Intuitionistic Fuzzy Multi Similarity Measure Based on Cotangent Function

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ABSTRACT

In this paper, the Similarity measure of Intuitionistic Fuzzy Multi sets (IFMS) based on Cotangent function is presented and analyzed. The properties of the similarity measure are proved and verified using the numerical evaluation of the proposed similarity measure. The unique feature of this proposed method is that it considers multi membership and non membership for the same element. As the proposed method is mathematically valid, it can be applied to any decision making problems, medical diagnosis, engineering problems, pattern recognition, etc. Finally, the application of medical diagnosis shows that the proposed similarity measures are much simpler, well suited one to use with linguistic variables.

KEY WORDS: Intuitionistic fuzzy set, Intuitionistic Fuzzy Multi sets, Similarity measure, Cotangent Function

INTRODUCTION

Krasssimir T. Atanassov [1], [2] proposed the Intuitionistic Fuzzy sets (*IFS*) as the generalisation of the Fuzzy set (*FS*) introduced by **Lofti A. Zadeh [3].** The *FS* allows the object to partially belong to a set with a membership degree (μ) between 0 and 1 whereas *IFS* represent the uncertainty with respect to both membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0,1]$) such that $\mu + \vartheta \leq 1$. The number $\pi = 1 - \mu - \vartheta$ is called the hesitiation degree or intuitionistic index.

The Multi set **[4]** allows the repeated occurrences of any element and hence the Fuzzy Multi set (*FMS*) can occur more than once with the possibly of the same or the different membership values was introduced by **R. R. Yager [5]**. Recently, the new concept Intuitionistic Fuzzy Multi sets (*IFMS*) was proposed by **T.K Shinoj and Sunil Jacob John [6].**

The study of distance and similarity measure of *IFS*s gives lots of measures, each representing specific properties and behaviour in real-life decision making and pattern recognition works. For measuring the degree of similarity between vague sets, **Chen and Tan [7]** proposed two similarity measures. The Hamming, Euclidean distance and similarity measures were introduced by **Szmidt and Kacprzyk [8], [9], [10], [11].** The Geometric distance and similarity measures were given by **Xu (12)**. Using the Cotangent function, a new similarity measure was proposed by **Wang et al (13)**. Later a new fuzzy cotangent similarity measure for *IFS*s was introduced by **Tian Maoying (14)**.

As the extension of the distance and similarity measure of *IFS*s to *IFMS*s **[15], [16]** are possible; in this paper we extend the fuzzy cotangent similarity measure of *IFS*s to *IFMS*s. The numerical results of the examples show that the developed similarity measures are well suited to use any linguistic variables.

The organization of this paper is as follows: In section 2, the Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets and similarity measures of *IFMS* are presented. The section 3 deals with the proposed Cotangent Similarity measure for the *IFMS*, along with the numerical evaluation. The application of medical diagnosis using IFMS is explained in detail in section 4.

II. PRELIMINARIES

Definition: 2.1

Let X be a nonempty set. A **fuzzy *set*** *A* in *X* is given by $A = \{\langle x, \mu_A(x)\rangle / \; x \in X\}$ -- (2.1)

where μ_A : X → [0, 1] is the membership function of the fuzzy set *A* (i.e.) $\mu_A(x) \in [0,1]$ is the membership of $x \in X$ in *A*. The generalizations of fuzzy sets are the Intuitionistic fuzzy *(IFS)* set proposed by **Atanassov [1], [2]** is with independent memberships and non memberships.

Definition: 2.2

An ***Intuitionistic fuzzy set*** *(IFS)*, *A* in *X* is given by $A = \{\langle x,\ \mu_A(x), \vartheta_A(x)\ \rangle / \ x \in X\}$ -- (2.2)

where $\mu_A : X \to [0,1]$ and $\vartheta_A : X \to [0,1]$ with the condition $0 \le \mu_A(x) + \vartheta_A(x) \le 1\ ,\ \forall\, x \in X$ Here $\mu_A(x)\ and\ \vartheta_A(x) \in [0,1]$ denote the membership and the non membership functions of the fuzzy set *A*; For each Intuitionistic fuzzy set in *X*, $\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0$ for all $x \in X$ that is

$\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is the hesitancy degree of $x \in X$ in *A*. Always $0 \le \pi_A(x) \le 1,\ \forall\, x \in X$.

The ***complementary set*** A^c of *A* is defined as $A^c = \{\langle x,\ \vartheta_A(x),\ \mu_A(x)\ \rangle / \ x \in X\}$ -- (2.3)

Definition: 2.3

Let X be a nonempty set. A ***Fuzzy Multi set (FMS)*** A in X is characterized by the count membership function Mc such that Mc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1]. The membership sequence is defined as

$(\ \mu_A^1(x)\ ,\mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ )$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$.

Therefore, A FMS A is given by $A = \{\langle\ x,\ (\ \mu_A^1(x)\ ,\mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ )\ \rangle / \ x \in X\ \}$ -- (2.4)

Definition: 2.4

Let X be a nonempty set. A ***Intuitionistic Fuzzy Multi set (IFMS)*** *A* in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that Mc : X → Q and NMc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1] whose membership sequence is defined as

$(\ \mu_A^1(x)\ ,\mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ )$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$ and the corresponding

non membership sequence NMc (x) is defined as $(\ \vartheta_A^1(x)\ ,\ \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x)\ )$ where the non membership can be either decreasing or increasing function. such that $0 \le \mu_A^i(x) + \vartheta_A^i(x) \le 1\ , \forall\, x \in X\ and\ i = 1,2, \dots p$. Therefore,

An ***IFMS*** *A* is given by $A = \{\langle\ x,\ \left(\ \mu_A^1(x)\ ,\mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ \right),\ (\ \vartheta_A^1(x)\ ,\vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x)\ )\ \rangle / \ x \in X\ \}$ -- (2.5)

where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$. The ***complementary set*** A^c of *A* is defined as

$$A^c = \{\langle x,\ (\ \vartheta_A^1(x)\ ,\vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x)\ ),\ \left(\ \mu_A^1(x)\ ,\mu_A^2(x), \dots \dots \dots \mu_A^p(x)\ \right), \rangle / \ x \in X\} \qquad -\ (2.6)$$

$$where\ \vartheta_A^1(x) \ge \vartheta_A^2(x) \ge \cdots \ge \vartheta_A^p(x)$$

Definition: 2.5

The **Cardinality** of the membership function Mc(x) and the non membership function NMc (x) is the length of an element x in an *IFMS* *A* denoted as η, defined as η = | Mc(x)| = |NMc(x)|

If A, B, C are the *IFMS* defined on *X* , then their cardinality η = Max { η(A), η(B), η(C) }.

Definition: 2.6

$\boldsymbol{Sim}\,(\boldsymbol{A},\boldsymbol{B})$ is said to be the **similarity measure** between A and B , where A, B ∈ X and X is an *IFMS*, as $Sim\,(A,\ B)$ satisfies the following properties

1. $Sim\,(A,B) \in [0,1]$

2. $Sim\,(A,B) = 1$ if and only if A = B

3. $Sim\,(A,B) = Sim\,(B,A)$

4. If A ⊆ B ⊆ C ∈ X , then $Sim\,(A,C) \le Sim\,(A,B)$ and $Sim\,(A,C) \le Sim\,(B,C)$

COTANGENT SIMILARITY MEASURE OF *IFS*s

The entropy measure of *IFS*s proposed by **Wang et al [13]** was as follows

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{1}{4}\pi + \frac{|\mu_A(x) - \vartheta_A(x)|\pi}{4(1+\pi_A(x))}\right\}$$

As there is a relationship between the Entropy and Similarity Measure, the fuzzy cotangent similarity measure of *IFS*s was proposed by **Tian Maoying [14]**

$$FCS(A,B) = \frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi + \pi(|\mu_A(x_i) - \mu_B(x_i)|\vee|\vartheta_A(x_i) - \vartheta_B(x_i)|)}{4}\right\}$$

where A, B ∈ IFS(X) and E be the entropy of IFSs, consisting of the membership and non membership functions. If there are three parameters like membership, non membership and hesitation function then the fuzzy cotangent similarity measure of *IFS*s becomes

$$FCS(A,B) = \frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi + \pi(|\mu_A(x_i) - \mu_B(x_i)|\vee|\vartheta_A(x_i) - \vartheta_B(x_i)|\vee|\pi_A(x_i) - \pi_B(x_i)|)}{4}\right\}$$

III PROPOSED COTANGENT SIMALRITY MEASURE OF *IFMS*s

In *IFS*, the Similarity measures are considered for the membership and non membership functions only once. But in *IFMS*, it should be considered more than once; because of their multi membership and non membership functions. And, their considerations are combined together by means of Summation concept based on their cardinality.

Definition: 3.1

$$IFMS(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left[\frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi + \pi\left(\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right|\vee\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right|\right)}{4}\right\}\right]$$

of the membership and non membership functions. And if there are three parameters like membership, non membership and hesitation function then the fuzzy multi cotangent similarity measure becomes

$$IFMS(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left[\frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi + \pi\left(\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right|\vee\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right|\vee\left|\pi_A^j(x_i) - \pi_B^j(x_i)\right|\right)}{4}\right\}\right]$$

PROPOSITION: 3.2

The defined similarity measure $IFMS(A,B)$ between *IFMS A* and *B* satisfies the following properties

D1. $0 \leq IFMS(A,B) \leq 1$

D2. $A = B$ if and only if $IFMS(A,B) = 0$

D3. $IFMS(A,B) = IFMS(B,A)$

D4. If $A \subseteq B \subseteq C$, for *A*, *B*, *C* are *IFMS* then

$IFMS(A,B) \leq IFMS(A,C)$ and $IFMS(B,C) \leq IFMS(A,C)$

Proof

D1. $\mathbf{0} \leq \boldsymbol{IFMS(A,B)} \leq \mathbf{1}$

As the membership and the non membership functions of the *IFMSs* lies between 0 and 1, the similarity measure based cotangent function also lies between 0 and 1. (Always cot function lies between 0 and 1)

D2. $\boldsymbol{A = B}$ **if and only if** $\boldsymbol{IFMS(A,B) = 0}$

(i) Let the two *IFMS* A and B be equal (i.e.) **A = B**. This implies for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ which states that $\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right|$ *and* $\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right| = 0$. Hence $\boldsymbol{IFMS(A,B) = 0}$

(ii) Let the $\boldsymbol{IFMS(A,B) = 0}$

The zero distance measure is possible only if both $\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right|$ *and* $\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right| = 0$, as the Cotangent similarity measure concerns with max operator **(** $\vee$ **)** of membership and non membership difference. This refers that $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ for all i, j values. Hence **A = B.**

D3. $\boldsymbol{IFMS(A,B) = IFMS(B,A)}$

It is obvious that $\mu_A^j(x_i) - \mu_B^j(x_i) \neq \mu_B^j(x_i) - \mu_A^j(x_i)$ and $\vartheta_A^j(x_i) - \vartheta_B^j(x_i) \neq \vartheta_B^j(x_i) - \vartheta_A^j(x_i)$

But $\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right| = \left|\mu_B^j(x_i) - \mu_A^j(x_i)\right|$ and $\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right| = \left|\vartheta_B^j(x_i) - \vartheta_A^j(x_i)\right|$

Hence $IFMS(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left[\frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi+\pi(|\mu_A(x_i)-\mu_B(x_i)| \vee |\vartheta_A(x_i)-\vartheta_B(x_i)|)}{4}\right\}\right]$

$$= \frac{1}{\eta}\sum_{j=1}^{\eta}\left[\frac{1}{n}\sum_{i=1}^{n} Cot\left\{\frac{\pi+\pi(|\mu_B(x_i)-\mu_A(x_i)| \vee |\vartheta_B(x_i)-\vartheta_A(x_i)|)}{4}\right\}\right] = IFMS(B,A)$$

D4. If $\boldsymbol{A \subseteq B \subseteq C}$ **for A, B, C are** $\boldsymbol{IFMS}$ **then** $\boldsymbol{IFMS(A,B) \leq IFMS(A,C)}$ **and** $\boldsymbol{IFMS(B,C) \leq IFMS(A,C)}$

Let $A \subseteq B \subseteq C$, then the assumption is $\mu_A^j(x_i) \leq \mu_B^j(x_i) \leq \mu_C^j(x_i)$ and $\vartheta_A^j(x_i) \geq \vartheta_B^j(x_i) \geq \vartheta_C^j(x_i)$ for every $x_i \in X$

Case (i) Let $\boldsymbol{\left|\mu_A^j(x_i) - \mu_C^j(x_i)\right| \geq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|}$ Then from the assumption of non membership function, we have $\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right|$ ---- (3.4.1)

Also $\left|\vartheta_B^j(x_i) - \vartheta_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right|$ ---- (3.4.2)

Now from the assumption of the membership, we have

$\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right|$ and $\left|\mu_B^j(x_i) - \mu_C^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right|$ ---- (3.4.3)

From (3.4.1, 3.4.2, 3.4.3) $\boldsymbol{IFMS(A,B) \leq IFMS(A,C)}$ **and** $\boldsymbol{IFMS(B,C) \leq IFMS(A,C)}$

Case (ii) Let $\boldsymbol{\left|\mu_A^j(x_i) - \mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|}$ Then from the assumption of membership function, we

have $\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|$ ---- (3.4.4)

Also $\left|\mu_B^j(x_i) - \mu_C^j(x_i)\right| \leq \left|\mu_A^j(x_i) - \mu_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|$ ---- (3.4.5)

Now from the assumption of the non membership, we have

$\left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|$ and $\left|\vartheta_B^j(x_i) - \vartheta_C^j(x_i)\right| \leq \left|\vartheta_A^j(x_i) - \vartheta_C^j(x_i)\right|$ -- (3.4.6)

From (3.4.4, 3.4.5, 3.4.6) $\boldsymbol{IFMS(A,B) \leq IFMS(A,C)}$ **and** $\boldsymbol{IFMS(B,C) \leq IFMS(A,C)}$

NUMERICAL EVUALATION: 3.3

EXAMPLE: 4.1

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2, A_3, A_4, A_5} and B ={ A_6, A_7, A_8, A_9, A_{10}} are the *IFMS* defined as

A = { $\langle A_1 : (0.6,0.4), (0.5, 0.5) \rangle$, $\langle A_2 : (0.5,0.3), (0.4, 0.5) \rangle$, $\langle A_3, (0.5, 0.2), (0.4, 0.4) \rangle$, $\langle A_4 : (0.3,0.2), (0.3, 0.2) \rangle$, $\langle A_5 : (0.2,0.1), (0.2, 0.2) \rangle$ }

B = { $\langle A_6 : (0.8,0.1), (0.4, 0.6) \rangle$, $\langle A_7 : (0.7,0.3), (0.4, 0.2) \rangle$, $\langle A_8, (0.4, 0.5), (0.3, 0.3) \rangle$ $\langle A_9 : (0.2,0.7), (0.1, 0.8) \rangle$, $\langle A_{10} : (0.2,0.6), (0, 0.6) \rangle$ }

Here, the cardinality η = 5 as | Mc(A)| = |NMc(A)| = 5 and | Mc(B)| = |NMc(B)| = 5 and the cotangent similarity measure is

$$\frac{1}{5}\sum_{j=1}^{5}\left[\frac{1}{2}\sum_{i=1}^{2} Cot\left\{\frac{\pi+\pi(|\mu_A(x_i)-\mu_B(x_i)|\vee|\vartheta_A(x_i)-\vartheta_B(x_i)|)}{4}\right\}\right] = \mathbf{0.5936}$$

EXAMPLE: 4.2

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2 } and B ={ A_9, A_{10}} are the *IFMS* defined as

A = { $\langle A_1 : (0.1,0.2) \rangle, \langle A_2 : (0.3,0.3) \rangle$ }, B = { $\langle A_9 : (0.1,0.2) \rangle, \langle A_{10} : (0.2,0.3) \rangle$ } Here, the cardinality η = 2 as | Mc(A)| = |NMc(A)| = 2 and | Mc(B)| = |NMc(B)| = 2 and the cotangent similarity measure is

$$\frac{1}{2}\sum_{j=1}^{2}\left[\frac{1}{1}\sum_{i=1}^{1} Cot\left\{\frac{\pi+\pi(|\mu_A(x_i)-\mu_B(x_i)|\vee|\vartheta_A(x_i)-\vartheta_B(x_i)|)}{4}\right\}\right] = \mathbf{0.9271}$$

EXAMPLE: 4.3

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2 } and B ={ A_3, A_4 } are the *IFMS* defined as

A = { $\langle A_1 : (0.4,0.2,0.1), (0.3, 0.1, 0.2), (0.2, 0.1, 0.2), (0.1, 0.4, 0.3) \rangle$, $\langle A_2 : (0.6,0.3,0), (0.4, 0.5, 0.1), (0.4, 0.3, 0.2), (0.2, 0.6, 0.2) \rangle$ }

B = { $\langle A_3 : (0.5,0.2,0.3), (0.4, 0.2, 0.3), (0.4, 0.1, 0.2), (0.1, 0.1, 0.6) \rangle$ $\langle A_4 : (0.4,0.6,0.2), (0.4, 0.5, 0), (0.3, 0.4, 0.2), (0.2, 0.4, 0.1) \rangle$ }

The cardinality η = 2 as | Mc(A)| = |NMc(A)| = | Hc(A)| = 2 and | Mc(B)| = |NMc(B)| = | Hc(B)| = 2

Hence, the cotangent similarity measure is

$$\frac{1}{2}\sum_{j=1}^{2}\left[\frac{1}{4}\sum_{i=1}^{4} Cot\left\{\frac{\pi+\pi(|\mu_A(x_i)-\mu_B(x_i)|\vee|\vartheta_A(x_i)-\vartheta_B(x_i)|\vee|\pi_A(x_i)-\pi_B(x_i)|)}{4}\right\}\right] = \mathbf{0.7450}$$

EXAMPLE: 4.4

Let X = {A_1, A_2, A_3, A_4..... A_n } with A = { A_1, A_2 } and B = {A_6} such that the IFMS A and B are

A = { $\langle A_1 : (0.6,0.2,0.2), (0.4, 0.3, 0.3), (0.1, 0.7, 0.2) \rangle$, $\langle A_2 : (0.7,0.1,0.2), (0.3, 0.6, 0.1), (0.2, 0.7, 0.1) \rangle$ }

B = { $\langle A_6 : (0.8,0.1,0.1), (0.2, 0.7, 0.1), (0.3, 0.5, 0.2) \rangle$ }

As | Mc(A)| = |NMc(A)| = | Hc(A)| =2 and | Mc(B)| = |NMc(B)| = | Hc(B)| =1,

their cardinality η = Max { η(A), η(B) } = max {2,1} = 2. Therefore the cotangent similarity measure is

$$\frac{1}{2}\sum_{j=1}^{2}\left[\frac{1}{3}\sum_{i=1}^{3} Cot\left\{\frac{\pi+\pi(|\mu_A(x_i)-\mu_B(x_i)|\vee|\vartheta_A(x_i)-\vartheta_B(x_i)|\vee|\pi_A(x_i)-\pi_B(x_i)|)}{4}\right\}\right] = \mathbf{0.7329}$$

IV MEDICAL DIAGNOSIS USING IFMS - COTANGENT MEASURE

As Medical diagnosis contains lots of uncertainties, they are the most interesting and fruitful areas of application for fuzzy set theory. In some situations, terms of membership function alone is not adequate. Hence, the Intuitionistic fuzzy set theory consisting of both the terms like membership and non membership function is considered to be the better one. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. Recently, there are various models of medical diagnosis under the general framework of fuzzy sets are proposed. In some practical situations, there is the possibility of each element having different membership and non membership functions. The proposed distance and similarity measure among the Patients Vs Symptoms and Symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi membership and non membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis.

Let P = { P_1, P_2, P_3, P_4 } be a set of Patients.

D = { Fever, Tuberculosis, Typhoid, Throat disease } be the set of diseases

and S = { Temperature, Cough, Throat pain, Headache, Body pain } be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

TABLE : 4.1 – IFMs Q : The Relation between Patient and Symptoms

Q	Temperature	Cough	Throat Pain	Head Ache	Body Pain
P_1	(0.6, 0.2) (0.7, 0.1) (0.5, 0.4)	(0.4, 0.3) (0.3, 0.6) (0.4, 0.4)	(0.1, 0.7) (0.2, 0.7) (0, 0.8)	(0.5, 0.4) (0.6, 0.3) (0.7, 0.2)	(0.2, 0.6) (0.3, 0.4) (0.4, 0.4)
P_2	(0.4, 0.5) (0.3, 0.4) (0.5, 0.4)	(0.7, 0.2) (0.6, 0.2) (0.8, 0.1)	(0.6, 0.3) (0.5, 0.3) (0.4, 0.4)	(0.3, 0.7) (0.6, 0.3) (0.2, 0.7)	(0.8, 0.1) (0.7, 0.2) (0.5, 0.3)
P_3	(0.1, 0.7) (0.2, 0.6) (0.1, 0.9)	(0.3, 0.6) (0.2, 0) (0.1, 0.7)	(0.8, 0) (0.7, 0.1) (0.8, 0.1)	(0.3, 0.6) (0.2, 0.7) (0.2, 0.6)	(0.4, 0.4) (0.3, 0.7) (0.2, 0.7)

Let the samples be taken at three different timings in a day (morning, noon and night)

TABLE : 4.2 – IFMs R : The Relation among Symptoms and Diseases

R	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8, 0.1)	(0.2, 0.7)	(0.5, 0.3)	(0.1, 0.7)
Cough	(0.2, 0,7)	(0.9, 0)	(0.3, 0,5)	(0.3, 0,6)
Throat Pain	(0.3, 0.5)	(0.7, 0.2)	(0.2, 0.7)	(0.8, 0.1)
Head ache	(0.5, 0.3)	(0.6, 0.3)	(0.2, 0.6)	(0.1, 0.8)
Body ache	(0.5, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.1, 0.8)

TABLE : 4.3 – The Cotangent Similarity Measure between IFMs Q and R :

Cotangent Similarity measure	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.7242	0.5152	**0.7711**	0.4868
P_2	0.5967	**0.7292**	0.6446	0.5327
P_3	0.5271	0.5777	0.6177	**0.7825**

The ***highest similarity measure*** from the table 4.3 gives the proper medical diagnosis.

Patient **P1** suffers from **Typhoid,** Patient **P2** suffers from **Tuberculosis** and Patient **P3** suffers from **Throat disease**.

V. CONCLUSION

In this paper, we have developed the method for deriving the cotangent similarity measure of *IFMS* from *IFS* theory. The prominent characteristic of this method is that it considers multi membership, non membership functions and this similarity measure also guarantee that the cotangent function of any two *IFMS*s equals to one if and only if the two *IFMS*s are the same. The **example 3.1, 3.2** shows that the new measure perform well in the case of membership and non membership function and **example 3.3, 3.4** depicts that the proposed measure is effective with three representatives of *IFMSs* – membership, non membership and hesitation functions. Hence, from the analysis and numerical evaluation, it is clear that this proposed method can be applied to any pattern recognitions and decision making problems. Finally, an illustrative example, the medical diagnosis has been given to show the efficiency of the developed cotangent similarity measure.

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