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HIGHER PRICES FOR ALL CONSUMERS;
HIGHER REVENUES FOR WHOM?**

By

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Introducing CO₂ Allowances, Higher Prices for All Consumers; Higher Revenues for Whom?

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Abstract

Introducing a ceiling on total carbon dioxide (CO₂) emissions and allowing polluting industries to buy and sell permits to meet it (known as a cap-and-trade system) affects investment strategies, generation quantities, and prices in electricity markets. In this paper we analyze these effects under the assumption of perfect competition and make a comparison with another potential way of reducing CO₂ emissions, namely a fixed carbon tax charged per unit emission. We deal with an energy only market and model it as a two-stage game where capacities are installed in the first stage and production takes place in the future spot market. For a stylized version of this model (with no network effects and deterministic demand), we show that at the equilibrium either one or a mixture of two technologies is used. Such a mixture consists of a relatively clean and a relatively dirty technology. In the absence of a ceiling on total emissions, marginal operating costs of different technologies form a fixed merit order; that is, the marginal costs are ordered in an ascending fashion. Based on the observed demand, this fixed merit order is used to determine the total number of technologies used so that all demand is satisfied. We show that, as long as there is enough capacity in the system, when a fixed maximum allowance level is introduced, different demand levels impose different prices for a unit of emission allowance, and consequently there is no fixed merit order on the technologies. Therefore, for different levels of observed demand one can find a different optimal mixture. We develop an algorithm for finding the induced optimal mixture in a systematic way. We show that the price of electricity and the price of allowances increase as the maximum allowance level decreases. When, in comparison, a fixed tax is charged for the emissions, the merit order is fixed for all demand levels and the first technology in the merit order is the only generating unit. By means of a numerical study, we consider a more general version of the model with stochastic demand and observe that a broader mixture of technologies is used to satisfy the uncertain demand. We show that if there is a shortage of transmission capacity in the system, only introducing financial incentives and instruments (such as taxation or a cap-and-trade system) neither is sufficient to curb CO₂ levels nor

necessarily induces investment in cleaner technologies.

Keywords: investment modeling in electricity markets, energy policy, carbon tax, emission allowances, perfect competition equilibrium

JEL code: C61, C63, H23, Q58

1 Introduction

These days, policy makers and businesses are setting goals in order to reduce the emission of carbon dioxide (CO_2). Especially power generators in electricity markets emit high levels of CO_2 . Since investments in polluting technologies such as coal are most profitable, profit-maximizing firms do not have the right incentives to invest in cleaner alternatives.

Encouraging firms to invest in cleaner technologies can be done by implementing financial incentives. We consider two actions governments may take to accomplish such incentives. One is imposing a maximum allowance level on the total emissions by power generators. Firms buy and trade permits on a secondary market, which results in a price for emission allowances that should be paid when allowances are scarce. This is what is called a cap-and-trade system. Another way to give firms financial incentives to invest in cleaner alternatives is to charge a fixed tax per unit emission. Our goal is to analyze the effects of introducing these incentives on investment and production quantities, and on consumer prices. Our analysis will be done in two parts, the first part focussing on the deterministic demand setting in order to get more stylized and analytical results, and the second part dealing with stochastic demand as to derive results that are closer to reality.

In order to analyze investments in an electricity market, a suitable framework is a two-stage game between firms where investments take place at the first stage, and production and dispatching to consumers at the second stage. Several two-stage models for electricity investments are available in the literature, and they can basically be separated into two streams, one dealing with imperfect competition (Murphy and Smeers (2005), Ralph and Smeers (2006), and Hu and Ralph (2007)) and the other dealing with perfect competition (Neuhoff et al. (2005), Ehrenmann and Smeers (2008), Zhao et al. (2010), and Gürkan et al. (2012)). When there are small number of firms competing, an imperfect oligopolistic framework is believed to be a better representation of reality. On the other hand, when market power is successfully mitigated by regulators, a perfect competition framework is better suited. In addition, from a modeling and analyzing perspective, perfect competition first and foremost presents a benchmark for imperfect competition models and leads to more analytically tractable results; that is, assuming imperfect competition in a two-stage framework could result in an equilibrium problem with equilibrium constraints (EPEC), which is known to often be analytically intractable. We thus choose to focus on a perfectly competitive electricity market and will use and expand the model as presented by Gürkan

et al. (2012). Their analysis specifically focusses on capacity investments and resource adequacy in both a deterministic and a stochastic setting, which poses a good starting point for incorporating and subsequently analyzing CO₂ regulation. For perfectly competitive electricity markets, emission allowances have been analyzed before in for example Zhao et al. (2010) and Neuhoff et al. (2005). However, these papers put more emphasis on the initial allocation of allowances and the effects thereof. In addition, the model in Zhao et al. (2010) is more general and is therefore used for deriving mostly numerical results, while we consider a more stylized framework in order to derive analytical results. The model in Neuhoff et al. (2005) is more stylized, but it takes emission allowance prices as given. Instead, we consider an endogenously determined allowance price that is determined by the market. Finally, Ehrenmann and Smeers (2008) deal with a perfect competition model including CO₂ emission allowances and endogenous allowance prices. The effects of different price caps in case of demand curtailment and of uncertainty in fuel prices and environmental policies are studied. While we adopt their way of modeling emission allowances, we will put more emphasis on the effects of the policies themselves and rather than taking fuel prices and policies as uncertain, we are going to deal with uncertain demand.

We first deal with a deterministic version of the model presented in Gürkan et al. (2012) and extend the model to include cap-and-trade and taxation. We show that under either policy the two-stage game can be reduced to a single optimization problem, similar to what has been shown for the original problem. For a stylized version of the single optimization problem we then analyze equilibria under both cap-and-trade and taxation. We make two simplifying assumptions. We consider a single node and assume that each firm is producing with a single and unique technology. Having a single node means that we can ignore network limitations that may affect the tractability of the results. Second, we assume that there is an order on the technologies; that is, we can order the technologies from lowest to highest marginal cost and assume that the cheapest technology is the most polluting, the second cheapest is the second most polluting, and so on. This assumption is both realistic, and, as we show, can be made without loss of generality. These assumptions allow us to systematically analyze the direct effects of cap-and-trade and taxation on investments. These effects on investments can best be explained via the notion of merit order. At the beginning of a period firms invest in certain technologies; this is the first stage. Then, at the second stage, firms use the installed capacities to generate power; a transmission system operator (TSO) will buy the power and dispatch it to the demand nodes. Power is dispatched to the demand nodes according to a merit order. A merit order is a sequence of technologies based on their marginal costs in an ascending fashion. When power is dispatched, power from the first technology in the merit order will be used until all its capacity is used up. Then, the next technology in the merit order will be used, and so on. That way a number of technologies will be used to satisfy demand. In each node, the market price is then set by the technology producing with the highest marginal cost. Regulation via either a cap-and-trade system or a fixed tax affects marginal costs since it comes with an additional cost

per unit production. Obviously, these marginal cost changes may affect the merit order and hence the dispatching order. Production quantities and hence investment decisions will change accordingly.

In case of cap-and-trade firms will have to pay the unit allowance price per unit emission. This allowance price is determined by the market, and as such is dependent on both the level of demand and the maximum emission allowance level. Therefore, marginal costs and hence the merit order changes with a change in demand or allowance level. We say that the merit order is not fixed; this analytical result coincides with numerical evidence found in Zhao et al. (2010), who carry out a numerical study and find that the merit order changes for certain (high) allowance prices. Since in our analysis we find the allowance price to be increasing when the allowance level goes down or when demand goes up, this indeed leads to a non-fixed merit order. We show that in our simplified setting either one or two technologies are first in a merit order, for given demand and allowance level. Three cases can be distinguished: First, the dirtiest and cheapest technology can satisfy the demand without violating the (relatively low) allowance level. This technology will be the first in the merit order and hence the only technology used at the market equilibrium. Total emissions will be below the allowance level resulting in the allowance price to be zero (free allowances). Second, even the cleanest firm cannot satisfy the demand while meeting the (very strict) allowance level. The most expensive technology will be first in the merit order and hence the only technology used. Electricity demand will not be satisfied and the electricity price will be set at a price cap. Third, when none of the first two cases occur, a combination of a relatively cheap and dirty and a relatively expensive and clean technology will be first in the merit order. The allowance price is then set in such a way that marginal costs of these two technologies are equal. We develop an algorithm to find the optimal technology mixture and the resulting allowance price in a systematic way. Using the algorithm, we show that electricity prices and the price for emission allowances increases when the emission allowance level decreases.

In case of a fixed tax, the merit order is fixed in the sense that it does not depend on the demand or the allowance level. Either one or two technologies are first in the merit order, which can easily be found by comparing effective marginal costs. A special case is when the taxation is set equal to the optimal unit allowance price that was found in case of cap-and-trade for a given allowance level. We find that multiple market equilibria exist, of which some do not satisfy the allowance level. Finally, we characterize technologies that will never be first in the merit order; that is, technologies for which there is no level of taxation such that it becomes the cheapest. We show that this, in turn, implies that those technologies will never be in the optimal mixture in case of cap-and-trade either.

After analyzing the effects on firms, we take a look at the extent to which costs for CO₂ emissions are passed through to consumers. Several results for perfect competition with inelastic demand have been shown in the literature; see for example Bonacina and Gullí (2007) and Chen et al. (2008), where it is argued that there is a 100% cost pass-through to consumers. Contrary to our study, they take the

price of emission allowances as exogenous to the model and in addition focus on the short-run without considering investment strategies. Even though an endogenously determined allowance price results in a different merit order for different demand levels, we show that both endogenous prices and the two-stage nature of the model have no effect on the cost pass-through rate as long as demand is not curtailed; that is, the cost pass-through is still 100%. If demand is curtailed we show that the pass-through even exceeds 100%.

The second part of our analysis deals with the two-stage investment model with stochastic exogenous demand. Demand is unknown to firms at the first stage and will be revealed at the second stage. We can now interpret the first stage as the long-run; that is, investment decisions are made once every period, for example a year, while not knowing the future demands. The second stage can be seen as the short-run, where each day there is a demand realization. Each day a second stage problem is solved, while investment decisions are made based on the expected outcome of these daily realizations. To the model as presented by Gürkan et al. (2012), we again add the cap-and-trade and fixed taxation. As allowances are typically set for a certain period rather than on a daily basis, the maximum emission allowance is going to be imposed at the first stage. As stochastic programs are obscured by the large dimension of the problem, instead of an analytical study we carry out a numerical study using sampling. We propose the sampled versions of the problem in the form of a large mixed complementarity problem (MCP). Using the PATH MCP-solver we can, for a small network, derive and analyze numerical results. In particular, we focus on the adequacy of cap-and-trade and taxation in the presence of limited network capacity.

For our numerical study we consider a small network with three supplying firms producing with either coal, which is relatively cheap and polluting, combined cycle gas turbine (CCGT), which is the cleanest available conventional technology we consider, and open cycle gas turbine (OCGT), which has the lowest investment cost. A key result is that under demand uncertainty we see broader technology mixtures than in the deterministic case. OCGT is in the mixture as the peak load technology, whereas coal and CCGT are used as base load technologies. When a cap-and-trade system is implemented, we see that the tighter the allowance level, the more coal is replaced by the cleaner CCGT. In case of a fixed tax we observe that for low levels mainly coal is used, up to a certain threshold tax for which we see a shift to CCGT. The implications of limited transmission capacity in combination with government regulation are the following. In case of a maximum emission allowance level, limited transmission capacity may induce cleaner mixtures in case emission allowances are scarce. However, it does not necessarily induce investments in cleaner technologies, since limited transmission capacity may block such investments. Hence, investments in network capacity may be necessary to achieve the goal of motivating investment in cleaner technologies. In case of a fixed tax per unit emission we find that for higher tax levels a dirty technology is replaced by a cleaner technology. A network capacity may put a limit on this replacement. Therefore, it may be necessary to invest in network capacity in order to curb CO₂ levels.

Finally, we establish a relation between the optimal outcome in case of cap-and-trade and the optimal outcome in case of taxation. We show that for a special case of taxation, that is, taking the taxation level equal to the optimal price of emission allowances derived in the cap-and-trade model, we either find the exact same solution, or we find multiple solutions of which the cap-and-trade solution is one. In case there are multiple solutions, we see a trade-off between minimizing pollution and maximizing the regulator's surplus. Some of the solutions violate the maximum emission allowance level, indicating that when the optimal emission allowance price is set as a fixed tax, in the absence of a cap-and-trade system there is no way to enforce firms to choose the solution with minimal pollution.

The rest of the paper is organized as follows. We begin with the introduction of the two-stage investment model with deterministic exogenous demand in Section 2. We sequentially reduce it to a single optimization problem. Section 3 introduces a stylized version of the model. We then derive our main findings concerning the effects of government regulations in the deterministic setting. In addition, an algorithm that finds the optimal mixture of technologies in case of a maximum emission allowance level is developed. In Section 4 we introduce the model with stochastic demand and provide our numerical study. Section 5 concludes.

2 The Investment Model - Deterministic Exogenous Demand

We deal with a perfectly competitive electricity market with deterministic exogenous demand; under perfect competition firms cannot exert market power and act as price takers. We discuss the basics of the electricity market and give an overview of the electricity market investment model as presented in Gürkan et al. (2012). Then, as an extension, we include either an emission constraint or a fixed carbon tax imposed by the environmental regulator.

The electricity market consists of a grid of supply and demand nodes connected by transmission lines. Typical to such an electricity network, compared to other networks treated in the literature, is first of all the non-storability of electricity. Secondly, power transmission between two nodes affects the capacities on all transmission lines in the network in accordance with Kirchhoff's voltage law; see for example Chao et al. (2000). At supply nodes, electricity producing firms are located, whereas consumers with a fixed exogenous demand are located at demand nodes. Decisions in the market are taken in two stages. At the first stage firms maximize their profits while choosing for each of their supply nodes the production capacities in the available technologies. All firms are assumed to take these investment decisions simultaneously without knowing the decisions of other firms and their effect on the electricity price. First stage profits depend on the equilibrium outcome of the second stage, where prices and production quantities are determined. At the second stage, firms determine their optimal production

quantity given their investment capacities from the first stage as to maximize their second stage profits. In addition, a transmission system operator (TSO) owning the electricity grid is taking care of the transmission of power, while maximizing its own profits. Finally, the market is cleared by means of market clearing conditions imposing that in each node supply of electricity should cover demand, and imposing a price cap in case of demand curtailment.

In addition, there is an environmental regulator that tries to curb and prevent high levels of CO₂ emissions in the electricity market. We focus on two main financial instruments available to such a regulator. One option is to impose a maximum allowance level for the total emissions. For each unit of CO₂ firms emit they should possess an allowance. Allowances can be traded on a secondary market. The "correct" price of an allowance depends on the number of allowances available and the demand for allowances. As soon as the total emissions hit the maximum allowed level, the price of the allowance becomes positive; otherwise it is zero. An alternative way for reducing the total emissions is to impose a fixed carbon tax per unit emission. With both instruments, the environmental regulator aims to reduce the production by the more polluting technologies and motivate firms to produce more with cleaner technologies, eventually inducing higher investment levels in cleaner technologies.

We call the resulting two-stage game containing all firms, the TSO, the market clearing conditions, and the environmental regulator, a perfect competition equilibrium problem. "Equilibrium" emphasizes that the firms optimize their own objective functions, and that their individual optimization problems are tied together by the market clearing conditions. When we are at a so-called perfectly competitive equilibrium, for none of the firms it is profitable to deviate.

A suitable mathematical framework for modeling the perfectly competitive electricity market is presented in Gürkan et al. (2012). However, that model does not include an environmental regulator or CO₂ regulation. In the next section, we introduce the two-stage game as presented in Gürkan et al. (2012) and incorporate the environmental regulator's problem. As shown in Gürkan et al. (2012), the original two-stage game can be reduced to a single optimization problem. The same reduction can be done when adding a cap-and-trade system or imposing taxation, as we will show in Section 2.2.

2.1 Introducing the Two-Stage Game

The sets, parameters, and variables we use are given below.

Sets:

- N : the set of demand nodes
- G : the set of firms
- I_g : the set of supply nodes of firm $g \in G$
- I : the set of all supply nodes ($I := \cup_g I_g$)
- K_g : the set of technologies of firm $g \in G$
- K : the set of all technologies ($K := \cup_g K_g$)
- L : the set of electricity transmission lines connecting nodes in the network.

Parameters:

- c_{ik}^g : unit production cost of firm g at supply node $i \in I_g$ for technology $k \in K_g$
- κ_{ik} : unit investment cost at supply node $i \in I$ for technology $k \in K$
- d_n : demand at demand node $n \in N$
- $PTDF_{l,j}$: power transmitted through line $l \in L$ due to one unit of power injection into node $j \in N \cup I$
- h_l : capacity limit of line $l \in L$
- $VOLL$: value of unserved energy or lost load
- E : total CO₂ emission allowed by the environmental regulator
- e_k : units of CO₂ emitted per unit production with technology $k \in K_g$

Variables:

- x_{ik}^g : generation capacity investment of firm g for technology $k \in K_g$ at supply node $i \in I_g$
- y_{ik}^g : quantity of power generated by firm g at supply node $i \in I_g$ by using technology $k \in K_g$
- f_j : net power flow dispatched by the TSO to node $j \in N \cup I$
- δ_j : unserved demand at node $j \in N \cup I$
- p_j : electricity price at node $j \in N \cup I$
- μ : unit allowance price.

For $g \in G$ we write $x^g = (x_{ik}^g)_{i \in I_g, k \in K_g}$ and $y^g = (y_{ik}^g)_{i \in I_g, k \in K_g}$, the vectors containing investment and production quantities, respectively, of firm g in all its technologies in all its supply nodes.

We next introduce the first and second stage problems. At the first stage, each firm $g \in G$ simultaneously decides on its optimal investment quantities x^g in all its available technologies in all its supply nodes. Per unit investment in technology $k \in K$ in supply node $i \in I$, firms pay investment cost κ_{ik} . Firms determine their optimal investment quantities by maximizing their optimal second stage profits that are dependent on their investment quantities, minus their first stage investment costs. The second stage profit per unit production of firm $g \in G$ with technology $k \in K_g$ in supply node $i \in I_g$ consists of the market price of electricity, p_i , minus the unit production cost c_{ik}^g , and minus the price paid for

emission allowances, $e_k\mu$. The objective function for firm $g \in G$ at stage one is

$$\max_{x^g \geq 0} \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c_{ik}^g - e_k\mu)y_{ik}^g(x^g) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_{ik}x_{ik}^g. \quad (1)$$

Here $y_{ik}^g(x^g)$ is the optimal production quantity of firm g at supply node $i \in I_g$ with technology $k \in K_g$ at the second stage. p_i , $i \in I$, and μ are taken as parameters, since firms are behaving as price takers and are not aware that by changing their investments they may influence the price of electricity and the allowance price. Both will be determined at the second stage as a result of market clearing conditions and the emission allowance constraint, as we explain below.

At the second stage, firms treat the investment quantities as parameters. Each firm $g \in G$ determines its production quantities y^g by optimizing

$$\begin{aligned} \Pi_g(x^g) := \max_{y^g \geq 0} \quad & \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c_{ik}^g - e_k\mu)y_{ik}^g \\ \text{s.t.} \quad & y_{ik}^g \leq x_{ik}^g \quad (\beta_{ik}^g) \quad \forall i \in I_g, k \in K_g \end{aligned} \quad (2)$$

where β_{ik}^g is the shadow price associated with the capacity constraint, representing the scarcity rent of technology $k \in K_g$ at supply node $i \in I_g$. The produced power is then dispatched by the transmission system operator (TSO) from the supply nodes to demand nodes. The TSO maximizes its profit from transmitting power while taking into account the network capacity. The electricity network consists of a set of transmission lines L in which each line $l \in L$ runs from one node to another node. The amounts transmitted, that is, the net flows into or out of each node $j \in N \cup I$, are denoted by f_j . $f_j > 0$ represents a flow into node j , whereas $f_j < 0$ represents a flow out of node j . For each unit of power flow into node j , some amount of power, given by the coefficient $PTDF_{l,j}$, is transmitted along transmission line $l \in L$. A power injection in one node typically affects the flows on all transmission lines (either positively or negatively). h_l is the capacity on transmission line $l \in L$, and total net power flow (which can be either negative or positive) on each line $l \in L$ must be between $-h_l$ and h_l . If there is limited capacity on some lines (h_l is finite for some $l \in L$), we call the network a capacitated network. The TSO's problem is formulated as follows:

$$\begin{aligned} \max_f \quad & \sum_{j \in N \cup I} p_j f_j \\ \text{s.t.} \quad & \sum_{j \in N \cup I} f_j = 0 \quad (\rho) \\ & h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^+) \quad \forall l \in L \\ & h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^-) \quad \forall l \in L. \end{aligned} \quad (3)$$

The first constraint, with corresponding dual variable ρ , is the flow balance constraint; that is, the

total amount the TSO buys from firms should equal the total amount the TSO dispatches to demand nodes. The second and third constraints take into account the limited positive and negative transmission capacity in the network and have dual variables λ_l^+ , $l \in L$, and λ_l^- , $l \in L$, respectively.

The environmental regulator determines the level of maximum emission allowance, E , which should be satisfied by the entire market and which is announced to the firms in advance. The price of emission allowances, μ , is then determined by the market. As long as the maximum allowance level is not reached, the price of an emission allowance will be zero; as soon as emissions hit the ceiling, μ will become positive as to create an incentive for firms to switch to cleaner technologies. This leads to the following complementarity condition:

$$0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g \quad \perp \quad \mu \geq 0. \quad (4)$$

Finally, there are two types of market clearing conditions. The market price of electricity is determined by the first type, which balances supply and demand in each node. In each demand node $n \in N$, where production is defined as $y_{nk}^g = 0$, $g \in G, k \in K$, the net flow f_n into the node should be at least the demand d_n , unless there is unsatisfied demand. In each supply node $i \in I$, where demand is defined as $d_i = 0$, the net flow f_i will in general be negative, meaning it is a flow out of node i . This flow can be at most equal to the total production in node i . Hence, for each node $j \in N \cup I$ we have a constraint on the flows, balancing supply and demand. Nodal prices are determined perpendicular to each of these constraints. A second type of condition puts a cap on the price in each node and is known as VOLL pricing in the literature; see, for example, Stoft (2002), or alternatively Ehrenmann and Smeers (2008). Whenever demand cannot be satisfied at node $j \in N \cup I$, unsatisfied demand δ_j will be positive. Then, the price of electricity at node j is set at $VOLL$, the value of lost load. $VOLL$ is a relatively large number; that is, in numerical experiments it is typical to assume $VOLL = 10.000$, whereas the regular nodal prices usually lie between 30 and 80. The market clearing conditions look as follows:

$$\begin{aligned} 0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g + \delta_j + f_j - d_j \quad \perp \quad p_j \geq 0 \quad \forall j \in N \cup I \\ 0 \leq VOLL - p_j \quad \perp \quad \delta_j \geq 0 \quad \forall j \in N \cup I. \end{aligned} \quad (5)$$

As mentioned earlier, an alternative way of reducing the amount of CO₂ emitted by energy companies, is to tax firms per unit emission without imposing a bound on the total amount of CO₂ emitted. In such a system the environmental regulator fixes a level of taxation, say $\bar{\mu}$. In the model the (optimal) price of emission allowances, μ , should be replaced by the fixed parameter $\bar{\mu}$ and the emission constraint (4) should be omitted. In the next section we analyze the two models in further detail.

2.2 Reduction to a Single Optimization Problem

In Gürkan et al. (2012), it is shown that the problem of finding a perfect competition equilibrium, excluding the emission constraint (4), between the firms and the TSO can be written as a single optimization problem. This result can be extended to the models which include either an emission constraint or taxation imposed by the environmental regulator. We do not discuss the derivations in detail here (see Gürkan et al. (2012)); however, we briefly elaborate on the results that are relevant to us.

First we show that there exists a single optimization problem that simultaneously solves the firms' second stage problems (2) and the TSO's second stage problem (3), under the emission constraint (4) and the market clearing conditions (5). As a result, we are left with a single optimization problem at the second stage. Then we show that the first and the second stage problems together can be written as a single optimization problem. Having been reduced to a single optimization problem, a perfect competition equilibrium can be easily found by using standard optimization techniques.

In order to show that there exists a single optimization problem that solves all second stage problems simultaneously, we first write the KKT optimality conditions for all the second stage problems introduced in Section 2.1 for given $x = (x^g)_{g \in G}$:

$$\begin{aligned}
0 \leq \beta_{ik}^{*g} - p_i^* + c_{ik}^g + e_k \mu^* &\perp y_{ik}^{*g} \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq x_{ik}^g - y_{ik}^{*g} &\perp \beta_{ik}^{*g} \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^* &\perp \delta_j^* \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g} + \delta_j^* + f_j^* - d_j &\perp p_j^* \geq 0 & \forall j \in N \cup I \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^* &\perp \lambda_l^{*+} \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^* &\perp \lambda_l^{*-} \geq 0 & \forall l \in L \\
0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^{*g} &\perp \mu^* \geq 0 \\
p_j^* - \rho^* + \sum_{l \in L} PTDF_{l,j} (\lambda_l^{*+} - \lambda_l^{*-}) = 0 & & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^* = 0. & &
\end{aligned} \tag{6}$$

A point which satisfies these KKT conditions (6) also solves (2) and (3) under the constraints (4) and (5).

The KKT conditions (6) correspond to a single optimization problem, referred to as the Optimal

Power Flow Problem (OPF):

$$\begin{aligned}
Z(x) &:= \min_{y, f, \delta} \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} c_{ik}^g y_{ik}^g + VOLL \sum_{j \in N \cup I} \delta_j \\
s.t. \quad & \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g + \delta_j + f_j \geq d_j & (p_j) \quad \forall j \in N \cup I \\
& \sum_{j \in N \cup I} f_j = 0 & (\rho) \\
& h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^+) \quad \forall l \in L \\
& h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^-) \quad \forall l \in L \\
& x_{ik}^g - y_{ik}^g \geq 0 & (\beta_{ik}^g) \quad \forall g \in G, i \in I_g, k \in K_g \\
& E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g \geq 0 & (\mu) \\
& y_{ik}^g \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
& \delta_j \geq 0 & \forall j \in N \cup I.
\end{aligned} \tag{7}$$

The basic idea of this derivation was originally introduced in Boucher and Smeers (2001) for a game between firms, consumers, and TSO, and can be used in more general settings such as the one we consider here. Solving the OPF problem clearly results in the optimal solution for the firms' problems at the second stage, the TSO's problem, and the environmental regulator's problem. The resulting optimal solution is a perfectly competitive equilibrium of the market at the second stage, since none of the players will have an incentive to deviate.

Next, we briefly outline how the problem in stage one, consisting of (1) for each firm, together with the OPF problem (7) at stage two, can be written as a single optimization problem. When demand is assumed to be deterministic, Lemma 1.1 of Gürkan et al. (2012) states that the optimal investment amount x_{ik}^{*g} is equal to the optimal production amount y_{ik}^{*g} for all $g \in G$, $i \in I_g$, $k \in K_g$, and Lemma 1.2 of Gürkan et al. (2012) states that for x_{ik}^{*g} to be positive at the equilibrium, β_{ik}^{*g} should be equal to κ_{ik} . These results are employed in Lemma 1.3 of Gürkan et al. (2012) to show that a point satisfying a particular set of KKT conditions is an equilibrium solution of the two-stage game consisting of (1) and (7). When we use the corresponding result in our setting, the resulting set of KKT conditions is the

following:

$$\begin{aligned}
& 0 \leq \kappa_{ik} - p_i^* + c_{ik}^g + e_k \mu^* \perp x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
& 0 \leq VOLL - p_j^* \perp \delta_j^* \geq 0 \quad \forall j \in N \cup I \\
& 0 \leq \sum_{g \in G} \sum_{k \in K_g} x_{jk}^{*g} + \delta_j^* + f_j^* - d_j \perp p_j^* \geq 0 \quad \forall j \in N \cup I \\
& 0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^* \perp \lambda_l^{*+} \geq 0 \quad \forall l \in L \\
& 0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^* \perp \lambda_l^{*-} \geq 0 \quad \forall l \in L \\
& 0 \leq E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k x_{ik}^{*g} \perp \mu^* \geq 0 \\
& p_j^* - \rho^* + \sum_{l \in L} PTDF_{l,j} (\lambda_l^{*+} - \lambda_l^{*-}) = 0 \quad \forall j \in N \cup I \\
& \sum_{j \in N \cup I} f_j^* = 0.
\end{aligned}$$

This set of KKT conditions corresponds to the following single optimization problem:

$$\begin{aligned}
& \min_{x, f, \delta} \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^g + \kappa_{ik}) x_{ik}^g + VOLL \sum_{j \in N \cup I} \delta_j \\
& s.t. \quad \sum_{g \in G} \sum_{k \in K_g} x_{jk}^g + \delta_j + f_j \geq d_j \quad (p_j) \quad \forall j \in N \cup I \\
& \quad \sum_{j \in N \cup I} f_j = 0 \quad (\rho) \\
& \quad h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^+) \quad \forall l \in L \\
& \quad h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 \quad (\lambda_l^-) \quad \forall l \in L \\
& \quad E - \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k x_{ik}^g \geq 0 \quad (\mu) \\
& \quad x_{ik}^g \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
& \quad \delta_j \geq 0 \quad \forall j \in N \cup I.
\end{aligned} \tag{8}$$

Note that (8) simultaneously solves the optimization problems (1), (2), and (3), while taking the emission constraint (4) and the market clearing conditions (5) into account. As a consequence, the optimal solution (x^*, δ^*, f^*) of (8) results in optimality for all firms at both stages.

We now turn our attention to the fixed tax model. As mentioned earlier, one needs to replace the (optimal) price of emission allowances μ by the fixed parameter $\bar{\mu}$, and omit the emission constraint. It then turns out that the two-stage game can again be written as a single optimization problem for this

model. Without going into details, the resulting optimization problem is:

$$\begin{aligned}
\min_{x,f,\delta} \quad & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^g + \kappa_{ik} + e_k \bar{\mu}) x_{ik}^g + VOLL \sum_{j \in N \cup I} \delta_j \\
s.t. \quad & \sum_{g \in G} \sum_{k \in K_g} x_{jk}^g + \delta_j + f_j \geq d_j & (p_j) \quad \forall j \in N \cup I \\
& \sum_{j \in N \cup I} f_j = 0 & (\rho) \\
& h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^+) \quad \forall l \in L \\
& h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j \geq 0 & (\lambda_l^-) \quad \forall l \in L \\
& x_{ik}^g \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
& \delta_j \geq 0 & \forall j \in N \cup I.
\end{aligned} \tag{9}$$

3 Equilibrium Analysis

The qualitative equilibrium analysis of the underlying two-stage game is obscured by the intractability of the problem in its current form because of the network effects due to the underlying network topology, the transmission line capacities, and the associated PTDFs. In order to understand the direct effects of the emission constraint or taxation on the decisions of the firms, we work with the following simplifying assumptions. There is a single node with a given demand d and with n producing firms. Each firm $g \in G$ possesses a single and unique technology $k \in K$. Hence, we compromise indices g, i , and k by simply k and denote by $K = \{1, \dots, n\}$ the set of all firms or technologies. In effect, having a single node means that we are focusing on a network with unlimited transmission capacities and that we can omit the flows and the PTDF-constraints. In addition, we write $a_k := c_k + \kappa_k$, the total cost per unit production in technology k and we make the following assumption on the parameters:

$$a_1 < a_2 < \dots < a_n \quad \text{and} \quad e_1 > e_2 > \dots > e_n.$$

That is, the technologies are ordered such that technology 1 is the cheapest and the dirtiest, and technology n is the most expensive and the cleanest technology. This assumption is reasonable, since most dirty technologies have relatively low long-run marginal cost, but have a relatively high level of CO₂ emission per unit production; later we will argue that this assumption can in fact be made without loss of generality, see Remark 1.

Let s be the slack variable in the emission constraint. The stylized version of model (8), with only one node and hence no network effects, is summarized as:

$$\begin{aligned}
\min_{x, \delta, s} \quad & \sum_{k=1}^n a_k x_k + VOLL\delta \\
s.t. \quad & \sum_{k=1}^n x_k + \delta = d \quad (p) \\
& -\sum_{k=1}^n e_k x_k - s = -E \quad (\mu) \\
& x_k \geq 0 \quad \forall k \in K \\
& \delta, s \geq 0.
\end{aligned} \tag{10}$$

Note that minus signs appear on both sides of the emission constraint. Obviously, it is possible to multiply this constraint by -1 , but this results in a non-positive dual variable. However, the current dual variable actually gives the "correct" price of the unit emission allowance. Since that will be more tricky to interpret, we will work with the current formulation (10).

The stylized version of the fixed tax model (9) is obtained in a similar way:

$$\begin{aligned}
\min_{x, \delta} \quad & \sum_{k=1}^n (a_k + e_k \bar{\mu}) x_k + VOLL\delta \\
s.t. \quad & \sum_{k=1}^n x_k + \delta = d \quad (p) \\
& x_k \geq 0 \quad \forall k \in K \\
& \delta \geq 0.
\end{aligned} \tag{11}$$

Next, we elaborate on the perfect competition equilibrium in both models (10) and (11). In Section 3.1, we derive that in case of a maximum allowance level one or two firms will be producing at the equilibrium. This will have an important consequence. When there is enough capacity in the system, different demand levels will impose different prices for a unit of emission allowance and there will not be a fixed merit order of the technologies. An algorithm for finding the optimal mixture in the equilibrium in a systematic way is proposed in Section 3.1.1. A proof showing that the algorithm finds the optimal solution, is included in Appendix A. In case of a fixed tax, there will be a fixed merit order and only one firm will be producing at the equilibrium; this and its implications are dealt with in Section 3.2. Section 3.3 provides a characterization of the technologies that can never be the first in the merit order; hence those technologies will permanently be dominated by other technologies. Finally, Section 3.4 analyzes the effects of cap-and-trade and taxation on consumer prices via the concept of consumers' surplus.

3.1 Characterizing the Equilibrium with a Cap on Total Emissions

As there are only two constraints in the stylized model (10), there will be only two basic variables. Therefore, at most two technologies will be producing at the equilibrium. Obviously, without an emission

constraint (that is, when $E = \infty$) only the cheapest technology will be used. On the other hand, when the emission constraint is very tight (that is, when E is extremely low), only the cleanest technology can be used. In all other cases, two technologies, a relatively cheap one and a relatively clean one, will be contained in the optimal mixture.

In order to find which technologies will be part of the optimal mix, we divide the set of firms into two sets; the first set, J , contains the relatively cheap and dirty firms, whereas the second set, H , contains the relatively expensive and clean firms. To be more precise, a firm is in J when it cannot satisfy the demand on its own without violating the emission constraint; a firm is in H otherwise. We must then find the cheapest combination of a firm in J and a firm in H , which can together satisfy the emission allowance constraint. Note that, when $J = N$, even the cleanest firm cannot satisfy demand without violating the emission constraint. On the other hand, when $H = N$, all demand can be produced by the cheapest firm without violating the emission constraint. The following proposition summarizes these results formally and lays down a property that the basic variables of the linear program (10) should satisfy at optimality when J and H are both nonempty.

Proposition 3.1. *Suppose that there are n firms with for which it holds that*

$$a_1 < a_2 < \dots < a_n \quad \text{and} \quad e_1 > e_2 > \dots > e_n.$$

Given E and d , define

$$J = \{1, \dots, i\} \quad \text{and} \quad H = \{i + 1, \dots, n\},$$

where $i \in K$ is such that $e_{i+1}d \leq E < e_id$. Then, at the perfect competition equilibrium, that is, at the optimal solution to (10), exactly one of the following holds:

(i) *The cheapest firm is able to satisfy all demand without violating the emission constraint. This is the case when $J = \emptyset$. At the equilibrium, $x_1^* = d$, $x_k^* = 0$ for $k = 2, \dots, n$, $\delta^* = 0$, and $s^* = E - e_1d$.*

or,

(ii) *No firm is able to satisfy all demand without violating the emission constraint. This is the case when $H = \emptyset$. At the equilibrium, $x_n^* = E/e_n$, $x_k^* = 0$ for $k = 1, \dots, n - 1$, $\delta^* = d - \frac{E}{e_n}$, and $s^* = 0$.*

or,

(iii) A combination of a relatively clean and a relatively dirty firm will satisfy demand without violating the emission constraint. This is the case when both J and H are non-empty. Firms $j \in J$ and $h \in H$ produce at the equilibrium if

$$a_k(e_j - e_h) + a_j(e_h - e_k) + a_h(e_k - e_j) \geq 0 \quad \forall k \in \{1, \dots, n\}. \quad (12)$$

At the equilibrium, $x_j^* = \frac{E - e_h d}{e_j - e_h}$, $x_h^* = \frac{e_j d - E}{e_j - e_h}$, $x_k^* = 0$ for $k \neq j, h$, $\delta^* = 0$, $s^* = 0$.

Proof: Let p be the price of electricity and μ the price of unit emission. That is, p and μ are the dual variables associated with the first and second constraint of (10). Then we can write the KKT optimality conditions of the linear program (10) as

$$\begin{aligned} 0 &\leq a_k - p^* + e_k \mu^* \quad \perp \quad x_k^* \geq 0 \quad \forall k \in \{1, \dots, n\} \\ 0 &\leq VOLL - p^* \quad \perp \quad \delta^* \geq 0 \\ 0 &\leq \sum_{k=1}^n x_k^* + \delta^* - d \quad \perp \quad p^* \geq 0 \\ 0 &\leq E - \sum_{k=1}^n e_k x_k^* \quad \perp \quad \mu^* \geq 0. \end{aligned} \quad (13)$$

If $J = \emptyset$, then $e_1 d \leq E$ and therefore x_1 and s are the basic variables in (10) with basis

$$\mathbf{c}_B = \begin{bmatrix} a_1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -e_1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = \mathbf{c}_B B^{-1} = [a_1 \quad 0].$$

The optimal solution is $x_1^* = d$, $x_k^* = 0$ for $k = 2, \dots, n$, $\delta^* = 0$, and $s^* = E - e_1 d$. This is case (i).

If $H = \emptyset$, then $e_n d > E$ and therefore x_n and δ are the basic variables in (10) with basis

$$\mathbf{c}_B = \begin{bmatrix} a_n & VOLL \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -e_n & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = \mathbf{c}_B B^{-1} = [VOLL \quad \frac{VOLL - a_n}{e_n}].$$

The optimal solution is $x_n^* = E/e_n$, $x_k^* = 0$ for $k = 1, \dots, n-1$, $\delta^* = d - \frac{E}{e_n} > 0$, and $s^* = 0$. This is case (ii).

If $J \neq \emptyset$ and $H \neq \emptyset$, then for some $j \in J$, $h \in H$, x_j and x_h are the basic variables in (10) with basis

$$\mathbf{c}_B = \begin{bmatrix} a_j & a_h \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -e_j & -e_h \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} d \\ -E \end{bmatrix}.$$

Hence,

$$[p^* \quad \mu^*] = c_B B^{-1} = \begin{bmatrix} -a_j e_h + a_h e_j & a_h - a_j \\ e_j - e_h & e_j - e_h \end{bmatrix}.$$

The optimal solution is $x_j^* = \frac{E - e_h d}{e_j - e_h}$, $x_h^* = \frac{e_j d - E}{e_j - e_h}$, $x_k^* = 0$ for $k \neq j, h$, $\delta^* = 0$, $s^* = 0$. When (12) holds, that is,

$$a_k(e_j - e_h) + a_j(e_h - e_k) + a_h(e_k - e_j) \geq 0 \quad \forall k \in \{1, \dots, n\},$$

we see that the first condition of (13) is satisfied since

$$0 \leq a_k - \frac{-a_j e_h + a_h e_j}{e_j - e_h} e_k = a_k - p^* + e_k \mu^* \quad \forall k \in \{1, \dots, n\}.$$

One can easily check that the other conditions in (13) are also satisfied, by noting that $j \in J$, $h \in H$, and $e_h d \leq E < e_j d$ holds. \square

Since determining the sets J and H is straightforward, it is easy to see which case in Proposition 3.1 holds. However, finding a $j \in J$ and $h \in H$ in case (iii) that satisfy (12) is not straightforward. In Section 3.1.1 we give an algorithm for finding the optimal $j \in J$ and $h \in H$. First, we provide an interpretation of the optimal price of emission allowances and the optimal electricity price.

The proof of Proposition 3.1 shows that it is possible to derive both μ^* and p^* explicitly at optimality. One can interpret μ^* as follows. In case (i), μ^* is zero, since one should not pay for an extra unit of CO₂ emission when the emission constraint is not active. In case (ii), since demand is not satisfied, *VOLL* pricing is in effect, and consequently μ^* is relatively high. When E decreases, one produces less with technology n and the unsatisfied demand δ^* increases. Let Δy denote the change in unsatisfied demand and ΔE denote the change in E . Since the new emission constraint should be satisfied, we need to have

$$-e_n \Delta y = \Delta E.$$

Therefore,

$$\Delta y = -\frac{\Delta E}{e_n}.$$

Then, one pay $\Delta y \cdot VOLL$ more and save $\Delta y \cdot a_n$. If we take $\Delta E = -1$, μ^* gives the extra cost for satisfying the new, lower emission allowance level $E + \Delta E$, as long as the optimal basis does not change. In other words, μ^* is equal to the price one pays to substitute the production of firm n by unsatisfied demand for a unit decrease in E :

$$\mu^* = (VOLL - a_n)\Delta y = \frac{VOLL - a_n}{e_n}.$$

In case (iii), μ^* represents the price to pay to substitute the dirty technology by the clean technology when E decreases. Let Δy denote the extra amount of the clean technology that is produced. This volume Δy of the substitution of the dirty technology by the clean technology should satisfy

$$-e_j\Delta y + e_h\Delta y = \Delta E,$$

in order to satisfy the emission constraint. $e_j\Delta y$ is the amount of CO₂ emission saved and $e_h\Delta y$ is the amount of CO₂ emitted instead. Hence, we get

$$\Delta y = \frac{\Delta E}{e_h - e_j}.$$

Due to the lower allowance level, one saves $a_j\Delta y$ and additionally pays $a_h\Delta y$. Again, if we take $\Delta E = -1$, then μ^* represents the price to pay for substituting dirty technology by clean technology as a result of a unit decrease in E :

$$\mu^* = (a_h - a_j)\Delta y = \frac{a_h - a_j}{e_j - e_h}.$$

A similar argument holds for p^* when we perturb demand d . In case (i), firm 1 can satisfy demand without violating the emission constraint. As long as the optimal basis does not change, a unit increase of demand costs $p^* = a_1$. In case (ii), demand is not satisfied; hence, VOLL pricing is in effect and an extra unit of demand causes the unsatisfied demand to increase by one unit. The extra cost involved equals $p^* = VOLL$. In case (iii), p^* represents the extra cost incurred (shared by firms j and h) in order to satisfy an extra unit of demand. In order to do so, they should take care that, with the new production amounts, the total emission does not increase; that is,

$$e_j(\Delta d - \Delta y) + e_h\Delta y = 0,$$

where Δy denotes the extra production by firm h , whereas Δd denotes the extra demand. Hence $\Delta d - \Delta y$

represents the change in firm j 's production. Rewriting gives

$$\Delta y = \frac{\Delta d e_j}{e_j - e_h}.$$

Furthermore, when demand increases with Δd , and given that the basis does not change, the total production cost changes by $a_h \Delta y$ and $a_j(\Delta d - \Delta y)$. If we take $\Delta d = 1$ then

$$p^* = a_j(1 - \Delta y) + a_h \Delta y = \frac{a_h e_j - a_j e_h}{e_j - e_h}.$$

3.1.1 Algorithms for Finding the Equilibrium

Given a maximum emission allowance level E and demand d , we formed sets of technologies $J = \{1, \dots, i\}$ and $H = \{i+1, \dots, n\}$ where $i \in K$ satisfies $e_{i+1}d \leq E < e_i d$. We next discuss two algorithms for finding $j \in J$ and $h \in H$ such that (12) is satisfied. Initially we generate candidate sets \bar{J} and \bar{H} for J and H , respectively. By starting with the smallest non-empty \bar{J} or \bar{H} and expanding or reducing the sets systematically we will reach to the optimal j and h in a finite number of iterations. In each iteration, we find a $j^* \in \bar{J}$ and an $h^* \in \bar{H}$ such that (12) is satisfied. Afterwards, we evaluate whether j^* and h^* belong to the sets J and H , respectively. If yes, the optimal solution is found. If not, the algorithm systematically expands one candidate set and reduces the other.

The forward version of the algorithm first checks whether J or H is empty. If so, we are in case (i) or (ii) of Proposition 3.1, respectively. If not, we let $j^* = 1$ and start with the smallest non-empty candidate set for J , $\bar{J} = \{1\}$. Since J is nonempty, firm 1 cannot satisfy all demand without violating the emission constraint. A cleaner and more expensive firm has to contribute. As a result, the total production cost will increase. The algorithm will then find $h^* \in \bar{H} = \{2, \dots, n\}$ such that this increase in production cost is minimized. By doing this, we guarantee that (12) is satisfied, as shown in the proof that can be found in the appendix. If $h^* \in H$, the optimal solution is found. If not, we have to do another iteration. We first expand \bar{J} by adding $\{j^* + 1, \dots, h^*\}$ and reduce \bar{H} accordingly. Then, we make $j^* = h^*$ and find again the firm in the new candidate set \bar{H} that minimizes the extra cost. The algorithm terminates when $h^* \in H$; that is, we found a $j^* \in J$ and $h^* \in H$ such that (12) is satisfied. The optimal solution to (10) then immediately follows from Proposition 3.1.

One may wonder why we pick $j^* = h^*$ in each new iteration. In the proof of the algorithm we show that if the new j^* is not h^* , then for at least one firm k (12) is violated.

The backward version of the algorithm has the same structure. It first checks if J or H is empty. If not, we let $h^* = n$ and start with the smallest non-empty candidate set for H , $\bar{H} = \{n\}$. Since H is nonempty, firm n is able to satisfy all demand without violating the emission constraint. However, we can reduce the objective function value by letting a more polluting and cheaper firm contribute. The algorithm will find the firm $j^* \in \bar{J}$ that maximizes the reduction of the total cost. Again by ensuring

this, (12) is automatically satisfied for all k . If $j^* \in H$, we can gain even more by letting another firm in $\bar{J} \setminus \{j^*, \dots, h^* - 1\}$ contribute instead of h^* . We hence do another iteration and update \bar{J} by removing $\{j^*, \dots, h^* - 1\}$ and update \bar{H} accordingly. Then we make $h^* = j^*$ and find the firm that maximizes the reduction of the total cost. The algorithm terminates when $j^* \in J$.

Algorithm 1. *The Forward Version*

Step 0:

Given E and d , define $J = \{1, \dots, i\}$ and $H = \{i+1, \dots, n\}$, where $i \in K$ is such that $e_{i+1}d \leq E < e_id$.

Step 1:

If $J = \emptyset$, STOP= 1, Output= $(1, 1)$.

Else if $H = \emptyset$, STOP= 1, Output= (n, n) .

Else STOP= 0, $j^* = 1$, $h^* = 1$.

Step 2:

While STOP= 0 do

Let $\bar{J} = \{1, \dots, h^*\}$, $\bar{H} = N \setminus \bar{J}$, $j^* = h^*$, $h^* = \arg \min_{h \in \bar{H}} \left\{ \frac{a_h - a_{j^*}}{e_{j^*} - e_h} \right\}$;

If $h^* \in H$, $j = j^*$, $h = h^*$, STOP= 1, Output= (j, h) ;

end.

Algorithm 2. *The Backward Version*

Step 0:

Given E and d , define $J = \{1, \dots, i\}$ and $H = \{i+1, \dots, n\}$, where $i \in K$ is such that $e_{i+1}d \leq E < e_id$.

Step 1:

If $J = \emptyset$, STOP= 1, Output= $(1, 1)$.

Else if $H = \emptyset$, STOP= 1, Output= (n, n) .

Else STOP= 0, $j^* = n$, $h^* = n$.

Step 2:

While STOP= 0 do

Let $\bar{H} = \{j^*, \dots, n\}$, $\bar{J} = N \setminus \bar{H}$, $h^* = j^*$, $j^* = \arg \max_{j \in \bar{J}} \left\{ \frac{a_{h^*} - a_j}{e_j - e_{h^*}} \right\}$;

If $j^* \in J$, $j = j^*$, $h = h^*$, STOP= 1, Output= (j, h) ;

end.

Given E, d , and the characteristics of the firms, we can apply either one of the algorithms to find the optimal (j, h) . A proof showing that the Forward algorithm finds the optimal solution is included in Appendix A; clearly one can write a proof for the Backward algorithm in a similar way. Also note that when $\text{Output} = (1, 1)$, firm 1 is the only producer (case (i) of Proposition 3.1) and when $\text{Output} = (n, n)$, firm n is the only producer (case (ii) of Proposition 3.1).

Remark 1. *From the arguments in Appendix A, it becomes clear that the assumption that $a_1 < a_2 < \dots < a_n$ and $e_1 > e_2 > \dots > e_n$ can in fact be made without loss of generality. If a technology would not obey this assumption, it would either dominate another technology or it would be dominated by another technology itself. A technology that is dominated by another technology will never be chosen by the algorithm since it is both more polluting and more expensive than the other one.*

The following proposition establishes the monotonicity relationship between the price of electricity, the price of emission allowances, and the maximum emission allowance level.

Proposition 3.2. *The price of electricity p^* and the price of emission allowances μ^* are weakly increasing as E decreases.*

Proof: Given E , let (j_E, h_E) be the producing firms at the perfect equilibrium. We let E decrease to $\tilde{E} < E$ and consider the effect. Define

$$\tilde{J} = \{1, \dots, \tilde{i}\} \quad \text{and} \quad \tilde{H} = \{\tilde{i} + 1, \dots, n\},$$

where $\tilde{i} \in K$ is such that $e_{\tilde{i}+1}d \leq \tilde{E} < e_{\tilde{i}}d$. Note that $\tilde{H} \subseteq H$; that is, since the emission constraint became tighter, we may need to choose a cleaner firm. Two things may happen.

- (1) $h_E \in \tilde{H}$: (j_E, h_E) is still the optimal mixture; no need for further action. p^* and μ^* remain the same.
- (2) $h_E \notin \tilde{H}$: Apply the Forward Algorithm with $j^* = h_E$, $\tilde{J} = \{1, \dots, h_E\}$, and $\tilde{H} = N \setminus \tilde{J}$ and find

$$h^* = \arg \min_{h \in \tilde{H}} \left\{ \frac{a_h - a_{h_E}}{e_{h_E} - e_h} \right\}.$$

In case this $h^* \in \tilde{H}$, the new optimal solution is found and we can compare the prices of allowances. First notice that, since j_E and h_E were producing at the equilibrium for the allowance level E , (12) holds for all k . In particular, for $k = h^*$ we have

$$a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E}) \geq 0. \quad (14)$$

Using this, we compare the prices of allowances and the prices of electricity:

$$\mu_{\tilde{E}}^* - \mu_E^* = \frac{a_{h^*} - a_{h_E}}{e_{h_E} - e_{h^*}} - \frac{a_{h_E} - a_{j_E}}{e_{j_E} - e_{h_E}} = \frac{a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E})}{(e_{h_E} - e_{h^*})(e_{j_E} - e_{h_E})} \geq 0.$$

Since the denominator and the numerator are positive by the fact that $e_{h^*} < e_{h_E} < e_{j_E}$ and inequality (14), respectively, the last inequality holds. Similarly,

$$p_{\tilde{E}}^* - p_E^* = \frac{e_{h_E} a_{h^*} - e_{h^*} a_{h_E}}{e_{h_E} - e_{h^*}} - \frac{e_{j_E} a_{h_E} - e_{h_E} a_{j_E}}{e_{j_E} - e_{h_E}} = e_{h_E} \frac{a_{j_E}(e_{h_E} - e_{h^*}) + a_{h_E}(e_{h^*} - e_{j_E}) + a_{h^*}(e_{j_E} - e_{h_E})}{(e_{h_E} - e_{h^*})(e_{j_E} - e_{h_E})} \geq 0.$$

In case $h^* \notin \tilde{H}$, continue the algorithm. Trivially, μ^* and p^* further increase. \square

3.2 Characterizing the Equilibrium with a Fixed Tax per Unit Emission

We have seen that there is no fixed merit order of technologies in (10). In order to satisfy the emission allowance, different technologies may be chosen in the optimal mixture for different levels of demand. In particular, for each demand realization we get a combination of a cheap but dirty technology and an expensive but clean technology; this combination depends on the level of demand. As a consequence, the entire industry is motivated to have several technologies available.

We now turn our attention to the situation where the environmental regulator taxes firms per unit emission. In the fixed tax model (11), there is a fixed merit order on the firms. Only the firm(s) with the lowest total marginal cost, being production cost plus investment cost plus tax on CO₂ emission, will be producing at the equilibrium. Hence, this system singles out one or, in case of equal marginal cost, several technologies and is independent of demand levels as long as there is sufficient capacity. In addition, we show the implications of taking the optimal price of emission allowances from the cap-and-trade model as the fixed tax. In this special case, the fixed tax model cannot guarantee that emissions stay below the maximum allowance level and hence may lead to a more polluting technology mixture.

To illustrate this result, suppose that, for given $\bar{\mu}$, we may reorder the firms such that

$$a_1 + \bar{\mu}e_1 \leq a_2 + \bar{\mu}e_2 \leq \dots \leq a_n + \bar{\mu}e_n. \quad (15)$$

This is, aside from possible equalities, the fixed merit order of the firms in the presence of taxation. In order to illustrate our results in a clear way, we additionally assume that either the first or the second inequality in (15) is strict. With this assumption, at the optimal solution to (11), either firm 1 or firm 1 and firm 2 are producing. Firm 1 is the only firm producing at the equilibrium when

$a_1 + \bar{\mu}e_1 < a_2 + \bar{\mu}e_2$; both firms might be producing when equality holds, since then both are equally cheap. Hence, independent of the level of demand, we know which firm(s) will be producing at the equilibrium.

In order to see that we cannot necessarily guarantee that the total amount of CO₂ emitted will remain below some maximum allowance level, we will consider a special case of taxation; that is, we choose $\bar{\mu} = \mu^*$. In order to distinguish between the cap-and-trade and the taxation solution we will give the corresponding variables a superscript E and T , respectively. First, recall that in the cap-and-trade model, for given E , firms j and h , with j the relatively dirty but cheap firm and h the relatively clean but expensive firm, are found as producing firms at the equilibrium in (10). By Proposition 3.1, we have $x_j^{E*} = \frac{E - e_h d}{e_j - e_h}$, $x_h^{E*} = \frac{e_j d - E}{e_j - e_h}$, $x_k^{E*} = 0$ for $k \neq j, h$, and

$$(p^{E*}, \mu^*) = \left(\frac{-a_j e_h + a_h e_j}{e_j - e_h}, \frac{a_h - a_j}{e_j - e_h} \right).$$

Now, we choose $\bar{\mu} = \mu^*$. Obviously, the optimal solution (x^{E*}, p^{E*}) is also an optimal solution to (11), since the set of KKT conditions to (11) is a subset of (13). Hence, firm j and firm h must be the first two firms in the merit order (15) and their effective marginal costs are equal, namely $a_j + \bar{\mu}e_j = a_h + \bar{\mu}e_h$. Therefore, any convex combination of x^T with $(x_j^T, x_h^T) = (d, 0)$ and x^T with $(x_j^T, x_h^T) = (0, d)$ is also a solution to the fixed tax problem (11); see Figure 1 where we draw the set of optimal solutions for firms j and h . In fact, there is a trade-off between producing with the cheaper technology and satisfying the emission constraint. The total emission will depend on this trade-off, and may exceed the maximum allowance level. To see this, note that for the solution $(x_j^T, x_h^T) = (d, 0)$, $e_j d$ is the total amount of CO₂ emitted; however, by the choice of j and h , we have $e_j d > E$. Hence, this solution would not have been allowed in (10). In particular, all points on the dotted line in Figure 1 would exceed the maximum allowance level, although they are optimal solutions of (11).

On the other hand, all points between $(x_j^T, x_h^T) = (x_j^{E*}, x_h^{E*})$ and $(x_j^T, x_h^T) = (0, d)$ do satisfy the emission constraint. This then raises the questions: Is one of the solutions that satisfy the emission allowance better than any other solution? Is there a way to distinguish between several solutions by means of some reasonable measures? We may for example focus on the total emissions, which is obviously lowest in case $(x_j^T, x_h^T) = (0, d)$, but we may also consider social welfare. As the environmental regulator (or government) takes action to reduce the amount of CO₂ emitted, it generates some income. This income consists of the fixed tax collected per unit emission. We will call the total amount the regulator earns from a certain action the regulator's surplus. The regulator's surplus equals

$$RS^T = \sum_k \bar{\mu} e_k x_k^{T*} = \bar{\mu} (e_j x_j^{T*} + e_h x_h^{T*}). \quad (16)$$

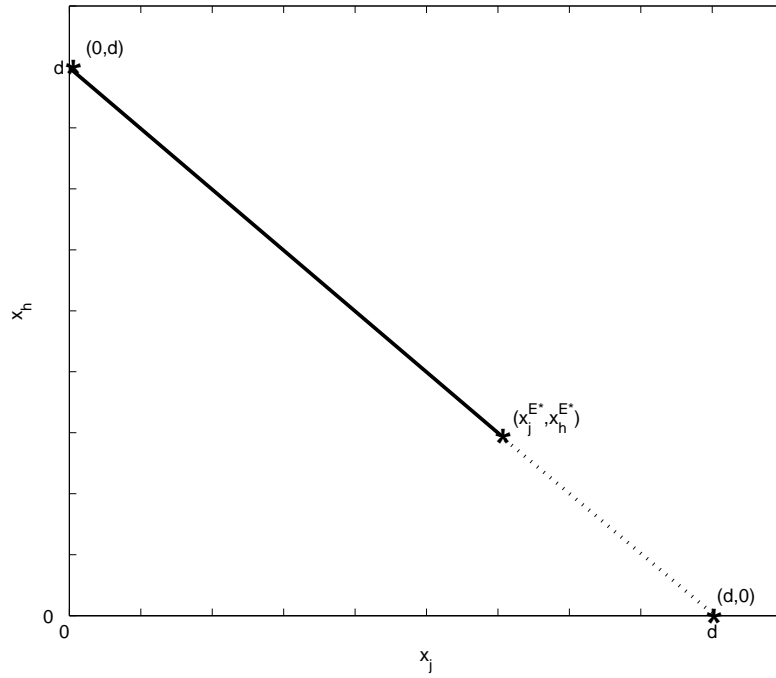


Figure 1: *Optimal production (x_j^{T*}, x_h^{T*}) when $\bar{\mu} = \mu^*$.*

We next discuss why this way of computing the regulator's surplus helps us in distinguishing between the multiple optima. Since multiple optima means that there is a range of values for x_j^{T*} and x_h^{T*} , there is a range of values for the regulator's surplus (16). Recall that we would like to exclude points that are on the dotted line of Figure 1; that is, all points where the maximum emission allowance level would be violated. Analyzing the remaining points, one can see the trade off between maximizing the regulator's surplus and minimizing the total emissions. In particular, (x_j^{E*}, x_h^{E*}) is the remaining point for which the regulator's surplus is maximized, whereas $(0, d)$ is the point for which the total emissions are minimized; obviously, the implications of these for "social welfare" are unclear. In addition, in reality there is no way to enforce one of these potentially preferred solutions without imposing additional conditions on the technology mixture (for example in the form of an emission allowance).

3.3 Characterizing Unused Technologies

As mentioned earlier, when ignoring possible equal marginal cost, any specific level of taxation, $\bar{\mu}$, induces a fixed merit order on the technologies. When a technology has the lowest total marginal cost, that is, when it is the first in the fixed merit order, it is used to satisfy (part of) the demand. This raises the following question: For any technology k , does there exist a level of fixed tax such that technology k is the first in the merit order? If the answer is "not affirmative", then there is no reason for that technology to exist with its current specifications; hence either something should be done to improve the specifications

or it should be discarded.

To answer this question, we try to characterize the technologies that can not be the first in the merit order for any level of taxation. We give sufficient conditions implicating when no fixed tax level exists such that a technology is first in the merit order. We show, by means of a counter-example, that the conditions are not necessary. In addition, we argue that a technology which satisfies the sufficient conditions is not used in the optimal mixture in case of a maximum emission allowance level either.

Proposition 3.3. *Suppose that there are n firms with n different technologies, for which it holds that*

$$a_1 < a_2 < \dots < a_n \quad \text{and} \quad e_1 > e_2 > \dots > e_n.$$

For each technology $k \in K$ define:

$$\gamma_k = \begin{cases} 0 & \text{for } k = 1, \\ \frac{a_k - a_1}{e_1 - e_k} & \text{for } k = 2, \dots, n, \end{cases} \quad \text{and} \quad \tau_k = \begin{cases} \frac{a_n - a_k}{e_k - e_n} & \text{for } k = 1, \dots, n - 1 \\ \infty & \text{for } k = n. \end{cases}$$

For any technology $k \in K$, if we have

$$\gamma_k > \gamma_i \quad \text{for at least one } i > k, \quad (17)$$

or

$$\tau_k < \tau_i \quad \text{for at least one } i < k, \quad (18)$$

then no $\bar{\mu}$ exists such that firm $k \in K$ is first in the merit order.

Before we prove the proposition, we first give an interpretation of the quantities γ_k and τ_k . Let MC_k denote the effective marginal costs of technology $k \in K$, that is, $MC_k = a_k + \bar{\mu}e_k$. Then we can think of γ_k as the level of taxation, $\bar{\mu}$, for which $MC_1 = MC_k$. Furthermore, by the assumption that $e_1 > e_k$ for $k \neq 1$, we have

$$\begin{aligned} MC_k < MC_1 & \quad \text{if } \bar{\mu} > \gamma_k, \\ MC_k > MC_1 & \quad \text{if } \bar{\mu} < \gamma_k. \end{aligned} \quad (19)$$

Similarly, τ_k can be interpreted as the level of fixed tax for which $MC_n = MC_k$. By the assumption that $e_n < e_k$ for $k \neq n$, we get

$$\begin{aligned} MC_k < MC_n & \quad \text{if } \bar{\mu} < \tau_k, \\ MC_k > MC_n & \quad \text{if } \bar{\mu} > \tau_k. \end{aligned} \quad (20)$$

In the proof we show that for no level of taxation technology $k \in K$ can have the lowest marginal cost if

it satisfies (17) or (18).

Proof: First, consider a technology $k \in K$ for which (17) holds; that is, for some $i > k$ we have $\gamma_k > \gamma_i$. We show that for all levels of taxation a different technology, with lower marginal cost than technology k , can be found; that is, technology k will always be dominated. In particular, we claim:

$$\begin{aligned} \text{for } 0 \leq \bar{\mu} < \gamma_i, & \quad MC_1 < MC_i \text{ and } MC_1 < MC_k; \\ \text{for } \gamma_i \leq \bar{\mu} < \gamma_k, & \quad MC_i \leq MC_1 < MC_k; \\ \text{for } \gamma_k \leq \bar{\mu}, & \quad MC_i < MC_k \leq MC_1. \end{aligned}$$

The first two statements follow immediately from (19). To see the third statement, note that we have $MC_i < MC_k$ with taxation levels between γ_i and γ_k . Observe that, because $i > k$ and hence $e_k > e_i$, an increase of $\bar{\mu}$ such that $\bar{\mu} \geq \gamma_k$ does not influence the sign of the inequality $MC_i < MC_k$.

Next, assume we have a technology, $k \in K$, for which (18) holds; that is, for some $i < k$ we have $\tau_k < \tau_i$. We again show that for all levels of taxation another technology with lower marginal cost can be found. This time, we will have:

$$\begin{aligned} \text{for } \tau_i \leq \bar{\mu}, & \quad MC_n \leq MC_i \text{ and } MC_n < MC_k; \\ \text{for } \tau_k \leq \bar{\mu} < \tau_i, & \quad MC_i < MC_n \leq MC_k; \\ \text{for } 0 \leq \bar{\mu} < \tau_k, & \quad MC_i < MC_k < MC_n. \end{aligned}$$

Again, the first two statements follow immediately from (20). The third statement is a consequence of the assumption $e_i > e_k$. \square

Next, by means of a numerical example, we show that (17) and (18) are not necessary conditions. That is, we have a situation in which none of the technologies satisfy (17) and (18); nevertheless, there is a technology for which no $\bar{\mu}$ can ensure that this technology will be the first in the merit order.

Example 3.1. *Table 1 contains the characteristics of five technologies and the corresponding γ - and τ -values; note that none of the technologies satisfies (17) and (18).*

Next, we compute the marginal costs of the firms for all levels of fixed tax. Table 2 shows that for no level of taxation technology 3 is first in the merit order. Hence, no level of fixed tax exists for which technology 3 has lowest marginal cost.

Hence, the conditions mentioned in Proposition 3.3 are not necessary. We remark that it is possible to show that for less than five technologies the conditions are necessary, but with five or more technologies counter examples can be found.

Table 1: Characteristics of five arbitrary technologies.

	a_k	e_k	γ_k	τ_k
1	10	0.9	0	175
2	18.8	0.8	88	204
3	20.5	0.79	95.45	205.17
4	25	0.75	100	220
5	80	0.5	175	∞

Table 2: Levels of fixed tax and the corresponding technology that appears first in the merit order.

Lower lim.	Upper lim.	First in merit order
0	88	Technology 1
88	124	Technology 2
124	220	Technology 4
220	∞	Technology 5

Finally, we argue that a technology which satisfies the sufficient conditions can not be chosen in the optimal mixture in case of a maximum emission allowance level.

Corollary 1. *Given an arbitrary maximum emission allowance level in (10), a technology which satisfies condition (17) or (18) is not chosen in the optimal mixture.*

Proof: Assume that (17) or (18) hold for that specific technology $k \in K$. Then, for no level of taxation technology k is first in the merit order. Hence, in the equilibrium solution to problem (11), the corresponding production quantity will be zero, independent of the level of taxation. Furthermore, if $\bar{\mu} = \mu^*$ is chosen, then each equilibrium solution (10) is a solution of (11). Since technology $k \in K$ will not be in the solution set of (11) for any $\bar{\mu}$, it will not be in the solution set of (10) for any E . \square .

3.4 Analyzing Effects for Consumers: CO₂ Cost Pass-Through

Reducing the total amount of CO₂ emitted, will obviously have a price. Since cheap and dirty technologies are replaced by costly clean technologies, total production cost and hence consumer prices increase. In this section we elaborate on to what extent additional production costs are passed through to the consumers. In particular, we consider changes in consumers' surplus as a result of regulator's actions in the form of a CO₂ emission cap or in the form of taxation.

Varian (1996) introduces consumers' surplus as follows. Each consumer is willing to pay a certain price for a good. In case of fixed, inelastic demand, the consumer is supposed to be willing to pay any price for the good. In an electricity market without demand side bidding, it is customary to assume that consumers are willing to pay any price up to VOLL to obtain the electricity. Every price below VOLL thus generates a surplus for the consumer. Hence, the consumers' surplus (CS) is taken as the difference between the price consumers are willing to pay and the actual price paid for each unit of demand. Given

the demand d and the price of electricity p^* , the consumers' surplus can be calculated as

$$CS = (VOLL - p^*)d. \quad (21)$$

The total production cost can also easily be computed by multiplying the effective marginal costs, consisting of investment, production, and emission cost, with the total production for all technologies. Hence we get

$$PC = \sum_{k=1}^n MC_k x_k^*. \quad (22)$$

We first consider the effect of going from one maximum emission allowance level to a lower maximum emission allowance level in (10). We distinguish between two possibilities. In the first one, both allowance levels are such that we are in case (i) or case (iii) of Proposition 3.1. In the second one, one allowance level is such that we are in case (i) or (iii) of Proposition 3.1 and the other allowance level is such that we are in case (ii) of Proposition 3.1, that is, the allowance level is so low that even the cleanest firm cannot satisfy demand without violating the emission constraint. We do not consider the possibility in which both allowance levels are such that we are in case (ii) of Proposition 3.1, since in this case a change of the allowance level has no effect on consumers. We show that the consumers' surplus is decreasing with the emission allowance level. We also show that in the first case the decrease in consumers' surplus equals the increase in production cost, whereas in the second case the decrease in consumers' surplus is larger than the additional production cost incurred.

We start with three different maximum allowance levels E_1, E_2 , and E_3 such that $E_1 > E_2 > E_3$. Assume that E_1 and E_2 are such that we are in case (i) or case (iii) of Proposition 3.1; that is, we are in the first alternative. We know that either one or two technologies are first in the merit order and hence the market prices p^{1*} and p^{2*} , corresponding to E_1 and E_2 , respectively, will be set by the effective marginal costs of these technologies. By Proposition 3.2, we know that $p^{1*} < p^{2*}$. Furthermore, the total output is d . Using (21) and (22), one can easily see that the decrease in consumers' surplus and the increase in production cost are:

$$\Delta CS = CS^{E_2} - CS^{E_1} = (VOLL - p^{2*})d - (VOLL - p^{1*})d = (p^{1*} - p^{2*})d$$

and

$$\Delta PC = PC^{E_2} - PC^{E_1} = (p^{2*} - p^{1*})d.$$

Since $p^{1*} < p^{2*}$, the consumers' surplus is decreasing as E decreases; the decrease in consumers' surplus is equal to the increase in total production cost, implying that the CO₂ cost pass-through to consumers is 100%.

Next, consider the second alternative: With the maximum allowance level E_2 we are in case (i) or

(iii) of Proposition 3.1, and with the maximum allowance level E_3 we move to case (ii) of Proposition 3.1. That is, when the allowance level equals E_2 , the consumer price and the marginal cost equal p^{2*} , and the total output equals d ; when the allowance level becomes E_3 , demand can no longer be satisfied. Then, the cleanest firm, by assumption firm n , is the only producer and is allowed to produce a quantity equal to $\frac{E_3}{e_n}$; VOLL-pricing is in effect and the price of allowances will be $\frac{VOLL - a_n}{e_n}$. Hence, the effective marginal cost of firm n equals $a_n + e_n \frac{VOLL - a_n}{e_n} = VOLL$. Using (21) and (22), the decrease in consumers' surplus and the increase in production cost are:

$$\Delta CS = CS^{E_3} - CS^{E_2} = (VOLL - VOLL) \frac{E_3}{e_n} - (VOLL - p^{2*})d = (p^{2*} - VOLL)d$$

and

$$\Delta PC = PC^{E_3} - PC^{E_2} = VOLL \frac{E_3}{e_n} - p^{2*}d.$$

Since $\frac{E_3}{e_n} < d$ by assumption, there is a negative gap between the increase in production cost and the decrease in consumers' surplus. All additional production costs are passed through to the consumers, but there is an extra loss for the consumers due to the lower total output. Obviously, this is a situation where E_3 is set to such a low value necessitating to curb the total production.

Next, we turn our attention to the fixed tax model (11). We consider two different levels of fixed tax, $\bar{\mu}^1$ and $\bar{\mu}^2$ with $\bar{\mu}^1 < \bar{\mu}^2$. Suppose only one firm is producing the entire quantity demanded, d ; hence, the corresponding consumer prices, p^{1*} and p^{2*} , respectively, will be set by the producing firm's effective marginal cost. Again, the corresponding decrease in consumers' surplus and increase in production cost can be found using (21) and (22), that is,

$$\Delta CS = CS^{\bar{\mu}^2} - CS^{\bar{\mu}^1} = (VOLL - p^{2*})d - (VOLL - p^{1*})d = (p^{1*} - p^{2*})d$$

and

$$\Delta PC = PC^{\bar{\mu}^2} - PC^{\bar{\mu}^1} = (p^{2*} - p^{1*})d.$$

It is easy to check that $p^{1*} < p^{2*}$. Hence, again the consumers' surplus is decreasing as $\bar{\mu}$ increases. Furthermore, the decrease in consumers' surplus is exactly equal to the increase in total production cost; hence the CO₂ cost pass-through to consumers is again 100%.

Similar to the results of Chen et al. (2008) and Bonacina and Gullí (2007), we see that under the assumption of deterministic demand and exogenous CO₂ costs (e.g. fixed tax), the CO₂ cost pass-through to consumers is 100% not only in the short run but also in a market with optimal generation capacities in the long run. In addition, we see that when CO₂ allowance prices are endogenously determined by the market, the CO₂ cost pass-through to consumers is again 100% except when the CO₂ allowance

cap is too low. When the cap is too low, demand is curtailed with additional cost and the CO₂ cost pass-through to consumers exceeds 100%.

4 The Investment Model - Stochastic Exogenous Demand

In previous sections, we considered the impact of a CO₂ allowance on the technology mixture and the CO₂ pass through rate to consumers in a deterministic setting. In reality, there are uncertainties in an electricity market related to future demand, fuel prices, and emission allowances set by the regulator. Hence, extending the deterministic framework by including uncertainty provides more insight into the consequences of CO₂ regulation in reality. In this paper we focus on uncertainty in demand. Realized demand is assumed to be unknown to the firms at the first stage, and will be revealed to the firms at the second stage. In particular, the first stage decisions can be seen as long term decisions, that is, capacity investments are made for a certain period, for example a year, and are based on the possible future outcomes of the second stage. The second stage decisions can then be seen as short term, for example hourly or daily, decisions. Uncertainty about the second stage outcomes may affect the choice of technology and its investment level at the first stage, that is, in order to deal with both peak and off-peak demand realizations, firms may want to invest in broader mixtures of technologies.

Whereas the deterministic setting allowed us to derive analytical results, the most convenient way to derive results in the stochastic setting is via a numerical study. When the random demand distribution is given, sampling is a handy tool for deriving numerical results. After introducing the general version of the model including stochastic demand, we will state the sampled problem. Then, we apply this on a small network and derive our results. We observe that indeed a broader portfolio of technologies will be used in the system. Furthermore, we investigate the adequacy of putting financial incentives, namely cap-and-trade and taxation, when the network capacity is limited. Finally we show a connection between the outcome of the cap-and-trade model and the taxation model.

In Section 4.1 we briefly introduce the altered investment model including cap-and-trade and introduce how to solve this model as a large MCP using sampling. The altered fixed tax model is introduced in Section 4.2. We finally apply the theory on a small network in our numerical study in Section 4.3.

4.1 Introducing the Two-Stage Game Including an Emission Allowance Level

We assume that demand is determined by a random process. The demand at node $n \in N$ is denoted by $d_n(\omega)$, which has a continuous joint distribution Ψ . Here $\omega \in \Omega$ is a random vector in Ω , the space of possible outcomes. The probability distribution and its possible outcomes are known to the firms at the first stage. The realized demand will be revealed to the firms at the second stage. For each $\omega \in \Omega$ there

may be a different optimal second stage outcome depending on the demand realization.

At the first stage, firms consider the expected optimal second stage profit based on the information they have on the probability distribution of demand. Hence, the objective function for firm $g \in G$ at stage one is defined as

$$\max_{x^g \geq 0} E_\omega \left[\sum_{i \in I_g} \sum_{k \in K_g} (p_i(\omega) - c_{ik}^g - e_k \mu) y_{ik}^g(x^g, \omega) \right] - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_{ik} x_{ik}^g, \quad (23)$$

where $y_{ik}^g(x^g, \omega)$, $g \in G$, $i \in I_g$, $k \in K_g$, and $p_i(\omega)$, $i \in I$, for a given realization $\omega \in \Omega$, are taken from the second stage. μ is the price of emission allowances, that will now be determined at the first stage; that is, since a maximum emission allowance level is typically set for a certain period, for example a year, the emission allowance constraint is going to be a first stage constraint. We impose that the expected (average) emission over all realizations of the second stage should be less than or equal to the maximum allowance level E , while the price of emission allowances will have to be perpendicular to that constraint, that is,

$$0 \leq E - E_\omega \left[\sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g(x^g, \omega) \right] \perp \mu \geq 0. \quad (24)$$

Next, we write the OPF problem that solves all second stage problems for given x and ω . As a result of imposing (24), firms pay μ for each unit of CO₂ they emit. Hence, contrary to the OPF problem (7) in the deterministic case, we get a term $e_k \mu$ in the OPF's objective function. For given $x = (x^g)_{g \in G}$ and $\omega \in \Omega$ we solve

$$\begin{aligned} Z(x, \omega) := & \\ \min_{y(\omega), f(\omega), \delta(\omega)} & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^g + e_k \mu) y_{ik}^g(\omega) + VOLL \sum_{j \in N \cup I} \delta_j(\omega) \\ \text{s.t.} & \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g(\omega) + \delta_j(\omega) + f_j(\omega) \geq d_j(\omega) & (p_j(\omega)) \quad \forall j \in N \cup I \\ & \sum_{j \in N \cup I} f_j(\omega) = 0 & (\rho(\omega)) \\ & \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \leq h_l & (\lambda_l^+(\omega)) \quad \forall l \in L \\ & - \sum_{j \in N \cup I} PTDF_{l,j} f_j(\omega) \leq h_l & (\lambda_l^-(\omega)) \quad \forall l \in L \\ & y_{ik}^g(\omega) \leq x_{ik}^g & (\beta_{ik}^g(\omega)) \quad \forall g \in G, i \in I_g, k \in K_g \\ & y_{ik}^g(\omega) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\ & \delta_j(\omega) \geq 0 & \forall j \in N \cup I. \end{aligned} \quad (25)$$

An equilibrium to the two-stage game can be found by solving the first stage problem (23) subject to (24), while solving for each possible realization the second stage problem (25). As the set of possible

realizations is often very large or even uncountable, we are going to use a sample of the given demand distribution as we explain in the next section.

4.1.1 Solving the Two-Stage Game as an MCP

In order to solve the two-stage game with random demand, we generate a random sample $\omega_1, \omega_2, \dots, \omega_M$ from Ω and define $d_n(\omega_m)$ as the demand at node $n \in N$ of realization ω_m for $m \in \{1, \dots, M\}$. As we have a random sample, the first stage problem (23) for firm $g \in G$ is approximated by

$$\max_{x^g \geq 0} \frac{1}{M} \sum_{m=1}^M \sum_{i \in I_g} \sum_{k \in K_g} (p_i(\omega_m) - c_{ik}^g - e_k \mu) y_{ik}^g(x^g, \omega_m) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_{ik} x_{ik}^g, \quad (26)$$

where $p_i(\omega_m)$, $i \in I$, and $y_{ik}^g(x^g, \omega_m)$, $g \in G$, $i \in I_g$, $k \in K_g$, are the price of electricity and the optimal production quantities of firm g in realization ω_m , taken from the second stage. The corresponding KKT condition of the sampled problem (26) is

$$0 \leq -\frac{1}{M} \sum_{m=1}^M \beta_{ik}^{*g}(\omega_m) + \kappa_{ik} \perp x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g, \quad (27)$$

as shown by Gürkan et al. (2012). (27) implies that the (sample) averaged scarcity rent should cover the unit investment costs. If that is not the case, no investments will be done. The sampled market clearing condition with respect to the emission allowance is

$$0 \leq E - \frac{1}{M} \sum_{m=1}^M \left[\sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} e_k y_{ik}^g(x^g, \omega_m) \right] \perp \mu^* \geq 0. \quad (28)$$

The interpretation of this condition is as follows. Each realization ω_m , $m = 1, \dots, M$, can be seen as a day; M is the length of a period, let's say a year. What we have between the brackets is then the yearly emission. The regulator then imposes a maximum allowance level, E , per day, that should be satisfied on average.

At the second stage we solve the OPF problem; that is, for given $x = (x^g)_{g \in G}$ and each realization ω_m , $m = 1, \dots, M$, we find a solution $y^*(\omega_m), \delta^*(\omega_m), p^*(\omega_m), \beta^*(\omega_m), \lambda^{*+}(\omega_m), \lambda^{*-}(\omega_m), \rho^*(\omega_m), f^*(\omega_m)$ to

the following set of KKT-conditions:

$$\begin{aligned}
0 \leq \beta_{ik}^{*g}(\omega_m) - p_i^*(\omega_m) + c_{ik}^g + e_k \mu^* &\perp y_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^*(\omega_m) &\perp \delta_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega_m) + \delta_j^*(\omega_m) + f_j^*(\omega_m) - d_j(\omega_m) &\perp p_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq x_{ik}^g - y_{ik}^{*g}(\omega_m) &\perp \beta_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\perp \lambda_l^{*+}(\omega_m) \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\perp \lambda_l^{*-}(\omega_m) \geq 0 & \forall l \in L
\end{aligned} \tag{29}$$

$$\begin{aligned}
p_j^*(\omega_m) - \rho^*(\omega_m) + \sum_{l \in L} PTDF_{l,j} (\lambda_l^{*+}(\omega_m) - \lambda_l^{*-}(\omega_m)) &= 0 & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^*(\omega_m) &= 0.
\end{aligned}$$

Solving (27), (28), and (29) for all realizations, results in an approximation of the equilibrium solution of the two-stage stochastic game. A large mixed complementarity problem (MCP) is solved, containing (29) for all realizations, (27), and (28). When the original (deterministic) problem is large (namely when we have a large network), solving the large MCP may become too time consuming. In our numerical experiments we therefore consider a small network.

4.2 Introducing the Two-Stage Game Including a Fixed Tax

The fixed tax model with stochastic demand is similar to the model defined by Gürkan et al. (2012). There is no emission constraint and the price per unit emission does not depend on the demand realization. Hence, in the above sampled version of the model we replace the variable μ^* by the parameter $\bar{\mu}$ and omit the emission constraint (24). The resulting sampled MCP is to find a solution $x^*, y^*(\omega_m), \delta^*(\omega_m), p^*(\omega_m), \beta^*(\omega_m), \lambda^{*+}(\omega_m), \lambda^{*-}(\omega_m), \rho^*(\omega_m), f^*(\omega_m)$, $m = 1, \dots, M$, satisfying

$$0 \leq -\frac{1}{M} \sum_{m=1}^M \beta_{ik}^{*g}(\omega_m) + \kappa_{ik} \perp x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g,$$

and for each realization ω_m , $m = 1, \dots, M$,

$$\begin{aligned}
0 \leq \beta_{ik}^{*g}(\omega_m) - p_i^*(\omega_m) + c_{ik}^g + e_k \bar{\mu} &\perp y_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_j^*(\omega_m) &\perp \delta_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega_m) + \delta_j^*(\omega_m) + f_j^*(\omega_m) - d_j(\omega_m) &\perp p_j^*(\omega_m) \geq 0 & \forall j \in N \cup I \\
0 \leq x_{ik}^{*g} - y_{ik}^{*g}(\omega_m) &\perp \beta_{ik}^{*g}(\omega_m) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\perp \lambda_l^{*+}(\omega_m) \geq 0 & \forall l \in L \\
0 \leq h_l + \sum_{j \in N \cup I} PTDF_{l,j} f_j^*(\omega_m) &\perp \lambda_l^{*-}(\omega_m) \geq 0 & \forall l \in L \\
p_j^*(\omega_m) - \rho^*(\omega_m) + \sum_{l \in L} PTDF_{l,j} (\lambda_l^{*+}(\omega_m) - \lambda_l^{*-}(\omega_m)) &= 0 & \forall j \in N \cup I \\
\sum_{j \in N \cup I} f_j^*(\omega_m) &= 0.
\end{aligned}$$

4.3 Numerical Experiments

In this section we consider a six-node example for analyzing the effect of an emission constraint and a fixed tax on the investments under demand uncertainty. In the deterministic setting we derived some results concerning the merit order and the number of technologies used at equilibrium. We show how stochastic demand results in broader technology mixtures. In addition, we investigate the adequacy of cap-and-trade and taxation when the network capacity is limited. We observe that, in order to curb CO₂ levels, investments in network capacity may be necessary. Finally we will establish a relationship between the optimal outcome of the cap-and-trade model and the taxation model. It will turn out that the result will partly coincide with the result derived for the deterministic setting in Section 3.2.

There are three supplying firms, each located in a different node and having a unique technology at their disposal; we therefore use a single index k to distinguish between the firms. The three technologies available are coal, open cycle gas turbine (OCGT), and closed cycle gas turbine (CCGT), used by firms 1, 2, and 3, respectively. In addition, there are three demand nodes, nodes 4, 5, and 6. The network, which was originally introduced by Chao and Peck (1998), is depicted in Figure 2. We assume infinite capacity on all transmission lines, except for lines $l = (1, 6)$ and $l = (2, 5)$, for which we assume there is a finite capacity later on. Table 3 contains the PTDFs representing the flows through lines $l = (1, 6)$ and $l = (2, 5)$ resulting from a power injection into nodes 1 through 5; node 6 is taken as the hub node and thus has coefficients 0.

Table 3: Power transmission distribution factors of lines $l = (1, 6)$ and $l = (2, 5)$.

	1	2	3	4	5
$l = (1, 5)$	0.625	0.5	0.5625	0.0625	0.125
$l = (2, 6)$	0.375	0.5	0.4375	-0.0625	-0.125

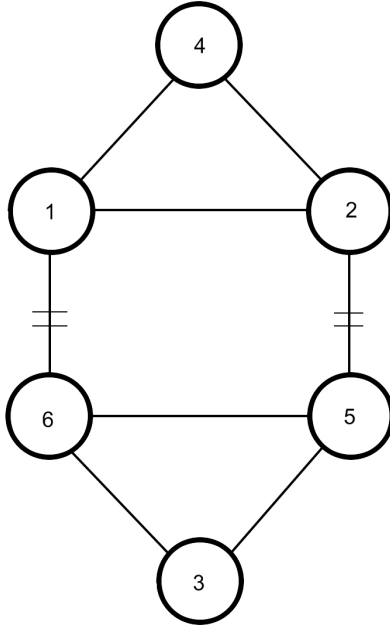


Figure 2: *The electricity network.*

Table 4 contains the characteristics of the technologies, consisting of per unit production costs (c_k), investment costs (κ_k), both in euros per MWh, and tons of CO₂ emission (e_k).

Table 4: Characteristics of the technologies.

	Coal	OCGT	CCGT
c_k	30	80	45
κ_k	18.3	6.8	9.1
e_k	1	0.6	0.35

These characteristics are taken from Ehrenmann and Smeers (2008). Notice that, in contrary to the deterministic demand case, to compute the effective marginal costs we cannot simply add the investment and production costs, since the investment quantity is not necessarily equal to the production amount in case of demand uncertainty.

Demand $d_n(\omega)$ in demand nodes $n = 4, 5, 6$ is assumed to be independently distributed. They are sampled from uniform distributions with lower bound a_n and upper bound b_n , as given in Table 5.

Table 5: Parameters for uniform demand distribution

	a_n	b_n
4	8	12
5	3	5
6	15	20

We take a sample of 3000 realizations and solve the resulting MCP using the PATH solver; see Ferris and Munson (2000).

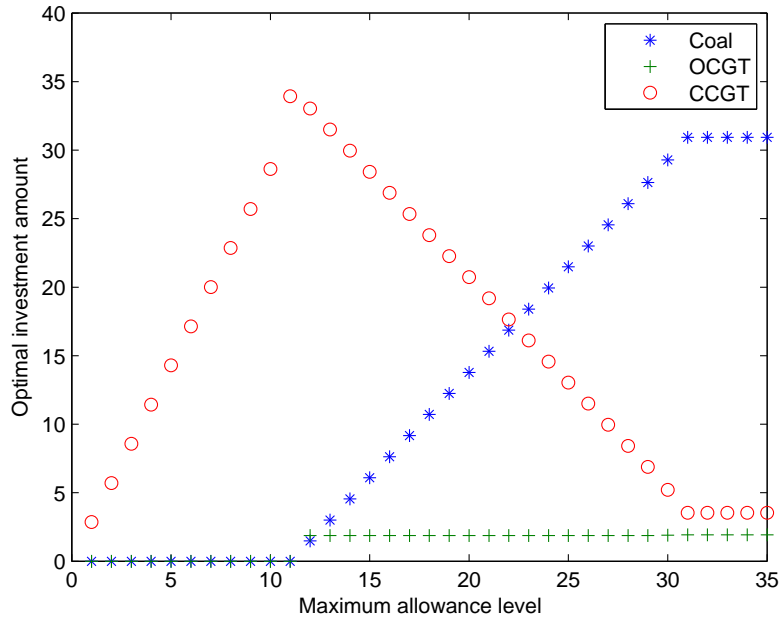


Figure 3: *The optimal investment quantities for different maximum emission allowance levels in case of infinite network capacity.*

4.3.1 The Effect of Maximum Allowance Level on Uncapacitated and Capacitated Networks

We consider optimal investment quantities for $E = 1, 2, \dots, 35$ in three different settings, namely an uncapacitated network, transmission line $l = (1, 6)$ has limited capacity, and transmission line $l = (2, 5)$ has limited capacity.

For the network without capacity constraints on the transmission lines, the optimal investment amounts in coal, OCGT, and CCGT are depicted in Figure 3. We observe that up to $E = 11$ only CCGT is used to satisfy the demand. Afterwards, up to $E = 31$, CCGT is gradually replaced by coal and OCGT since a more polluting mix of technologies is allowed. We observe that, in comparison to the deterministic demand case, broader mixtures are used; OCGT would never be used in the deterministic setting due to the high production cost. However, since its investment cost is low, a positive investment in OCGT turns out to be profitable in order to satisfy peak demand realizations. For $E \geq 31$ the emission constraint is not active and hence investments will be unaffected.

Next, we consider a network with a capacity of 5 on line $l = (1, 6)$. As a result, the PTDF-constraints will be of importance and prices and hence investment decisions will be influenced; the optimal investment amounts are depicted in Figure 4. Up to the level $E = 22$, Figure 4 and Figure 3 look very similar. However, starting at the point $E = 22$ higher levels of production via coal would result in exceeding the transmission capacity of the line $l = (1, 6)$; consequently, no more CCGT is replaced by coal beyond that level. In fact the network is cleaner due to its limited transmission capacity. One would say that the

network does the cleaning here, but since the allowance level is not binding this is not very interesting when it comes to CO₂ emission reduction. In addition, depending on the network structure, one may even observe more polluting mixtures in the absence of an allowance level due to limited transmission capacities.

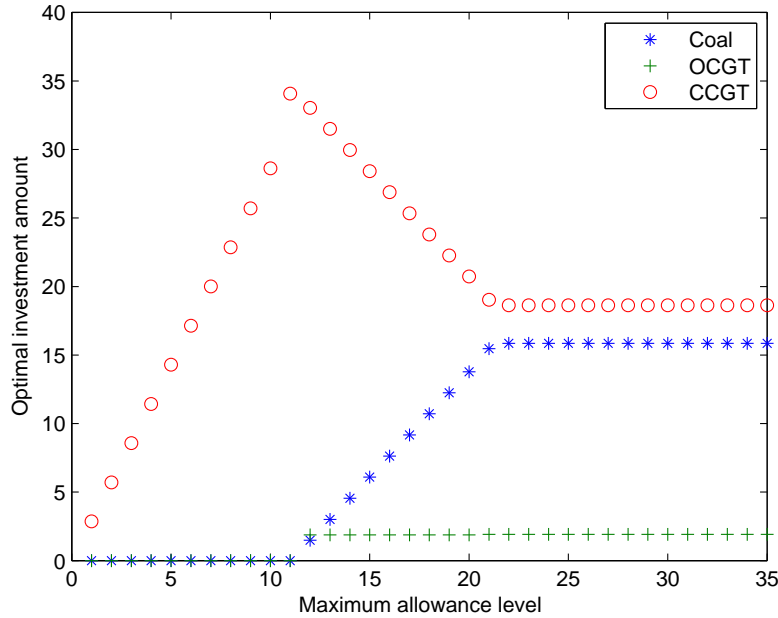


Figure 4: *The optimal investment quantities for different maximum emission allowance levels in case there is a transmission capacity of 5 on line $l = (1, 6)$.*

A similar behavior is observed when we choose a transmission capacity of 4 on line $l = (2, 5)$ and no limit on line $l = (1, 6)$. The optimal investment amounts are depicted in Figure 5. We observe two major differences between Figure 5 and Figure 3. At $E = 11$ up to $E = 14$, a strictly positive investment is made in OCGT. The transmission capacity of line $l = (2, 5)$ causes this behavior. The mixture of technologies chosen ensures that both the emission constraint and the transmission capacity constraint are not violated. Apparently, in this case the extra constraint associated with the transmission line capacity blocks investments in CCGT and instead motivates investments in OCGT. Clearly, this is a typical example illustrating that the shortage of transmission line capacity is preventing the cap-and-trade system to reach one of its main goals, namely to induce investments in cleaner technologies. While the regulator is creating financial incentives for firms to invest in cleaner technologies, investments in more polluting technologies are continued due to the limited network capacity. Hence, to really induce investments in cleaner technologies, investments in network capacity may also become necessary.

The other main difference between Figure 5 and Figure 3 occurs at $E = 26$. Starting at this point, higher levels of production via coal would result in exceeding the transmission capacity of line $l = (2, 5)$; therefore the replacement of CCGT by coal cannot continue in the same way as observed in Figure 3.

This actually causes a reduction in the total emissions starting at $E = 26$; in fact, a similar behavior was observed in Figure 4 at $E = 22$. These are examples showing that the limited network transmission capacity by itself may lower the total emissions. However, as noted before, it may also induce more investments in more polluting technologies, depending on the network structure.

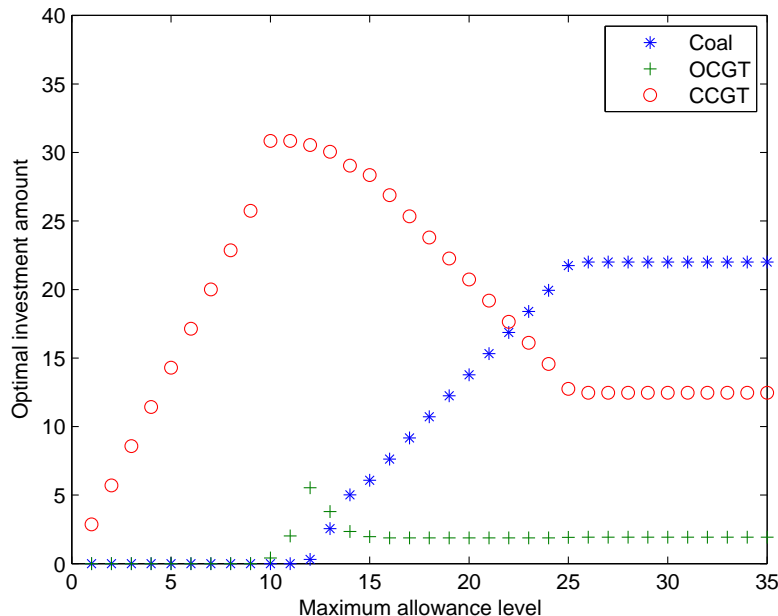


Figure 5: The optimal investment quantities for different maximum emission allowance levels in case there is a transmission capacity of 4 on line $l = (2, 5)$.

4.3.2 The Effect of Taxation Level on Uncapacitated and Capacitated Networks

In this section we discuss the effect of a fixed tax on the investment quantities. In order to analyze the impact of transmission capacity on the optimal mixture of technologies and the total emissions, we again distinguish between three different settings, namely an uncapacitated network, transmission line $l = (1, 6)$ has limited capacity, and transmission line $l = (2, 5)$ has limited capacity. We take $\bar{\mu} = 0, \dots, 20$ for the first two settings and $\bar{\mu} = 0, \dots, 75$ for the third setting. Note that $\bar{\mu} = 0$ corresponds to an electricity market without an emission limit ($E = \infty$).

The optimal investment quantities in the network without capacities on the transmission lines are depicted in Figure 6. At $\bar{\mu} = 0$, the optimal investment quantities in coal, CCGT, and OCGT coincide with the optimal investment quantities in Figure 3 when $E \geq 34$, because the emission constraint is not binding. As $\bar{\mu}$ increases from $\bar{\mu} = 0$ to $\bar{\mu} = 8$, we observe that the investment in coal is slowly decreasing and replaced by investments in CCGT. That is caused by the increasing cost per unit production as a result of the increasing taxation. Since coal based generation emits more CO_2 per unit generation, its marginal cost increases more rapidly with $\bar{\mu}$ than the marginal cost of CCGT based generation.

Therefore, as the fixed tax level increases, investments in CCGT become more attractive. Note that investments in CCGT are done although the sum of the marginal production cost, the tax paid for the emissions, and the investment cost, in other words what we have been calling the "effective" marginal cost, of CCGT may not be lowest. Recall that in the deterministic case only investments in the cheapest technology were done. In the stochastic demand case it may still be optimal to invest in a technology with higher "effective" marginal cost when the unit investment cost is relatively low. Such investments are optimal when the corresponding capacity is mostly used for peak demand realizations.

We observe that for high levels of fixed tax, from $\bar{\mu} = 9$ to $\bar{\mu} = 20$, all coal got replaced by CCGT. As we mentioned before, when the tax per unit emission increases, the "effective" marginal cost of coal increases more rapidly than the marginal cost of CCGT. Beyond $\bar{\mu} = 9$ coal becomes more expensive than CCGT, making investment in CCGT more attractive than investment in coal-based generation. Finally, we see that the investments in OCGT are at a constant level throughout to serve the peak demand.

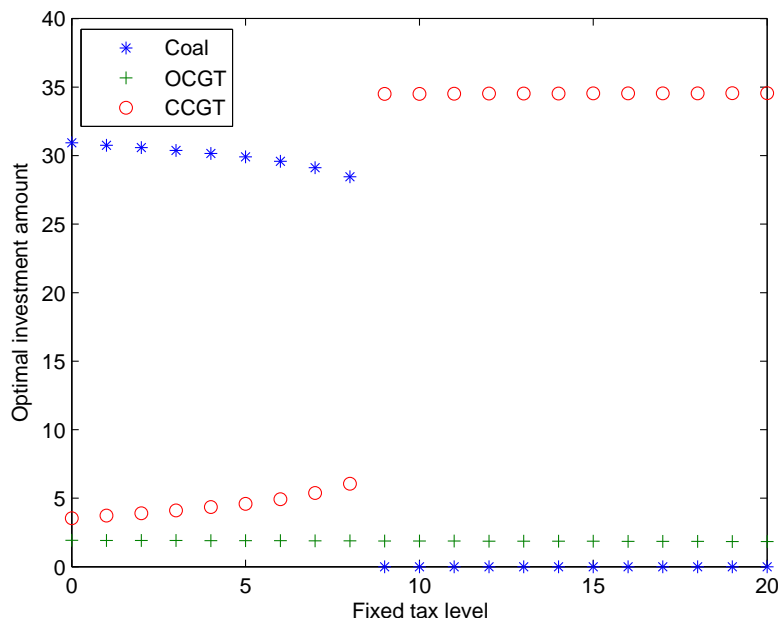


Figure 6: *The optimal investment quantities for different values of fixed tax in case the network capacity is infinite.*

We next consider a network with a capacity of 5 on line $l = (1, 6)$; the resulting optimal investments are depicted in Figure 7. There is one major difference between Figure 7 and Figure 6. Up to $\bar{\mu} = 8$, the investment in coal is at a much lower level, whereas the investment in CCGT is higher; this is obviously caused by the network capacity. The limited network capacity thus leads to a cleaner mixture. Beyond $\bar{\mu} = 9$, the curves look similar.

We finally put a capacity of 4 on line $l = (2, 5)$; the results are shown in Figure 8. We extend the x-axis to $\bar{\mu} = 75$. This would not be interesting in the previous two cases, since results beyond $\bar{\mu} = 20$ would be the same. However, when $h_{(2,5)} = 4$, we observe different behavior. Comparing Figure 8 with

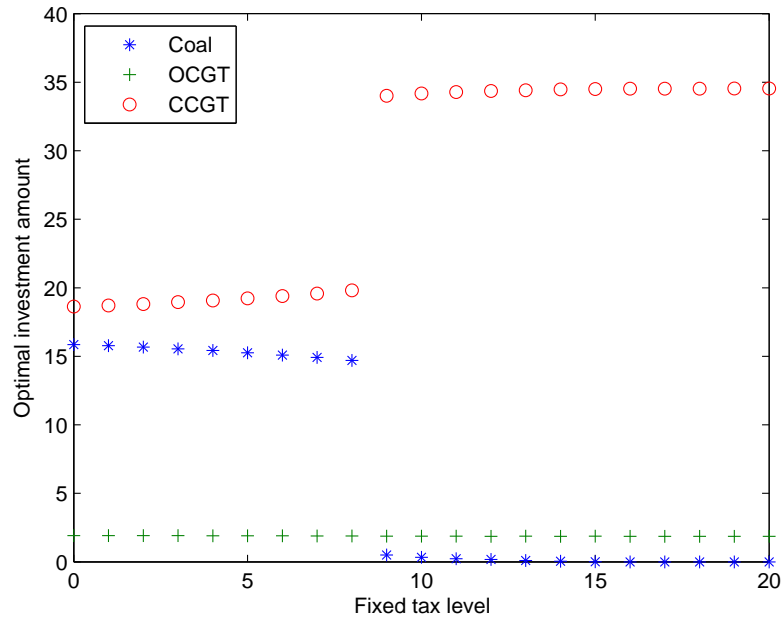


Figure 7: The optimal investment quantities for different values of fixed tax in case there is a transmission capacity of 5 on line $l = (1, 6)$.

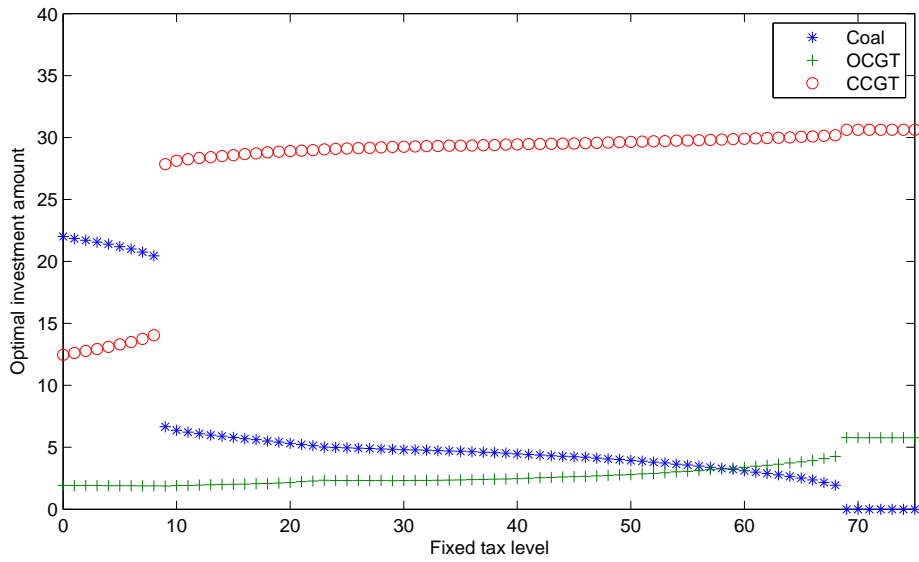


Figure 8: The optimal investment quantities for different values of fixed tax in case there is a transmission capacity of 4 on line $l = (2, 5)$.

Figure 6, we notice that up to $\bar{\mu} = 8$ production with coal is somewhat lower in Figure 8; this is again due to the limited network capacity. At $\bar{\mu} = 9$, CCGT starts dominating coal in both figures. However, in this case since the transmission capacity of the network is not sufficient to replace all coal based generation by CCGT based generation, investments in coal remain at a positive level beyond $\bar{\mu} = 9$. In other words, for high levels of carbon tax the reduction in the total emissions would be higher if line $l = (2, 5)$ had more capacity. This example illustrates that a financial incentive like the carbon tax may not be sufficient to curb the CO₂ levels when there is insufficient transmission capacity.

In addition, we observe that beyond $\bar{\mu} = 20$ investments in OCGT are slowly increasing and replacing investments in coal. Although coal has lower marginal production cost, the increasing taxation causes the effective marginal cost to increase up to a point where the effective marginal cost of coal and OCGT are equal; that is, at $\bar{\mu} = 69$. Starting from that point, it is less costly to invest in OCGT than in coal. We did not see such behavior before. This can be explained by the fact that in the other two examples coal was replaced by the cheaper and less polluting CCGT. Since with the current transmission capacity this is not feasible, OCGT is used. Still, since OCGT is more polluting than CCGT, the transmission capacity induces higher total emissions and hence investments in transmission capacity may be necessary to curb CO₂ levels.

4.3.3 Establishing a Relationship between Maximum Allowance Level and Taxation

In Section 3.2 we considered a special case of taxation, namely the fixed tax $\bar{\mu}$ equal to the optimal price of emission allowance μ^* for some maximum allowance level E . We found that for this taxation level multiple optima exist and that the optimal cap-and-trade solution is one of them. Between the multiple optima, a trade-off exists between minimizing pollution and maximizing regulator's surplus. A similar result can be observed when the demand is stochastic.

We assume stochastic demand and let μ^* be the optimal price of emission allowance for some given E . Next, we take $\bar{\mu} = \mu^*$ and find a solution to the taxation model. We observe two possible outcomes. Either we find a single optimum which then coincides with the optimum found in the cap-and-trade model, or, similar to what we found when demand is deterministic, we find multiple optima, of which the cap-and-trade solution is one. The latter occurs when μ^* induces two technologies with equal effective marginal cost. Our numerical results show that this is often the case. We next show an example of both possible outcomes.

In Figure 9 we depict the optimal allowance price for a range of E in a network with infinite capacities. We fix an E , take the corresponding optimal price of emission allowances as the fixed tax, and then compare the optimal investment quantities of both models. We first take $E = 30$. In Figure 9 we observe that the corresponding price of emission allowances is $\mu^* = 6.66$. Taking $\bar{\mu} = 6.66$ results in a single optimum, see Figure 6. This optimum coincides with the optimal investment quantities in the

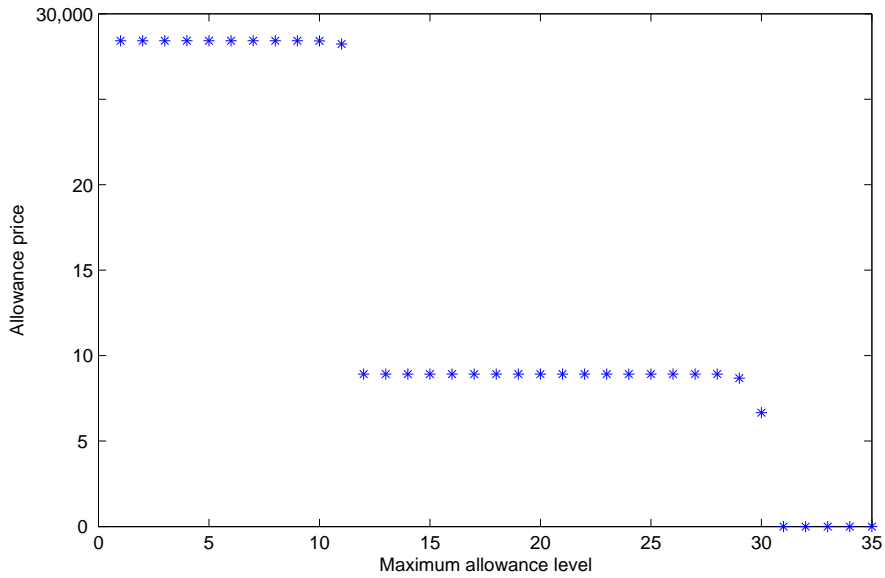


Figure 9: *The optimal price of emission allowance for different maximum emission allowance levels in case of infinite network capacity.*

cap-and-trade model when $E = 30$, as can be seen in Figure 3.

Next we consider $E = 15$. In Figure 9 we observe that the corresponding price of emission allowances is $\mu^* = 8.92$. Taking $\bar{\mu} = 8.92$ will make the effective marginal cost of coal and CCGT equal. As a result a central decision maker would be indifferent between the two technologies. When solving the taxation model with this particular taxation level, we find a range of optimal solutions. This can be seen in Figure 6, where at a certain point a jump occurs. This jump occurs exactly at $\bar{\mu} = 8.92$; all points in between represent optimal investment quantities. We thus have multiple optima, of which some may violate the allowance level of 15 and some may be cleaner. Similar to the deterministic case, as discussed in Section 3.2, there exists a trade-off between minimizing pollution and maximizing regulator's surplus. One of the multiple solutions results in a total emission of exactly 15. That solution coincides with the cap-and-trade solution.

Concluding, taking the optimal price of emission allowances as the fixed tax either results in the same unique optimal solution, or results in multiple optima of which the cap-and-trade solution is one.

5 Conclusions

In this paper we address the effect of two possible actions at the disposal of a regulator to curb CO₂ emission levels and to give power generating firms incentives to invest in cleaner technologies. In a stylized version of the investment model with no network effects and deterministic inelastic demand, we show that it is optimal to use a mixture of a relatively clean and a relatively dirty technology to satisfy

the demand under the cap-and-trade system. For a fixed ceiling on the total emissions and for different demand levels, there is a different optimal mixture of technologies. We also propose an algorithm that finds such an optimal mixture. Furthermore, we analytically show that the price of electricity and the price of allowances increase as the ceiling on the total emissions decreases; and the extra production costs incurred are fully passed through to the consumers.

In comparison, when a fixed carbon tax per unit emission is charged, we observe a fixed merit order on the firms. We give a characterization of technologies for which no fixed tax level exists, such that they are first in the merit order. Consequently, these technologies will never be used in the optimal technology mixture. We show that these technologies will not be in the optimal mixture in case of a cap-and-trade system either.

We also analyze the investment model with network effects and stochastic inelastic demand through a numerical study and discussed the implications of limited network capacity. We find that due to demand uncertainty a broader mix of technologies is used in the optimal mixture, both with cap-and-trade and carbon tax. We observe that in case of cap-and-trade, limited network capacity may cause that investments in dirty technologies are necessary to satisfy the demand without violating the transmission constraints. Hence, cleaner mixtures of technologies are not necessarily induced when there is limited network capacity. In case a carbon tax per unit emission is charged, we observe that limited transmission capacity puts a limit on the replacement of dirty technology by clean technology. In other words, the reduction of the total emissions due to taxation would be higher if there was more available transmission capacity. Hence, in order to curb CO₂ levels, investments in network capacity may be necessary. Finally, we establish a connection between the equilibria in both models and find that, when taking the optimal price of emission allowances as the fixed tax, multiple optima may exist. When this is the case, some optima violate the emission allowance constraint, and one of the optima coincides with the cap-and-trade solution.

Appendix: Proof of the Forward Algorithm

In this appendix we will show the following for the Forward Algorithm:

1. Given a j^* , choosing $h^* \in \bar{H}$ which minimizes the given quotient in Step 2 guarantees that (12) is satisfied.
2. Given a j_E , if we find a corresponding $h_E \notin H$ in Step 2, then we need to update $j^* = h_E$. Else, for at least one firm, (12) is violated.

Proof of 1. Suppose $J \neq \emptyset$ and define $\bar{J} = \{1\}$ and $\bar{H} = \{2, \dots, n\}$. In Step 2 of the algorithm we choose

$$h^* = \arg \min_{h \in \bar{H}} \left\{ \frac{a_h - a_1}{e_1 - e_h} \right\}.$$

For the sake of clarity, assume throughout the proof that the minimum found in this step is unique. If not, one can still find an optimal solution as we argue in the *Observation* below. By the choice of h^* we obtain for every $k \in \bar{H}$,

$$0 \leq \frac{a_k - a_1}{e_1 - e_k} - \frac{a_{h^*} - a_1}{e_1 - e_{h^*}} = \frac{a_k(e_1 - e_{h^*}) + a_1(e_{h^*} - e_k) + a_{h^*}(e_k - e_1)}{(e_1 - e_k)(e_1 - e_{h^*})}.$$

Since the denominator is positive, (12) follows. Note that equality holds for $k = h^*$.

Suppose we have $h^* \notin H$. Hence we define new candidate sets $\bar{J} = \{1, \dots, h^*\}$ and $\bar{H} = \{h^* + 1, \dots, n\}$.

We take $j^* = h^*$ and find the new h^* , denoted by h^{**} , as

$$h^{**} = \arg \min_{h \in \bar{H}} \left\{ \frac{a_h - a_{h^*}}{e_{h^*} - e_h} \right\}.$$

Next, we prove that by this choice (12) is satisfied for all k . Notice that, since we altered j^* and h^* , (12) will have the following form:

$$a_k(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*}) \geq 0 \quad \forall k \in \{1, \dots, n\}. \quad (30)$$

We will show that (30) holds in two parts; first for $k \in \bar{H}$, then for $k \notin \bar{H}$.

For every $k \in \bar{H}$

$$0 \leq \frac{a_k - a_{h^*}}{e_{h^*} - e_k} - \frac{a_{h^{**}} - a_{h^*}}{e_{h^*} - e_{h^{**}}} = \frac{a_k(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*})}{(e_{h^*} - e_k)(e_{h^*} - e_{h^{**}})},$$

and (30) follows for $k \in \bar{H}$, since the denominator is positive.

For every $k \notin \bar{H}$, first observe that (12) holds for every k when $j = 1$ and $h = h^*$, as found in the first iteration. That is,

$$a_k(e_1 - e_{h^*}) + a_1(e_{h^*} - e_k) + a_{h^*}(e_k - e_1) \geq 0 \quad \forall k \in \{1, \dots, n\}. \quad (31)$$

In particular, for $k = h^{**}$, we have

$$a_{h^{**}}(e_1 - e_{h^*}) + a_1(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_1) \geq 0. \quad (32)$$

Note that (31) implies

$$a_k \geq \frac{a_1(e_k - e_{h^*}) + a_{h^*}(e_1 - e_k)}{(e_1 - e_{h^*})} \quad \forall k \notin \bar{H}. \quad (33)$$

We will finally use this together with (32) to show that (30) holds for $k \notin \bar{H}$:

$$\begin{aligned} & a_k(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*}) \geq \\ & \frac{a_1(e_k - e_{h^*}) + a_{h^*}(e_1 - e_k)}{(e_1 - e_{h^*})}(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_k) + a_{h^{**}}(e_k - e_{h^*}) = \\ & \frac{e_k - e_{h^*}}{e_1 - e_{h^*}}(a_{h^{**}}(e_1 - e_{h^*}) + a_1(e_{h^*} - e_{h^{**}}) + a_{h^*}(e_{h^{**}} - e_1)) \geq 0. \end{aligned}$$

The first inequality follows from (33) and $e_{h^*} > e_{h^{**}}$, and the last inequality follows from (32) and by the fact that $e_k \geq e_{h^*} \forall k \notin \bar{H}$. This shows that (30) holds for every k . \square

Proof of 2. Given a j^* , say j_E , suppose we found a corresponding h_E with $h_E \notin H$. Then, we define the new candidate sets $\bar{J} = \{1, \dots, h_E\}$ and $\bar{H} = \{h_E + 1, \dots, n\}$. Suppose in the next iteration we do not choose $j^* = h_E$, but $j^* = k_0$ for some $k_0 \in \{j_E + 1, \dots, h_E - 1\}$ and find the corresponding new h^* as $m_0 \in \bar{H}$ using Step 2. We show that with this choice (12) will be violated for $k = h_E$, that is

$$a_{h_E}(e_{k_0} - e_{m_0}) + a_{k_0}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) \leq 0. \quad (34)$$

First observe that, since for j_E we found h_E , (12) holds for every k . In particular, for $k = k_0$ and $k = m_0$; that is,

$$a_{k_0}(e_{j_E} - e_{h_E}) + a_{j_E}(e_{h_E} - e_{k_0}) + a_{h_E}(e_{k_0} - e_{j_E}) \geq 0 \quad (35)$$

and

$$a_{m_0}(e_{j_E} - e_{h_E}) + a_{j_E}(e_{h_E} - e_{m_0}) + a_{h_E}(e_{m_0} - e_{j_E}) \geq 0. \quad (36)$$

Furthermore, (35) implies

$$a_{k_0} \geq \frac{a_{j_E}(e_{k_0} - e_{h_E}) + a_{h_E}(e_{j_E} - e_{k_0})}{(e_{j_E} - e_{h_E})}. \quad (37)$$

We finally use this together with (36) to show (34):

$$\begin{aligned} & a_{h_E}(e_{k_0} - e_{m_0}) + a_{k_0}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) \leq \\ & a_{h_E}(e_{k_0} - e_{m_0}) + \frac{a_{j_E}(e_{k_0} - e_{h_E}) + a_{h_E}(e_{j_E} - e_{k_0})}{(e_{j_E} - e_{h_E})}(e_{m_0} - e_{h_E}) + a_{m_0}(e_{h_E} - e_{k_0}) = \\ & \frac{e_{h_E} - e_{k_0}}{e_{j_E} - e_{h_E}}(a_{m_0}(e_{j_E} - e_{h_E}) + a_{h_E}(e_{m_0} - e_{j_E}) + a_{j_E}(e_{h_E} - e_{m_0})) \leq 0. \end{aligned}$$

The first inequality follows from (37) and the fact that $e_{m_0} < e_{h_E}$, and the last inequality follows from (36) and the fact that $e_{h_E} < e_{k_0} < e_{j_E}$. This shows that (34) holds. Hence, (12) is violated for $k = h_E$.

□

Observation: As a final remark, notice that when taking the argmin in Step 2 of the algorithm, the minimum may not be unique. Suppose that for a j^* we find that both h_1^* and h_2^* , with $h_1^* < h_2^*$, satisfy

$$h^* = \arg \min_{h \in \bar{H}} \left\{ \frac{a_h - a_{j^*}}{e_{j^*} - e_h} \right\}.$$

Next we show that

$$\frac{a_{h_1^*} - a_{j^*}}{e_{j^*} - e_{h_1^*}} = \frac{a_{h_2^*} - a_{j^*}}{e_{j^*} - e_{h_2^*}} = \frac{a_{h_2^*} - a_{h_1^*}}{e_{h_1^*} - e_{h_2^*}}. \quad (38)$$

Notice that these quantities are actually the resulting prices of emission allowances, μ^* , if we would choose one of the pairs in $\{j^*, h_1^*, h_2^*\}$ in the optimal mixture. This means that for all pairs in $\{j^*, h_1^*, h_2^*\}$, μ^* is equal and as a consequence p^* is equal as well. Hence, if for one pair (12) is satisfied, it is automatically satisfied for the other pairs in case the argmin finds more than one minimizer. The first equality in (38) follows immediately since both h_1^* and h_2^* give a minimum. Rewriting the first equality gives

$$e_{j^*} - e_{h_1^*} = \frac{a_{h_1^*} - a_{j^*}}{a_{h_2^*} - a_{j^*}} (e_{j^*} - e_{h_2^*}). \quad (39)$$

The second equality in (38) is derived as follows:

$$\begin{aligned} \frac{a_{h_1^*} - a_{j^*}}{e_{j^*} - e_{h_1^*}} - \frac{a_{h_2^*} - a_{h_1^*}}{e_{h_1^*} - e_{h_2^*}} &= \frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{h_1^*}) + a_{h_2^*}(e_{h_1^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} = \\ &= \frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{j^*}) + (a_{h_2^*} - a_{j^*})(e_{h_1^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} = \\ &= \frac{a_{h_1^*}(e_{j^*} - e_{h_2^*}) + a_{j^*}(e_{h_2^*} - e_{j^*}) + (a_{h_1^*} - a_{j^*})(e_{h_2^*} - e_{j^*})}{(e_{j^*} - e_{h_1^*})(e_{h_1^*} - e_{h_2^*})} = 0. \end{aligned}$$

The first and second equality follow by rewriting, whereas the third equality follows by replacing $(e_{h_1^*} - e_{j^*})$ according to (39). Hence, for all pairs the fraction is equal and any pair can be chosen as an optimal pair.

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