Cambridge University Press
978-0-521-81747-9 - Introduction to Strings and Branes
Peter West
Frontmatter
More information

Introduction to Strings and Branes

Supersymmetry, strings and branes are believed to be essential ingredients in a single unified consistent theory of physics. This book gives a detailed, step-by-step introduction to the theoretical foundations required for research in strings and branes.

After a study of the different formulations of the bosonic and supersymmetric point particles, the classical and quantum bosonic and supersymmetric string theories are presented. This book contains accounts of brane dynamics including D-branes and the M5-brane as well as the duality symmetries of string theory. Several different accounts of interacting strings are presented; these include the sum over world-sheets approach and the original S-matrix approach. More advanced topics include string field theory and Kac–Moody symmetries of string theory.

The book contains pedagogical accounts of conformal quantum field theory, supergravity theories, Clifford algebras and spinors, and Lie algebras. It is essential reading for graduate students and researchers wanting to learn strings and branes.

Peter West is a Professor at King's College London and a Fellow of the Royal Society. He is a pioneer in the development of supersymmetry and its application to strings and branes.

Cambridge University Press
978-0-521-81747-9 - Introduction to Strings and Branes
Peter West
Frontmatter
More information

Introduction to Strings and Branes

PETER WEST
King's College London

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521817479

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-81747-9 Hardback

Contents

	Preface	*page* xi
1	**The point particle**	1
1.1	The bosonic point particle	1
1.1.1	The classical point particle and its Dirac quantisation	1
1.1.2	The BRST quantization of the point particle	5
1.2	The super point particle	8
1.2.1	The spinning particle	9
1.2.2	The Brink–Schwarz superparticle	17
1.2.3	Superspace formulation of the point particle	21
1.3	The twistor approach to the massless point particle	24
1.3.1	Twistors in four and three dimensions	25
1.3.2	The twistor point particle actions	29
2	**The classical bosonic string**	36
2.1	The dynamics	36
2.1.1	The closed string	42
2.1.2	The open string	46
2.2	The energy-momentum and angular momentum of the string	49
2.3	A classical solution of the open string	50
3	**The quantum bosonic string**	52
3.1	The old covariant method	54
3.1.1	The open string	55
3.1.2	The closed string	62
3.2	The BRST approach	64
3.2.1	The BRST action	64
3.2.2	The world-sheet energy-momentum tensor and BRST charge	72
3.2.3	The physical state condition	77
4	**The light-cone approach**	81
4.1	The classical string in the light-cone	81
4.2	The quantum string in the light-cone	89
4.3	Lorentz symmetry	92
4.4	Light-cone string field theory	99

Cambridge University Press
978-0-521-81747-9 - Introduction to Strings and Branes
Peter West
Frontmatter
More information

5 Clifford algebras and spinors 100
5.1 Clifford algebras 100
5.2 Clifford algebras in even dimensions 101
5.3 Spinors in even dimensions 106
5.4 Clifford algebras in odd dimensions 111
5.5 Central charges 114
5.6 Clifford algebras in space-times of arbitrary signature 116

6 The classical superstring 120
6.1 The Neveu–Schwarz–Ramond (NS–R) formulation 121
6.1.1 The open superstring 125
6.1.2 The closed superstring 130
6.2 The Green–Schwarz formulation 133

7 The quantum superstring 143
7.1 The old covariant approach to the open superstring 144
7.1.1 The NS sector 146
7.1.2 The R sector 148
7.2 The GSO projector for the open string 150
7.3 The old covariant approach to the closed superstring 153

8 Conformal symmetry and two-dimensional field theory 160
8.1 Conformal transformations 161
8.1.1 Conformal transformations in D dimensions 161
8.1.2 Conformal transformations in two dimensions 163
8.2 Conformally invariant two-dimensional field theories 171
8.2.1 Conformally invariant two-dimensional classical theories 171
8.2.2 Conformal Ward identities 173
8.3 Constraints due to global conformal transformations 181
8.4 Transformations of the energy-momentum tensor 184
8.5 Operator product expansions 187
8.6 Commutators 189
8.7 Descendants 192
8.8 States, modes and primary fields 196
8.9 Representations of the Virasoro algebra and minimal models 199

9 Conformal symmetry and string theory 210
9.1 Free field theories 210
9.1.1 The free scalar 210
9.1.2 The free fermion 219
9.2 First order systems 222
9.3 Application to string theory 227
9.3.1 Mapping the string to the Riemann sphere 227
9.3.2 Construction of string theories 232
9.4 The free field representation of the minimal models 235

10 String compactification and the heterotic string 240
10.1 Compactification on a circle 240
10.2 Torus compactification 247
10.3 Compactification in the presence of background fields 253
10.4 Description of the moduli space 257
10.5 Heterotic compactification 261
10.6 The heterotic string 264

11 The physical states and the no-ghost theorem 272
11.1 The no-ghost theorem 272
11.2 The zero-norm physical states 281
11.3 The physical state projector 285
11.4 The cohomology of Q 287

12 Gauge covariant string theory 293
12.1 The problem 294
12.2 The solution 300
12.3 Derivation of the solution 306
12.4 The gauge covariant closed string 310
12.5 The gauge covariant superstring 315

13 Supergravity theories in four, ten and eleven dimensions 320
13.1 Four ways to construct supergravity theories 321
13.1.1 The Noether method 323
13.1.2 The on-shell superspace method 331
13.1.3 Gauging of space-time groups 339
13.1.4 Dimensional reduction 342
13.2 Non-linear realisations 346
13.3 Eleven-dimensional supergravity 361
13.4 The IIA supergravity theory 366
13.5 The IIB supergravity theory 374
13.5.1 The algebra and field content 375
13.5.2 The equations of motion 378
13.5.3 The SL(2, **R**) version 380
13.6 Symmetries of the maximal supergravity theories in dimensions less than ten 383
13.7 Type I supergravity and supersymmetric Yang–Mills theories in ten dimensions 390
13.8 Solutions of the supergravity theories 392
13.8.1 Solutions in a generic theory 392
13.8.2 Brane solutions in eleven-dimensional supergravity 408
13.8.3 Brane solutions in the ten-dimensional maximal supergravity theories 411
13.8.4 Brane charges and the preservation of supersymmetry 413

14 Brane dynamics 420
14.1 Bosonic branes 420
14.2 Types of superbranes 424
14.3 Simple superbranes 430

14.4 D-branes 434
14.5 Branes in M theory 435
14.6 Solutions of the 5-brane of M theory 444
14.6.1 The 3-brane 445
14.6.2 The self-dual string 448
14.7 Five-brane dynamics and the low energy effective action of the $N = 2$ Yang–Mills theory 452

15 D-branes 460
15.1 Bosonic D-branes 461
15.2 Super D-branes in the NS–R formulation 469
15.3 D-branes in the Green–Schwarz formulation 475

16 String theory and Lie algebras 485
16.1 Finite dimensional and affine Lie algebras 485
16.1.1 A review of finite-dimensional Lie algebras and lattices 485
16.1.2 Representations of finite dimensional semi-simple Lie algebras 502
16.1.3 Affine Lie algebras 509
16.2 Kac–Moody algebras 512
16.3 Lorentzian algebras 516
16.4 Very extended and over-extended Lie algebras 519
16.5 Weights and inverse Cartan matrix of E_n 524
16.6 Low level analysis of Lorentzian Kac–Moody algebras 526
16.6.1 The adjoint representation 526
16.6.2 All representations 528
16.7 The Kac–Moody algebra E_{11} 532
16.7.1 E_{11} at low levels 532
16.7.2 The l_1 representation of E_{11} 536
16.7.3 The Cartan involution invariant subalgebra of a Kac–Moody algebra 539
16.8 String vertex operators and Lie algebras 541

17 Symmetries of string theory 550
17.1 T duality 550
17.2 Electromagnetic duality 556
17.3 S and U duality 569
17.4 M theory 573
17.5 E theory 581
17.5.1 The eleven-dimensional theory 581
17.5.2 The IIA and IIB theories 586
17.5.3 The common origin of the eleven-dimensional, IIA and IIB theories 595
17.5.4 Theories in less than ten dimensions 598
17.5.5 Duality symmetries and conditions 601
17.5.6 Brane charges, the l_1 representation and generalised space-time 605
17.5.7 Weyl transformations of E_{11} and the non-linear realisation of its Cartan sub-algebra 609

18 String interactions 612
18.1 Duality, factorisation and the origins of string theory 612
18.2 The path integral approach 633
18.3 The group theoretic approach 647
18.4 Interacting open string field theory 657
18.4.1 Light-cone string field theory 657
18.4.2 Mapping the interacting string 662
18.4.3 A brief discussion of interacting gauge covariant string field theory 664

Appendix A The Dirac and BRST methods of quantisation 666
A.1 The Dirac method 666
A.2 The BRST method 668

Appendix B Two-dimensional light-cone and spinor conventions 673
B.1 Light-cone coordinates 673
B.2 Spinor conventions 674

Appendix C The relationship between S^2 and the Riemann sphere 676

Appendix D Some properties of the classical Lie algebras 679
D.1 The algebras A_{n-1} 679
D.2 The algebras D_n 680
D.3 The algebra E_6 681
D.4 The algebra E_7 681
D.5 The algebra E_8 682
D.6 The algebras B_n 682
D.7 The algebras C_n 683

Chapter quote acknowledgements 684
References 685
Index 706

Cambridge University Press
978-0-521-81747-9 - Introduction to Strings and Branes
Peter West
Frontmatter
More information

Preface

If we have told lies you have told half lies. A man who tells lies merely hides the truth, but a man who tells half truths has forgotten where he put it.

The British consul to Laurence of Arabia before he arrived with the Arab army in Damascus.

In the late 1960s a small group of theorists concluded that quantum field theory could not provide a suitable description for the main problem of the time, that is, to account for hadronic physics. As a result, they began a quest to find an S-matrix that had certain preordained properties. The search culminated in the discovery of such an S-matrix for four, and then any number of, spin-0 particles. By using physical principles and mathematical consistency it was found that these S-matrix elements were part of a larger theory that possessed an infinite number of particles. Remarkably, the early pioneers found the scattering amplitudes for any number, and any type, of these particles; they even found these results at any loop order. It was subsequently realised that this was the theory of string scattering and that the theory was more suited to describe fundamental, rather than hadronic, physics.

Supersymmetry was unearthed from the world-sheet action for the ten-dimensional string and also found by independent quantum field theory considerations in Russia. Supersymmetry is entwined with string theory, but it is an independent subject. Hopefully, it will be found at the Large Hadron Collider at CERN, but even if it is, this is unlikely to be direct evidence for string theory. Supersymmetry and string theory are believed to be essential ingredients in a unified consistent theory of all physics. It was thought initially that this theory would just be the theory of ten-dimensional strings, but we now realise that it must also include branes on an equal footing. We are quite far from having a systematic understanding of the quantum properties of branes and what the underlying theory is remains unclear. Indeed, even the concepts on which it is based may be quite different to those we know now. Ironically when string theory was first discovered it was not called string theory, but the dual model, as researchers were unaware of its stringy origins, where as nowadays all discoveries on fundamental physics involving supersymmetry and supergravity are also packaged up in the term string theory. As the subject has developed sometimes string theory, and sometimes supersymmetry, has provided the dominant insights, but it remains to be seen what the mix of ideas will be in the final theory. In this book I have tried to reflect this.

The aim of this book is to provide a systematic and, hopefully, pedagogical account of the essential topics in the subject known as string theory. Almost all of the computations are carried out explicitly. The book also contains some more advanced topics; these have been selected on the basis that I know something about them and I have a wish to explain them. There are also some pedagogical chapters such as those on Clifford algebras and Lie algebras that students should know and which could well play an even more important role in future developments. There are several very important topics that are missing: Calabi–Yau compactifications, string based black hole entropy computations and the AdS–CFT duality. However, these are rapidly developing and perhaps not yet ready for a systematic, or complete, treatment. There is also a long chapter on supergravity theories reflecting the important role they have played in the subject; this includes the methods used to construct them, their symmetries and the properties of these theories in ten and eleven dimensions. Many aspects of supersymmetric theories which are not discussed in this book can be found in my book *Introduction to Supersymmetry and Supergravity* [1.11].

This book has evolved over more than 25 years and some of the calculations were performed many years ago. Although almost certainly correct when first derived they may have developed transcription errors since then. As such, if you find a factor of 2, or a minus sign out, or some other defect in the occasional place you could be correct. Hopefully, these can be corrected in a second edition.

I have tried to reference the original papers in order to give the reader a better guide to the literature and in particular access to some of the best accounts of the material presented. I have studied quite a number of the papers that I had not read before, but I may well have missed some references. For this I apologise, and I hope to put such mistakes right in the future.

The reader who wants to get to grips with the basics of string theory in the quickest possible time could take the following path: first sections 1.1–1.2.3, then chapters 2, 3, 4, 5, 7, then sections 8.1–8.3, followed by the chapters 9 and 10, then sections 13.3–13.8.4, chapter 14, and finally sections 18.1 and 18.2.

I wish to thank Paul Cook for designing the cover and Pascal Anastasopoulos for drawing and helping to construct the figures. I also wish to thank Andreas Braun, Lars Brink, Lisa Carbone, Paul Cook, Finn Guaby, Arthur Greenspoon, Joanna Knapp, Neil Lambert, Sakura Shafer-Nameki, Duncan Steele and Arkardy Tseytlin for help with proof reading sections and references. My thanks also goes to the staff and students of the Department of Mathematics, King's College London and the Technical University of Vienna for many useful comments on my lectures which were taken from this book.