



## Introducing Space into MIMO Capacity Calculations

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**Abstract.** The large spectral efficiencies promised for multiple-input multiple-output (MIMO) wireless fading channels are derived under certain conditions which do not fully take into account the spatial aspects of the channel. Spatial correlation, due to limited angular spread or insufficient antenna spacing, significantly reduces the performance of MIMO systems. In this paper we explore the effects of spatially selective channels on the capacity of MIMO systems via a new capacity expression which is more general and realistic than previous expressions. By including spatial information we derive a closed-form expression for ergodic capacity which uses the physics of signal propagation combined with the statistics of the scattering environment. This expression gives the capacity of a MIMO system in terms of antenna placement and scattering environment and leads to valuable insights into the factors determining capacity for a wide range of scattering models.

**Keywords:** MIMO systems, wireless, capacity, spatial correlation, multipath channels, non-isotropic scattering

### 1. Introduction

Multiple-Input Multiple-Output (MIMO) communication systems using multi-antenna arrays simultaneously during transmission and reception have generated significant interest in recent years. Theoretical work of [Foschini and Gans, 13; Telatar, 41] showed the potential for significant capacity increases in wireless channels utilizing spatial diversity. Consider a system employing  $n_T$  transmit antennas and  $n_R$  receive antennas in a narrowband flat fading channel. It was shown in [Foschini and Gans, 13] that as  $\min(n_T, n_R)$  tends to infinity then the capacity of the system grows proportionally to  $\min(n_T, n_R)$  for fixed transmit power, provided that the fading between antennas is independent and identically distributed (i.i.d.) Rayleigh. This linear capacity growth has emerged as one of the most promising solutions for overcoming the demand for higher bit rates in wireless communications. In reality, however, the capacity is significantly reduced when the fades are not independent, but correlated due to insufficient antenna separation or angular spread of the scatterers surrounding the arrays [Foschini and Gans, 13; Shiu et al., 38].

A significant hurdle in analyzing the capacity of wireless fading MIMO systems is the random nature of the channel. The ergodic, or mean, capacity is often used to

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characterize the random channel capacity. However, the ergodic calculation requires extensive simulations which limits analysis into the physical factors determining MIMO capacity. A closed form expression for the ergodic capacity is derived in [Telatar, 42] for the special case of uncorrelated Rayleigh fading. However, as mentioned above, i.i.d. Rayleigh fading models an unrealistic environment not often seen in practice.

Capacity results for various correlation models using Monte Carlo simulations are studied in [Chizhik et al., 8; Chuah et al., 9; Driessen and Foschini, 12; Gorokhov, 19; Pedersen et al., 31; Shiu et al., 38], and asymptotic results and bounds on the effects of correlated channels are presented in [Chen-Nee et al., 6; Gorokhov, 19; Loyka and Kouki, 27; Pedersen et al., 31; Sengupta and Mitra, 37; Shiu et al., 38]. However, these simulations, bounds and asymptotic results have been for a limited set of channel realizations and/or antenna configurations. For single antenna systems it is sufficient to only consider received signal power and/or the time varying amplitude distribution of the channel. However, for systems employing multiple antennas, consideration must also be given to the angle of arrival (AOA) of the impinging signals as well as the spatial geometry of the array. Most channel models do not include spatial information (antenna locations and scattering environment) explicitly. Although spatial information is represented by the correlation between channel matrix elements there is no direct realizable physical representation, and, therefore does not easily lend itself to insightful capacity results. In particular, of interest is the effect on channel capacity of antenna placement, particularly in the realistic case when antenna arrays are restricted in size, along with non-isotropic scattering environments.

In contrast, the contribution of this paper is an expression for MIMO capacity which overcomes these limitations, that is, with additional theory for modelling scattering environments which we refine here, we derive a model which can be readily reconciled with a multitude of scattering distributions and antenna configurations and allows us to derive a closed form expression for the MIMO capacity.

In this paper, we exploit the convergence of ergodic capacity as the number of transmit antennas is increased to derive a closed-form channel capacity expression which depends on the correlation between the receive antennas. To model the correlation we generalize the *one-ring* model [Lee, 24; Shiu et al., 38], to develop a closed form expression which depends on the antenna spacing and placement along with the mean angle-of-arrival (AOA) and angular spread for a wide range of common scattering distributions. This allows for a capacity expression which can be evaluated for any general spatial scenario, giving significant insights into the capacity of such systems, without the need for multiple correlation models or extensive simulations.

The remainder of this paper is organized as follows. In section 2, we derive a closed form expression for the ergodic capacity of a MIMO system and discuss limitations to capacity scaling. In section 3, we develop a generalized 3D multipath channel model which includes antenna locations and environmental factors to characterize the MIMO wireless channel. Using this model, a closed-form spatial correlation expression for general scattering environments is derived, and we summarize some common scattering distributions. MIMO capacity is then analyzed in section 4 for a variety of physically re-

alistic spatial scenarios, and spatial limitations to capacity growth are discussed. Finally, we make some concluding remarks in section 5.

## 2. Convergence of ergodic capacity

Consider a MIMO system consisting of  $n_T$  transmit antenna and  $n_R$  receive antennas. When the transmitted signal vector is composed of statistically independent equal power components, each with a Gaussian distribution, the ergodic channel capacity was shown to be [Telatar, 41; Foschini and Gans, 13],

$$C_{\text{erg}} = E_H \left\{ \log \left| \mathbf{I}_{n_R} + \frac{\eta}{n_T} \mathbf{H} \mathbf{H}^\dagger \right| \right\}, \quad (1)$$

where  $\mathbf{H}$  is the  $n_R \times n_T$  random flat fading channel matrix, assumed known at the receiver, and normalized such that  $E\{|h_{rt}|^2\} = 1$ , where  $h_{rt}$  is the channel gain from the  $t$ th transmitter to the  $r$ th receiver. Note that the logarithm is base 2 and gives the capacity in bits-per-second-per-Hertz (bps/Hz).

Let  $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \dots \ \mathbf{h}_{n_T}]$ , where  $\mathbf{h}_t$  is the  $n_R \times 1$  complex zero-mean Gaussian vector of channel gains corresponding to the  $t$ th transmit antenna, then the correlation matrix at the receiver is defined as  $\mathbf{R}_{\text{rx}} \triangleq E\{\mathbf{h}_t \mathbf{h}_t^\dagger\}$ , where the expectation is over all transmitters and channel realizations, then

$$\mathbf{R}_{\text{rx}} = \begin{bmatrix} \rho_{11} & \rho_{12} & \dots & \rho_{1n_R} \\ \rho_{21} & \rho_{22} & & \\ \vdots & & \ddots & \vdots \\ \rho_{n_R 1} & & \dots & \rho_{n_R n_R} \end{bmatrix}, \quad (2)$$

with elements  $\mathbf{R}_{\text{rx}}|_{rr'} = \rho_{rr'}$  corresponding to the spatial correlation between two sensors  $r$  and  $r'$  at the receiver.

Consider the situation where the transmit array has uncorrelated transmit branches corresponding to independent  $\mathbf{h}_t$  vectors. This can occur when the transmit antennas are sufficiently separated for a given angular spread of the scatterers surrounding the transmit array. For example, using the correlation expression developed in section 3, a two dimensional isotropic scattering environment with antennas separated by  $0.35\lambda$  gives correlation of 0.1. However, for more realistic scattering environments, when the transmitter is usually mounted high above the scatterers, the angular spread is considerably less. For angular spread of  $3^\circ$  the antennas must be separated by around  $9\lambda$  to achieve the same level of correlation. With sufficient transmit antenna spacing the vectors  $\mathbf{h}_t$  are independent and the sample correlation matrix, defined as

$$\widehat{\mathbf{R}}_{\text{rx}} \triangleq \frac{1}{n_T} \sum_{t=1}^{n_T} \mathbf{h}_t \mathbf{h}_t^\dagger, \quad (3)$$

converges to  $\mathbf{R}_{\text{rx}}$  for large numbers of transmit antennas ( $n_T \rightarrow \infty$ ). Observing that the channel matrix product can be expressed as

$$\mathbf{H}\mathbf{H}^\dagger = \sum_{t=1}^{n_T} \mathbf{h}_t \mathbf{h}_t^\dagger, \quad (4)$$

then for a large number of sufficiently separated transmit antennas the ergodic capacity converges to the capacity  $C$ ,

$$\lim_{n_T \rightarrow \infty} C_{\text{erg}} = C \triangleq \log |\mathbf{I}_{n_R} + \eta \mathbf{R}_{\text{rx}}|. \quad (5)$$

The convergence error of the ergodic capacity to the expression (5) is shown in figure 1 for increasing numbers of uncorrelated transmit antennas and various numbers of receiver antennas. It is important to observe that the ergodic capacity approaches the capacity  $C$  for finite numbers of transmit antennas, with faster convergence for smaller numbers of receive antennas. Therefore, the capacity expression will be accurate for many practical fixed wireless scenarios, where the receiver has a small number of antennas whilst the base station is less restricted in geometrical size and is able to provide a sufficient number of uncorrelated transmit branches. Therefore, provided the channel does not contain keyholes<sup>1</sup> and the transmit antennas are sufficient in number and sep-

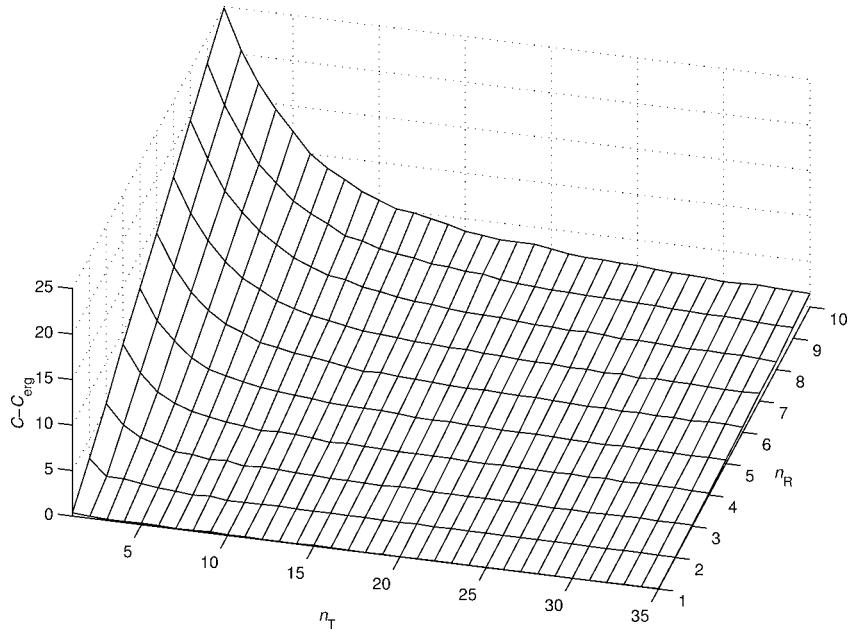


Figure 1. Convergence error of ergodic capacity  $C_{\text{erg}}$  (1) to bound  $C$  (5) with increasing number of transmit antennas for various numbers of receive antennas and SNR 10 dB.

<sup>1</sup> It has been shown theoretically that some channels may exhibit low capacity even though there is no spatial correlation at the transmitter or receiver [Chizhik et al., 7; Gesbert et al., 15]. However, no observations of

aration, the capacity  $C$  provides a good estimate of the ergodic capacity  $C_{\text{erg}}$ . Note that a bound similar to (5) has also been derived in [Abdi and Kaveh, 2] to demonstrate a space–time cross-correlation model for a von-Mises scattering environment.

### 2.1. Capacity scaling limits

The capacity given by (5) is maximized when there is no correlation between the receive antennas, i.e.,  $\mathbf{R}_{\text{rx}} = \mathbf{I}_{n_{\text{R}}}$ , giving

$$C_{\text{max}} = n_{\text{R}} \log(1 + \eta). \quad (6)$$

Therefore, in the idealistic situation of zero correlation at both transmitter and receiver arrays (corresponding to the i.i.d. case) we see the maximum capacity scaling is linear in the number of receive antennas. In this case, the system achieves the equivalent of  $n_{\text{R}}$  independent nonfading subchannels, each with SNR  $\eta$ . This result agrees with the traditional capacity formulation [Telatar, 42] which is widely used to advocate the use of MIMO systems.

Conversely, when each pair of antenna elements at the receiver are fully correlated, the correlation matrix becomes the  $n_{\text{R}} \times n_{\text{R}}$  matrix of ones,  $\mathbf{R}_{\text{rx}} = \mathbf{1}_{n_{\text{R}}}$ , and the capacity of the MIMO system will be minimized to

$$C_{\text{min}} = \log(1 + n_{\text{R}}\eta). \quad (7)$$

Here the logarithmic capacity growth with increasing receiver antennas is due to an effective increase in the average SNR of the single antenna case, due to the assumption of independent noise at each receiver, and is widely known as a receiver diversity array gain effect.

The capacity (5) provides an expression for the ergodic capacity without the need for extensive simulations. In contrast to current simulation studies presented in the literature, which are difficult to relate to physical factors of the system, in the next section it is shown that for essentially all common scattering distributions, and any array configurations, it is possible to compute a closed form capacity expression.

## 3. Receiver spatial correlation for general distributions of Farfield scatterers

### 3.1. Channel model

The *one-ring* model was initially proposed in [Lee, 24] to model fixed wireless communications systems where the base station is elevated and not obstructed by local scattering, whilst the user is uniformly surrounded by scatterers. In [Shiu et al., 38] this model was extended to include multiple transmit and receive antennas, and the capacity was studied for various transmit angular spreads and receive antenna geometries. In

keyhole (pinhole) degenerate channel effects from practical measurements have appeared in the literature. Therefore, in this paper we will assume nondegenerate channels only.

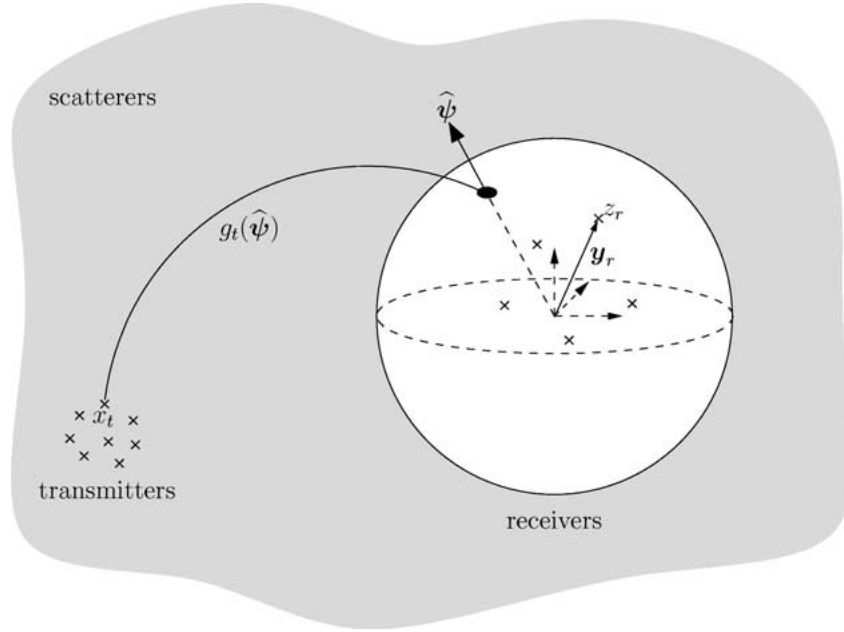


Figure 2. Scattering model for a flat fading MIMO system.  $g_t(\hat{\psi})$  represents the effective random complex gain of the scatterers for a transmitted signal  $x_t$  arriving at the receiver array from direction  $\hat{\psi}$  via any number paths through the scattering environment. The sphere surrounding the receive antennas contains no scatterers and is assumed large enough that any scatterers are farfield to all receive antennas located within.

this paper we model the multipath propagation and fading correlation using a generalized 3D *one-ring* model that allows for more general scattering environments at both the transmit and receive arrays. This model allows for greater insights into the spatial factors determining channel capacity, in particular, closed-form capacity expressions can be computed for a wide range of scattering environments and antenna geometries.

Consider the narrowband transmission of  $n_T$  statistically independent uniform power signals  $\{x_1, x_2, \dots, x_{n_T}\}$  through a general 3D flat fading scattering environment with scatterers assumed distributed in the farfield from the receiver antennas, as shown in figure 2, then the incoming signal from direction  $\hat{\psi}$  at the receiver is given by

$$\Phi(\hat{\psi}) = \sum_{t=1}^{n_T} x_t g_t(\hat{\psi}), \quad (8)$$

where  $g_t(\hat{\psi})$  is the effective random complex gain of the scatters for the transmitted signal  $x_t$  from the  $t$ th transmit antenna arriving at the receiver array from direction  $\hat{\psi}$ . Since the scatterers are assumed farfield to the receiver antennas, signals impinging on the receiver array will be plane waves, therefore, the received signal at the  $r$ th sensor

located at  $\mathbf{y}_r$  is given by

$$z_r = \int_{\Omega} \Phi(\hat{\boldsymbol{\psi}}) e^{-ik\mathbf{y}_r \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}) = \sum_{t=1}^{n_T} x_t \int_{\Omega} g_t(\hat{\boldsymbol{\psi}}) e^{-ik\mathbf{y}_r \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}), \quad (9)$$

where  $k = 2\pi/\lambda$  is the wavenumber with  $\lambda$  the wavelength, and  $d\Omega(\hat{\boldsymbol{\psi}})$  is a surface element of the unit sphere  $\Omega$ . Therefore, the channel gain  $h_{rt}$  from the  $t$ th antenna to the  $r$ th receiver is given by

$$h_{rt} = \int_{\Omega} g_t(\hat{\boldsymbol{\psi}}) e^{-ik\mathbf{y}_r \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}), \quad (10)$$

with normalization  $\int_{\Omega} E\{|g_t(\hat{\boldsymbol{\psi}})|^2\} d\Omega(\hat{\boldsymbol{\psi}}) = 1$ .

### 3.2. Correlation of the received complex envelopes

Define the normalized spatial correlation function between the complex envelopes of two received signals  $r$  and  $r'$  located at positions  $\mathbf{y}_r$  and  $\mathbf{y}_{r'}$ , respectively, as

$$\rho_{rr'} = \frac{E\{z_r \bar{z}_{r'}\}}{\sqrt{E\{z_r \bar{z}_r\} E\{z_{r'} \bar{z}_{r'}\}}}, \quad (11)$$

where  $\bar{x}$  denotes the complex conjugate of  $x$ , and  $z_r$  denotes the noiseless received signal at receiver  $r$ .

From (9) the covariance between signals at sensors  $r$  and  $r'$  is given by

$$\begin{aligned} E\{z_r \bar{z}_{r'}\} &= E\left\{ \sum_{t=1}^{n_T} x_t \int_{\Omega} g_t(\hat{\boldsymbol{\psi}}) e^{-ik\mathbf{y}_r \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}) \sum_{t'=1}^{n_T} \bar{x}_{t'} \int_{\Omega} \overline{g_{t'}(\hat{\boldsymbol{\psi}}')} e^{ik\mathbf{y}_{r'} \cdot \hat{\boldsymbol{\psi}}'} d\Omega(\hat{\boldsymbol{\psi}}') \right\} \\ &= \sigma_T^2 \int_{\Omega} \sum_{t=1}^{n_T} E\{g_t(\hat{\boldsymbol{\psi}}) \overline{g_t(\hat{\boldsymbol{\psi}}')}\} e^{-ik(\mathbf{y}_r \cdot \hat{\boldsymbol{\psi}} - \mathbf{y}_{r'} \cdot \hat{\boldsymbol{\psi}}')} d\Omega(\hat{\boldsymbol{\psi}}) d\Omega(\hat{\boldsymbol{\psi}}'), \end{aligned} \quad (12)$$

where  $\sigma_T^2 = E\{|x_t|^2\} \forall t$  is the transmit power for each antenna, and it is assumed the transmitted symbols are independent across antennas and independent of the scattering environment. Assuming a zero-mean uncorrelated scattering environment, the scattering channel is characterized by the second-order statistics of the scattering gain function  $g_t(\hat{\boldsymbol{\psi}})$ , given by

$$E\{g_t(\hat{\boldsymbol{\psi}}) \overline{g_t(\hat{\boldsymbol{\psi}}')}\} = G_t(\hat{\boldsymbol{\psi}}) \delta(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}'), \quad (13)$$

where  $G_t(\hat{\boldsymbol{\psi}}) = E\{|g_t(\hat{\boldsymbol{\psi}})|^2\}$ , then (12) simplifies to

$$E\{z_r \bar{z}_{r'}\} = \sigma_T^2 \int_{\Omega} \sum_{t=1}^{n_T} G_t(\hat{\boldsymbol{\psi}}) e^{-ik(\mathbf{y}_r - \mathbf{y}_{r'}) \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}). \quad (14)$$

Substitution of (14) into (11) gives the correlation between the two receiver sensors as

$$\rho_{rr'} = \int_{\Omega} \mathcal{P}(\hat{\boldsymbol{\psi}}) e^{-ik(\mathbf{y}_r - \mathbf{y}_{r'}) \cdot \hat{\boldsymbol{\psi}}} d\Omega(\hat{\boldsymbol{\psi}}), \quad (15)$$

where  $\mathcal{P}(\hat{\boldsymbol{\psi}})$  is the normalized average power received from direction  $\hat{\boldsymbol{\psi}}$ , defined by

$$\mathcal{P}(\hat{\boldsymbol{\psi}}) \triangleq \frac{\sum_{t=1}^{n_T} G_t(\hat{\boldsymbol{\psi}})}{\int_{\Omega} \sum_{t=1}^{n_T} G_t(\hat{\boldsymbol{\psi}}) d\Omega(\hat{\boldsymbol{\psi}})}. \quad (16)$$

In two dimensional scattering environments the power distribution (16) is commonly known as the power azimuth spectrum (PAS) [Kermoal et al., 23], or power azimuth distribution (PAD) [Gesbert et al., 18].

To highlight the factors which effect spatial correlation the Jacobi-Anger plane wave expansion is employed [Colton and Kress, 10],

$$e^{ik\mathbf{y} \cdot \hat{\boldsymbol{\psi}}} = \sum_{m=0}^{\infty} i^m (2m+1) j_m(k\|\mathbf{y}\|) P_m(\cos \xi), \quad (17)$$

where  $\xi = \angle(\mathbf{y}, \hat{\boldsymbol{\psi}})$  denotes the angle between  $\mathbf{y}$  and  $\hat{\boldsymbol{\psi}}$ ,  $j_m(\cdot)$  are the spherical Bessel functions of the first kind, and  $P_m(\cdot)$  are the Legendre polynomials of degree  $m$ . To further separate the effects of the scattering and the sensor positioning, consider the identity [Arfken, 4, p. 694]

$$P_m(\cos \xi) = \frac{4\pi}{2m+1} \sum_{n=-m}^m \overline{Y_m^n(\hat{\mathbf{y}})} Y_m^n(\hat{\boldsymbol{\psi}}), \quad (18)$$

where  $\hat{\mathbf{y}} = \mathbf{y}/\|\mathbf{y}\|$  and  $Y_m^n(\cdot)$  represent spherical harmonics [Colton and Kress, 10, p. 25], then the spatial correlation (15) can be expressed as

$$\rho_{rr'} = 4\pi \sum_{m=0}^{\infty} \sum_{n=-m}^m (-i)^m \alpha_m^n j_m(k\|\mathbf{y}_r - \mathbf{y}_{r'}\|) Y_m^n\left(\frac{\mathbf{y}_{r'} - \mathbf{y}_r}{\|\mathbf{y}_r - \mathbf{y}_{r'}\|}\right), \quad (19)$$

where coefficients  $\alpha_m^n$  characterize the scattering environment,

$$\alpha_m^n = \int_{\Omega} \mathcal{P}(\hat{\boldsymbol{\psi}}) \overline{Y_m^n(\hat{\boldsymbol{\psi}})} d\Omega(\hat{\boldsymbol{\psi}}), \quad (20)$$

and are independent of the antenna positions. The spatial correlation (19) is now composed of a summation of terms, where each term has independent factors characterizing the scattering environment and the antenna locations. Therefore, unlike previous models, where the antenna locations and the scattering environment are coupled, the separate effects of antenna geometry and scattering environment on the channel capacity can now be studied.



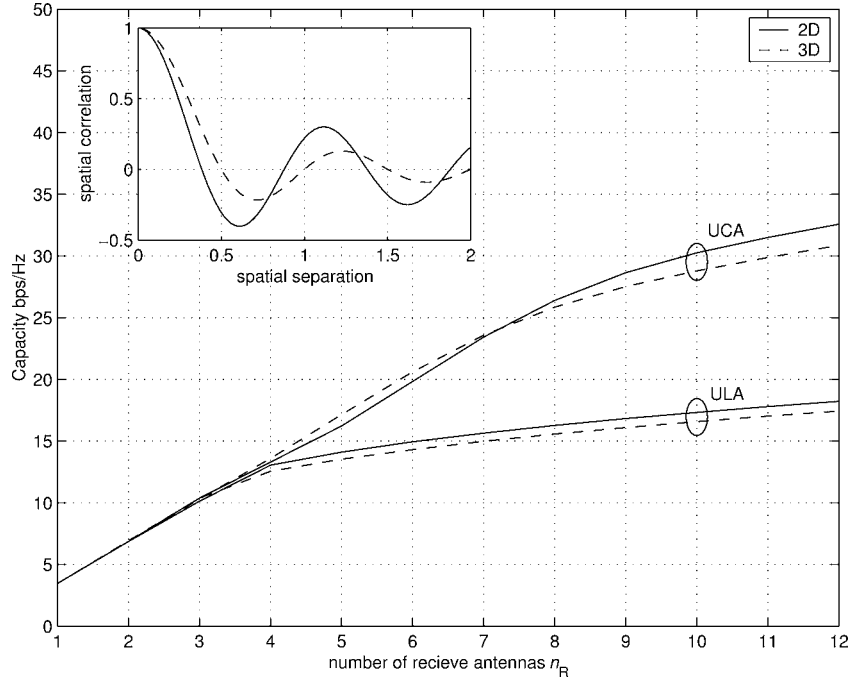


Figure 3. Capacity of 2D and 3D isotropic scattering environments for fixed length aperture ( $1\lambda$ ) uniform linear (ULA) and uniform circular (UCA) arrays for increasing number of receive antennas. Insert: spatial correlation between two antennas against spatial separation for the 2D and 3D isotropic scattering environments.

### 3.2.1. Isotropic scattering environment

For the special case of isotropic scattering (omnidirectional diffuse fields) the power distribution is given by the constants,  $\mathcal{P}(\boldsymbol{\psi}) = 1/2\pi^2$  and  $\mathcal{P}(\psi) = 1/2\pi$ , for the 3D and 2D scattering environments, respectively.<sup>2</sup> In this case, the summation in (19) reduces to a single term and the 3D and 2D spatial correlation can be shown to be:

$$\rho_{rr'}^{3D} = j_0(k\|\mathbf{y}_r - \mathbf{y}_{r'}\|), \quad (21)$$

$$\rho_{rr'}^{2D} = J_0(k\|\mathbf{y}_r - \mathbf{y}_{r'}\|), \quad (22)$$

which agree with the classical results [Cook et al., 11; Jakes, 20], where  $J_n(\cdot)$  are the Bessel functions of order  $n$ , and by definition  $j_0(\cdot) = \text{sinc}(\cdot)$ . With these analytic forms for the spatial correlation we can compute the capacity (5). Figure 3 shows the theoretical capacity for increasing antenna numbers for a fixed aperture uniform linear (ULA) and circular (UCA) arrays in 2D and 3D isotropic scattering with SNR of 10 dB. It is clear from comparison of the 2D and 3D capacities that any elevation spread has little effect on the capacity of an array in the horizontal plane. This can be seen in the insert

<sup>2</sup> The 2D scattering environment is a special case of the 3D case when the signals arrive from the azimuthal plane only.

of figure 3 where the spatial correlation for increasing spatial separation is shown for the 2D and 3D isotropic scattering environments, here the two functions  $J_0(\cdot)$  and  $j_0(\cdot)$  are qualitatively similar, particularly for low spatial separation. Therefore, without loss of generality, we focus on scattering environments where there is negligible power arriving from elevation angles.

### 3.3. Two-dimensional scattering environment

Consider a 2D scattering environment where the signals arrive only from the azimuthal plane, then  $\mathbf{y} = (\|\mathbf{y}\|, \theta_y)$  and  $\hat{\boldsymbol{\psi}} = (1, \psi)$  in polar coordinates, and (17) can be shown to reduce to,

$$e^{ik\mathbf{y}\cdot\hat{\boldsymbol{\psi}}} = J_0(k\|\mathbf{y}\|) + 2 \sum_{m=1}^{\infty} i^m J_m(k\|\mathbf{y}\|) \cos(m\xi) = \sum_{m=-\infty}^{\infty} J_m(k\|\mathbf{y}\|) e^{-im(\theta_y - \pi/2)} e^{im\psi}. \quad (23)$$

Substitution of (23) into (15) gives the spatial correlation for a 2D environment as

$$\rho_{rr'} = \sum_{m=-\infty}^{\infty} \alpha_m J_m(k\|\mathbf{y}_r - \mathbf{y}_{r'}\|) e^{im\theta_{rr'}}, \quad (24)$$

where  $\theta_{rr'}$  is the angle of the vector connecting  $\mathbf{y}_r$  and  $\mathbf{y}_{r'}$ . The coefficients  $\alpha_m$  characterize any possible 2D scattering environment surrounding the receiver and are given by

$$\alpha_m = \int_0^{2\pi} \mathcal{P}(\psi) e^{-im\psi} d\psi, \quad (25)$$

where  $\mathcal{P}(\psi)$  is the average angular power distribution, or power azimuth distribution (PAD).

Bessel functions  $J_n(x)$ ,  $|n| > 0$  exhibit spatially high pass behavior, that is, for fixed order  $n$ ,  $J_n(x)$  starts small and becomes significant for arguments  $x \approx O(n)$ . Therefore, to compute the spatial correlation for points closely located in space, only a few terms in the sum (24) need to be evaluated in order to give a very good approximation [Jones et al., 21]. Thus, closed-form solutions for the correlation can be found provided closed-form expressions exist for the scattering coefficients (25). As summarized next, for many common scattering distributions, there exists closed-form expressions for the scattering coefficients  $\alpha_m$  and thus the capacity can be computed for a wide range of realistic scattering environments.

### 3.4. Non-isotropic scattering environments

The spatial correlation as a function of receive antenna separation depends on the scattering distribution surrounding the receiver. One of the most commonly used distributions is the isotropic scattering model, where the power is assumed to be uniform over all AOA [Jakes, 20]. However, as discussed in [Braun and Dersch, 5;

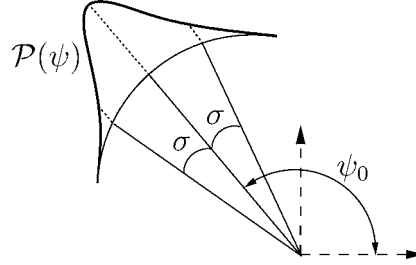


Figure 4. Multipath signal energy modelled as a non-isotropic scattering distribution  $\mathcal{P}(\psi)$  with mean AOA  $\psi_0$  and angular spread  $\sigma$  (defined as the standard deviation of the distribution).

Patzold et al., 29, 30; Sadowsky and Katedziski, 35], and verified via experimental measurement campaigns [Fuhl et al., 14; Lee, 25; Lo and Litva, 26; Rossi et al., 34; Spencer et al., 39; Wang et al., 45], many realistic scattering environments have nonuniform AOA distributions.

Non-isotropic scattering distributions model multipath as energy arriving from a particular direction with angular spread related to the non-isotropy parameter of the distribution, as shown in figure 4. The non-isotropic distributions are characterized by the mean AOA  $\psi_0$  and the angular spread  $\sigma$ , defined as the standard deviation of the distribution. Several distributions have been proposed for modelling the non-isotropic scattering environment [Abdi et al., 1; Kalkan and Clarke, 22; Pedersen et al., 32; Salz and Winters, 36; Spencer et al., 39; Vaughan, 44], in the following we outline several common distributions along with the scattering parameter effects on spatial correlation and capacity.

### 3.4.1. Uniform limited distributed field

When the energy arrives uniformly from a restricted range of azimuth angles  $\pm\Delta$  around a mean AOA,  $\psi_0 \in [-\pi, \pi)$ , we have the uniform limited distribution

$$\mathcal{P}(\psi) = \begin{cases} K_U, & |\psi - \psi_0| \leq \Delta, \\ 0, & \text{elsewhere,} \end{cases} \quad (26)$$

where  $K_U$  is a normalization constant such that  $\int_0^{2\pi} \mathcal{P}(\psi) d\psi = 1$ . In this case  $K_U = 1/2\Delta$  and the scattering environment coefficients (25) are given by

$$\alpha_m = \frac{\sin(m\Delta)}{m\Delta} e^{-im\psi_0}, \quad (27)$$

which gives the equivalent correlation expression to that derived in [Salz and Winters, 36]. For  $\Delta = \pi$  (isotropic scattering) (24) is given by a single term, and the correlation coefficient becomes  $J_0(k\|\mathbf{y}_r - \mathbf{y}_{r'}\|)$ , which agrees with earlier results.

### 3.4.2. Truncated Gaussian distributed field

The Gaussian distribution was proposed in [Kalkan and Clarke, 22] for modelling the distribution of scatterers as

$$\mathcal{P}(\psi) = K_G e^{-((\psi - \psi_0)/(\sqrt{2}\sigma_G))^2}, \quad \psi \in [-\pi, \pi), \quad (28)$$

where  $\sigma_G$  is the standard deviation of the nontruncated distribution and is related to the angular spread.  $K_G$  is a normalization constant, which can be shown to be

$$K_G = \frac{1}{\sqrt{2\pi}\sigma_G \operatorname{erf}(\pi/\sqrt{2}\sigma_G)}, \quad (29)$$

where  $\operatorname{erf}(x)$  is the error function, defined as  $\operatorname{erf}(x) = \int_0^x e^{-t^2} dt$ . In this case (25) is given by

$$\alpha_m = \frac{\Re\{\operatorname{erf}((\pi/2 + im\sigma_G^2)/(\sqrt{2}\sigma_G))\}}{\operatorname{erf}(\pi/\sqrt{2}\sigma_G) e^{m^2\sigma_G/2}} e^{-im\psi_0}, \quad (30)$$

which is well approximated by  $\alpha_m \approx e^{-m^2\sigma_G/2} e^{-im\psi_0}$  for narrow angular spread [Teal et al., 40].

### 3.4.3. Von-Mises distributed field

Another recently proposed non-isotropic scattering model is the von-Mises distribution [Abdi et al., 1],

$$\mathcal{P}(\psi) = K_v e^{\kappa \cos(\psi - \psi_0)}, \quad \psi \in [-\pi, \pi), \quad (31)$$

where  $\kappa \geq 0$  represents the degree of non-isotropy and is related to the angular spread of the AOA. Here the normalization constant  $K_v$  is given by

$$K_v = \frac{1}{2\pi I_0(\kappa)}, \quad (32)$$

where  $I_m(\cdot)$  is the modified Bessel function of the first kind. For  $\kappa = 0$  (isotropic scattering) the distribution becomes  $\mathcal{P}(\psi) = 1/2\pi$ , while for small angular spread,  $\kappa = \infty$ , the distribution is the Dirac delta function  $\mathcal{P}(\psi) = \delta(\psi - \psi_0)$ . For the truncated von-Mises field the scattering environment coefficients are given by

$$\alpha_m = \frac{I_{-m}(\kappa)}{I_0(\kappa)} e^{-im\psi_0}. \quad (33)$$

### 3.4.4. Truncated Laplacian distributed field

For some scenarios the Laplacian distribution has been proposed as a good model of the scattering distribution [Pedersen et al., 32; Spencer et al., 39]. The Laplacian distribution is defined as

$$\mathcal{P}(\psi) = K_L e^{-\sqrt{2}|\psi - \psi_0|/\sigma_L}, \quad \psi \in [-\pi, \pi), \quad (34)$$

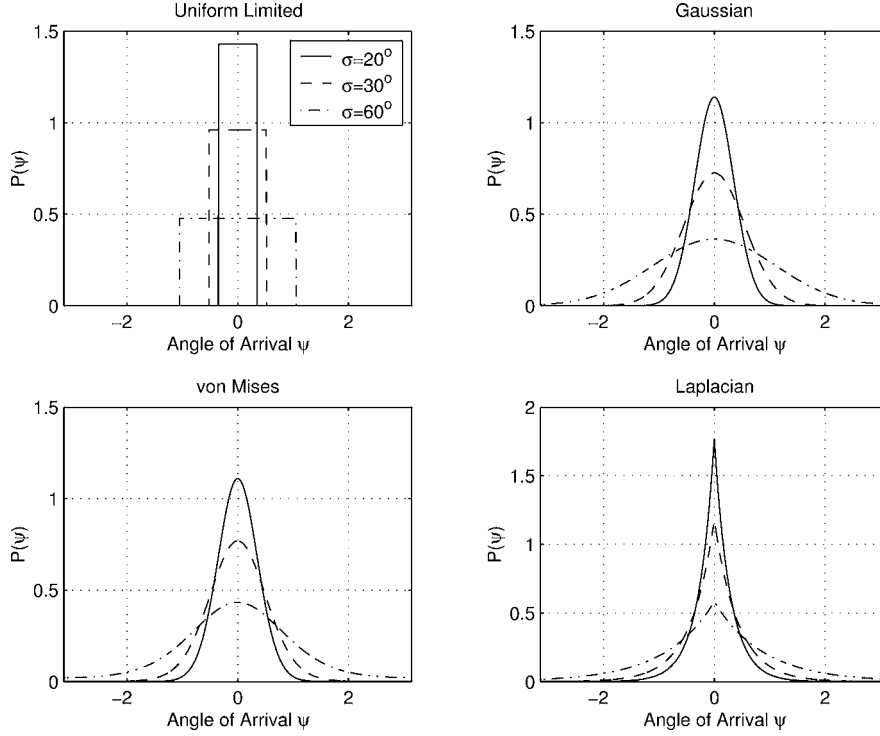


Figure 5. Comparison of common scattering distributions: uniform, Gaussian, von-Mises and Laplacian, for angular spread  $\sigma = \{20^\circ, 30^\circ, 60^\circ\}$ .

where  $\sigma_L$  is the standard deviation of the nontruncated distribution and is related to the angular spread of the AOA, and the normalization constant  $K_L$  is given by

$$K_L = \frac{1}{\sqrt{2}\sigma_L(1 - e^{-\sqrt{2}\pi/\sigma_L})}. \quad (35)$$

Here the scattering coefficients (25) can be expressed as [Teal et al., 40],

$$\alpha_m = \frac{(1 - (-1)^{\lceil m/2 \rceil} e^{-\pi/\sqrt{2}\sigma_L F_m})}{(1 + \sigma_L^2 m^2/2)(1 - e^{-\pi/\sqrt{2}\sigma_L})}, \quad (36)$$

where  $F_m = 1$  for even  $m$ , and  $F_m = m\sigma/\sqrt{2}$  for odd  $m$ . In this case, the correlation agrees with the recent derivation in [Tsai et al., 43], where the spatial correlation is derived for a uniform circular array within a Laplacian distribution scattering environment.

The above distributions are shown in figure 5 for various angle spreads  $\sigma = \{20^\circ, 30^\circ, 60^\circ\}$  about the mean AOA  $\psi_0 = 0$ , where the angle spread is defined as the standard deviation of the truncated distribution and is related to the nonisotropy parameter  $\Delta$ ,  $\sigma_G$ ,  $\kappa$ , or  $\sigma_L$ .

We now explore the effects on spatial correlation of angle spread and mean AOA for the above distributions. In order to compare the spatial correlation we set the angle

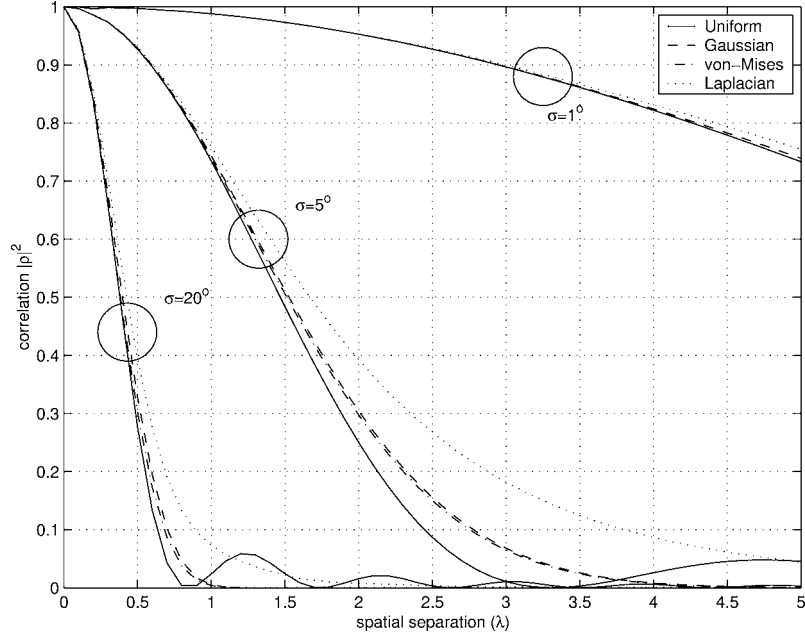


Figure 6. Spatial correlation between two antennas for mean AOA  $90^\circ$  (broadside) against spatial separation for uniform, Gaussian, von-Mises and Laplacian scattering distributions and angular spreads  $\sigma = \{1^\circ, 5^\circ, 20^\circ\}$ .

spread to  $\sigma = \{1^\circ, 5^\circ, 10^\circ\}$  for each distribution and increase the separation distance between the antennas, located on the  $x$ -axis. The spatial correlation for mean AOA  $\psi_0 = 90^\circ$  (broadside) is shown in figure 6. As shown, the spatial correlation decreases as the antenna spacing and/or the angular spread increases.<sup>3</sup> Here we also see that the scattering models all give similar spatial correlation for the same angular spread, particularly for small spatial separations, indicating that the choice of non-isotropic distribution is unimportant as the distribution variance dominates correlation. However, due to the higher concentration of energy about the mean for the Laplacian distribution,<sup>4</sup> for large spatial separation the Laplacian model generally gives higher correlation than the other three distributions for all angular spreads considered. Finally, we observe that in all cases the von-Mises distribution gives spatial correlation nearly identical to that of the Gaussian model. This agrees with observations made in [Abdi et al., 1], where it was noted that the von-Mises distribution resembles a Gaussian pdf for large  $\kappa$  (small angular spread) [Mardia, 28, p. 61].

In figure 7 the spatial correlation is shown for mean AOA  $\psi_0 = 30^\circ$  ( $60^\circ$  from broadside). Similar results are seen as for the broadside case, however, we see significant increase in spatial correlation for all angle spreads and distributions for the same spatial

<sup>3</sup> It is important to note that the correlation does not decrease monotonically with antenna separation, therefore, in certain scenarios increasing the antenna spacing may actually increase spatial correlation.

<sup>4</sup> When compared with the other models for identical angular spread.

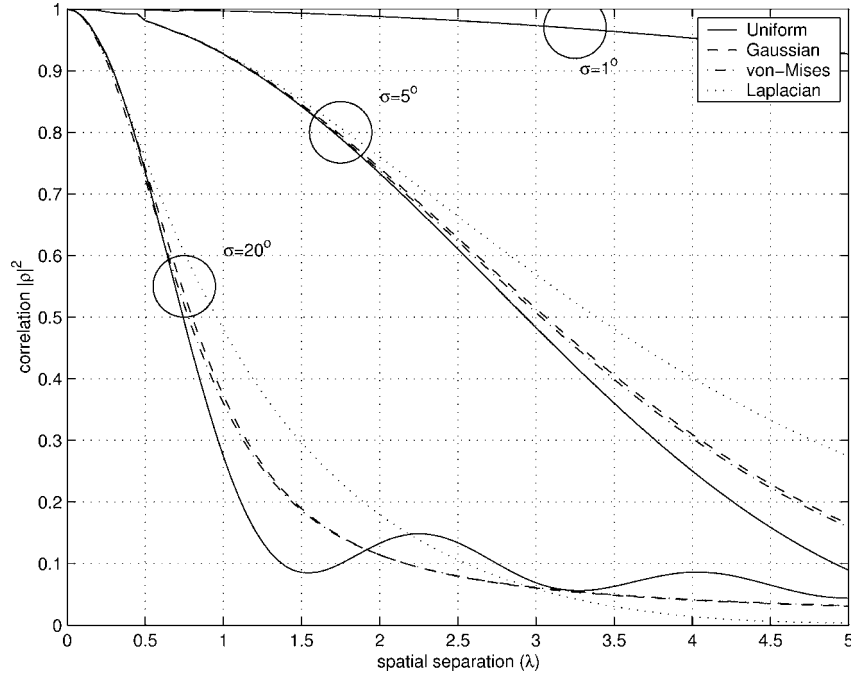


Figure 7. Spatial correlation between two antennas on the  $x$ -axis for mean AOA  $30^\circ$  ( $60^\circ$  from broadside) against spatial separation for uniform, Gaussian, von-Mises and Laplacian scattering distributions and angular spreads  $\sigma = \{1^\circ, 5^\circ, 20^\circ\}$ .

separation as before. Therefore, due to the reduction of resolvable angular spread at the antennas, the spatial correlation is increased as the mean AOA moves away from the broadside angle, where the broadside angle defined as the angle perpendicular to the line connecting the two antennas.

#### 4. Capacity results

In this section we study the spatial effects of non-isotropic scattering and antenna geometry on the capacity of MIMO systems. Using (5) and (24) along with the expressions for the non-isotropic scattering coefficients given in the previous section, we compute the capacity bound for a variety scattering environments for the uniform linear (ULA) and uniform circular (UCA) arrays. In particular, of interest is the notion of linear in antenna number capacity scaling widely used to advocate the use of MIMO systems [Foschini and Gans, 13; Telatar, 42], and how this is effected by non-isotropic scattering and dense array configurations. For all scenarios we consider a signal-to-noise ratio of  $\eta = 10$  dB. For comparison, the maximum (6) and minimum (7) capacities are also shown, corresponding to no spatial correlation and full spatial correlation capacities, respectively.

First we consider the capacity of an 8 antenna fixed aperture (length/diameter)  $3.5\lambda$  array for each scattering distribution against the non-isotropy factor and for mean AOA

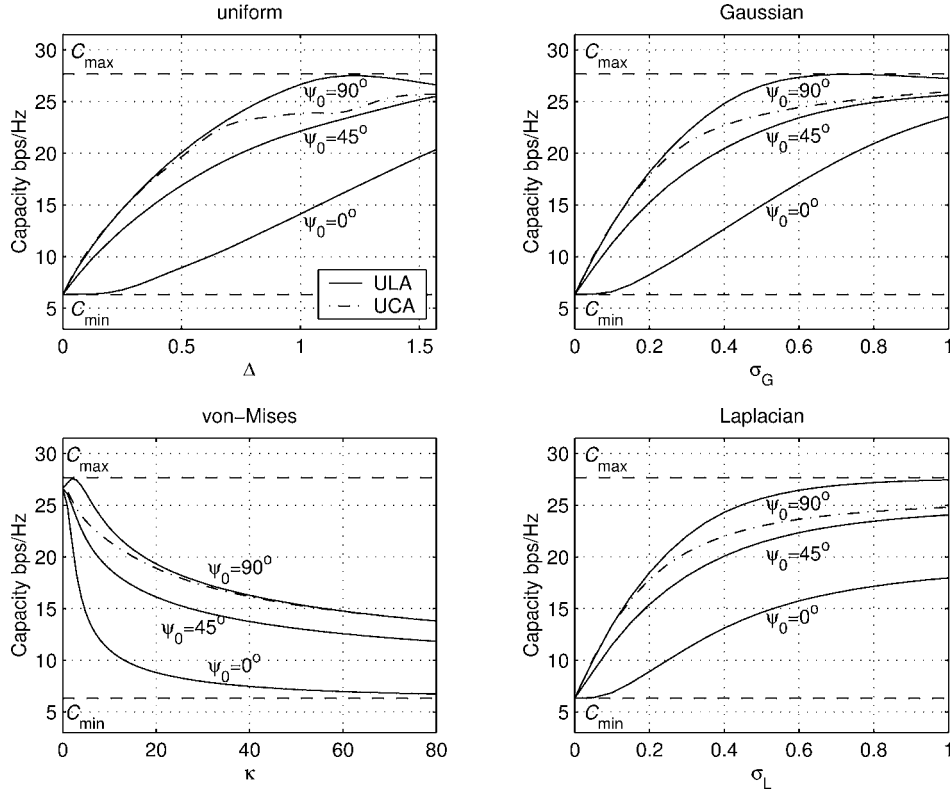


Figure 8. Capacity for non-isotropic distributed scattering with mean AOA  $\psi_0 = \{0^\circ, 45^\circ, 90^\circ\}$  and increasing nonisotropy factor, for the 8 antenna uniform linear (ULA) and uniform circular (UCA) arrays of aperture width (length/diameter)  $3.5\lambda$ .

$\psi_0 = \{0^\circ, 45^\circ, 90^\circ\}$ . As shown in figure 8, the capacity increases for increasing nonisotropy factor for the uniform, Gaussian, and Laplacian distributions, corresponding to an increase in angle spread surrounding the receiver. For the von-Mises distribution, the capacity decreases as  $\kappa$  increases, corresponding to a reduction in angular spread at the receiver. For all distributions, the reduction in capacity with decreasing angular spread is most pronounced for the ULA when the mean AOA is inline with the array ( $\psi_0 = 0^\circ$ ). However, for the UCA the capacity is unaffected by mean AOA, advocating the use of 2D array configurations, which are less sensitive to the mean direction of signal arrival, compared to the 1D linear arrays.

We now consider the capacity scaling of a MIMO system as the number of receive antennas increases whilst the array aperture size remains fixed. First we consider the ULA and UCA of aperture (length/diameter)  $D = \{0.4\lambda, 0.6\lambda, 0.8\lambda\}$  in an isotropic scattering environment, shown in figure 9.

It can be observed from figure 9 that the capacity scales almost linearly with the number of antennas before reducing to logarithmic growth after some saturation point. The saturation point is clearly related to the aperture of the array for both the ULA and



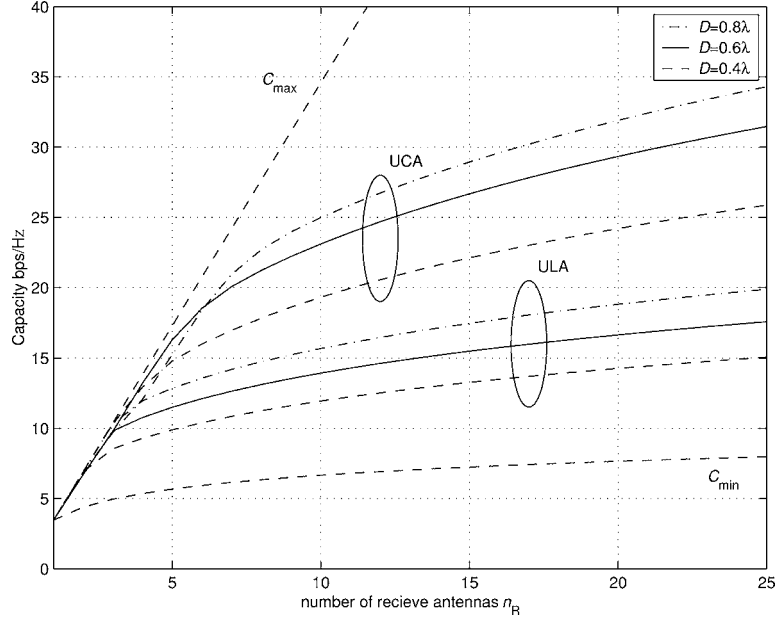


Figure 9. Capacity scaling of the uniform linear (ULA) and uniform circular (UCA) arrays with fixed aperture (length/diameter)  $D = \{0.4\lambda, 0.6\lambda, 0.8\lambda\}$  in an isotropic scattering environment.

UCA, where smaller apertures saturate for lower numbers of receive antennas. The capacity scaling of the ULA and UCA is shown in figures 10 and 11, respectively, for fixed aperture  $4\lambda$  with angular spread  $\sigma = \{1^\circ, 5^\circ, 20^\circ\}$  of the various scattering distributions. Again, for both ULA and UCA saturation is observed in the capacity scaling, where the growth reduces from approximately linear to logarithmic after the number of antennas reaches a distinct threshold. Note that the array aperture is sufficiently large so any saturation is due to the reduction of angular spread at the receiver, which is clearly seen in both figures.

To emphasize the effects of saturation with antenna numbers seen above, we can write the capacity (5) as

$$C = C_{\max} + \Delta C, \quad (37)$$

where  $C_{\max}$  is the maximum antenna capacity (6), and  $\Delta C = \log |\mathbf{\Gamma}_{\text{rx}}|$ , where  $\mathbf{\Gamma}_{\text{rx}}$  is the  $n_{\text{R}} \times n_{\text{R}}$  matrix

$$\mathbf{\Gamma}_{\text{rx}} = \begin{bmatrix} 1 & \frac{\eta}{1+\eta}\rho_{12} & \cdots & \frac{\eta}{1+\eta}\rho_{1n_{\text{R}}} \\ \frac{\eta}{1+\eta}\rho_{21} & 1 & & \vdots \\ \vdots & & \ddots & \\ \frac{\eta}{1+\eta}\rho_{n_{\text{R}}1} & \cdots & & 1 \end{bmatrix}. \quad (38)$$

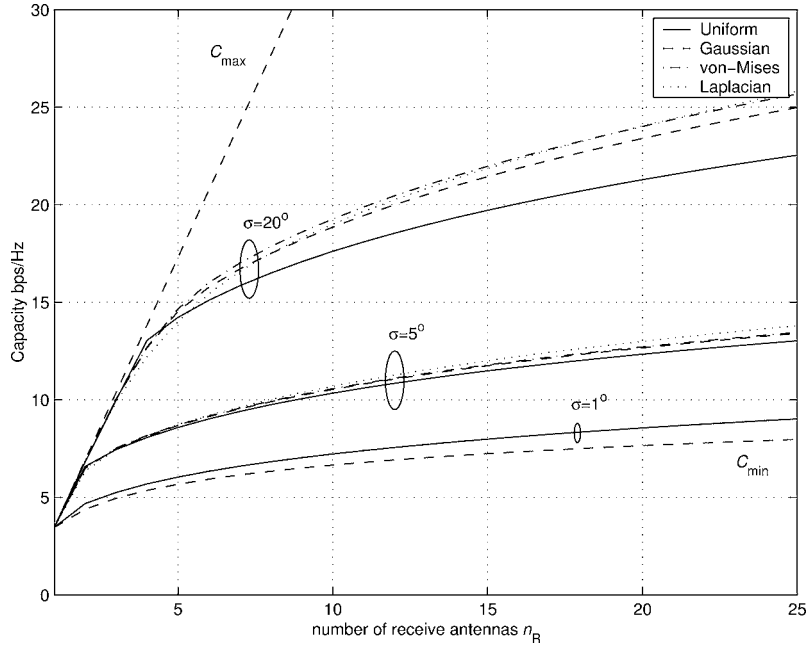


Figure 10. Capacity scaling of the broadside uniform linear array with fixed aperture  $4\lambda$  for angular spread  $\sigma = \{1^\circ, 5^\circ, 20^\circ\}$  of the various scattering distributions.

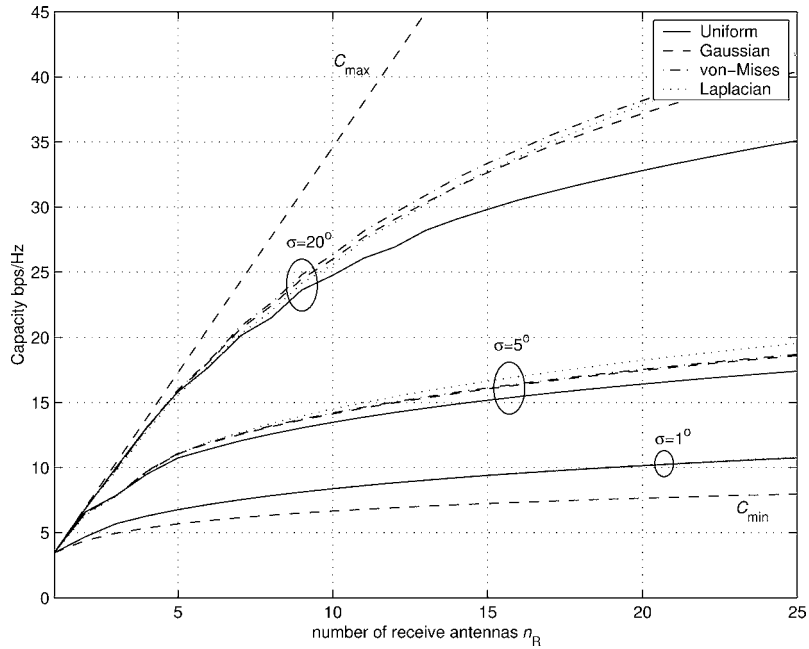


Figure 11. Capacity scaling of the uniform circular array with fixed aperture  $4\lambda$  for angular spread  $\sigma = \{1^\circ, 5^\circ, 20^\circ\}$  of the various scattering distributions.

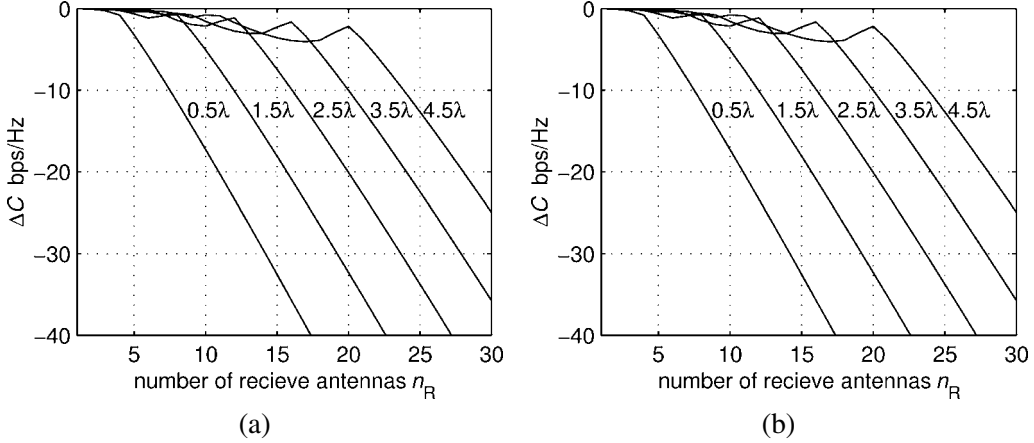


Figure 12. Capacity loss due to correlation of the broadside uniform linear array for: (a) fixed aperture  $D = \{0.5\lambda, 1.5\lambda, 2.5\lambda, 3.5\lambda, 4.5\lambda\}$  in an isotropic scattering environment. (b) fixed aperture  $4\lambda$  for angular spreads  $\sigma = \{1^\circ, 5^\circ, 10^\circ, 20^\circ\}$  of the various scattering distributions.

Note that  $\mathbf{\Gamma}_{rx}$  is a positive semi-definite matrix with  $0 \leq |\Gamma_{rx}| \leq 1$ , thus  $\Delta C \leq 0$ , therefore  $\Delta C$  represents the loss of capacity due to antenna correlation at the receiver. The loss of capacity due to correlation is shown in figure 12(a) for a ULA for fixed aperture  $D = \{0.5\lambda, 1.5\lambda, 2.5\lambda, 3.5\lambda, 4.5\lambda\}$  in an isotropic scattering environment. Here the distinct saturation point in antenna number is clearly visible with increasing aperture. Likewise, the loss in capacity is significant beyond an antenna saturation point for increasing angular spread as shown in figure 12(b) for a ULA of fixed aperture  $4\lambda$ .

Using a singular value decomposition of the channel matrix, the capacity of a MIMO system can be shown to be equivalent to the sum of the capacities of  $\min(n_T, n_R)$  subchannels, each with independent power gains corresponding to the eigenvalues of the channel matrix product  $\mathbf{H}\mathbf{H}^\dagger$  [Telatar, 41]. The subchannel gains depend on the correlation between channel branches, and as the correlation increases some subchannels have gains too small to convey information at any significant rate [Shiu et al., 38]. Therefore, as seen here, as the number of antennas increases for a fixed aperture, or angular spread, due to increased channel branch correlation there exists a saturation point at which the subchannels generated by any addition antennas have negligible gains and do not increase the capacity, other than logarithmic array gain.

This saturation effect has significant implications for realizable MIMO systems, as the saturation point gives the optimal number of antennas required to maximize capacity, after which there are negligible gains. Asymptotic results for the fixed aperture ULA in isotropic scattering are studied in [Wei et al., 46], with identical independent results in [Gesbert et al., 17]. However, the saturation point for fixed aperture arrays has not been addressed so far, except for the special case of the UCA [Pollock et al., 33]. Promising results from a spatial model presented in [Abhayapala et al., 3] indicate the throughput of a MIMO system is limited by the radius of the region containing the antenna arrays, agreeing with observations in this paper. Likewise, to the authors' knowl-

edge, no analytical results exist for the capacity scaling saturation due to insufficient angular spreads, although the effect on capacity due to limited angular spread at the transmitter has been reported in [Shiu et al., 38], and observed elsewhere, e.g., [Gesbert et al., 16], via simulation.

## 5. Conclusions

In i.i.d. Rayleigh fading channels the capacity of a multi-antenna system has been shown to grow linearly with antenna numbers. However, in realistic propagation environments, the fading is correlated due to insufficient antenna spacing or angular spread, and the capacity is often significantly lower than that predicted for i.i.d. fading. By assuming a sufficient number of sufficiently separated transmit antennas we have derived a closed-form expression for the capacity of a MIMO random fading channel. Using a novel channel model we separate the effects of the scattering environment and antenna array configuration on the spatial correlation at the receiver, allowing for capacity evaluation of systems for a wide variety of scattering environments and antenna placement.

We show that the angular spread surrounding the receiver dominates the spatial correlation and thus capacity, rather than the choice of scattering distribution. However, this may not hold for multi-modal distributions which were not discussed here, but are fully captured by our analytical framework. For 1D arrays it was observed that the mean AOA has a significant impact on the capacity of the system, advocating the use of 2D arrays, which are less sensitive to the mean direction of signal arrival.

The capacity was also shown to suffer from a saturation effect in the number of antennas for a fixed angular spread or array aperture. This saturation point, at which the capacity scaling is reduced from linear to logarithmic increase with increasing antenna numbers, is an important factor in the design of practical MIMO systems and needs further investigation.

With the improved spatial channel modelling and capacity expressions presented in this paper, it is hoped the spatial factors influencing the capacity of wireless multi-antenna systems can be better exploited to aid in the design of realistic high-capacity/high-quality wireless communication systems.

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