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Jerrold E. Marsden Tudor S. Ratiu

Introduction to Mechanics and Symmetry

A Basic Exposition of Classical Mechanical Systems

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To Barbara and Lilian for their love and support

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM).*

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

Symmetry and mechanics have been close partners since the time of the founding masters: Newton, Euler, Lagrange, Laplace, Poisson, Jacobi, Hamilton, Kelvin, Routh, Riemann, Noether, Poincaré, Einstein, Schrödinger, Cartan, Dirac, and to this day, symmetry has continued to play a strong role, especially with the modern work of Kolmogorov, Arnold, Moser, Kirillov, Kostant, Smale, Souriau, Guillemin, Sternberg, and many others. This book is about these developments, with an emphasis on concrete applications that we hope will make it accessible to a wide variety of readers, especially senior undergraduate and graduate students in science and engineering.

The geometric point of view in mechanics combined with solid analysis has been a phenomenal success in linking various diverse areas, both within and across standard disciplinary lines. It has provided both insight into fundamental issues in mechanics (such as variational and Hamiltonian structures in continuum mechanics, fluid mechanics, and plasma physics) and provided useful tools in specific models such as new stability and bifurcation criteria using the energy–Casimir and energy–momentum methods, new numerical codes based on geometrically exact update procedures and variational integrators, and new reorientation techniques in control theory and robotics.

Symmetry was already widely used in mechanics by the founders of the subject, and has been developed considerably in recent times in such diverse phenomena as reduction, stability, bifurcation and solution symmetry breaking relative to a given system symmetry group, methods of finding explicit solutions for integrable systems, and a deeper understanding of special systems, such as the Kowalewski top. We hope this book will provide a reasonable avenue to, and foundation for, these exciting developments.

Because of the extensive and complex set of possible directions in which one can develop the theory, we have provided a fairly lengthy introduction. *It is intended to be read lightly at the beginning and then consulted from time to time as the text itself is read.*

This volume contains much of the basic theory of mechanics and should prove to be a useful foundation for further, as well as more specialized, topics. Due to space limitations we warn the reader that many important topics in mechanics are not treated in this volume. We are preparing a second volume on general reduction theory and its applications. With luck, a little support, and yet more hard work, it will be available in the near future.

Solutions Manual. A solution manual is available for instructors. It contains complete solutions to many of the exercises, as well as other supplementary comments. For further information, see

`http://www.cds.caltech.edu/~marsden/books/`.

Internet Supplements. To keep the size of the book within reason, we are making some material available (free) on the Internet. These are a collection of sections whose omission does not interfere with the main flow of the text. See `http://www.cds.caltech.edu/~marsden/books/`. Updates and information about the book can also be found at this website.

What Is New in the Second Edition? In this second edition, the main structural changes are the creation of a solutions manual (along with many more exercises in the text) and the Internet supplements. The Internet supplements contain, for example, the material on the Maslov index that was not needed for the main flow of the book. As for the substance of the text, much of the book was rewritten throughout to improve the flow of material and to correct inaccuracies. Some examples: The material on the Hamilton–Jacobi theory was completely rewritten, a new section on Routh reduction (§8.9) was added, Chapter 9 on Lie groups was substantially improved and expanded. The presentation of examples of coadjoint orbits (Chapter 14) was improved by stressing matrix methods throughout.

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December, 1998

About the Authors

Jerrold E. Marsden is professor of control and dynamical systems at Caltech. He received his B.Sc. at Toronto in 1965 and his Ph.D. from Princeton University in 1968, both in applied mathematics. He has done extensive research in mechanics, with applications to rigid-body systems, fluid mechanics, elasticity theory, plasma physics, as well as to general field theory. His primary current interests are in the area of dynamical systems and control theory, especially how it relates to mechanical systems with symmetry. He is one of the founders in the early 1970s of reduction theory for mechanical systems with symmetry, which remains an active and much studied area of research today. He was the recipient of the prestigious Norbert Wiener prize of the American Mathematical Society and the Society for Industrial and Applied Mathematics in 1990, and was elected a fellow of the American Academy of Arts and Sciences in 1997. He has been a Carnegie Fellow at Heriot–Watt University (1977), a Killam Fellow at the University of Calgary (1979), recipient of the Jeffrey–Williams prize of the Canadian Mathematical Society in 1981, a Miller Professor at the University of California, Berkeley (1981–1982), a recipient of the Humboldt Prize in Germany (1991), and a Fairchild Fellow at Caltech (1992). He has served in several administrative capacities, such as director of the Research Group in Nonlinear Systems and Dynamics at Berkeley, 1984–86, the Advisory Panel for Mathematics at NSF, the Advisory committee of the Mathematical Sciences Institute at Cornell, and as director of the Fields Institute, 1990–1994. He has served as an editor for Springer-Verlag's *Applied Mathematical Sciences* series since 1982 and serves on the editorial boards of several journals in mechanics, dynamics, and control.

Tudor S. Ratiu is professor of mathematics at the University of California, Santa Cruz and the Swiss Federal Institute of Technology, in Lausanne. He received his B.Sc. in mathematics and M.Sc. in applied mathematics, both at the University of Timişoara, Romania, and his Ph.D. in mathematics at Berkeley in 1980. He has previously taught at the University of Michigan, Ann Arbor, as a T. H. Hildebrandt Research Assistant Professor (1980–1983) and at the University of Arizona, Tucson (1983–1987). His research interests center on geometric mechanics, symplectic geometry, global analysis, and infinite-dimensional Lie theory, with applications to integrable systems, nonlinear dynamics, continuum mechanics, plasma physics, and bifurcation theory. He was a National Science Foundation Postdoctoral Fellow (1983–86), a Sloan Foundation Fellow (1984–87), a Miller Research Professor at Berkeley (1994), and a recipient of of the Humboldt Prize in Germany (1997). Since his arrival at UC Santa Cruz in 1987, he has been on the executive committee of the Nonlinear Sciences Organized Research Unit. He is managing editor of the AMS *Surveys and Monographs* series and on the editorial board of the *Annals of Global Analysis* and the *Annals of the University of Timişoara*. He was also a member of various research institutes such as MSRI in Berkeley, the Center for Nonlinear Studies at Los Alamos, the Max Planck Institute in Bonn, MSI at Cornell, IHES in Bures-sur-Yvette, The Fields Institute in Toronto (Waterloo), the Erwin Schroödinger Institute for Mathematical Physics in Vienna, the Isaac Newton Institute in Cambridge, and RIMS in Kyoto.

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