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Short Note

Intrinsic and layer-induced vertical transverse isotropy

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INTRODUCTION

Anisotropy caused by fine layering is often considered responsible for the differences between velocities obtained in sonic-log and seismic experiments. Understanding the link between the two is critical, especially in the current era of most wells being (highly) deviated.

Vertical sonic velocities may be upscaled according to Backus (1962) to deduce interval vertical velocities for seismic frequencies. However, few experimental observations relate complete properties of fine layers to seismic anisotropy observed at seismic scale, which is necessary if we want to predict and explain reflection moveout for both P- and S-waves (Sams et al., 1993; Sams and Williamson, 1994; Kebaili and Schmitt, 1996; Vernik and Fisher, 2001). Partly this is caused by fine layers themselves being anisotropic (e.g., shales). For a medium with two constituents (such as sand and shale), the anisotropic parameters of both layers should be known to predict the properties of the effective compound (Backus, 1962). In addition, effective medium averaging is a nonlinear procedure which gives little insight into what to expect.

A discussion was sparked by Thomsen's (1986) paper on how seismically measured anisotropy values relate to the properties of the thin layers and their intrinsic anisotropy (Levin, 1988). Although numerical schemes have existed for a long time, the physics behind them was not always clear. The situation is exemplified by Frank Levin's comment that "predicting the delta of a transversely isotropic solid from component delta's is not easy" (Levin, 1988).

This note intends to improve understanding of how each Thomsen coefficient for effective transversely isotropic media with a vertical symmetry axis depends on the parameters of the individual constituents and the elastic contrasts between layers.

EXACT AVERAGING

Within our current abilities, most sedimentary rocks can be described by vertical transverse isotropy (VTI) (Thomsen, 1986). Each VTI constituent is defined by a stiffness matrix with five independent elements:

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \tag{1}$$

where $c_{66} = (c_{11} - c_{12})/2$. The composite effective medium is also VTI according to the following averaging equations (Backus, 1962; Molotkov and Khilo, 1985):

$$c_{11} = \langle c_{13}/c_{33}\rangle^2/\langle 1/c_{33}\rangle - \langle c_{13}^2/c_{33}\rangle + \langle c_{11}\rangle, \tag{2}$$

$$c_{12} = c_{11} - \langle c_{11}\rangle + \langle c_{12}\rangle, \tag{3}$$

$$c_{13} = \langle c_{13}/c_{33}\rangle/\langle 1/c_{33}\rangle, \tag{4}$$

$$c_{33} = \langle 1/c_{33}\rangle^{-1}, \tag{5}$$

$$c_{44} = \langle 1/c_{44}\rangle^{-1}, \tag{6}$$

where $c_{66} = (c_{11} - c_{12})/2 = \langle c_{66}\rangle$. Here, $\langle\cdot\rangle$ denotes the thickness-weighted average of corresponding parameters of individual constituents, for example, $\langle\alpha\rangle = \phi_1\alpha_1 + \phi_2\alpha_2$ with ϕ_1 and $\phi_2 = 1 - \phi_1$ being their relative thicknesses.

Once the stiffnesses are obtained, they can be recast into Thomsen notation commonly used in reflection seismology:

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}}, \qquad V_{S0} \equiv \sqrt{\frac{c_{44}}{\rho}}, \tag{7}$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \tag{8}$$

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$$\delta \equiv \frac{(c_{13}+c_{44})^2-(c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})}, \tag{9}$$

$$\gamma \equiv \frac{c_{66}-c_{44}}{2c_{44}}, \tag{10}$$

where V_{P0} and V_{S0} are vertical velocities of P- and S-waves, ρ is the density, and ϵ, δ, and γ are the dimensionless Thomsen (1986) anisotropic parameters.

WEAK-ANISOTROPY AND WEAK-CONTRAST APPROXIMATION

It is difficult to develop physical intuition for understanding and predicting the outcome of the exact equations (2)–(6) and (7)–(10). Even for isotropic constituents, the results are not intuitive. Several studies have attempted to derive some conclusions from numerical calculations and analysis of special cases (Berryman, 1979; Brittan et al., 1995; Shapiro and Hubral, 1996; Anno, 1997; Berryman et al., 1999; Werner and Shapiro, 1999) considering only isotropic constituents.

We will analyze the case of VTI constituent layers. We first make two simplifying assumptions before proceeding with analysis:

1) The anisotropy of each constituent is weak. In the case of VTI layers, this means the Thomsen parameters are much smaller than unity ($|\epsilon| \ll 1$, $|\delta| \ll 1$, and $|\gamma| \ll 1$).
2) There is a weak contrast between the two constituents, $|\Delta c_{33}/\bar{c}_{33}| \ll 1$ and $|\Delta c_{44}/\bar{c}_{44}| \ll 1$.

The average stiffness $\bar{c}_{33} = 1/2(c_{33}^{(1)} + c_{33}^{(2)})$ and the difference $\Delta c_{33} = c_{33}^{(2)} - c_{33}^{(1)}$ are expressed as functions of stiffnesses of the first ($c_{33}^{(1)}$) and second ($c_{33}^{(2)}$) constituents. Similar quantities are defined for the shear stiffness c_{44}. Note that the whole range $0 < c_{33}^{(2)}/c_{33}^{(1)} < \infty$ is mapped into $-2 < \Delta c_{33}/\bar{c}_{33} < 2$.

Here we are making weak-anisotropy and weak-contrast assumptions. To utilize them together, we also assume that the Thomsen parameters ϵ, δ, and γ, along with normalized jumps $\Delta c_{33}/\bar{c}_{33}$ and $\Delta c_{44}/\bar{c}_{44}$, are small quantities of the same order. These assumptions are quite reasonable for many sedimentary sequences and are often used in anisotropic processing and AVO analysis (Thomsen, 1993; Rüger, 2002).

Linearization of the effective stiffness matrix in these small quantities leads to the interesting result that the effective anisotropic parameters ϵ, δ, and γ depend only on the corresponding Thomsen coefficients of the constituents:

$$\epsilon = \langle \epsilon \rangle, \tag{11}$$

$$\delta = \langle \delta \rangle, \tag{12}$$

$$\gamma = \langle \gamma \rangle. \tag{13}$$

Such results may be expected from the physics of wave propagation: in the limit of zero frequency, the effective media properties are independent of the order of layers for any number of constituents. For media with two constituents, this means the effective elastic properties should be independent of the signs of Δc_{33} and Δc_{44} and, thus, may not contain linear terms in contrasts.

Additional physical meaning of these results is most easily illustrated for γ. For isotropic constituents ($c_{66}^{(1)} = c_{44}^{(1)}, c_{66}^{(2)} = c_{44}^{(2)}$), the overall anisotropy γ is proportional to $\langle c_{44}\rangle - \langle 1/c_{44}\rangle^{-1}$. One can verify with simple algebra that if averaged quantities are different by some small amount Δ, then to the first order in Δ the geometric mean average is equivalent to the arithmetic mean average. Thus, we conclude that (1) to the first order in elastic parameter contrasts isotropic layering does not produce effective anisotropy; (2) effective anisotropy arises only if the constituents have intrinsic anisotropy; and (3) each effective anisotropic parameter is the thickness-weighted average of the corresponding parameters of the constituent layers.

To verify the accuracy of formulas (11)–(13), we compare them with the exact Thomsen parameters computed using equations (2)–(10). For two examples from Table 1, we focus only on predicting the effective Thomsen parameters ϵ, δ, and γ because the vertical velocities of P- and S-waves can be computed easily from logs using exact equations (5)–(7). Figure 1 shows that, despite substantial values of anisotropic parameters and contrasts reaching 30%, the maximum error does not exceed 0.03.

SECOND-ORDER APPROXIMATION (STRONGER CONTRASTS)

What happens if the property variation among the constituents is not small? The elastic parameter contrast lumps together the density and velocity contrasts. Consider the approximate linear relation derived from equation (7):

$$\frac{\Delta c_{33}}{\bar{c}_{33}} = \frac{\Delta\rho}{\bar{\rho}} + \frac{2\Delta V_{P0}}{\bar{V}_{P0}}. \tag{14}$$

If, for example, we have a 20% density and 15% velocity contrast, this may result in a $\Delta c_{33}/\bar{c}_{33}$ of about 50%. In this case the linearizations described above lead to erroneous predictions because the anisotropy caused by vertical heterogeneity is nonnegligible. To include these effects, we will obtain second-order approximations with respect to both intrinsic anisotropy and the contrasts in the elastic moduli. In so doing, we can gain useful insight into how effective anisotropy is influenced by intrinsic anisotropy, anisotropy induced by vertical heterogeneity, and by their interaction.

This exercise has two objectives: (1) to gain clear understanding of the main factors that control the magnitude of effective anisotropy in the case of stronger contrast between the VTI constituents and (2) to develop useful approximations allowing us to compute each Thomsen coefficient using fewer input parameters.

For a second-order approximation each Thomsen parameter is given by a simple equation of the following form:

$$\epsilon = \langle \epsilon \rangle + \epsilon_{is} + \epsilon_{is-an} + \epsilon_{an}, \tag{15}$$

where $\langle \epsilon \rangle$ is the first-order term that depends on intrinsic anisotropy only, ϵ_{is} is the second-order isotropic term from

Table 1. Models used to test the accuracy of equations (11)–(13). For the first constituent, $V_{P0} = 3$ km/s, $V_{S0} = 1.5$ km/s, and $\rho = 2.4$ g/cm^3. (Any other set of parameters can be used that leads to the same $c_{33} = \rho V_{P0}^2$ and $c_{44} = \rho V_{S0}^2$.)

Case	$\frac{\Delta c_{33}}{\bar{c}_{33}}$	$\frac{\Delta c_{44}}{\bar{c}_{44}}$	ϵ_1	ϵ_2	δ_1	δ_2	γ_1	γ_2
1	30%	−30%	0.05	0.25	0.0	0.20	0.05	0.25
2	25%	30%	0.05	0.25	0.0	0.20	0.05	0.25

vertical heterogeneity obtained by replacing the VTI constituents by isotropic layers with the same vertical velocities, ϵ_{is-an} is the second-order crossterm arising from the coupling of vertical heterogeneity and intrinsic anisotropy, and ϵ_{an} is the second-order term from intrinsic anisotropy only.

In the remainder of this section, we analyze the second-order approximation in more detail. Inspection of isotropic terms reveals that when the shear modulus is constant, then $\delta_{is} = \epsilon_{is} = \gamma_{is} = 0$. This is a well-known result for isotropic layers (Postma, 1955) and is indeed a special case of more general fact that the effective models made up of isotropic constituents with a constant shear modulus are always isotropic irrespective of their shape (Hill, 1963). Analysis also shows that when $c_{44}/c_{33} = (V_{S0}/V_{P0})^2 = \text{const}$, then $\delta_{is} = 0$. This case was first discussed by Krey and Helbig (1956).

Effective parameter γ

The effective γ is represented by

$$\gamma = \langle\gamma\rangle + \gamma_{is} + \gamma_{is-an} + \gamma_{an}, \tag{16}$$

where

$$\gamma_{is} = \frac{1}{2}\phi_1\phi_2\left(\frac{\Delta c_{44}}{\bar{c}_{44}}\right)^2, \tag{17}$$

$$\gamma_{is-an} = \phi_1\phi_2\frac{\Delta c_{44}}{\bar{c}_{44}}\Delta\gamma, \tag{18}$$

$$\gamma_{an} = 0. \tag{19}$$

In the case of $|\Delta c_{44}/c_{44}| \gg |\Delta\gamma|$, the third term γ_{is-an} can be neglected and

$$\gamma \approx \langle\gamma\rangle + \gamma_{is}. \tag{20}$$

As follows from equation (17), $\gamma_{is} > 0$, which is a strict constraint for isotropic constituent layers.

Equation (20) explains the results of Werner and Shapiro (1999), who have found that the contributions of intrinsic anisotropy and anisotropy caused by vertical variations of the elastic moduli must be summed up to obtain the effective anisotropic parameter γ. However, they consider only the case of intrinsic anisotropy constant for all layers ($\gamma_1 = \gamma_2 = \langle\gamma\rangle$), which is a special case of equation (20).

Effective parameter δ

The effective δ is represented by

$$\delta = \langle\delta\rangle + \delta_{is} + \delta_{is-an} + \delta_{an}, \tag{21}$$

where

$$\delta_{is} = 2\phi_1\phi_2\frac{\bar{c}_{44}}{\bar{c}_{33}}\left[\frac{\Delta c_{33}}{\bar{c}_{33}} - \frac{\Delta c_{44}}{\bar{c}_{44}}\right]\frac{\Delta c_{44}}{\bar{c}_{44}}, \tag{22}$$

$$\delta_{is-an} = 0, \tag{23}$$

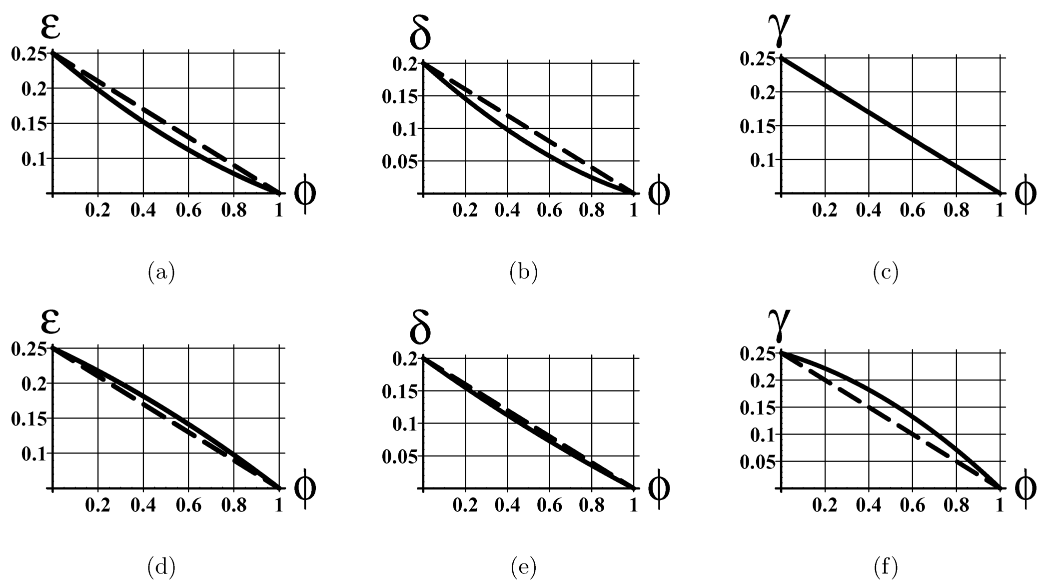
$$\delta_{an} = -\frac{1}{2}\phi_1\phi_2\frac{(\Delta\delta)^2}{\left(1 - \dfrac{\bar{c}_{44}}{\bar{c}_{33}}\right)}. \tag{24}$$

For most subsurface boundaries we do not expect the contrast $|\Delta\delta|$ to be higher than 0.1–0.2, which implies that $|\delta_{an}| < 0.025$ (for $\bar{c}_{44}/\bar{c}_{33} = 1/4$). Clearly, δ_{an} may be neglected in practice because it is smaller than 0.03–0.04, which is the minimum expected uncertainty in estimating interval δ from field data (Grechka et al., 2002).

Therefore, we can use a simplified approximation similar to equation (20),

$$\delta \approx \langle\delta\rangle + \delta_{is}. \tag{25}$$

The effective δ is a simple sum of averaged intrinsic anisotropy $\langle\delta\rangle$ and a purely isotropic contribution δ_{is} related to fluctuations



(a) (b) (c) (d) (e) (f)

FIG. 1. Thomsen parameters of two-component VTI media as functions of the fraction of the first constituent $\phi = \phi_1$ ($\phi_2 = 1 - \phi_1$). Shown are the exact solutions (solid lines) and weak-anisotropy, weak-contrast approximations (dashed) [equations (11)–(13)]. Parameters are listed in Table 1; plots (a)–(c) correspond to case 1, whereas (d)–(f) correspond to case 2.

in vertical elastic parameters. The last term in equation (25) has been extensively studied for isotropic components (Anno, 1997; Berryman et al., 1999; Brittan et al., 1995), and conclusions may be readily transferred and applied to the case of VTI constituents.

Both approximation (25) and the exact equation (9) show that no information about constituents ϵ (or c_{11}) is required. The presence of the isotropic term can make the effective δ smaller than $min(\delta_1, \delta_2)$ or larger than $max(\delta_1, \delta_2)$, as noted by Levin (1988).

Therefore, approximation (25) resolves the issue of predicting the effective δ for VTI constituents raised by Levin (1988) in his reply to Thomsen's original paper (1986). Indeed, simple inspection of equations (22) and (25) leads to the following conclusions for two possible cases.

In the first case, the second-order isotropic term δ_{is} is small compared to $\langle\delta\rangle$. This happens when

1) the contrasts are small enough to neglect the second-order term δ_{is}, which corresponds to the first-order approximation considered above.
2) $\Delta c_{44}/\bar{c}_{44} = 0$. In this case of a constant shear modulus, δ_{is} is always zero irrespective of how large $\Delta c_{33}/\bar{c}_{33}$ is.
3) $\Delta c_{44}/\bar{c}_{44} = \Delta c_{33}/\bar{c}_{33}$. This is equivalent to the case of a constant V_{S0}/V_{P0} or c_{44}/c_{33}. It can be recognized by acknowledging that to the first order,

$$\frac{c_{44}^{(2)}}{c_{33}^{(2)}} = \frac{c_{44}^{(1)}}{c_{33}^{(1)}}\left(1 + \frac{\Delta c_{44}}{\bar{c}_{44}} - \frac{\Delta c_{33}}{\bar{c}_{33}}\right). \quad (26)$$

Again, δ_{is} is always zero for any contrasts in the elastic moduli.

For all three cases above we only need to know the constituent δ to estimate the effective δ, and we always have $min(\delta_1, \delta_2) < \delta < max(\delta_1, \delta_2)$. This is a significant simplification compared to the exact Backus equation (9), where three stiffnesses (or V_{P0}, V_{S0}, and δ) must be known to predict the effective δ.

In the second case, the second-order isotropic term δ_{is} is comparable to $\langle\delta\rangle$. This might happen only for sizeable contrasts between the constituents that also exhibit strong variations in the V_{S0}/V_{P0} ratio so that the term $\Delta c_{44}/\bar{c}_{44} - \Delta c_{33}/\bar{c}_{33}$ may become considerable. As to the sign of δ_{is}, Anno (1997) notices that a typically observed positive correlation between V_{S0} and V_{S0}/V_{P0} leads to negative effective δ (δ_{is}) for purely isotropic layering. Utilizing relation (26), such a correlation may be recast in our notation as

$$\frac{\Delta c_{44}}{\bar{c}_{44}} \geq 0 \quad \text{and} \quad \frac{\Delta c_{33}}{\bar{c}_{33}} - \frac{\Delta c_{44}}{\bar{c}_{44}} \leq 0, \quad (27)$$

or

$$\frac{\Delta c_{44}}{\bar{c}_{44}} \leq 0 \quad \text{and} \quad \frac{\Delta c_{33}}{\bar{c}_{33}} - \frac{\Delta c_{44}}{\bar{c}_{44}} \geq 0. \quad (28)$$

For either case (27) or (28), equation (22) predicts negative δ_{is}. As suggested by Anno (1997), the sign of the effective δ can indeed discriminate the cases of isotropic layering (characterized by negative δ) from the cases of shale intrinsic anisotropy, which typically exhibit positive δ. In more realistic situations of interleaving isotropic and VTI layers, one should apply equation (25) to interpret effective δ.

Effective parameter ϵ

Effective ϵ is represented by

$$\epsilon = \langle\epsilon\rangle + \epsilon_{is} + \epsilon_{is-an} + \epsilon_{an}, \quad (29)$$

where

$$\epsilon_{is} = 2\phi_1\phi_2\left(\frac{\bar{c}_{44}}{\bar{c}_{33}}\right)^2\left[\frac{\bar{c}_{33}}{\bar{c}_{44}}\frac{\Delta c_{33}}{\bar{c}_{33}} - \frac{\Delta c_{44}}{\bar{c}_{44}}\right]\frac{\Delta c_{44}}{\bar{c}_{44}}, \quad (30)$$

$$\epsilon_{is-an} = \phi_1\phi_2\frac{\bar{c}_{44}}{\bar{c}_{33}}\left[2\frac{\Delta c_{44}}{\bar{c}_{44}}\Delta\delta + \frac{\bar{c}_{33}}{\bar{c}_{44}}\frac{\Delta c_{33}}{\bar{c}_{33}}(\Delta\epsilon - \Delta\delta)\right], \quad (31)$$

$$\epsilon_{an} = -\frac{1}{2}\phi_1\phi_2(\Delta\delta)^2. \quad (32)$$

Similar to the analysis for δ, $\epsilon_{an} \sim (\Delta\delta)^2$ and may be safely neglected in most cases of practical importance. Also, as in the case of δ, the isotropic term (30) vanishes when either the shear modulus is constant or when $\Delta c_{44}/\bar{c}_{44} = \bar{c}_{33}/\bar{c}_{44}\Delta c_{33}/\bar{c}_{33}$.

When the constituents δ are unknown but small, we can approximate equation (29) as

$$\epsilon \approx \langle\epsilon\rangle + \epsilon_{is} + \tilde{\epsilon}_{is-an}, \quad (33)$$

where $\tilde{\epsilon}_{is-an}$ is a simplified coupling term (31) expressed as

$$\tilde{\epsilon}_{is-an} = \phi_1\phi_2\frac{\Delta c_{33}}{\bar{c}_{33}}\Delta\epsilon. \quad (34)$$

In most cases, however, the whole crossterm ϵ_{is-an} is small and may also be neglected, yielding a simple approximation similar to those for γ and δ:

$$\epsilon = \langle\epsilon\rangle + \epsilon_{is}. \quad (35)$$

Both approximations (33) and (35) allow us to compute the effective ϵ without knowledge of the constituents δ.

CONCLUSIONS: MOST LIKELY ROCK PHYSICS APPROXIMATION

In practice, one usually has some a priori rock physics information about velocities and anisotropies. Therefore, utilizing expressions (15) and (17)–(32), one can quantify plausible contributions of various terms and select an appropriate approximation. In most practical cases, it should be either the linear approximation [equations (11)–(13)] or simplified second-order approximation [equations (16)–(20), (25), (33), or (35)], depending on the fluctuations of c_{33} and c_{44}.

The linear approximation is particularly attractive because it does not require any information on the vertical velocities apart from the fact that their variation is small (say, $\Delta c_{33}/\bar{c}_{33}$ and $\Delta c_{44}/\bar{c}_{44}$ are less than 30%). Moreover, each linearized effective Thomsen parameter depends only on the corresponding Thomsen parameters of the constituents. It is straightforward to prove that the linear approximation remains valid for any number of constituent layers. Its generalization to more complex anisotropy will be discussed in a forthcoming paper.

Approximations introduced in this study may also be useful to analyze VTI rocks permeated with sets of parallel fractures. For example, Bakulin et al. (2000, 2002) show that if set(s) of vertical fractures are added to VTI (finely layered) background, then anisotropic parameters of effective orthorhombic media can be conveniently approximated as the sum of intrinsic VTI anisotropy parameters and isotropic fracture-induced contributions. Complementing this result with current conclusions, we may further decompose effective orthorhombic coefficients to the sum of intrinsic Thomsen parameters of individual fine layers and fracture-induced contributions.

The second-order approximation becomes necessary when the contrasts in elastic moduli become significant (>30–40%). Full second-order approximations do not give any computational advantage because they require the same input as the exact equations. However, they are instructive in analyzing the role of individual contributions into the overall anisotropy. Such analysis leads to further simplifications [equations (33) and (35)] that reduce the number of input parameters; for example, the effective ϵ can be found without using the constituent δs.

Let us compare the performance of equations (16), (20), (25), (33), and (35) with that of the exact equations with the following objectives: (1) to show that the second-order approximation is likely to be sufficient in describing the overall subsurface anisotropy and (2) to demonstrate the value of approximations that require fewer input parameters. In the first two models (Table 2) we consider the extreme behavior of correlated (both V_{P0} and V_{S0} increase) and anticorrelated (V_{P0} increases while V_{S0} decreases) P- and S-velocities. Figures 2a–f demonstrate that in both cases the maximum error of the second-order approximations is of reasonable size and does not exceed 0.07 when three terms are used. The parameters of model 3 are taken from the real case study by Vernik and Fisher (2001), who analyze sand–shale sequences from the deepwater Gulf of Mexico. Figures 2g–i show that the maximum error while using all suggested approximations does not exceed 0.015.

Table 2. Models used to test the accuracy of the second-order weak-anisotropy, weak-contrast approximations. For the first constituent layer, V_{P0} = 3.2 km/s, V_{S0} = 1.55 km/s, and ρ = 2.45 g/cm^3. (Any other set of parameters can be used that leads to the same $c_{33} = \rho V_{P0}^2$ and $c_{44} = \rho V_{S0}^2$.)

Model	$\frac{\Delta c_{33}}{\bar{c}_{33}}$ $\left(\frac{c_{33}^{(2)}}{c_{33}^{(1)}}\right)$	$\frac{\Delta c_{44}}{\bar{c}_{44}}$ $\left(\frac{c_{44}^{(2)}}{c_{44}^{(1)}}\right)$	ϵ_1	ϵ_2	δ_1	δ_2	γ_1	γ_2
1	90% (2.64)	70% (2.08)	0.05	0.25	0.20	0.0	0.05	0.25
2	30% (1.35)	−80% (0.43)	0.05	0.2	0.05	0.15	0.05	0.2
3	−45% (0.63)	−27% (0.76)	0.05	0.0	0.02	0.0	0.15	0.0

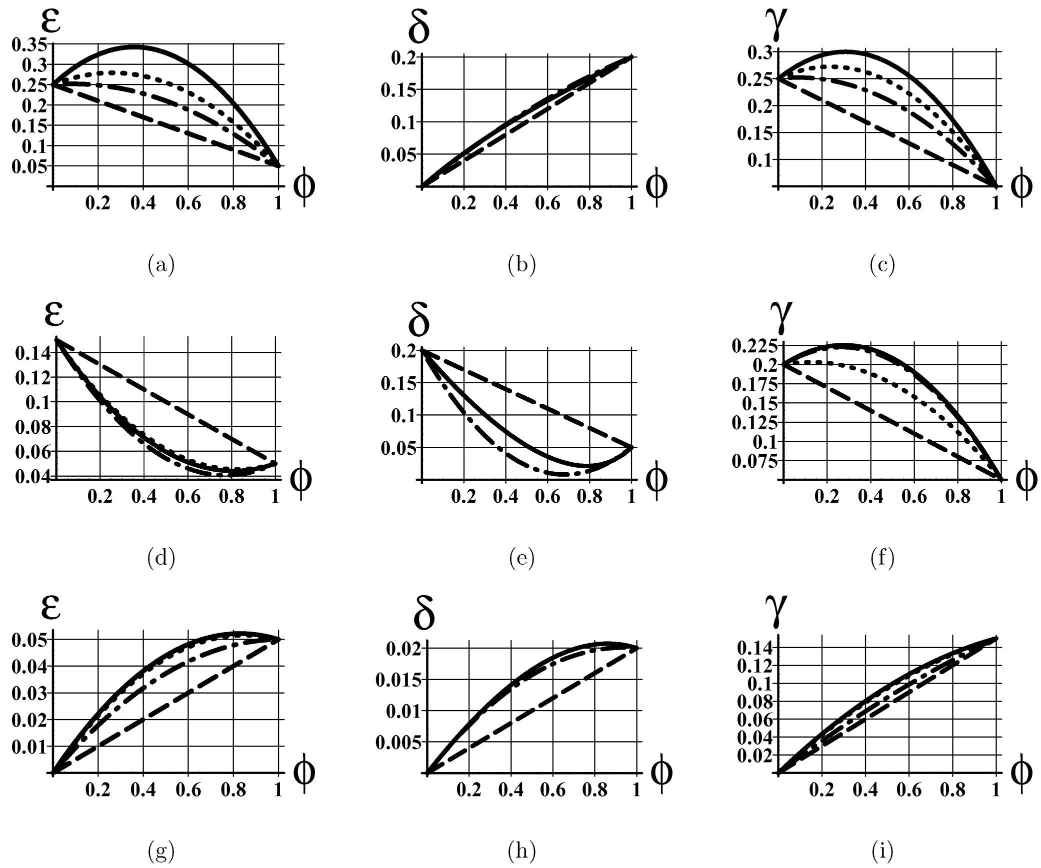


FIG. 2. Effective Thomsen parameters of two-component VTI media as functions of the fraction of the first constituent $\phi = \phi_1$ ($\phi_2 = 1 - \phi_1$). Shown are the exact solutions (solid lines), first-order (dashed) [equations (11)–(13)], and simplest second-order (dash-dotted) [equations (20), (25), (35)] weak-anisotropy, weak-contrast approximations with only the isotropic second-order term. The dotted lines for γ and ϵ correspond to second-order approximations (16) and (33) with extra terms. Parameters are listed in Table 2: plots (a)–(c) correspond to model 1, (d)–(f) to model 2, (g)–(i) to model 3.

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