)$ average to zero.

There are two polarization terms; namely, one resulting from the laser medium and the other resulting from the parametric medium. Their sum is given in equation (21), from which $C_j(t)$, $S_j(t)$ can be calculated. By substituting their values in equations (20), (20′) we obtain time-dependent expressions for the amplitudes and phases of the resonant waves.

a) The polarization caused by laser's field is determined by [7]

$$C_j^{\mathrm{L}}(t) = \varepsilon_j^{\mathrm{L}}\, \chi_j'\, E_j(t) \tag{23}$$

$$S_j^{\mathrm{L}}(t) = \varepsilon_j^{\mathrm{L}}\, \chi_j''\, E_j(t) \tag{24}$$

where $\varepsilon_j^{\mathrm{L}}(j = \mathrm{p})$ are dielectric constants of the laser medium. We assume that the signal and idler frequencies are well-removed from any transition of the laser medium, so that contributions to them are essentially negligible ($j \neq \mathrm{i}, \mathrm{s}$ in Eqs. (23), (24)). For the pump, χ_{p}'' accounts for gain and saturation due to the laser medium; χ_{p}' results in frequency shifts mode-pulling and -pushing effects.

b) Now let us consider polarization resulting from the parametric medium

$$\mathbf{P}^{\mathrm{p}}(r, t) = \chi : \mathbf{E}(r, t)\, \mathbf{E}(r, t)\,. \tag{25}$$

Using equations (2), (21) and (22) we get

$$\overline{P}_{\mathrm{i}}^{\mathrm{p}}(r, t) = \sum_{j,k} \mathbf{e}_{\mathrm{i}}\, \chi : \mathbf{e}_j\, \mathbf{e}_k\, E_j\, E_k\, \mathrm{e}^{i(\omega_j t + \phi_j)}\, \mathrm{e}^{i(\omega_k + \phi_k)} \times \times \frac{1}{\pi W_{0\mathrm{i}}^2\, l_{\mathrm{i}}} \int \mathrm{d}r^3\, U_j\, U_{\mathrm{i}}\, U_k\,. \tag{26}$$

Thus we must evaluate the integral of the triple product $U_{\mathrm{i}} . U_{\mathrm{s}} . U_{\mathrm{p}}$ over the parametric medium. Using (3) we get

$$\int \mathrm{d}r^3\, U_{\mathrm{p}}\, U_{\mathrm{i}}\, U_{\mathrm{s}} = \int \mathrm{d}r^3\, u_{\mathrm{p}}^*\, u_{\mathrm{i}}\, u_{\mathrm{s}}\,. \tag{27}$$

Substituting (11) and (18) into (27) we have, after integration,

$$\int \mathrm{d}r^3\, U_{\mathrm{p}}(r)\, U_{\mathrm{i}}(r)\, U_{\mathrm{s}}(r) = \int \mathrm{d}r^3\, u_{\mathrm{p}}^*(r)\, u_{\mathrm{i}}(r)\, u_{\mathrm{s}}(r) = = \frac{\pi}{2} W_{0\mathrm{p}}^2 \frac{b l_j}{1 + \mu} \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{28}$$

where

$$\overline{h}(\overline{B}, \mu, \sigma, \xi) = (2\,\xi)^{-1} \int_{-\xi}^{\xi} \mathrm{d}\tau \times \times \int_{-\xi}^{\tau} \mathrm{d}\tau' \frac{\mathrm{e}^{-j\sigma(\tau - \tau')}\, F(\overline{B}, \mu, \xi)}{1 - \frac{j}{2}\left(\frac{1 + \mu}{1 - \mu} + \frac{1 - \mu}{1 + \mu}\right)(\tau - \tau') + \tau\tau'} \tag{29}$$

and

$\xi = \dfrac{l}{b}$ is the focusing parameter

$l_{\mathrm{s}} = l =$ crystal length

$\sigma =$ phase-mismatching parameter

$\overline{B} = \dfrac{\rho}{2}(l k_{\mathrm{p}})^{1/2}$ is the double-refraction parameter.

$F(\overline{B}, \mu, \xi)$ is determined by

$$F(\overline{B}, \mu, \xi) = \exp\left\{ -\frac{\overline{B}}{\xi} \left(\frac{\tau'^2}{1 + j\tau'} \times \times \left(1 - \frac{j(1 - j\tau')(\tau - \tau')}{(1 - \mu)[\ ]_1}\right) + \frac{\tau^2}{1 - j} - \left(\frac{\tau'(1 - j\tau')}{[\ ]_1} - \frac{\tau}{1 - j\tau}\right)^2 \cdot \frac{(1 + \tau^2)[\ ]_1}{2(1 + \mu)\{\ \}_4}\right)\right\}, \tag{30}$$

where

$$[\ ]_1 = 1 + j\frac{1 + \mu}{1 - \mu}(\tau - \tau') + \tau\tau'$$

$$\{\ \}_4 = 1 - \frac{j}{2}\left(\frac{1 + \mu}{1 - \mu} + \frac{1 + \mu}{1 - \mu}\right)(\tau - \tau') + \tau\tau'\,.$$

Using the symmetric relation [12], we have

$$\chi = \mathbf{e}_{\mathrm{p}}\, \chi : \mathbf{e}_{\mathrm{i}}\, \mathbf{e}_{\mathrm{s}} = \mathbf{e}_{\mathrm{s}}\, \chi : \mathbf{e}_{\mathrm{i}}\, \mathbf{e}_{\mathrm{p}} = \mathbf{e}_{\mathrm{i}}\, \chi : \mathbf{e}_{\mathrm{p}}\, \mathbf{e}_{\mathrm{s}} = 2\, d_{\mathrm{eff}}\,. \tag{31}$$

Using (28), (31) in conjunction with (26) and then rewriting equation (26) as a sequence of sine and cosine functions, by comparing them with equation (21) we obtain coefficients $C_{\mathrm{i}}^{\mathrm{p}}(t)$, $S_{\mathrm{i}}^{\mathrm{p}}(t)$

$$S_{\mathrm{i}}^{\mathrm{p}}(t) = \frac{W_{0\mathrm{p}}^2}{W_{0\mathrm{i}}^2} \frac{l^2}{l_{\mathrm{i}}} \frac{d_{\mathrm{eff}}}{4(1 + \mu)} \frac{E_j\, E_k}{\xi}\, \delta_{\mathrm{i}}\, \Re\mathrm{e}\, \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{32}$$

$$C_{\mathrm{i}}^{\mathrm{p}}(t) = -\frac{W_{0\mathrm{p}}^2}{W_{0\mathrm{i}}^2} \frac{l^2}{l_{\mathrm{i}}} \frac{d_{\mathrm{eff}}}{4(1 + \mu)} \frac{E_j\, E_k}{\xi}\, \delta_{\mathrm{i}}\, \Im\mathrm{m}\, \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{33}$$

with $\mathrm{i} = \mathrm{s}, \mathrm{p}$ and $\delta_{\mathrm{i}} = \begin{cases} 1 \text{ when } \mathrm{i} = \mathrm{p} \\ -1 \text{ when } \mathrm{i} = \mathrm{s}. \end{cases}$

Substituting (19) into (33) and (32), we obtain the values of $C_{\mathrm{p}}^{\mathrm{p}}$, $C_{\mathrm{s}}^{\mathrm{p}}$ and $S_{\mathrm{p}}^{\mathrm{p}}$, $S_{\mathrm{s}}^{\mathrm{p}}$ as functions of $E_{\mathrm{p}}(t)$, $E_{\mathrm{s}}(t)$. Carrying these values into (20), (21) and with the aid of (23), (24) we obtain the following wave equations :

$$\dot{E}_{\mathrm{p}}(t) = \frac{1}{2}\left(\frac{\omega_{\mathrm{p}}}{Q_{\mathrm{p}}}\right) E_{\mathrm{p}}(t) + \frac{1}{2}\omega_{\mathrm{p}}\, \chi_{\mathrm{p}}''\, E_{\mathrm{p}}(t) - \frac{4\,\pi^2\, \omega_{\mathrm{i}}\, \omega_{\mathrm{p}}}{c n_{\mathrm{i}}\, n_{\mathrm{p}}} \times \times d_{\mathrm{eff}}^2 \frac{l^2}{\xi L_{\mathrm{p}}} \frac{E_{\mathrm{p}}(t)\, E_{\mathrm{s}}^2(t)}{1 + \mu} \times \Re\mathrm{e}\, \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{34}$$

$$\dot{E}_{\mathrm{s}}(t) = -\frac{1}{2}\left(\frac{\omega_{\mathrm{s}}}{Q_{\mathrm{s}}}\right) E_{\mathrm{s}}(t) + \frac{4\,\pi^2\, \omega_{\mathrm{i}}\, \omega_{\mathrm{s}}}{c n_{\mathrm{i}}\, n_{\mathrm{s}}} d_{\mathrm{eff}}^2 \frac{l}{\xi} \frac{\mu}{1 + \mu} \times \times E_{\mathrm{p}}^2(t)\, E_{\mathrm{s}}(t)\, \Re\mathrm{e}\, \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{35}$$

$$\dot{\phi}_{\mathrm{p}} = -\frac{1}{2}\omega_{\mathrm{p}}\, \chi_{\mathrm{p}}' - \frac{4\,\pi^2\, \omega_{\mathrm{i}}\, \omega_{\mathrm{p}}}{c n_{\mathrm{i}}\, n_{\mathrm{p}}} d_{\mathrm{eff}}^2 \times \times \frac{l^2}{\xi L_{\mathrm{p}}} \frac{E_{\mathrm{p}}(t)\, E_{\mathrm{s}}^2(t)}{1 + \mu} \Im\mathrm{m}\, \overline{h}(\overline{B}, \mu, \sigma, \xi) \tag{36}$$

$$\dot{\phi}_s = -\frac{1}{2}\omega_s \chi_s' + \frac{4\pi^2 \omega_i \omega_s}{cn_i n_s} d_{eff}^2 \frac{l}{\xi}\frac{\mu}{1+\mu} \times \times E_p^2(t) E_s(t) \Im m\, \bar{h}(\bar{B}, \mu, \sigma, \xi) \quad (36')$$

in consideration of $\nu_p = \omega_p$ and $\nu_s = \omega_s$.

Studying gain coefficients and saturation effects of the pump laser, Oshman [7] has used Lamb's approximations and has found the following equation which determines χ_p'' :

$$\frac{\omega_p L_p}{c}\chi_p'' = -g_0(1 - \beta E_p^2) \quad (37)$$

where g_0 is the single-pass unsaturated power gain and β is a parameter accounting for saturation effects [7]. Equation (37) holds to a good approximation if saturation effects are too small to be observed and if [19]

$$\left(\frac{g_0}{\alpha_p}\right)^2 - 1 \ll 1 \quad (38)$$

$\alpha_p = L_p/c.\omega_p/Q_p$ is the single-pass power loss for the pump mode. In practice some lasers have a small gain g_0 (YAG-Nd^{3+}) and in the case of I.O.P.O., the signal and idler waves « drain » much of the laser power, so that the pump cannot saturate fully. Substituting equation (37) into (34), (35), (36) with a new time variable τ_1 given by

$$\tau_1 = \frac{ct}{L_p}; \quad \tau_2 = \frac{ct}{l}$$

and

$$\alpha_s = \frac{l_s}{c}\frac{\omega_s}{Q_s}$$

where α_s is the single-pass power loss for the signal mode. We obtain the following equations of motion :

$$\frac{dP_p}{d\tau_1} = \left(-\alpha_p + g_0\left(1 - \frac{16\beta}{cn_p W_{0p}^2}P_p\right) - K_1 \frac{\omega_i \omega_p}{W_{0p}^2 \xi} l^2 P_s h^{(1)}(\bar{B}, \mu, \sigma, \xi)\right) P_p \quad (39)$$

$$\frac{dP_s}{d\tau_2} = \left(-\alpha_s + K_1 \frac{\omega_i \omega_s}{W_{0p}^2 \xi} l^2 P_p h^{(1)}(\bar{B}, \mu, \sigma, \xi)\right) P_s \quad (40)$$

$$\frac{d\phi_p}{d\tau_1} = -\frac{1}{2}\frac{L_p \omega_p}{c}\chi_p' - \frac{1}{2}K_1 \frac{\omega_i \omega_p}{W_{0p}^2 \xi} l^2 P_s \times \times \Im m\, \bar{h}(\bar{B}, \mu, \sigma, \xi) \quad (41)$$

$$\frac{d\phi_s}{d\tau_1} = -\frac{1}{2}\frac{L_p \omega_s}{c}\chi_s' + \frac{1}{2}K_1 \frac{\omega_i \omega_s}{W_{0p}^2 \xi} \times \times L_p l P_p \Im m\, \bar{h}(\bar{B}, \mu, \sigma, \xi) \quad (42)$$

where

$$K_1 = \frac{128\pi^2 d_{eff}^2}{c^3 n_i n_p n_s}\frac{\mu}{1+\mu} \quad (43)$$

and the function $h^{(1)}(\bar{B}, \mu, \sigma, \xi)$ is determined from (29)

$$h^{(1)}(\bar{B}, \mu, \sigma, \xi) = \Re e\, \bar{h}(\bar{B}, \mu, \sigma, \xi) = \Re e (2\xi)^{-1} \int_{-\xi}^{\xi} d\tau \times \times \int_{-\xi}^{\tau} d\tau' \frac{e^{-j\sigma(\tau-\tau')} F(\bar{B}, \mu, \xi)}{1 - \frac{j}{2}\left(\frac{1+\mu}{1-\mu} + \frac{1-\mu}{1+\mu}\right)(\tau - \tau')\tau\tau'} \quad (44)$$

and the power of Gaussian waves are determined from

$$P_{p,s} = \frac{1}{16} cn_{p,s} W_{0p,s}^2 \mid E_{p,s}^{(t)} \mid^2 . \quad (45)$$

The equations (39) to (42) illustrate the time-dependence of the power and the phase of the resonance waves, equation (14) determines the power of the non-resonant parametric wave (idler wave) when all characteristics of the resonant waves are known. In the general case, these equations cannot be resolved analytically. They can be resolved only with the aid of a computer.

The function $h^{(1)}(\bar{B}, \mu, \sigma, \xi)$ is a focusing function which allows the optimal power of the non-resonant wave to be obtained. This function was calculated numerically and the result is given in figure 2, it reaches a maximum at the values of ξ and σ which correspond to a maximal parametric interaction between the waves.

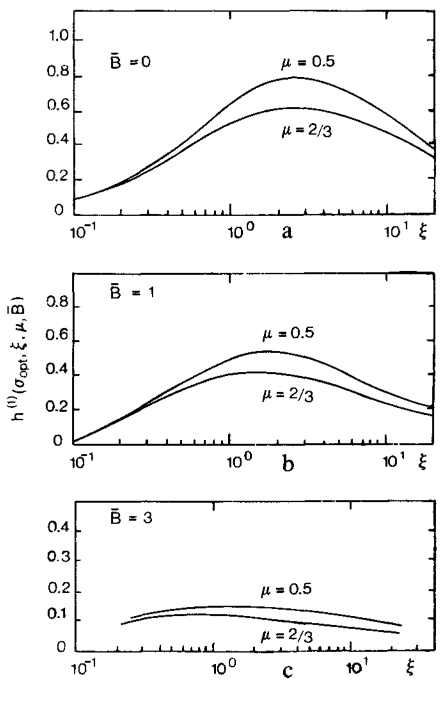

Fig. 2. — Plot of $h^{(1)}(\sigma_{opt.}, \mu, \bar{B}, \xi)$ as a function of ξ. a. The case of $\bar{B} = 0$; $\mu = 0.5$ and $\mu = 2/3$. b. The case of $\bar{B} = 1$; $\mu = 0.5$ and $\mu = 2/3$. c. The case of $\bar{B} = 3$; $\mu = 0.5$ and $\mu = 2/3$.

3. Powers of the parametric interacting waves in the steady-state (c.w.-regime). Discussion.

In steady-state conditions, we have

$$- \alpha_p + g_0\left(1 - \frac{16\beta}{cn_p W_{0p}^2} P_p\right) - K_1 \frac{\omega_i \omega_p l^2}{W_{0p}^2 \xi} \times$$

$$\times P_s h^{(1)}(\bar{B}, \mu, \sigma, \xi) = 0 \tag{46}$$

$$- \alpha_s + K_1 \frac{\omega_i \omega_s l^2}{W_{0p}^2 \xi} P_p h^{(1)}(\bar{B}, \mu, \xi) = 0 \tag{47}$$

$$\frac{d}{d\tau_1}(\phi_p \phi_s) = 0 \rightarrow \phi_i = \phi_p - \phi_s + \frac{\pi}{2} = \text{Cte}. \tag{48}$$

Equation (48) is identical to (1).

a) *In the absence of the parametric effect* ($K_1 = 0$); from (46) the power of the laser in the resonator becomes

$$P_p = P_{0p} = \frac{g_0 - \alpha_p}{16 g_0 \beta} cn_p W_{0p}^2. \tag{49}$$

Equation (49) corresponds to the case in which the mirrors of the laser resonator have a reflection coefficient which is 100 % at the laser power.

b) *Presence of the parametric effect* ($K_1 \neq 0$); from (47) we have

$$P_p = \frac{\alpha_s}{K_1 \omega_i \omega_s l k_p h^{(1)}(\bar{B}, \mu, \sigma, \xi)}. \tag{50}$$

The pump power P_p is a simple hyperbolic function of the non-linear coefficient $K_1 = 128 \pi^2 d_{eff}^2/c^3 n_i \times n_p n_s . \mu/1 + \mu$. With equations (46), (49), (50) we can calculate the signal wave's power within the resonator, and we have

$$P_s = (P_{0p} - P_p) \frac{16 g_0 \beta}{l K_1 \omega_i \omega_p cn_p b} \frac{1}{h^{(1)}(\bar{B}, \mu, \sigma, \xi)}. \tag{51}$$

The power of the non-resonant parametric wave (idler wave) is

$$P_i = \frac{cn}{8\pi} \int_{-\infty}^{\infty} dX\, dY \,|\, E_i(X, Y, Z = 0, t) \,|^2.$$

From (14) we have, in the case of absence of walk-off ($\bar{B} = 0$), the total value of P_i

$$P_i = \frac{128 \pi^2 \omega_i^2 d_{eff}^2}{c^3 n_i n_p n_s} \frac{2\mu}{1 + \mu} \frac{P_p}{W_{0p}^2} P_s b^2 \int_0^\xi d\tau' \times$$

$$\times \int_0^\xi d\tau'' \frac{e^{-j\sigma(\tau' - \tau'')}}{1 - \frac{j}{2}\left(\frac{1 + \mu}{1 - \mu} + \frac{1 - \mu}{1 + \mu}\right)(\tau' - \tau'') + \tau' \tau''}. \tag{52}$$

Using (49) and (51), equation (52) becomes

$$P_i = 4(P_{0p} - P_p) \frac{P_p}{P_{0p}} (g_0 - \alpha_p)(1 - \mu) \times$$

$$\times \frac{H^{(1)}(\sigma, \mu, \xi)}{h^{(1)}(\sigma, \mu, \xi)} \tag{53}$$

where

$$H^{(1)}(\sigma, \mu, \xi) = \int_0^\xi d\tau' \times$$

$$\times \int_0^\xi d\tau'' \frac{e^{-j\sigma(\tau' - \tau'')}}{1 - \frac{j}{2}\left(\frac{1 + \mu}{1 - \mu} + \frac{1 - \mu}{1 + \mu}\right)(\tau' - \tau'') + \tau' \tau''}. \tag{54}$$

We see that the power of the non-resonant wave (idler wave) is a simple parabolic function of pump power P_p.

c) *Discussion.*

In steady-state, if we want to have parametric generation, it is necessary that $P_i > 0$. From (53) we have

$$g_0 > \alpha_p$$

and

$$P_{0p} \geqslant P_p$$

or

$$K_1 \geqslant K_{th} = \frac{\alpha_s}{\omega_i \omega_s k_p l h^{(1)}(\sigma, \mu, \xi) \xi} \cdot \frac{1}{P_{0p}}. \tag{55}$$

With a given pump laser, the non-linear coefficient of the crystal must be higher than a threshold value determined by (55). We can reduce K_{th} by following methods, as shown by equation (55)

— increase of the pump power (increase P_{0p}),

— increase $h^{(1)}$ by the optimal focusing method to obtain a maximum value of $h^{(1)}$.

Figure 3 shows the dependence of P_i on pump power P_p. The curve reaches a maximum when

$$P_p = \frac{P_{0p}}{2} = \frac{(g_0 - \alpha_p) cn_p W_{0p}^2}{16 g_0 \beta} \tag{56}$$

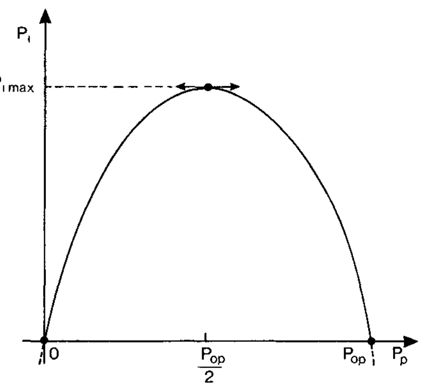

Fig. 3. — Plot of P_i as a function of P_p.

and this maximum value is equal to

$$P_{i\max} = P_{0p}(g_0 - \alpha_p)(1 - \mu)\frac{H^{(1)}(\sigma, \mu, \xi)}{h^{(1)}(\sigma, \mu, \xi)}. \quad (57)$$

The power P_i is zero when $P_p = 0$ and $P_p = P_{0p}$. This corresponds to an absence of pump laser, and to the condition $K_1 \leqslant K_{th}$. Substituting (50) into (53) and using (55) we have the value of power P_i as a function of K_1

$$P_i = 4\,P_{0p}\left(\frac{1}{K_1} - \frac{K_{th}}{K_1^2}\right)K_{th}(g_0 - \alpha_p)(1 - \mu) \times \frac{H^{(1)}(\sigma, \mu, \xi)}{h^{(1)}(\sigma, \mu, \xi)}. \quad (58)$$

Equation (58) illustrates that P_i is a simple function of K_1.

Figure 4 shows the dependence of P_i on K_1 (curve (c)). The curve (c) representing $P_i(K_1)$ cuts the x-axis at the point $P_i = 0$

$$K_1 = K_{th} = \frac{1}{P_{0p}} \cdot \frac{\alpha_s}{\omega_i\,\omega_s\,k_p\,lh^{(1)}(\sigma, \mu, \xi)} \quad (59)$$

as in equation (55) $K_1 \geqslant K_{th}$ is a necessary condition for parametric oscillation.

The curve (c) has its maximum at $dP_i/dK_1 = 0$. Letting the derivative of (58) with respect to K_1 tend to zero, we find

$$K_1 = 2K_{th} = \frac{2}{P_{0p}} \cdot \frac{\alpha_s}{\omega_i\,\omega_s\,k_p\,lh^{(1)}(\sigma, \mu, \xi)} \quad (60)$$

as $K_1 = 2\,K_{th}$, P_i reaches it maximum and its value is

$$P_{i\max} = P_{0p}(g_0 - \alpha_p)(1 - \mu)\frac{H^{(1)}(\sigma, \mu, \xi)}{\xi h^{(1)}(\sigma, \mu, \xi)}. \quad (61)$$

We again find equation (57).

As equation (58) shows, when K_1 tends to infinity, P_i reaches an asymptotic value along the x-axis.

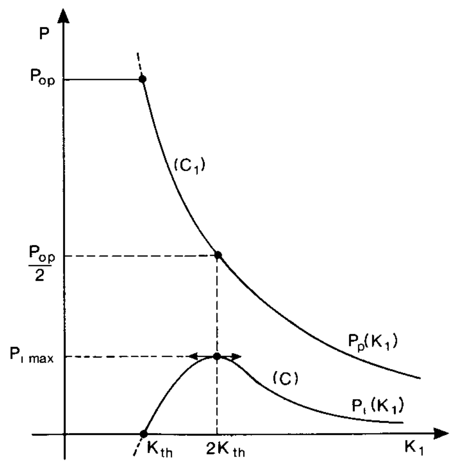

Fig. 4. — Plots of P_i and P_p as functions of K_1 : curve (c) shows P_i as a function of K_1, curve (c_1) shows P_p as a function of K_1.

We have plotted in figure 4 the curve (c_1) illustrating the dependence of P_p upon K_1. It has two parts. The first represents the pump laser power without parametric oscillation. The second is the intracavity power of the laser in the presence of the parametric generation. Using the value $K_1 = 2\,K_{th}$ with the aid of (60) and (50) we have

$$P_p(K_1 = 2\,K_{th}) = \frac{P_{0p}}{2} \quad (62)$$

which is equation (56). We can explain the power variations as follows. As the parametric generation increases, the pump power continually decreases; simultaneously, the non-resonance parametric wave power first increases, goes through a maximum, and begins to decrease. Quantitatively, this can be understood as the result of the fact that the parametric generation appears to be an increasing loss to the laser. Eventually, the total available power from the gain mechanism goes through a maximum and begins to decrease, in the same way that increasing output coupling losses to a laser can produce a similar maximum in output power [19]. We shall demonstrate that, in this case and at the degeneracy operation regime, these « non-linear output coupling losses » are equal to the maximum output power of the laser at the optimal transmission of the output mirror in the absence of the optional parametric oscillator.

In the case of a weak focusing ($\xi \ll 1$), we have [6]

$$H^{(1)}(\sigma, \mu, \xi) \simeq \xi^2$$
$$h^{(1)}(\sigma, \mu, \xi) \simeq \xi\,.$$

At the degeneracy regime μ has the value : $\mu = 1/2$. Inserting these values into (61), we have

$$P_{i\max} = \frac{P_{0p}}{2}(g_0 - \alpha_p)\,. \quad (63)$$

So the « non-linear transmission coefficient » of the output mirror is

$$\begin{aligned} T &= \frac{P_{i\max}}{P_{0p}} = \frac{1}{2}(g_0 - \alpha_p) \\ &= (\sqrt{g_0} - \sqrt{\alpha_p}) \times \left(\frac{\sqrt{g_0} + \sqrt{\alpha_p}}{\sqrt{\alpha_p}}\right)\sqrt{\alpha_p} \\ &= (\sqrt{g_0\,\alpha_p} - \alpha_p)\left(\frac{\sqrt{g_0}}{\sqrt{\alpha_p}} + 1\right)\frac{1}{2}. \end{aligned} \quad (64)$$

Using the condition (38) we have

$$\frac{1}{2}\left(\frac{\sqrt{g_0}}{\sqrt{\alpha_p}} + 1\right) \simeq 1\,. \quad (65)$$

Inserting (65) into (64) we have

$$T = \frac{P_{2\max}}{P_{0p}} = \sqrt{g_0\,\alpha_p} - \alpha_p\,.$$

Rigrod [19] has demonstrated that the mirror's transmission $T_{opt.}$ that results in maximum power output from a laser oscillator is given by

$$T_{opt.} = \sqrt{g_0 \alpha_p} - \alpha_p .$$

So we have demonstrated that, at the maximum I.S.R.O. power condition, the output power of the non-resonant parametric wave is equal to the maximum pump laser output power in the absence of O.P.O. (i.e. the converting coefficient η is 100 %).

In a previous work [14] it has been shown that, below threshold, the signal and idler powers remain zero, whereas above the threshold, the pump limits at the threshold level and additional pump power is converted to signal and idler power, thus the second part of the curve (c_1) illustrates the dependence of the threshold power on K_1. When the power of the non-resonant parametric wave is at a maximum ($\eta = 100\%$) we have (Eq. (62)).

$$\frac{P_{0p}}{P_p} = \frac{P_{0p}}{P_{th}} = 2$$

i.e. the pump power is twice the threshold value. In comparison with the results of previous work based on external single resonance optical parametric oscillator, the converting efficiency is equal to 100 % when [5]

$$\frac{P_{0p}}{P_{th}} = \left(\frac{\pi}{2}\right)^2 = 2.46 .$$

From (60) we see that with a given laser, we can choose the value of parameter K_1 to get a maximum value of the non-resonant parametric wave power P_i. Alternatively with a given laser and a given non-linear crystal we can choose the value of the function $h^{(1)}$ so that the equation (60) is satisfied for getting a maximum value of the output power P_i.

d) *Numerical results.*

Let us consider an intracavity single resonance optical parametric oscillator based on a pump c.w.-YAG-laser and a non-linear $Ba_2NaNb_5O_{15}$ crystal of length of $l = l_s = 4$ mm; $d_{eff} = 14.56 \times 10^{-8}$ e.s.u. [20]; $\alpha_s = 0.02$. The c.w.-YAG-laser has the following characteristics : $g_0 = 0.1$; $\alpha_p = 0.08$; $L_p = 50$ cm.

The intracavity power of laser is equal to 35 watts at wavelength $\lambda_p = 1.06\ \mu m$. The I.S.R.O. operates in the degeneracy regime, i.e. ($\mu = 0.5$) $\lambda_i = \lambda_s = 2\lambda_p = 2.12\ \mu m$. The refractive indices of the non-linear crystal are : $n_i = n_s = n_p = 2.3$ [20]. Using equation (60) we have the value of the function $h^{(1)}(\sigma_{opt.}, \mu = 1/2, \xi) = 0.4$. Figure 2a gives $\xi = 0.4$ (at $h^{(1)} = 0.4$), i.e. $b = l/\xi = 10$ mm. To obtain the maximum power of the non-resonant parametric wave, the pump laser beam must be focused into the crystal with a confocal parameter of 10 mm. We have weak focusing ($\xi = 0.4 < 1$); from equation (63) we have the condition

$$P_{i\,max} = 0.35\ \text{W} .$$

4. Conclusion.

The theory of c.w.-I.S.R.O. has been developed. This leads us to conclude that with the optimal focusing method of a Gaussian laser beam the I.S.R.O. can be carried out experimentally to produce a c.w. tunable laser.

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