FOCUS ISSUE: Control and Synchronization in Chaotic Dynamical Systems

Introduction: Control and synchronization in chaotic dynamical systems

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The last ten years have seen remarkable developments in the research of chaotic dynamics, particularly with respect to the interaction of chaotic dynamics with other fields of research and with applications. There is now a developed science of chaos that has as an essential underpinning the strong interaction of theory and experiment. This is a departure from earlier times in which theoretical work existed largely in the absence of substantial experimental realizations. Along with this new orientation has come increased appreciation and concern for the implications of chaotic dynamics in practical applications. Issues in topics such as the active control of chaotic systems in a broad variety of situations, the use of chaos for communication, and the synchronization of chaotic dynamics for various purposes, are at the forefront of recent application topics in nonlinear science. The common thread through those topics is the marriage between knowledge of the basic mathematical properties of chaos and specific practical considerations of various applications.

This Focus Issue resulted from a six-week event at the Max Planck Institute for Physics of Complex Systems in Dresden in the Fall of 2001. During that Worshop/Seminar, especially interesting and challenging topics on control and synchronization were addressed. We believe that successes in the research work coming out from that program will have far-reaching technological and economical impact for a broad area of important practical systems ranging from lasers, via engineering to neuroscience and medicine.

This issue focuses on *Control and Synchronization in Chaotic Dynamical Systems*. The fundamentals and the major concepts involved in this area were reviewed in *Chaos* in a Focus Issue in December 1997 [*Chaos* **7** (4)].[1] Since that time, the then novel topics and applications have matured, making this area well-established within nonlinear science. Elements and concepts from the theory of systems control and the theory of communication have been brought in, giving the whole topic a firmer foundation. Therefore, the papers in this issue have brought together researchers from the fields of nonlinear dynamics, statistical physics, applied mathematics, information science, engineering, experimental physics, chemistry, and neuroscience. Questions such as how to improve the performance of chaos control under different conditions involving especially noise and high-dimensionality, how to control patterns, what are the theoretical concepts for understanding synchronization of chaos, or how to combine methods of chaos control and synchronization are addressed. Recent and possible new applications are highlighted in several contributions.

CONTROLLING CHAOS

The presence of chaos in physical systems has been extensively demonstrated and is very common. The main property of chaotic dynamics is its critical sensitivity to initial conditions, which is responsible for initially neighboring trajectories separating from each other exponentially in the course of time. For many years, this feature made chaos undesirable, insofar as the sensitivity to initial conditions of chaotic systems reduces their predictability over long time scales. On the other hand, the capability of chaotic dynamics to amplify small perturbations improves their utility for reaching specific desired states with very high flexibility and low energy cost. Indeed, large alterations to achieve a desired behavior are in general experimentally unpractical, nor can a typical system suffer them without substantially changing its main dynamical properties. In contrast, the process of controlling chaos is directed to improving a desired behavior by making only small time-dependent perturbations in an accessible system parameter or dynamical variable. The key observation is that a chaotic attractor typically has embedded within it an infinite, and in fact dense, set of unstable periodic orbits. This means that the system is able to realize an infinite number of possible different kinds of periodic motion apart from chaotic motion. The theoretical and experimental work done on this topic shows that the various methods of control are capable of yielding improved performance in a

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wide variety of situations.[2] This Focus Issue collects several contributions that continue and extend the existing work to test its effectiveness under different conditions involving noise, high dimensionality, and experimental data, by describing refinements of open loop methods and of continuous time-delayed feedback techniques, as well as by testing methods in spatially extended systems for the control of localized and solitary structures in optical fields.

Controlled chaos is also useful for communication. The naturally occurring chaotic orbits on the attractor can be utilized to carry information by making the symbolic dynamics of the experimental system follow a prescribed symbol sequence: thus one can encode and transmit a prescribed message using chaotic wave form. This process may offer practical advantages over the usual periodic carriers, such as the possibility of real time reconstructions of signal dropouts in the communication, or the possibility of warranting higher security levels for confidential communication, or even the possibility of multiplexing.

SYNCHRONIZATION WITH CHAOS

Synchronization is one basic feature in nonlinear science. Historically, the analysis of synchronization phenomena has been a subject of active investigation since the early days of physics. The famous Dutch researcher Christiaan Huygens was the first to observe and describe this phenomenon in mutually coupled periodic oscillators; namely for two pendulum clocks hanging from a common support, a wooden beam in a half-timber house.[3] Since the early 1990s, there has been strongly increasing interest in the synchronization of chaotic systems. In fact, one can distinguish among different types of synchronization for chaotic systems, from complete or identical synchronization, to phase synchronization, lag synchronization, generalized synchronization, and anticipating synchronization. Complete synchronization implies a perfect linking of the chaotic trajectories, so as they remain in step with each other in the course of the time. Generalized synchronization implies the hooking of the output of one system to a given function of the output of the other system.

A weaker form of synchrony is represented by the phase synchronization regime, wherein a perfect locking of the phases from the oscillating subsystems is realized already for small coupling, while the two amplitudes remain almost uncorrelated. To get this kind of synchronization, only low energy is needed; it is abundant in science, nature, engineering, or social life.

A third type of synchronization is lag synchronization, occurring for a stronger coupling of the oscillating subsystems until they become identical in phases and amplitudes, but shifted in time of a lag time. In the same spirit, anticipating synchronization exploits unidirectional delayed coupling schemes to produce a collective behavior wherein the output of the response system anticipates the output of the drive one.

Recently, evidence and verification of these theoretical findings have been offered in various experimental as well as natural systems, and theoretical attempts are in progress to extend these concepts to spatially extended pattern forming systems, as well as to structurally nonequivalent systems.[4,5] This Focus Issue collects various papers presenting new achievements in the theoretical understanding of chaotic synchronization, including a geometric theory on phase synchronization, noise-induced effects, the role of unstable attractors in forming synchronized dynamics, the use of the concept of observability for the design of synchronized systems, and controlling synchronization. There are several contributions on experimental work, such as the first experimental evidence of imperfect phase synchronization, lasers and electrochemical systems, but also on natural systems, such as synchronization for sensory encoding of the crayfish or the directionality in the human cardiovascular system.

CONCLUDING REMARKS

Control and synchronization of chaotic dynamics have been established as a central topic in nonlinear science. This topic is strongly evolving, and we will not intend to forecast future developments. However, we are confident that it will continue to offer exciting challenges for mathematics and physics which will lead to new and unexpected insights in nonlinear science. We are also sure that this topic will find many more applications in various fields. We hope that this Focus Issue will be of relevance for specialists as well attract those who do not yet work in this field.

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