Quantum Statistical Mechanics is concerned with many-particle systems that interact with the external world. This leads to a quantum mechanical description of thermal equilibrium in terms of mixed states that follow a Gibbs distribution. The external world acts as a heat bath that fixes the temperature. This situation should be contrasted to closed, or isolated, systems, which are described by pure states subject to the time dependent Schrödinger equation. As we always may combine our system of interest with its environment, it is clear that Quantum Statistical Mechanics must somehow arise out of such time-evolving pure states. How and in what sense this happens is a very interesting question first investigated by von Neumann in the early days of quantum theory [1]. Unfortunately it is extremely challenging to experimentally realize many-particle systems, in which unitary time evolution can be observed on appreciable time scales. This greatly inhibited further developments for several decades, although a number of theoretical works appeared that are nowadays recognized as ground breaking [2]. In the early noughties the situation changed dramatically, when it became possible to investigate the non-equilibrium evolution of cold atomic systems that were to a good approximation isolated on long time scales [3]. These experimental advances opened the door to exploring new regions of Hilbert space and answer fundamental questions about relaxation and equilibration in many-particle quantum systems. Exciting results have kept on coming ever since, e.g. while this volume was being completed it was shown by a direct measurement [4], that entanglement creates local entropy that validates the use of statistical physics for local observables.

Quantum integrable models have played a important role in these developments for a number of reasons. First, some of the systems that have been explored experimentally are described by integrable theories with small perturbations. Second, integrable models have the attractive feature of allowing one to derive exact results. This in turn has proved extremely useful for revealing general features of non-equilibrium dynamics. For example it was understood by considering non-interacting theories that relaxation in isolated quantum systems occurs at the level of local properties, which can be described by appropriate statistical ensembles [7]. Finally, the non-equilibrium dynamics of (almost) integrable models has been found to differ in interesting ways from that of generic systems. For example, the pioneering quantum Newton's cradle experiment of Kinoshita, Wenger, and Weiss [5] demonstrated that while (non-integrable) two- and three-dimensional gases relax swiftly to an equilibrium Gibbs distribution, a one-dimensional (almost integrable) gas evolves slowly towards a non-thermal stationary state. For the purpose of this introduction we define quantum integrable models rather loosely as being characterized by having an infinite number of conservation laws that possess certain locality properties. A consequence of this structure is that the scattering of elementary excitations in these models is purely elastic. In this volume we will encounter many examples of integrable theories, ranging from systems of non-interacting particles to quantum spin chains to conformal field theories. Their behaviours out of equilibrium are strongly affected by their conservation laws, which constrain their dynamics and provide selection rules for the stationary states they relax to [6]. The locality properties of conservation laws in integrable theories play a crucial role in this context [8], as in fact any Hamiltonian H has a huge number of non-local conservation laws (such as the projectors on its eigenstates or arbitrary powers of H). The constraints imposed by the conservation laws on the evolution of integrable models out of equilibrium have the attractive consequence that such systems can relax to states of matter that are impossible to realize in equilibrium. In presence of weak integrability breaking perturbations these states become metastable, giving rise to the phenomenon of prethermalization.

Our aim in putting together this Special Issue of JSTAT was to provide a self-contained introduction to key aspects of the non-equilibrium dynamics in (almost) integrable theories. Nine reviews focus on a range of theoretical aspects in both lattice and continuum theories, while one review provides an overview of the current state of the art in cold atom experiments. The topics covered are largely complementary to previous reviews on quantum quenches and thermalisation in the literature [9, 10, 11, 12].

The contents of the Special Issue is as follows:

1. Essler and Fagotti open the volume with a pedagogical introduction to quantum quenches in integrable quantum spin chains, focussing in particular on the role played by local conservation laws.

2. Calabrese and Cardy review the imaginary time path integral approach to the quench dynamics in conformal field theories, and discuss the application of these results to condensed matter and cold atoms systems.

3. Cazalilla and Chung survey results on quantum quenches in the Luttinger model and its close relatives.

4. Bernard and Doyon review the transport properties of conformal field theories both in one and higher dimensions.

5. Caux presents an introduction to the quench action method, which is an effective representation for calculating time-dependent expectation values of physical operators following a generic out-of-equilibrium protocol.

6. Ilievski, Medenjak, Prosen, and Zadnik discuss the importance of quasilocal charges in integrable lattice systems both for quenches and transport phenomena.

7. Vidmar and Rigol report on the Generalized Gibbs Ensemble in integrable lattice models and how to measure its predictions in numerical simulations.

8. Langen, Gasenzer, and Schmiedmayer review the concept of prethermalization in near-integrable quantum systems with special emphasis on the experiments where these phenomena have been observed.

9. Vasseur and Moore review the non-equilibrium dynamics of many-body quantum systems after a quantum quench with spatial inhomogeneities, focusing on integrable and many-body localized systems.

10. De Luca and Mussardo close the volume with a manuscript on classical integrable field theories at a finite energy density, with a time evolution that starts from initial conditions far from equilibrium.

Even a volume as substantial as this one cannot give a complete account of all the recent exciting developments in the non-equilibrium dynamics of integrable models. Some topics like ramps and Kibble-Zurek physics have been omitted because reviews already exists [24], while others had to be left out due to space constraints. We nevertheless hope that this Special Issue can serve as a useful and substantial introduction for newcomers in the field, as well as a convenient and reasonably complete reference for experts.

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