Introduction to Nonlinear Finite Element Analysis

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To my family

Preface

The finite element method (FEM) is one of the numerical methods for solving differential equations that describe many engineering problems. The FEM, originated in the area of structural mechanics, has been extended to other areas of solid mechanics and later to other fields such as heat transfer, fluid dynamics, and electromagnetism. In fact, FEM has been recognized as a powerful tool for solving partial differential equations and integrodifferential equations, and in the near future, it may become the numerical method of choice in many engineering and applied science areas. One of the reasons for FEM's popularity is that the method results in computer programs versatile in nature that can solve many practical problems with least amount of training.

The availability of undergraduate- and advanced graduate- level FEM courses in engineering schools has increased in response to the growing popularity of the FEM in industry. In the case of linear structural systems, the methods of modeling and solution procedure are well established. Nonlinear systems, however, take different modeling and solution procedures based on the characteristics of the problems. Accordingly, the modeling and solution procedures are much more complicated than that of linear systems, although there are advanced topics in linear systems such as complex shell formulations.

Researchers who have studied and applied the linear FEM cannot apply the linearized method to more complicated nonlinear problems such as elastoplastic or contact problems. However, many textbooks in the nonlinear FEMs strongly emphasize complicated theoretical parts or advanced topics. These advanced textbooks are mainly helpful to students seeking to develop additional nonlinear FEMs. However, the advanced textbooks are oftentimes too difficult for students and researchers who are learning the nonlinear FEM for the first time.

One of the biggest challenges to the instructor is finding a textbook appropriate to the level of the students. The objective of this textbook is to simply introduce the nonlinear finite element analysis procedure and to clearly explain the solution procedure to the reader. In contrast to the traditional textbooks which treat a vast amount of nonlinear theories comprehensively, this textbook only addresses the representative problems, detailed theories, solution procedures, and the computer implementation of the nonlinear FEM. Especially by using the MATLAB programming language to introduce the nonlinear solution procedure, those readers who are not familiar with FORTRAN or C++ programming languages can easily understand and add his/her own modules to the nonlinear analysis program.

The textbook is organized into five chapters. The objective of Chap. 1 is to introduce basic concepts that will be used for developing nonlinear finite element formulations in the following chapters. Depending on the level of the students or prerequisites for the course, this chapter or a part of it can be skipped. Basic concepts in this chapter include vector and tensor calculus in Sect. 1.2, definition of stress and strain in Sect. 1.3, mechanics of continuous bodies in Sect. 1.4, and linear finite element formulation in Sect. 1.5. A MATLAB code for three-dimensional finite element analysis with solid elements will reinforce mathematical understanding.

Chapter 2 introduces nonlinear systems of solid mechanics. In Sect. 2.1, fundamental characteristics of nonlinear problems are explained in contrast to linear problems, followed by four types of nonlinearities in solid mechanics: material, geometry, boundary, and force nonlinearities. Section 2.2 presents different methods of solving a nonlinear system of equations. Discussions on convergence aspects, computational costs, load increment, and force-controlled vs. displacement-controlled methods are provided. In Sect. 2.3, step-by-step procedures in solving nonlinear finite element analysis are presented. Section 2.4 introduces NLFEA, a MATLAB code for solving nonlinear finite element equations. NLFEA can handle different material models, such as elastic, hyperelastic, and elastoplastic materials, as well as large deformation. Section 2.5 summarizes how commercial finite element analysis programs control nonlinear solution procedures. This section covers Abaqus, ANSYS, and NEi Nastran programs.

Chapter 3 presents theoretical and numerical formulations of nonlinear elastic materials. Since nonlinear elastic material normally experiences a large deformation, Sect. 3.2 discusses stress and strain measures under large deformation. Section 3.3 shows two different formulations in representing large deformation problems: total Lagrangian and updated Lagrangian. In particular, it is shown that these two formulations are mathematically identical but different in computer implementation and interpreting material behaviors. Critical load analysis is introduced in Sect. 3.4, followed by hyperelastic materials in Sect. 3.5. Different ways of representing incompressibility of elastic materials are discussed. The continuum form of the nonlinear variational equation is discretized in Sect. 3.6, followed by a MATLAB code for a hyperelastic material model in Sect. 3.7. Section 3.8 summarizes the usage of commercial finite element analysis programs to solve nonlinear elastic problems, particularly for hyperelastic materials. In hyperelastic materials, it is important to identify material parameters. Section 3.9 presents curve-fitting methods to identify hyperelastic material parameters using test data.

Different from elastic materials, some materials, such as steels or aluminum alloys, show permanent deformation when a force larger than a certain limit (elastic limit) is applied and removed. This behavior of materials is called plasticity.

When the total strain is small (infinitesimal deformation), it is possible to assume that the total strain can be additively decomposed into elastic and plastic strains. Sections 4.2 and 4.3 are based on infinitesimal elastoplasticity. In a large structure, even if the strain is small, the structure may undergo a large rigid-body motion due to accumulated deformation. In such a case, it is possible to modify infinitesimal elastoplasticity to accommodate stress calculation with the effect of rigid-body motion. Since the rate of Cauchy stress is not independent of rigid-body motion, different types of rates, called objective stress rates, are used in the constitutive relation, which is discussed in Sect. 4.4. When deformation is large, the assumption of additive decomposition of elastic and plastic strains is no longer valid. A hyperelasticity-based elastoplasticity is discussed in Sect. 4.5, in which the deformation gradient is multiplicatively decomposed into elastic and plastic parts and the stress–strain relation is given in the principal directions. This model can represent both geometric and material nonlinearities during large elastoplastic deformation. Section 4.6 is supplementary to Sect. 4.5, as it derives several expressions used in Sect. 4.5. Section 4.7 summarizes the usage of commercial finite element analysis programs to solve elastoplastic problems.

When two or more bodies collide, contact occurs between two surfaces of the bodies so that they cannot overlap in space. Metal formation, vehicle crash, projectile penetration, various seal designs, and bushing and gear systems are only a few examples of contact phenomena. In Sect. 5.2, simple one-point contact examples are presented in order to show the characteristics of contact phenomena and possible solution strategies. In Sect. 5.3, a general formulation of contact is presented based on the variational formulation. Section 5.4 focuses on finite element discretization and numerical integration of the contact variational form. Three-dimensional contact formulation is presented in Sect. 5.5. From the finite element point of view, all formulations involve use of some form of a constraint equation. Because of the highly nonlinear and discontinuous nature of contact problems, great care and trial and error are necessary to obtain solutions to practical problems. Section 5.6 presents modeling issues related to contact analysis, such as selecting slave and master bodies, removing rigid-body motions, etc.

This textbook details how the nonlinear equations are solved using practical computer programs and may be considered an essential course for those who intend to develop more complicated nonlinear finite elements. Usage of commercial FEA programs is summarized at the end of each chapter. It includes various examples in the text using Abaqus, ANSYS, NEi Nastran, and MATLAB program. Depending on availability and experience of the instructor, any program can be used as part of homework assignments and projects. The textbook website will maintain up-to-date examples with the most recent version of the commercial programs. Each chapter contains a comprehensive set of homework problems, some of which require commercial FEA programs.

Prospective readers or users of the text are graduate students in mechanical, civil, aerospace, biomedical, and industrial engineering and engineering mechanics as well as researchers and design engineers from the aforementioned fields.

The author is thankful to the students who took Advanced Finite Element Analysis course at the University of Florida and used the course package that had the same material as in this book. The author is grateful for their valuable suggestions especially regarding the example and exercise problems. Finally, special thanks to my daughter, Hyesu Grace Kim, for her outstanding work editing the manuscript.

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