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## Introductory Quantum Optics

This book provides an elementary introduction to the subject of quantum optics, the study of the quantum-mechanical nature of light and its interaction with matter.

The presentation is almost entirely concerned with the quantized electromagnetic field. Topics covered include single-mode field quantization in a cavity, quantization of multimode fields, quantum phase, coherent states, quasi-probability distribution in phase space, atom–field interactions, the Jaynes–Cummings model, quantum coherence theory, beam splitters and interferometers, nonclassical field states with squeezing etc., tests of local realism with entangled photons from down-conversion, experimental realizations of cavity quantum electrodynamics, trapped ions, decoherence, and some applications to quantum information processing, particularly quantum cryptography. The book contains many homework problems and a comprehensive bibliography.

This text is designed for upper-level undergraduates taking courses in quantum optics who have already taken a course in quantum mechanics, and for first- and second-year graduate students.

A solutions manual is available to instructors via [solutions@cambridge.org](mailto:solutions@cambridge.org).

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Laboratory



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C. C. G. dedicates this book to his son, Eric.  
P. L. K. dedicates this book to his wife Chris.

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