Open access • Journal Article • DOI:10.1103/PHYSREVA.44.5401

Intrinsic decoherence in quantum mechanics — Source link

Gerard J. Milburn

Institutions: University of Queensland

Published on: 01 Nov 1991 - Physical Review A (American Physical Society)

Topics: Quantum decoherence, Hamiltonian (quantum mechanics), Coherence (physics), Schrödinger equation and Free particle

PHYSICAL REVIEW A VOLUME 44, NUMBER 9 1 NOVEMBER 1991

Intrinsic decoherence in quantum mechanics

G. J. Milburn
Department of Physics, The University of Queensland, St. Lucia, Queensland 4072, Australia
(Received 18 October 1990)

A model for intrinsic decoherence in quantum mechanics is proposed, based on a simple modification of unitary Schrödinger evolution. On sufficiently small time scales the system evolves by a random sequence of unitary phase changes generated by the Hamiltonian. The Schrödinger equation is obtained to zeroth order in the expansion parameter. Higher-order corrections lead to a loss of coherence in the energy basis. The rate of coherence loss becomes very large as the energy scale of the system is increased. The expansion parameter determines an uncertainty in the time step on very short times scales. A number of testable consequences are derived including anomalous dispersion of a free particle, decay of oscillatory systems, destruction of interference-fringe visibility, and a phase shift of interference fringes.

PACS number(s): 03.65.Bz, 03.65.Sq, 03.65.Ca

I. INTRODUCTION

The feature of quantum mechanics that most distinguishes it from classical mechanics is the coherent superposition of distinct physical states. This feature is at the heart of the less intuitive aspects of the theory. It is the basis for the concern about measurement in quantum mechanics [1], and it is the explanation for the nonappearance of chaos in systems that classically would be chaotic [2]. Apparently, however, the superposition principle does not operate on macroscopic scales, although nothing in the present formulation of quantum mechanics would indicate this.

The problem for standard quantum mechanics is to provide an explanation for the nonappearance of macroscopically distinguishable states. At least two approaches have been adopted in the literature. The first approach seeks an explanation entirely within standard quantum mechanics supplemented with reasonable statistical arguments. One traces the decoherence of macroscopic systems to their characteristic multiplicity of degrees of freedom, the idea being the quantum coherence rapidly becomes spread over many more degrees of freedom than an observer can have access to [3]. The statistics of the few observables that are accessible is easily described by a reduced density operator that is rapidly diagonalized in some preferred basis.

A second approach to the problem seeks to modify Schrödinger's equation in such a way that coherence is automatically destroyed as the physical properties of the system approach a macroscopic level. This might be called "intrinsic" decoherence. A number of models have been proposed [4–10] the most widely known, perhaps, being that of Ghirardi, Rimini, and Weber [4]. In their model, each element of a system, at Poisson distributed times, undergoes a sudden localization over some range. The model contains two new parameters defining the frequency and the spatial extent of the localization. A related model is discussed in Ref. [5]. Diosi [6] has attempted to remove the need for new parameters by linking the localization process to variations in gravitational potential. As discussed in Ref. [7] however, the model of Diosi leads to an unacceptably large increase in the growth of the energy of microsystems. This problem can be overcome, albeit with the introduction of a new parameter [7]. Nonetheless, the energy will not be a constant of motion in any of these models.

Another approach linking decoherence to gravitational, in fact quantum gravitational, effects has recently been discussed by Ellis, Mohanty, and Nanopoulos [8,9]. In fact, each of these papers offers a different model based on wormhole effects. In Ref. [9], the loss of coherence can be traced to a new gauge-invariant, nonlocal interaction wormhole-matter coupling, which leads to a decay of coherence in a two-state SQUID system that goes exponentially as $-t^2$, rather than exponentially as $-t$, in other models. This suggests that the decoherence cannot be represented by a linear, Markovian quantum stochastic process. In this context, however, it should be noted that two measurement models [10,11] show a t^2 dependence at short times but a linear time dependence on larger time scales.

In this paper a simple modification of the standard quantum dynamics is proposed. The central postulate is that on sufficiently short time steps the system does not evolve continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations. Effectively this introduces a minimum time step in the universe. The inverse of this time step is the mean frequency of the unitary steps, γ, which becomes an expansion parameter. If the frequency of the time steps is large enough, the evolution appear approximately continuous on laboratory time scales. To zeroth order the Schrödinger equation is recovered. The first-order correction leads to a decay of coherence in the energy eigenstate basis.

One feature of this model is that constants of the motion remain constants of the motion and thus stationary states remain stationary states, a feature not found in the model of Ghirardi, Rimini, and Weber. This is also characteristic of the wormhole model of Ellis, Mohanty, and Nanopoulos [8] and of an earlier interesting model by Bedford and Wang [12]. The model, however, does have a surprising consequence if all orders of magnitude of the expansion parameter are included; oscillatory systems become frozen at very high energies. This is a direct consequence of introducing a minimum time step. One cannot produce an oscillator with a period shorter than the minimum time step. In the case of a free particle, the new evolution equation introduces an energy-dependent diffusion term to the position dispersion. Whether or not these consequences are observable depends on the size of γ.

II. GENERALIZED SCHRÖDINGER EVOLUTION

In standard quantum mechanics, the change in the state of a system in a time interval $(t,t+\tau)$ is given by

$$\hat{\rho}(t+\tau)=\exp\left[\frac{-i\hat{H}\tau}{\hbar}\right]\hat{\rho}(t)\exp\left[\frac{i\hat{H}\tau}{\hbar}\right] \tag{2.1}$$

independent of the size of τ. We replace this with the following postulates.

(i) On a sufficiently small time scale the change in the state of the system is uncertain. The probability that the system changes is $p(\tau)$.

(ii) Given that the state of the system changes, it changes by

$$\begin{aligned}\hat{\rho}(t+\tau)&=\exp\left[-\frac{i}{\hbar}\theta(\tau)\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar}\theta(\tau)\hat{H}\right]\\&\equiv\mathcal{J}(\tau)\hat{\rho}(t)\ .\end{aligned} \tag{2.2}$$

In standard quantum mechanics, $p(\tau)=1$ and $\theta(\tau)=\tau$. However, in the generalization considered here we only require $p(\tau)\to 1$ and $\theta(\tau)\to\tau$ for τ sufficiently large. Finally we postulate

(iii)

$$\lim_{\tau\to 0}\theta(\tau)=\theta_0\ , \tag{2.3}$$

i.e., there is some minimum unitary phase change. One possible choice is

$$\theta(\tau)=\tau/p(\tau)\ .$$

Now divide the interval $(0,t)$ into K steps of length τ. Thus $t=K\tau$. From elementary arguments we see that

$$\begin{aligned}\hat{\rho}(t)&=\sum_{k=0}^{K}\begin{bmatrix}K\\K\end{bmatrix}p(\tau)^k[1-p(\tau)]^{K-k}\mathcal{J}(\tau)^k\hat{\rho}(0)\\&=[1+p(\tau)\mathcal{U}(\tau)]^K\hat{\rho}(0)\ ,\end{aligned} \tag{2.4}$$

where $\mathcal{U}(\tau)\equiv\mathcal{J}(\tau)-1$. A similar equation to Eq. (2.4) has been used to describe the sub-Poissonian pumped laser [13]. The rate of change of $\hat{\rho}(t)$ is then given by

$$\frac{d\hat{\rho}(t)}{dt}=\frac{1}{\theta(\tau)p(\tau)}\ln[1+p(\tau)\mathcal{U}(\tau)]\hat{\rho}(t)\ . \tag{2.5}$$

In the limit that $\tau\to 0$, this reduces to

$$\frac{d\hat{\rho}(t)}{dt}=\frac{1}{\theta_0 p_0}\ln(1+p_0\mathcal{U}_0)\hat{\rho}(t)\ . \tag{2.6}$$

We now define a "Poisson model" for the stochastic time steps by setting $p_0\to 0$. Then

$$\frac{d\hat{\rho}}{dt}=\gamma\left[\exp\left[\frac{-i}{\hbar\gamma}\hat{H}\right]\hat{\rho}\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right]-\hat{\rho}\right]\ , \tag{2.7}$$

where $\gamma\equiv\theta_0^{-1}$. This equation is equivalent to the assumption that on a sufficiently short time scale the probability the system evolves is given by $\gamma\tau$ [14].

Equation (2.7) is the proposed generalized evolution equation. Expanding to first order in γ^{-1}, we find

$$\frac{d\hat{\rho}}{dt}=\frac{-i}{\hbar}[\hat{H},\hat{\rho}]-\frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}]]\ . \tag{2.8}$$

Thus Schrödinger's equation is recovered in the limit $\gamma\to\infty$ (that is, $\theta_0=0$). The first-order correction in Eq. (2.8) leads to a diagonalization of the density operator in the energy eigenstate basis,

$$\begin{aligned}\frac{\partial}{\partial t}\langle E'|\hat{\rho}|E\rangle&=\frac{-i}{\hbar}(E'-E)\langle E'|\hat{\rho}|E\rangle\\&\quad-\frac{1}{2\hbar^2\gamma}(E'-E)^2\langle E'|\hat{\rho}|E\rangle\ .\end{aligned} \tag{2.9}$$

Note that the rate of diagonalization depends on the square of the energy separation of the superposed states. Thus coherence between states that are widely separated in energy compared to Planck's constant decay rapidly. This type of double commutator has appeared in many models for coherence decay [3,4–10]. Most recently, Mohanty, Ellis, and Nanopoulos derived a similar term from a quantum gravitational effect. A similar term also arises in models of continuous measurement [10,15].

Complementary to the diagonalization of the density operator, the first-order term in Eq. (2.8) induces diffusion in variables that do not commute with the Hamiltonian. However, all constants of motion commute with the Hamiltonian and thus remain unaffected. In fact, this is a general feature of the complete evolution (2.7). Furthermore, the coherence between superposed degenerate energy eigenstates does not decay and thus no "splitting" can occur in the energy spectrum.

The model discussed above is intended to apply both to individual systems prepared initially in pure states or to large collections of systems perhaps better described initially as mixed states. Nonetheless, a density operator description is necessary in both cases. The reason is as follows. A single system, prepared in a pure state, evolving as indicated above and for which the sequence of exact times of occurrence of each unitary step is known, will always be described by a pure state. The essential point of the model, however, is that this sequence of times is, in principle, unknowable.

III. EFFECT ON MOMENTS

We now investigate the modification to the equations of the first- and second-order moments required by Eq. (2.7) for various systems.

For any operator $\hat{A}$ we have from Eq. (2.7)

$$\frac{d}{dt}\langle \hat{A} \rangle = \gamma \left\langle \exp\left[\frac{i}{\hbar\gamma}\hat{H}\right] \hat{A} \exp\left[\frac{-i\hat{H}}{\hbar\gamma}\right] - \hat{A} \right\rangle . \tag{3.1}$$

In the case of a free particle of mass m the Hamiltonian is

$$\hat{H} = \hat{p}^2/2m ,$$

where $\hat{p}$ is the momentum. The first-order moments are unchanged from their Schrödinger form. The position variance, however, does behave differently,

$$V(\hat{q}(t)) = V(\hat{q}(0)) + \frac{t^2}{m^2} V(\hat{p}(0)) + \frac{2t}{m}\langle \hat{q}(0), \hat{p}(0) \rangle_s + \frac{2E_0 t}{m\gamma} , \tag{3.2}$$

where $E_0 = \langle \hat{p}^2(0) \rangle /2m$ and $\langle \hat{A}, \hat{B} \rangle_s \equiv \frac{1}{2}\langle \hat{A}\hat{B} + \hat{B}\hat{A} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle$. The first terms in Eq. (2.10) are the standard Schrödinger result representing dispersion of the position variance. The final term represents an additional energy-dependent diffusion. A possible test of the generalization proposed here would consist in finding such a term. We shall return to this point in Sec. IV.

Consider now a harmonic oscillator with frequency ω_0. In terms of the raising and lowering operators $a^\dagger, a$, the Hamiltonian is $\hat{H} = \hbar\omega_0(a^\dagger a + \frac{1}{2})$. The mean complex amplitude of the oscillator is given by

$$\langle a(t) \rangle = \exp[-\gamma t(1 - e^{-i\omega_0/\gamma})] \langle a(0) \rangle . \tag{3.3}$$

When $\omega_0/\gamma \ll 1$ this reduces to

$$\langle a(t) \rangle \simeq \langle a(0) \rangle e^{-i\omega_0 t - (\omega_0^2/2\gamma)t} . \tag{3.4}$$

The effect of the generalized evolution equation when $\gamma \gg \omega_0$ is to induce a decay of the complex amplitude. In fact, this result is a direct reflection of the phase diffusion arising from the first-order term in Eq. (2.8) [16]. If, however, ω_0 is sufficiently large that $\omega_0/\gamma = 2n\pi$,

$$\langle a(t) \rangle = \langle a(0) \rangle$$

and the oscillator dynamics becomes frozen. It is thus impossible to produce an oscillator that has a period smaller than the fundamental time step γ^{-1}. In general, when very large energies are involved the dynamics arising from the generalized evolution is very different from Schrödinger dynamics.

A similar result occurs in a precessing spin system. For example, if $\hat{H} = \hbar\omega_0 \hat{J}_z$ the mean value of the x and y components of angular momentum are given by

$$\langle \hat{J}_x(t) \rangle = \tfrac{1}{2}\langle J_x(0) \rangle (e^{-\gamma t(1-e^{-i\mu})} + e^{\gamma t(1-e^{i\mu})}) - \frac{i}{2}\langle \hat{J}_x(0) \rangle (e^{-\gamma t(1-e^{-i\mu})} - e^{-\gamma t(1-e^{i\mu})}) , \tag{3.5}$$

$$\langle \hat{J}_y(t) \rangle = \tfrac{1}{2}\langle \hat{J}_y(0) \rangle (e^{-\gamma t(1-e^{-i\mu})} + e^{-\gamma t(1-e^{i\mu})}) + \frac{i}{2}\langle \hat{J}_x(0) \rangle (e^{-\gamma t(1-e^{-i\mu})} - e^{-\gamma t(1-e^{i\mu})}) , \tag{3.6}$$

where $\mu = \omega_0/\gamma$. If $\omega_0/\gamma \ll 1$,

$$\langle \hat{J}_x(t) \rangle = [\langle \hat{J}_x(0) \rangle \cos(\omega_0 t) - \langle \hat{J}_y(0) \rangle \sin(\omega_0 t)] e^{-\omega_0^2 t/2\gamma} , \tag{3.7}$$

$$\langle \hat{J}_y(t) \rangle = [\langle \hat{J}_x(0) \rangle \sin(\omega_0 t) + \langle \hat{J}_y(0) \rangle \cos(\omega_0 t)] e^{-\omega_0^2 t/2\gamma} . \tag{3.8}$$

As in the case of the harmonic oscillator, ordinary Schrödinger dynamics is recovered with the addition of a slow decay. The origin of this decay is also a "phase diffusion" of the angular momentum vector about the z axis [16].

Quite apart from the effect on quantum coherence, the above results indicate that the generalized evolution would cause significant deviation in behavior in the moments. As it is usually much easier to measure moments than to construct possibly macroscopic interference devices, such deviations should be easier to test. However, the crucial question is the time scale on which these effects become important and this is determined by the size of γ^{-1}. Previous studies [10,15] on the effect of nonunitary terms on interference effects have indicated that the time scale for such experiments is not determined by γ^{-1} alone but rather the product of γ^{-1} and the square of some variable representing the "separation" in the superposed variables monitored by the interference experiment. Thus if γ^{-1} is too small, quantum interference experiments may be the only way to test the model.

IV. EFFECT ON POSITION INTERFERENCE

In this section we consider the effect of the generalized evolution equation on the interference fringes on the position probability density for a harmonic oscillator prepared in an initial superposition state.

Consider a harmonic oscillator prepared in a linear superposition of two coherent states centered on opposite sides of the potential and separated by a distance $2X_0$.

$$|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle) , \tag{4.1}$$

where $|\alpha_0\rangle$ is a coherent state and

$$\mathcal{N} = \frac{1}{\sqrt{2}}(1 + e^{-2|\alpha_0|^2})^{-1/2} .$$

The position X and momentum P are determined by the complex parameter α,

$$\alpha = \left(\frac{2\hbar}{m\omega_0}\right)^{-1/2} X + i(2\hbar m\omega_0)^{-1/2} P ,$$

where m is the oscillator mass. We take $\alpha_0 = x_0$ purely real. The parameter x_0 is a dimensionless position.

Define the matrix elements of the density operator in

the energy basis by

$$\rho_{nm}(t)\equiv\langle n|\hat{\rho}(t)|m\rangle \ .$$

Then to first order in γ^{-1}

$$\frac{\partial}{\partial t}\rho_{nm}=\left[-i\omega(n-m)-\frac{\omega_0^2}{2\gamma}(n-m)^2\right]\rho_{nm}(t) \quad (4.2)$$

with the solution

$$\rho_{nm}(t)=\exp\left[-i\omega_0 t(n-m)-\frac{\omega_0^2 t}{2\gamma}(n-m)^2\right]\rho_{nm}(0) \ . \quad (4.3)$$

The position probability density for the initial state is given by

$$P(x,t)=\mathcal{N}^2\{P_+(x,t)+P_-(x,t)+2\,\mathrm{Re}[I(x,t)]\} \quad (4.4)$$

with

$$P_\pm(x,t)=\left[\frac{2}{\pi}\right]^{1/2}e^{-2x^2-x_0^2}\sum_{n,m=0}^{\infty}e^{-\omega_0^2 t(n-m)^2/2\gamma}\frac{[\pm\alpha^*(t)]^n}{n!}\frac{(\pm\alpha(t)]^m}{m!}2^{-(n+m)/2}H_n(\sqrt{2}x)H_m(\sqrt{2}x) \ , \quad (4.5)$$

$$I(x,t)=\left[\frac{2}{\pi}\right]^{1/2}e^{-2x^2-x_0^2}\sum_{n,m=0}^{\infty}e^{-\omega_0^2 t(n-m)^2/2\gamma}\frac{[\alpha^*(t)]^n}{n!}\frac{[-\alpha(t)]^m}{m!}2^{-(n+m)/2}H_n(\sqrt{2}x)H_m(\sqrt{2}x) \ , \quad (4.6)$$

and

$$\alpha(t)=\alpha_0 e^{-i\omega_0 t} \ .$$

For a free oscillator the maximum interference occurs when the Gaussian wave packets overlap at the origin. This occurs at $t=\pi/2\omega_0$ and we henceforth only consider the position probability at this time.

The sums in Eqs. (4.5) and (4.6) are difficult to evaluate exactly. However, one easily shows that

$$I(0,\pi/2\omega_0)=P_\pm(0,\pi/2\omega_0) \ . \quad (4.7)$$

The degree of interference is quantified by defining the fringe visibility

$$\mathcal{V}(x)=\frac{|I(x,\pi/2\omega_0)|}{P_\pm(x,\pi/2\omega_0)} \ . \quad (4.8)$$

When $\gamma\to\infty$, $\mathcal{V}(x)=1$ for all values of x, indicating maximum interference, i.e., maximum quantum coherence. In general, $\mathcal{V}(x)$ is a complicated function of x. Near $x=0$, $\mathcal{V}(x)$ decays from unity with a rate that depends on x_0. Thus to determine the effect of finite γ on quantum coherence we now find an approximate expression for $\mathcal{V}(x)$ near $x=0$.

Using the integral representation for the Hermite polynomial and expanding the exponential to first order, Eqs. (4.5) and (4.6) become

$$P_\pm[x,\pi/2\omega_0)\simeq\left[\frac{2}{\pi}\right]^{1/2}e^{-2x^2}[1-4\lambda x_0^2(1-4x^2)] \quad (4.9)$$

$$I(x,\pi/2\omega_0)\simeq\left[\frac{2}{\pi}\right]^{1/2}e^{-2x^2-4ixx_0}(1-4\lambda x_0^2+4i\lambda x x_0) \ , \quad (4.10)$$

where

$$\lambda\equiv\frac{\omega_0\pi}{4\gamma} \ . \quad (4.11)$$

The additional complex term proportional to λ in Eq. (4.10) indicates a frequency shift, i.e., a shift in the fringe pattern. Using Eqs. (4.9) and (4.10) the visibility to lowest order in $(x_0)^2$ becomes

$$\mathcal{V}(x)\simeq 1-16\lambda x_0^2 x^2$$

$$=1-\frac{4\omega_0\pi}{\gamma}x_0^2x^2 \ . \quad (4.12)$$

The important point to note here is that the decay rate of $\mathcal{V}(x)$ from one near $x=0$ is proportional to x_0^2. This dependence is characteristic of the effect of nonunitary evolution on quantum coherence features [17,18]. Such a dependence ensures that quantum coherence is suppressed as the macroscopic level is approached while remaining apparent at microscopic scales. Equation (4.12) is written in terms of the dimensionless position. In terms of the real position variables X and X_0,

$$\mathcal{V}(x)=1-\frac{\pi m^2\omega_0^3}{\gamma\hbar^2}X_0^2X^2 \ . \quad (4.13)$$

V. DISCUSSION AND CONCLUSION

There are number of reasons why destruction of coherence in the energy basis may be sufficient to avoid macroscopic consequences of the superposition principle. First, many schemes to search for a macroscopic quantum superposition state involve time evolution as an essential component. In coherent quantum tunneling an initial state localized in some macroscopic state evolves first to a superposition of two localized states and finally to a state localized in a state macroscopically distinct from the initial state. For such a process to occur, the time evolution must preserve coherence in the energy basis. For example, consider a particle moving in a symmetric bistable, quartic potential. A localized state in one well could simply be a superposition of the two lowest-energy eigenstates. Tunneling will only be possible if coherence be-

tween these states is maintained as a π phase shift develops between them.

Another example is the EPR experiment in which a superposition state of two-particle system is formed at some location and which evolves to a state in which the particles are spatially separated. In order for nonclassical correlations to be found in measurements on each of the particles, it is essential that during the time of flight of the particles coherence is maintained between the initially superposed states.

As a final example, there is the case of the quantum suppression of chaos in certain classically chaotic system. This is due to partial reconstruction of initial states as time evolution retains coherence between the energy eigenstates contributing to the initial state. The easiest way to prevent this suppression and permit the appearance of diffusion associated with classical chaos is to suppress coherence in the energy basis.

From another point of view, however, it seems plausible that a model that simply destroys coherence in the energy basis will not be sufficient to remove all the unwanted consequences of the superposition principle. In fact this must be the case if the model of this paper is to be made consistent with relativity; what appears as random time jumps in one inertial frame may well appear as a random position fluctuation in another frame. Thus it may be best to regard the model given here as only a part of a more comprehensive treatment that would also include spatial fluctuations, perhaps of the kind proposed by Ghirardi, Rimini, and Weber.

Where should one look to find evidence for a stochastic time evolution? Two possible consequences of the model will be considered: the additional energy-dependent diffusion of a free particle and the frequency shifts in oscillatory systems.

Consider a free particle initially prepared in a state in which the position and momentum are uncorrelated. The variance in position at time t is given by Eq. (4.3) as

$$V(\hat{q}(t))=\sigma_q^2+\Delta_F(t)+\Delta_D(t)\ , \tag{5.1}$$

where

$$\Delta_F(t)=\frac{t^2}{m^2}\sigma_p^2 \tag{5.2}$$

is the free-particle dispersion in position,

$$\Delta_D(t)=2Dt \tag{5.3}$$

is the additional energy-dependent diffusion, and σ_q^2 and σ_p^2 are the initial variance in position and momentum, respectively. The diffusion constant D is given by

$$D=\frac{E_0}{m\gamma}\ . \tag{5.4}$$

At long times the quadratic time dependence of free dispersion $\Delta_F(t)$ will dominate; anomalous dispersion is a short-time phenomenon. The two terms are equal at time t^* where

$$t^*=\frac{2E_0 m}{\gamma\sigma_p^2} \tag{5.5}$$

at which time

$$\Delta_F(t^*)=\Delta_D(t^*)=\left[\frac{2E_0}{\gamma\sigma_p}\right]^2\ . \tag{5.6}$$

For the anomalous dispersion to become apparent, we require t^* to be comparable to the time of flight of the free particle over the extent of the experiment. To increase t^* we can increase the kinetic energy of the particles or decrease the initial momentum uncertainty. If the energy is too large, relativistic corrections must dominate and the present nonrelativistic treatment is inadequate. We shall return to this point at the end of this section.

Experiments on neutron interference [19] do not provide an adequate constraint on γ as the energies of thermal neutrons are too low. Typically in such experiments $E_0\sim kT$ and $\sigma_p^2\sim mkT$. Thus $t^*\sim\gamma^{-1}$. Neutron interference experiments are conducted over length scales of the order of one meter, i.e., over time scales of the order of $(kT/m)^{-1/2}\simeq 10^{-4}$ s and γ^{-1} is certainly much smaller than this.

Neutron interferometry, while not likely to provide evidence for anomalous dispersion, may yet be useful in searching for the modification of interference fringes discussed in Sec. IV. In particular, it may be possible to observe the small fringe shift in a single slit neutron experiment. A fringe shift in such an experiment would appear as an anomalous slit width when the results were matched to theory.

Another possible test of the model is to search for the decay of first-order moments in oscillatory systems. The coherent excitation of a two-level atomic transition is well described in terms of the precession of a spin half particle. The precession frequency is the Rabi frequency for the transition, which is proportional to the strength of the applied field. The Hamiltonian describing this process, in a frame rotating at the transition frequency, is [20]

$$\hat{H}=\hbar\Omega\hat{J}_x\ , \tag{5.7}$$

where Ω is the Rabi frequency. For typical optical transitions, $\Omega\simeq 10^7\ \mathrm{s}^{-1}$. From Sec. II we see that the decay rate of $\langle\hat{J}_y(t)\rangle$, the polarization, is determined by Ω^2/γ. However, the natural linewidth is $\simeq 10^7\ \mathrm{s}^{-1}$. Thus for the effect of decoherence to become apparent we require $\Omega^2>10^7\gamma$. No experiment performed to date has detected such an additional decay of the atomic dipole. This indicates that $\gamma^{-1}<10^{-7}$ s. An attempt to force a tighter constraint on γ^{-1} by increasing the Rabi frequency will face serious problems as the dynamics for the atom under very intense fields is not well described by the simple Hamiltonian of Eq. (5.7) (among other problems the rotating-wave approximation will not be valid). Thus laser spectroscopic tests of the generalized evolution equation will be very difficult, though perhaps not impossible.

[1] H. D. Zeh, Found. Phys. **1**, 69 (1970).
[2] G. Casati and L. Molinair, Prog. Theor. Phys. Suppl. No. **98**, 287 (1989); B. C. Sanders and G. J. Milburn, Z. Phys. B **77**, 497 (1989).
[3] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982).
[4] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[5] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990).
[6] L. Diosi, Phys. Rev. A **40**, 1165 (1989).
[7] G. C. Ghirardi, R. Grassi, and A. Rimini, Phys. Rev. A **42**, 1057 (1990).
[8] J. Ellis, S. Mohanty, and D. Nanopoulos, Phys. Lett. B **221**, 113 (1989).
[9] J. Ellis, S. Mohanty, and D. V. Nanopoulos, Phys. Lett. B **235**, 305 (1990).
[10] C. M. Caves and G. J. Milburn, Phys. Rev. D **36**, 5543 (1987).
[11] D. F. Walls, M. J. Collett, and G. J. Milburn, Phys. Rev. D **32**, 3208 (1985).
[12] D. Bedford and D. Wang, Nuovo Cimento **26**, 313 (1975).
[13] F. Haake, S. M. Tan, and D. F. Wall, Phys. Rev. A **40**, 7121 (1989).
[14] G. J. Milburn, Phys. Rev. A **36**, 744 (1987).
[15] A. Barchielli, Nuovo Cimento B **74**, 113 (1983).
[16] B. C. Sanders, Phys. Rev. A **40**, 2417 (1989).
[17] A. J. Leggett, S. Chavkravorky, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[18] D. F. Walls and G. J. Milburn, Phys. Rev. A **35**, 2403 (1985).
[19] G. Badurek, H. Rauch, and D. Tuppinger, in *New Techniques and Ideas in Quantum Measurement Theory* (The New York Academy of Sciences, New York, 1986).
[20] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).