# Intuitionistic Fuzzy Soft Matrix Theory And Its Application In Decision Making 

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#### Abstract

Soft set theory is a newly emerging mathematical tool to deal with uncertain problems.In this paper,we proposed intuitionistic fuzzy soft matrices and defined different types of intuitionistic fuzzy soft matrices and some operations.Finally we extend our approach in application of these matrices in decision making problems.


Key words Soft set,fuzzy soft set, intuitionistic fuzzy soft set, intuitionistic fuzzy soft matrix.

## 1.Introduction

In real life situation, most of the problems in economics, social science, environment etc,have various uncertainties. However most of the existing mathematical tools for formal modeling, reasoning and computing are crisp deterministic and precise in character. There are theories viz,theory of probability, evidence, fuzzy set,intuitionistic fuzzy set, vague set, interval mathematics, rough set for dealing with uncertainties. These theories have their own difficulties as pointed out by Molodtsov[1].In 1999, Molodtsov[1] initiated a novel concept of soft set theory, which is completely new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems in economics, social science, medical science etc. Later on Maji et al [2] have studied the theory of fuzzy soft set.Majumdar et al[3] have further generalised the concept of fuzzy soft sets.Maji et al[4] extended soft sets to intuitionistic fuzzy soft sets.

Matrices play an important role in the broad area of science and engineering. However ,the classical matrix theory sometimes fails to solve the problems involving uncertainties.In [5],Yong et al initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems.In[6] Borah et al extended fuzzy soft matrix theory and its application.

In this paper,we proposed intuitionistic fuzzy soft matrices and defined different types of intuitionistic fuzzy soft matrices and some operations.Finally we extend our approach in application of these matrices in decision making problems.

## 2.Preliminaries

In this section,we recall some basic notion of fuzzy soft set theory and fuzzy soft matrices
2.1.Soft set [1] Suppose that $U$ is an initial universe set and $E$ is a set of parameters, let $P(U)$ denotes the power set of $U$.A pair $(F, E)$ is called a soft set over $U$ where $F$ is a mapping given by $F$ : $\mathrm{E} \rightarrow \mathrm{P}(\mathrm{U})$.Clearly, a soft set is a mapping from parameters to $\mathrm{P}(\mathrm{U})$, and it is not a set, but a parameterized family of subsets of the Universe.

Example 2.1.Suppose that $U=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}\right\}$ is a set of students and $\mathrm{E}=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3}\right\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $E$ to the set of all subsets of power set U.Then soft set ( $F, E$ ) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.
$(\mathrm{F}, \mathrm{E})=\left\{\left\{\right.\right.$ result $\left.=\mathrm{s}_{1}, \mathrm{~s}_{3}, \mathrm{~s}_{4}\right\} \quad\left\{\right.$ conduct $\left.=\mathrm{s}_{1}, \mathrm{~s}_{2}\right\} \quad\left\{\right.$ sports performances $\left.\left.=\mathrm{s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}\right\}\right\}$
We can represent a soft set in the form of Table1

| U | $\operatorname{Result}\left(\mathrm{e}_{1}\right)$ | Conduct $\left(\mathrm{e}_{2}\right)$ | Sports(e $\left.\mathrm{e}_{3}\right)$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{s}_{1}$ | 1 | 1 | 0 |
| $\mathrm{~s}_{2}$ | 0 | 1 | 1 |
| $\mathrm{~s}_{3}$ | 1 | 0 | 1 |
| $\mathrm{~s}_{4}$ | 1 | 0 | 1 |

Table1

### 2.2. Fuzzy soft set [ 2]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair ( $\mathrm{F}, \mathrm{A}$ ) is called fuzzy soft set over $U$ where $F$ is a mapping given by $F: A \rightarrow I^{U}$, where $I^{U}$ denotes the collection of all fuzzy subsets of $U$.

Example 2.2.Consider the example2.1, in soft $\operatorname{set}(\mathrm{F}, \mathrm{E})$, if $\mathrm{s}_{1}$ is medium in studies, we cannot expressed with only the two numbers 0 and 1, we can characterize it by a membership function instead of the crisp number 0 and 1 , which associates with each element a real number in the interval [0,1].Then fuzzy soft set can describe as where $A=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}\right\}$

$$
(\mathrm{F}, \mathrm{~A})=\left\{\mathrm{F}\left(\mathrm{e}_{1}\right)=\left\{\left(\mathrm{s}_{1}, 0.9\right),\left(\mathrm{s}_{2}, 0.3\right),\left(\mathrm{s}_{3}, 0.8\right),\left(\mathrm{s}_{4}, 0.9\right)\right\}, \mathrm{F}\left(\mathrm{e}_{2}\right)=\left\{\left(\mathrm{s}_{1}, 0.8\right),\left(\mathrm{s}_{2}, 0.9\right),\left(\mathrm{s}_{3}, 0.4\right),\left(\mathrm{s}_{4}, 0.3\right)\right\}\right\}
$$

We can represent a fuzzy soft set in the form of table 2

| U | Result $\left(\mathrm{e}_{1}\right)$ | Conduct $\left(\mathrm{e}_{2}\right)$ |
| :---: | :---: | :---: |
| $\mathrm{s}_{1}$ | 0.9 | 0.8 |
| $\mathrm{~s}_{2}$ | 0.3 | 0.9 |
| $\mathrm{~s}_{3}$ | 0.8 | 0.4 |
| $\mathrm{~s}_{4}$ | 0.9 | 0.3 |

Table2

### 2.3.Fuzzy Soft Matrices[6]

Let $\mathrm{U}=\left\{\mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{c}_{3} \ldots \mathrm{c}_{\mathrm{m}}\right\}$ be the Universal set and E be the set of parameters given by $\mathrm{E}=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3} \ldots \mathrm{e}_{\mathrm{n}}\right\}$.Let $\mathrm{A} \subseteq \mathrm{E}$ and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E).Then fuzzy soft set (F,A) in a matrix form as $A_{m \times n}=\left[a_{i j}\right]_{m \times n}$ or

$$
A=\left[a_{i j}\right] \quad i=1,2, \ldots m, j=1,2,3, \ldots . . n
$$

Where $\mathrm{a}_{\mathrm{ij}}=\left\{\begin{array}{ll}\mu_{j}\left(c_{i}\right) & \text { if } \mathrm{e}_{j} \in A \\ 0 & \text { if } \mathrm{e}_{j} \notin A\end{array} \quad \mu_{j}\left(c_{i}\right)\right.$ represents the membership of $\mathrm{c}_{\mathrm{i}}$ in the fuzzy set
$\mathrm{F}\left(\mathrm{e}_{\mathrm{j}}\right)$.
Example 2.3.Consider the example2.2, the matrix representation is

$$
\left(\begin{array}{lll}
0.9 & 0.8 & 0 \\
0.3 & 0.9 & 0 \\
0.8 & 0.4 & 0 \\
0.9 & 0.3 & 0
\end{array}\right)
$$

### 2.4.Intuitionistic Fuzzy soft set [ 4]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair ( $\mathrm{F}, \mathrm{A}$ ) is called intuitionistic fuzzy soft set over U where F is a mapping given by F : $\mathrm{A} \rightarrow \mathrm{I}^{\mathrm{U}}$, where $\mathrm{I}^{\mathrm{U}}$ denotes the collection of all intuitionistic fuzzy subsets of $U$.

Example 2.4. Suppose that $U=\left\{s_{1}, s_{2}, s_{3}, s_{4}\right\}$ is a set of students and $E=\left\{e_{1}, e_{2}, e_{3}\right\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U.Then soft set (F,A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.Consider, $A=\left\{e_{1}, e_{2}\right\}$ then intuitionistic fuzzy soft set is

$$
\begin{aligned}
(\mathrm{F}, \mathrm{~A})=\{ & F\left(\mathrm{e}_{1}\right)=\left\{\left(\mathrm{s}_{1}, 0.8,0.1\right),\left(\mathrm{s}_{2}, 0.3,0.6\right),\left(\mathrm{s}_{3}, 0.8,0.2\right),\left(\mathrm{s}_{4}, 0.9,0.0\right)\right\}, \\
& \mathrm{F}\left(\mathrm{e}_{2}\right)=\left\{\left(\mathrm{s}_{1}, 0.8,0.1\right),\left(\mathrm{s}_{2}, 0.9,0.1\right),\left(\mathrm{s}_{3}, 0.4,0.5\right),\left(\mathrm{s}_{4}, 0.3,0.6\right)\right\}
\end{aligned}
$$

## 3. Intuitionistic Fuzzy Soft Matrix Theory

### 3.1 Intuitionistic Fuzzy Soft Matrix(IFSM)

Let $U=\left\{c_{1}, \mathrm{c}_{2}, \mathrm{c}_{3} \ldots \mathrm{c}_{\mathrm{m}}\right\}$ be the Universal set and E be the set of parameters given by $\mathrm{E}=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3} \ldots \mathrm{e}_{\mathrm{n}}\right\}$. Let A $\subseteq \mathrm{E}$ and (F,A) be a intuitionistic fuzzy soft set in the fuzzy soft class (U,E).Then intuitionistic fuzzy soft set (F,A) in a matrix form as $\quad A_{m \times n}=\left[a_{i j}\right]_{m \times n}$ or $A=\left[a_{i j}\right] \quad i=1,2, \ldots m, j=1,2,3, \ldots . n$

Where $\mathrm{a}_{\mathrm{ij}}= \begin{cases}\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right) & \text { if } \mathrm{e}_{j} \in A \\ (0,1) & \text { if } \mathrm{e}_{j} \notin A\end{cases}$
$\mu_{j}\left(c_{i}\right)$ represents the membership of $\mathrm{c}_{\mathrm{i}}$ in the intuitionistic fuzzy set $\mathrm{F}\left(\mathrm{e}_{\mathrm{j}}\right)$.
$v_{j}\left(c_{i}\right)$ represents the non-membership of $\mathrm{c}_{\mathrm{i}}$ in the intuitionistic fuzzy set $\mathrm{F}\left(\mathrm{e}_{\mathrm{j}}\right)$.

Example 3.1 Suppose that $U=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}\right\}$ is a set of students and $\mathrm{E}=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3}\right\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U.Then soft set (F,A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.Consider, $A=\left\{e_{1}, e_{2}\right\}$ then intuitionistic fuzzy soft set is

$$
\begin{aligned}
(F, A)=\{ & F\left(e_{1}\right)=\left\{\left(s_{1}, 0.8,0.1\right),\left(s_{2}, 0.3,0.6\right),\left(s_{3}, 0.8,0.2\right),\left(s_{4}, 0.9,0.0\right)\right\}, \\
& F\left(e_{2}\right)=\left\{\left(s_{1}, 0.8,0.1\right),\left(s_{2}, 0.9,0.1\right),\left(s_{3}, 0.4,0.5\right),\left(s_{4}, 0.3,0.6\right)\right\} .
\end{aligned}
$$

We would represent this intuitionistic fuzzy soft set in matrix form as

$$
\left(\begin{array}{lll}
(0.8,0.1) & (0.8,0.1) & (0.0,1.0) \\
(0.3,0.6) & (0.9,0.1) & (0.0,1.0) \\
(0.8,0.2) & (0.4,0.5) & (0.0,1.0) \\
(0.9,0.0) & (0.3,0.6) & (0.0,1.0)
\end{array}\right)
$$

### 3.2 Intuitionistic Fuzzy Soft sub Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{B}=\left[\mathrm{b}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, Then A is a intuitionistic fuzzy soft submatrix of B, denoted by $A \subseteq B \quad$ if $\quad \mu_{\mathrm{A}} \leq \mu_{\mathrm{B}}$ and $v_{\mathrm{A}} \geq v_{\mathrm{B}} \forall i, j$

### 3.3 Intuitionistic Fuzzy Soft null (zero)Matrix

An intuitionistic fuzzy soft matrix of order mxn is called intuitionistic fuzzy soft null (zero)matrix if all its elements are $(0,1)$ It is denoted by $\boldsymbol{\Phi}$.

### 3.4Intuitionistic Fuzzy Soft universal Matrix

An intuitionistic fuzzy soft matrix of order mxn is called intuitionistic fuzzy soft universal matrix if all its elements are (1,0).It is denoted by $\mathbf{U}$.

### 3.5 Intuitionistic Fuzzy Soft equal Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{B}=\left[\mathrm{b}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, Then A is equal to B ,denoted by $A=B$ if
if $\mu_{\mathrm{A}}=\mu_{\mathrm{B}}$ and $v_{\mathrm{A}}=v_{\mathrm{B}} \forall i, j$

### 3.6 Intuitionistic Fuzzy Soft transpose Matrix

Let $A=\left[a_{i j}\right] \in \mathbf{I F S M}_{m \times n}$ Then $A^{T}$ is a intuitionistic fuzzy soft transpose matrix of $A$ if $A^{T}=\left[a_{j i}\right]$

### 3.7 Intuitionistic Fuzzy Soft rectangular Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in$ IFSM $_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft rectangular Matrix if $\mathrm{m} \neq \mathrm{n}$.

### 3.8Intuitionistic Fuzzy Soft square Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft rectangular Matrix if $\mathrm{m}=\mathrm{n}$.

### 3.9 Intuitionistic Fuzzy Soft rowMatrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft row Matrix if $\mathrm{m}=1$.

### 3.10 Intuitionistic Fuzzy Soft columnMatrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in$ IFSM $_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft rectangular Matrix if $\mathrm{n}=1$.

### 3.11 Intuitionistic Fuzzy Soft diagonal Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft diagonal Matrix if $\mathrm{m}=\mathrm{n}$ and $a_{i j}=(0,1)$. for all $\mathrm{i} \neq \mathrm{j}$

### 3.12 Intuitionistic Fuzzy Soft scalar Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft scalar Matrix if $\mathrm{m}=\mathrm{n}$ and $a_{i j}=(0,1)$ for all $\mathrm{i} \neq \mathrm{j}$ and $a_{i j}=(\alpha, \beta), \alpha \in[0,1], \beta \in[0,1] \forall \mathrm{i}=\mathrm{j}$

### 3.13 Intuitionistic Fuzzy Soft upper triangular Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft upper triangular Matrix if $\mathrm{m}=\mathrm{n}$ and $a_{i j}=(0,1)$ for all $\mathrm{i}>\mathrm{j}$.

### 3.14 Intuitionistic Fuzzy Soft lower triangular Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then A is called a Intuitionistic Fuzzy Soft upper triangular Matrix if $\mathrm{m}=\mathrm{n}$ and $a_{i j}=(0,1)$ for all $\mathrm{i}<\mathrm{j}$.

### 3.15 Intuitionistic Fuzzy Soft triangular Matrix

A Intuitionistic Fuzzy Soft Matrix is said to be triangular if it is either Intuitionistic Fuzzy Soft lower triangular Matrix or Intuitionistic Fuzzy Soft upper triangular Matrix.

### 3.16 Addition of Intuitionistic Fuzzy Soft Matrices

If $A=\left[a_{i j}\right] \in \mathbf{I F S M}_{\text {mxn }}, B=\left[b_{i j}\right] \in \mathbf{I F S M}_{m \times n}$, then we define $A+B$,addition of $A$ and $B$ as

$$
\begin{aligned}
\mathrm{A}+\mathrm{B} & =\left[\mathrm{c}_{\mathrm{ij}}\right]_{\mathrm{mxn}} \\
& =\left(\max \left(\mu_{\mathrm{A}}, \mu_{\mathrm{B}}\right), \min \left(v_{\mathrm{A}}, v_{\mathrm{B}}\right)\right) \forall i, j
\end{aligned}
$$

## Example 3.2

$$
\text { Consider } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2} \quad \text { and } \mathrm{B}=\left(\begin{array}{ll}
(0.6,0.3) & (0.8,0.2) \\
(0.7,0.3) & (0.5,0.5)
\end{array}\right)_{2 \times 2}
$$

are two intuitionistic fuzzy soft matrices, then the sum of these two is

$$
A+B=\left(\begin{array}{ll}
(0.8,0.1) & (0.8,0.2) \\
(0.7,0.3) & (0.5,0.5)
\end{array}\right)_{2 \times 2}
$$

### 3.17 Subtraction of Intuitionistic Fuzzy Soft Matrices

If $A=\left[a_{i j}\right] \in$ IFSM $_{\text {mxn }}, B=\left[b_{i j}\right] \in \mathbf{I F S M}_{m \times n}$, then we define $A-B$,subtraction of $A$ and $B$ as

$$
\begin{aligned}
\mathrm{A}-\mathrm{B}= & {\left[\mathrm{c}_{\mathrm{ij}}\right]_{\mathrm{mxn}} } \\
& =\left(\min \left(\mu_{\mathrm{A}}, \mu_{\mathrm{B}}\right), \max \left(v_{\mathrm{A}}, v_{\mathrm{B}}\right)\right) \forall i, j
\end{aligned}
$$

## Example 3.2

$$
\text { Consider } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2} \quad \text { and } \mathrm{B}=\left(\begin{array}{ll}
(0.6,0.3) & (0.8,0.2) \\
(0.7,0.3) & (0.5,0.5)
\end{array}\right)_{2 \times 2}
$$

are two intuitionistic fuzzy soft matrices, then the subtraction of these two is

$$
A-B=\left(\begin{array}{cc}
(0.6,0.3) & (0.4,0.5) \\
0.7,0.3) & (0.4,0.6
\end{array}\right)
$$

### 3.17 Product of Intuitionistic Fuzzy Soft Matrices

If $A=\left[a_{i j}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, B=\left[\mathrm{b}_{\mathrm{jk}}\right] \in \mathbf{I F S M}_{\mathrm{nxp}}$, then we define $\mathrm{A} * \mathrm{~B}$, multiplication of A and B as

$$
\begin{aligned}
\mathrm{A} * \mathrm{~B} & =\left[\mathrm{c}_{\mathrm{i} k}\right]_{\operatorname{mxp}} \\
& =\left(\max \min \left(\mu_{A j}, \mu_{B j}\right), \operatorname{minmax}\left(v_{A j}, v_{B j}\right)\right) \forall \mathrm{i}, \mathrm{j}
\end{aligned}
$$

## Example 3.2

$$
\text { Consider } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2} \quad \text { and } B=\left(\begin{array}{ll}
(0.6,0.3) & (0.8,0.2) \\
(0.7,0.3) & (0.5,0.5)
\end{array}\right)_{2 \times 2}
$$

are two intuitionistic fuzzy soft matrices, then the product of these two matrices is
$A * B=\left(\begin{array}{ll}(0.6,0.3) & (0.8,0.2) \\ (0.6,0.3) & (0.7,0.3)\end{array}\right)_{2 X 2}$
Remark: A*B $\neq \mathrm{B}^{*} \mathrm{~A}$

### 3.1Proposition

Let $A=\left[a_{i j}\right] \in \mathbf{I F S M}_{m \times n}, B=\left[b_{i j}\right] \in \mathbf{I F S M}_{m \times n}, \mathbf{C}=\left[\mathrm{c}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$ then
i) $\Phi \subseteq \mathrm{A}$
ii) $\mathrm{A} \subseteq \mathrm{U}$
iii) $A \subseteq \mathrm{~A}$
iv) $\mathrm{A} \subseteq B, \mathrm{~B} \subseteq \mathrm{C} \Rightarrow \mathrm{A} \subseteq \mathrm{C}$

PROOF: It follows from the definition.

### 3.2Proposition

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{B}=\left[\mathrm{b}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{C}=\left[\mathrm{c}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$ then
i) $A+\Phi=\mathrm{A}$
ii) $\mathrm{A}+\mathrm{U}=U$
iii) $A+\mathrm{B}=\mathrm{B}+\mathrm{A}$
iv) $(\mathrm{A}+B)+C=\mathrm{A}+(\mathrm{B}+\mathrm{C})$

PROOF: It follows from the definition.

### 3.18 Intuitionistic Fuzzy Soft Complement Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in$ IFSM $_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then $\mathrm{A}^{\mathrm{C}}$ is called a Intuitionistic Fuzzy Soft Complement Matrix if $A^{C}=\left[b_{i j}\right]_{m \times n}$

$$
b_{i j}=\left(v_{j}\left(c_{i}\right), \mu_{j}\left(c_{i}\right)\right) \forall \mathrm{i}, \mathrm{j} .
$$

## Example 3.3

$$
\text { let } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2}
$$

be intuitionistic fuzzy soft matrix, then the complement of this matrix is

$$
A^{c}=\left(\begin{array}{ll}
(0.1,0.8) & (0.5,0.4) \\
(0.3,0.7) & (0.6,0.4)
\end{array}\right)_{2 \times 2}
$$

### 3.3Proposition

i) $\left(A^{c}\right)^{c}=\mathrm{A}$
ii) $\Phi^{c}=U$
iii) $(A+U)^{c}=\Phi$
iv) $(\mathrm{A}+B)^{c}=(B+A)^{c}$

### 3.19 Scalar multiple of Intuitionistic Fuzzy Soft Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in$ IFSM $_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then scalar multiple of Intuitionistic Fuzzy Soft Matrix A by a scalar k is defined by $\mathrm{kA}=\left[\mathrm{k} \mathrm{a}_{\mathrm{ij}}\right]_{\mathrm{mxn}}$ where $0 \leq k \leq 1$.

## Example 3.4

$$
\text { let } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2}
$$

be intuitionistic fuzzy soft matrix, then the scalar multiple of this matrix by $\mathrm{k}=0.5$ is

$$
k A=\left(\begin{array}{ll}
(0.40,0.05) & (0.2,0.25) \\
(0.35,0.15) & (0.2,0.30)
\end{array}\right)_{2 \times 2}
$$

### 3.4 Proposition

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. if $\mathrm{m}, \mathrm{n}$ are two scalars such that $0 \leq m, n \leq 1$,then
i) $\mathrm{m}(\mathrm{n} A)=(\mathrm{mn}) \mathrm{A}$
ii) $\mathrm{m} \leq \mathrm{n} \Rightarrow m A \leq n A$
iii) $A \subseteq B \Rightarrow m A \subseteq m B$

### 3.19 Trace of Intuitionistic Fuzzy Soft Matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, where $\mathrm{m}=\mathrm{n}, a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then trace of Intuitionistic Fuzzy Soft Matrix A is $\operatorname{trA}=\sum_{i=1}^{m} a_{i i}=\sum_{i=1}^{m} \mu_{i i}-v_{i i}$.

## Example 3.5

$$
\text { let } A=\left(\begin{array}{ll}
(0.8,0.1) & (0.4,0.5) \\
(0.7,0.3) & (0.4,0.6)
\end{array}\right)_{2 \times 2}
$$

be intuitionistic fuzzy soft matrix, then trace of this matrix is

$$
\operatorname{trA}=0.8--0.1+0.4--0.6=0.5
$$

### 3.5Proposition

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{i} j}\right] \in \mathbf{I F S M}_{\mathrm{mxm}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. if m isa two scalar such that $0 \leq m \leq 1$,then
i) $\operatorname{tr}(\mathrm{kA})=\mathrm{k} \operatorname{tr} \mathrm{A}$
ii) $(k A)^{T}=k A^{T}$

## 4. Intuitionistic Fuzzy Soft Matrix Theory in Decision Making

### 4.1Value matrix

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in$ IFSM $_{\mathrm{mxn}}$, where $a_{i j}=\left(\mu_{j}\left(c_{i}\right), v_{j}\left(c_{i}\right)\right)$. Then we define the Value matrix of Intuitionistic Fuzzy Soft Matrix A is $\mathrm{V}(\mathrm{A})=\left[a_{i j}\right]=\left[\mu_{j}\left(c_{i}\right)-v_{j}\left(c_{i}\right)\right] \mathrm{i}=1,2, \ldots \mathrm{~m}, \mathrm{j}=1,2, \ldots \mathrm{n}$

### 4.2Score matrix

If $A=\left[a_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{B}=\left[\mathrm{b}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$, then we define Score matrix of A and B as
$\mathbf{S}_{(\mathrm{A}, \mathrm{B})}=\left[\mathrm{d}_{\mathrm{ij}}\right]_{\mathrm{mxn}}$ where $\left[\mathrm{d}_{\mathrm{ij}}\right]=\mathrm{V}(\mathrm{A})-\mathrm{V}(\mathrm{B})$

### 4.2Total Score

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}, \mathrm{B}=\left[\mathrm{b}_{\mathrm{ij}}\right] \in \mathbf{I F S M}_{\mathrm{mxn}}$. Let the corresponding Value matrices be $\mathrm{V}(\mathrm{A}), \mathrm{V}(\mathrm{B})$ and their score matrix is $\mathbf{S}_{(\mathbf{A}, \mathbf{B})}=\left[\mathrm{d}_{\mathrm{ij}}\right]_{\mathrm{mxn}}$ then we define Total Score for each $\mathrm{c}_{\mathrm{i}}$ in U is $\mathrm{S}_{\mathrm{i}}=\sum_{j=1}^{n} d_{i j}$

## Methodology

Suppose U is a set of candidates appearing in an interview for appointment in Manager post in a company.Let E is a set of parameters related to managerial level of candidates.We construct IFSS
( $\mathrm{F}, \mathrm{E}$ )over U represent the selection of candidate by field expert X , where F is a mapping $\mathrm{F}: \mathrm{E} \rightarrow \mathrm{IF}^{\mathrm{U}}, \mathrm{IF}^{\mathrm{U}}$ is the collection of all intuitionistic Fuzzy subsets of U. We further construct another IFSS (G,E)over U represent the selection of candidate by field expert Y,where $G$ is a mapping $\mathrm{G}: \mathrm{E} \rightarrow \mathrm{IF}^{\mathrm{U}}, \mathrm{IF}^{\mathrm{U}}$ is the collection of all intuitionistic Fuzzy subsets of U.The matricesA and B corresponding to the intuitionistic Fuzzy softsets (F,E) and (G,E) are constructed,we compute the complements (F,E) ${ }^{\mathrm{c}}$ and $(\mathrm{G}, \mathrm{E})^{\mathrm{c}}$ and their matrices $A^{c}$ and $B^{c}$ corresponding to ( $\left.\mathrm{F}, \mathrm{E}\right)^{\mathrm{c}}$ and $(\mathrm{G}, \mathrm{E})^{\mathrm{c}}$ respectively. compute $\mathrm{A}+\mathrm{B}$ which is the maximum membership of selection of candidates by the judges. compute $\mathrm{A}^{\mathrm{c}}+\mathrm{B}^{\mathrm{c}}$ which is the maximum membership of non selection of candidates by the judges. using def (4.1), Compute $\mathrm{V}(\mathrm{A}+\mathrm{B}), \mathrm{V}\left(\mathrm{A}^{\mathrm{c}}+\mathrm{B}^{\mathrm{c}}\right)$ $S_{\left((A+B),\left(A^{c}+B^{c}\right)\right)}$ and the total score $S_{i}$ for each candidate in U.Finally find $S_{j}=\max \left(S_{i}\right)$, then conclude that the candidate $c_{j}$ has selected by the judges.If $S_{j}$ has more than one value the process is repeated by reassessing the parameters.

## 5.ALGORITHM

Step1:Input the intuitionistic fuzzy soft set (F,E), (G,E) and obtain the intuitionistic fuzzy soft matrices $\mathrm{A}, \mathrm{B}$ corresponding to (F,E) and (G,E) respectively .

Step2: Write the intuitionistic fuzzy soft complement set $(\mathrm{F}, \mathrm{E})^{\mathrm{c}},(\mathrm{G}, \mathrm{E})^{\mathrm{c}}$ and obtain the intuitionistic fuzzy soft matrices $\mathrm{A}^{\mathrm{c}}, \mathrm{B}^{\mathrm{c}}$ corresponding to $(\mathrm{F}, \mathrm{E})^{\mathrm{c}}$ and $(\mathrm{G}, \mathrm{E})^{\mathrm{c}}$ respectively .

Step3:Compute $(A+B),\left(A^{c}+B^{c}\right), V(A+B), V\left(A^{c}+B^{c}\right)$ and $S_{\left((A+B),\left(A^{c}+B\right)\right)}{ }^{c}$
Step4:Compute the total score $\mathrm{S}_{\mathrm{i}}$ for each $\mathrm{c}_{\mathrm{i}} \mathrm{inU}$.
Step5:Find c for which max $\left(\mathrm{S}_{\mathrm{i}}\right)$.
Then we conclude that the candidate $\mathrm{c}_{\mathrm{i}}$ is selected for the post .
Incase max $S_{i}$ occurs for more than one value, then repeat the process by reassessing the parameters.

## 6.CASE STUDY

Let (F,E) and (G,E) be two intuitionistic fuzzy soft set representing the selection of four candidates from the universal set $U=\left\{c_{1}, c_{2}, c_{3}, c_{4}\right\}$ by the experts X, and Y.Let $E=\left\{e_{1}, e_{2}, e_{3}\right\}$ be the set of parameters which stand for confident , presence of mind and willingness to take risk.

$$
\begin{aligned}
&(F, E)=\{ \left.F\left(e_{1}\right)=\left\{\left(c_{1}, 0.7,0.1\right)\right),\left(c_{2}, 0.5,0.5\right),\left(c_{3}, 0.1,0.8\right),\left(c_{4}, 0.4,0.6\right)\right\} \\
&\left.F\left(e_{2}\right)=\left\{\left(c_{1}, 0.6,0.3\right)\right),\left(c_{2}, 0.4,0.6\right),\left(c_{3}, 0.5,0.4\right),\left(c_{4}, 0.7,0.3\right)\right\} \\
&\left.\left.F\left(e_{3}\right)=\left\{\left(c_{1}, 0.5,0.4\right)\right),\left(c_{2}, 0.7,0.2\right),\left(c_{3}, 0.6,0.3\right),\left(c_{4}, 0.5,0.4\right)\right\}\right\} \\
&(G, E)=\left\{G\left(e_{1}\right)=\left\{\left(c_{1}, 0.6,0.2\right)\right),\left(c_{2}, 0.6,0.4\right),\left(c_{3}, 0.2,0.7\right),\left(c_{4}, 0.6,0.4\right)\right\} \\
&\left.G\left(e_{2}\right)=\left\{\left(c_{1}, 0.6,0.3\right)\right),\left(c_{2}, 0.5,0.5\right),\left(c_{3}, 0.6,0.4\right),\left(c_{4}, 0.8,0.1\right)\right\} \\
&\left.\left.G\left(e_{3}\right)=\left\{\left(c_{1}, 0.5,0.5\right)\right),\left(c_{2}, 0.8,0.1\right),\left(c_{3}, 0.7,0.1\right),\left(c_{4}, 0.5,0.4\right)\right\}\right\}
\end{aligned}
$$

These two intuitionistic fuzzy soft sets are represented by the following intuitionistic fuzzy soft matrices respectively

$$
\mathrm{A}=\begin{gathered}
e_{1} \\
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{gathered} \begin{array}{cccc}
(0.7,0.1) & \mathrm{e}_{2} & \mathrm{e}_{3} & \\
(0.6,0.3) & (0.5,0.4) \\
(0.5,0.5) & (0.4,0.6) & (0.7,0.2) \\
(0.1,0.8) & (0.5,0.4) & (0.6,0.3) \\
(0.4,0.6) & (0.7,0.3) & (0.5,0.4)
\end{array} \quad \mathrm{c}=c_{1} \begin{gathered}
\mathrm{e}_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{gathered}\left(\begin{array}{cccc}
(0.6,0.2) & (0.6,0.3) & (0.5,0.5) \\
(0.6,0.4) & (0.5,0.5) & (0.8,0.1) \\
(0.2,0.7) & (0.6,0.4) & (0.7,0.1) \\
(0.6,0.4) & (0.8,0.1) & (0.5,0.4)
\end{array}\right)
$$

Then the intuitionistic fuzzy soft complement matrices are

$$
\mathrm{A}^{c}=\begin{gathered}
e_{1} \\
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{gathered}\left(\begin{array}{ccc}
(0.1,0.7) & \mathrm{e}_{2} & \mathrm{e}_{3} \\
(0.3,0.3) & (0.4,0.5) \\
(0.5,0.5) & (0.6,0.4) & (0.2,0.7) \\
(0.6,0.4) & (0.4,0.5) & (0.3,0.6) \\
(0.8,0.1) & (0.4,0.5)
\end{array}\right) \quad \mathrm{B}^{c}=c_{1} \begin{gathered}
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{gathered}\left(\begin{array}{cccc}
(0.2,0.6) & (0.3,0.6) & (0.5,0.5) \\
(0.4,0.6) & (0.5,0.5) & (0.1,0.8) \\
(0.7,0.2) & (0.4,0.6) & (0.1,0.7) \\
(0.4,0.6) & (0.1,0.8) & (0.4,0.5)
\end{array}\right)
$$

Then the addition matrices are

$$
\begin{aligned}
& \begin{array}{lllllll}
e_{1} & \mathrm{e}_{2} & \mathrm{e}_{3} & e_{1} & \mathrm{e}_{2} & \mathrm{e}_{3}
\end{array} \\
& \mathrm{~A}+\mathrm{B}=\begin{array}{c}
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{array}\left(\begin{array}{lll}
(0.7,0.1) & (0.6,0.3) & (0.5,0.4) \\
(0.6,0.4) & (0.5,0.5) & (0.8,0.1) \\
(0.2,0.7) & (0.6,0.4) & (0.7,0.1) \\
(0.6,0.4) & (0.8,0.1) & (0.5,0.4)
\end{array}\right) \quad \mathrm{A}^{c}+\mathrm{B}^{c}=c_{1} \begin{array}{c}
c_{2} \\
c_{3} \\
c_{4}
\end{array}\left(\begin{array}{lll}
(0.2,0.6) & (0.3,0.6) & (0.5,0.5) \\
(0.5,0.5) & (0.5,0.4) & (0.2,0.7) \\
(0.6,0.1) & (0.4,0.5) & (0.3,0.6) \\
(0.6) & (0.3,0.7) & (0.4,0.5)
\end{array}\right)
\end{aligned}
$$

Calculate the score matrix and the total score for selection

$$
\begin{aligned}
& e_{1} \\
& \mathrm{e}_{2} \mathrm{e}_{3} \\
& c_{1} \\
& c_{\left((\mathrm{A}+\mathrm{B}),\left(A^{c}+B^{c}\right)\right)}
\end{aligned} \begin{aligned}
& c_{2} \\
& c_{3} \\
& c_{4}
\end{aligned}\left(\begin{array}{rrl}
1.0 & 0.6 & 0.1 \\
0.2 & -.1 & 1.2 \\
-1.2 & 0.2 & 0.9 \\
0.0 & 1.1 & 0.2
\end{array}\right)
$$

We see that the first candidate has the maximum value and thus conclude that from both the expert's opinion, candidate $\mathrm{c}_{1}$ is selected for the post.

## 7.Conclusion

In this paper,we proposed intuitionistic fuzzy soft matrices and defined different types of intuitionistic fuzzy soft matrices and some operations.Finally we extend our approach in application of these matrices in decision making problems.Our work is infact an attempt to extend the notion of intuitionistic fuzzy soft matrix.Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

## 8.REFERENCES

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