Introduction to Random Signals and Noise

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Preface

Random signals and noise are present in several engineering systems. Practical signals seldom lend themselves to a nice mathematical deterministic description. It is partly a consequence of the chaos that is produced by nature. However, chaos can also be man-made, and one can even state that chaos is a *conditio sine qua non* to be able to transfer information. Signals that are not random in time but predictable contain no information, as was concluded by Shannon in his famous communication theory.

To deal with this randomness we have to nevertheless use a characterization in deterministic terms; i.e. we employ probability theory to determine characteristic descriptions such as mean, variance, correlation, etc. Whenever chaotic behaviour is time-dependent, as is often the case for random signals, the time parameter comes into the picture. This calls for an extension of probability theory, which is the theory of stochastic processes and random signals. With the involvement of time, the phenomenon of frequency also enters the picture. Consequently, random signal theory leans heavily on both probability and Fourier theories. Combining these subjects leads to a powerful tool for dealing with random signals and noise.

In practice, random signals may be encountered as a desired signal such as video or audio, or it may be an unwanted signal that is unintentionally added to a desired (information bearing) signal thereby disturbing the latter. One often calls this unwanted signal noise. Sometimes the undesired signal carries unwanted information and does not behave like noise in the classical sense. In such cases it is termed as interference. While it is usually difficult to distinguish (at least visually) between the desired signal and noise (or interference), by means of appropriate signal processing such a distinction can be made. For example, optimum receivers are able to enhance desired signals while suppressing noise and interference at the same time. In all cases a description of the signals is required in order to be able to analyse their impact on the performance of the system under consideration. In communication theory this situation often occurs. The random time-varying character of signals is usually difficult to describe, and this is also true for associated signal processing activities such as filtering. Nevertheless, there is a need to characterize these signals using a few deterministic parameters that allow a system user to assess system performance.

This book deals with stochastic processes and noise at an introductory level. Probability theory is assumed to be known. The same holds for mathematical background in differential and integral calculus, Fourier analysis and some basic knowledge of network and linear system theory. It introduces the subject in the form of theorems, properties and examples. Theorems and important properties are placed in frames, so that the student can easily summarize them. Examples are mostly taken from practical applications. Each chapter concludes with a summary and a set of problems that serves as practice material. The book is well suited for dealing with the subject at undergraduate level. A few subjects can be skipped if they do not fit into a certain curriculum. Besides, the book can also serve as a reference for the experienced engineer in his daily work.

In Chapter 1 the subject is introduced and the concept of a stochastic process is presented. Different types of processes are defined and elucidated by means of simple examples.

Chapter 2 gives the basic definitions of probability density functions and includes the time dependence of these functions. The approach is based on the 'ensemble' concept. Concepts such as stationarity, ergodicity, correlation functions and covariance functions are introduced. It is indicated how correlation functions can be measured. Physical interpretation of several stochastic concepts are discussed. Cyclo-stationary and Gaussian processes receive extra attention, as they are of practical importance and possess some interesting and convenient properties. Complex processes are defined analogously to complex variables. Finally, the different concepts are reconsidered for discrete-time processes.

In Chapter 3 a description of stochastic processes in the frequency domain is given. This results in the concept of power spectral density. The bandwidth of a stochastic process is defined. Such an important subject as modulation of stochastic processes is presented, as well as the synchronous demodulation. In order to be able to define and describe the spectrum of discrete-time processes, a sampling theorem for these processes is derived.

After the basic concepts and definitions treated in the first three chapters, Chapter 4 starts with applications. Filtering of stochastic processes is the main subject of this chapter. We confine ourselves to linear, time-invariant filtering and derive both the correlation functions and spectra of a two-port system. The concept of equivalent noise bandwidth has been defined in order to arrive at an even more simple description of noise filtering in the frequency domain. Next, the calculation of the spectrum of random data signals is presented. A brief resumé of the principles of discrete-time signals and systems is dealt with using the z-transform and discrete Fourier transform, based on which the filtering of discrete-time processes is described both in time and frequency domains.

Chapter 5 is devoted to bandpass processes. The description of bandpass signals and systems in terms of quadrature components is introduced. The probability density functions of envelope and phase are derived. The measurement of spectra and operation of the spectrum analyser is discussed. Finally, sampling and conversion to baseband of bandpass processes is discussed.

Thermal noise and its impact on systems is the subject of Chapter 6. After presenting the spectral densities we consider the role of thermal noise in passive networks. System noise is considered based on the thermal noise contribution of amplifiers, the noise figure and the influence of cascading of systems on noise performance.

Chapter 7 is devoted to detection and optimal filtering. The chapter starts by considering hypothesis testing, which is applied to the detection of a binary signal disturbed by white Gaussian noise. The matched filter emerges as the optimum filter for optimum detection performance. Finally, filters that minimize the mean squared error (Wiener filters) are derived. They can be used for smoothing stored data or portions of a random signal that arrived in the past. Filters that produce an optimal prediction of future signal values can also be designed.

Finally, Chapter 8 is of a more advanced nature. It presents the basics of random point processes, of which the Poisson process is the most well known. The characteristic function plays a crucial role in analysing these processes. Starting from that process several shot noise processes are introduced: the homogeneous Poisson process, the inhomogeneous Poisson process, the Poisson impulse process and the random-pulse process. Campbell's theorem is derived. A few application areas of random point processes are indicated.

The appendices contain a few subjects that are necessary for the main material. They are: signal space representation and definitions of attenuation, phase shift and decibels. The rest of the appendices comprises basic mathematical relations, a summary of probability theory, definitions of special functions, a list and properties of Fourier transform pairs, and a few mathematical and physical constants.

Finally, I would like to thank those people who contributed in one way or another to this text. My friend Rajan Srinivasan provided me with several suggestions to improve the content. Also, Arjan Meijerink carefully read the draft and made suggestions for improvement.

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