Intuitionistic Fuzzy Theory for Soft-Computing:

More Appropriate Tool Than Fuzzy Theory

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Abstract

The entire work reported here is a philosophy based theoretical presentation. In this work it has been justified that intuitionistic fuzzy theory is more appropriate tool than fuzzy theory for soft-computing. Any association of computing methodologies centered on fuzzy set theory is well regarded as one kind of Soft-computing. Soft computing with fuzzy theory involves fluent use of both $\mu(x)$ and $\mu^c(x)$ i.e. $\nu(x)$ of the elements of all the universes of the concerned decision problem. The role of both $\mu(x)$ and $\nu(x)$ are very fluent in solving decision problems in almost all application areas of fuzzy theory. Recent literatures reveal that fuzzy set theory may not be an appropriate model to deal with ill-defined large size decision problems. In this work the author very precisely unearths the weakness of fuzzy theory in a further dimension, which is not caused due to $\mu(x)$ but due to the other part $\nu(x)$ of the coin. Doing a rigorous exercise with few real life examples it is observed that the fuzzy set theory is inappropriate not only for large size decision problems but also for many decision problems, irrespective of its size, large or small, in real life environment. Consequently, a set of necessary eligibility conditions is proposed at least one of which is to be mandatorily satisfied by any fuzzy decision maker before using 'Fuzzy Set Theory' in Soft-Computing. The initial content of this paper deals with a basic question: Is 'Fuzzy Theory' really always good for soft-computing? In the Theory of CIFS it is justified that while solving any decision making problem, the selection of a suitable soft-computing set theory or crisp theory is made by the concerned decision maker by his own choice and own knowledge, which functions at the outer sphere of the cognition system of the decision maker. It is analogous to the case of a computer programmer who solves a problem by writing codes in a

higher level language of his own choice, and can interact with the screen from the outer sphere only while he executes his program in CPU in machine language. But it is the CIFS which mandatorily functions at the innermost sphere of the brain of the concerned decision maker by default (not by any choice of him), irrespective of what soft-computing set theory or crisp theory is used by the decision maker (be it a human being or an animal or a bird or any living thing which has a physical brain; excluding the cases of robot/machine/software which work with artificial intelligence). It is analogous to the case of execution of machine language codes in the innermost sphere of the CPU irrespective of what higher level language was used by the programmer by his own choice at the outermost sphere of the CPU. Different programmers use different languages by their respective own choice, but the execution in CPU is of a unique common language which is machine language, for the case of all the programmers, for the case of all the higher level languages, for the case of all the problems under consideration for writing codes for solution. And that is analogous to what the Theory of CIFS says about the case of a decision making process.

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1 Introduction

The work in this paper is a kind of "Achieving **Reality** by **Imagination**". According to Prof. Zadeh, Soft computing is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for soft computing is the human mind. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost and solve the fundamental problem associated with the current technological development. Soft-computing is not a methodology, but it is a partnership of methodologies that function effectively in an environment of imprecision and/or uncertainty. Thus soft computing aims to exploit the tolerance for imprecision and uncertainty in achieving solutions to ill-defined decision problems. One of the principal components of soft computing is regarded to be the fuzzy logic according to the presently agreed concept. To the world scientists and researchers, the most popular soft computing set theories ([1-10], [22-27], [30-43]) are : fuzzy set theory, intuitionistic fuzzy set theory (vague sets are nothing but intuitionistic fuzzy sets, as justified and reported by many authors), i-v fuzzy set theory, i-v intuitionistic fuzzy set theory, L-fuzzy set theory, type-2 fuzzy set theory, flou set theory, L-set theory and also rough set theory, soft set theory, etc. Which soft-computing set theory (theories) is to be used for solving a decision problem is the personal choice of the concerned decision maker. In such theories, the value of $\mu(x)$ for every x of the universe U is proposed by the concerned decision maker by his best possible judgment. In this

book the author makes a rigorous analysis to visualize whether Fuzzy Set theory is really good for soft-computing. In the recent literatures [15,16] it is justified in length that Fuzzy Set theory is not appropriate for large size decision problems. It is justified in [15] that whatever be the soft-computing set theory used by a decision maker by his own choice, the internal execution at the kernel of the cognition system of the decision maker is according to the "Theory of CIFS" which is not by choice, but automatic by default. The phrase CIFS stands for "Cognitive Intuitionistic Fuzzy System", the theory of which is based upon the Atanassov philosophy about soft-computing. By a 'decision maker' in the Theory of CIFS we mean a human being or an animal or a bird or any living thing which has at least one physical brain (excluding the case of robot, machine, equipment or software which has artificial intelligence). For evaluating the value of $\mu(x)$, the initialization at the kernel of the cognition system is always Atanassov Initialization irrespective of the nature of the soft-computing set theory chosen by the decision maker by his own choice. But for this, the decision maker need not be knowledgeable or aware of the Intuitionistic Fuzzy Set theory (see [15]) or even need not be literate at all as per our common definition of literacy. For example, a tiger is a decision maker (an excellent decision maker in his own society), but he is not aware of fuzzy theory or any soft-computing set theory. His brain does automatically process the CIFS logic mandatorily, but not by any choice of him. He may be called by us as an 'illiterate', but he is certainly having his own logic which is unknown to us and consequently he is very much literate by his own logic. To know the details of the Theory of CIFS one could view the work in [15].

The work in this paper is sequel to the earlier work reported in the books [15,16]. It is analyzed and explained in this paper by several examples that in most of the cases 'Fuzzy Set Theory' may lead to huge amount of error in the final results of decision making problems, deviating far from the reality, deviating far from the truth. The dangerous and shocking fact is that this error deeply penetrates in the results and conclusions in a completely hidden way without providing any prior information to the concerned fuzzy decision maker (as shown by several examples here). The initial content of this book is basically a kind of analysis for "achieving **Reality** by **Imagination",** but in fact the author guarantees that no amount of imagination is made or required here to cater to any unacceptable or illogical hypothesis.

2 Theory of CIFS: a brief survey

In this chapter a brief survey of the existing literature is done, mainly from the work reported in the book[15,16], before proceeding for the actual content here. In [15] a new theory called by "Theory of CIFS" (Cognitive Intuitionistic Fuzzy System) is developed and then it has been established that fuzzy set theory is not an appropriate tool to solve large size imprecise problems. In fact the quality of

decision becomes worst and multifold if there are many universes in the concerned decision problem. By large size we mean that there are large number of elements in the universe of discourse or there are many universes in the decision problem under consideration. The work in [15] is based on philosophical, logical as well as mathematical views on the subject of decoding the 'progress' of decision making process in the Human/Animal/Bird cognition systems while evaluating the membership value $\mu(x)$ in a fuzzy set or in an intuitionistic fuzzy set or in any such soft computing set model or even in a crisp set. By 'cognition system' of a decision maker it is meant here the cognition system of a human being or of a living animal or of a bird or of any living thing which has brain (ignoring the machines, robots, or software which have artificial intelligence).

While a hungry leopard finds one cow or one bull or one buffalo (or any other animal of his own food list) in his forest, he decides a lot by his best possible judgment on a number of significant parameters before he starts to chase his food. He also continuously decides during the real time period of his chasing. Even during the course of chasing he sometimes decides whether to give up the chasing or to continue chasing without any problem of his own security, etc. During the course of chasing he decides about what to do whenever the cow changes her direction, what to do if there is a wide drain in front of his chasing path, etc. He takes these types of important real time decisions by his best possible judgment using his own logic/theory, the logic which is not known to us. But whatever be the different type of logic/theory be used by different kind of decision makers in various decision problems by their own choice, the kernel of the brain of every decision maker executes an absolutely unique common logic of CIFS [15] by default, irrespective of their intellectual capabilities, irrespective of their knowledge, irrespective of their literacy elements possessed, irrespective of what kind of living thing they are (human or animal or bird etc.). This was fact during stone age period of earth, and will remain so for ever on this earth. The Theory of CIFS developed in [15] says that a crisp decision maker or a fuzzy decision maker (or any soft decision maker) can not decide upon any decision making issue without using intuitionistic fuzzy set (IFS) theory, but he does not necessarily need to have any knowledge of intuitionistic fuzzy set theory (for instance, a tiger does not know Intuitionistic Fuzzy Set theory). The permanent residence of the 'Theory of IFS' inside the brain (CPU) of every living thing (i.e. every decision maker) is a hidden truth, not by any choice of the concerned living thing, but by default. In fact the 'Theory of IFS' is a permanent and hidden resident inside the kernel (i.e. at the lowest level) of the processor/brain of every cognition system (be of human or of animal or of bird or of any living thing) in the form of like a 'in-built system-software' in the Operating System. Consider the case of a FORTRAN programmer who chooses the tool 'FORTRAN language' by his own choice and executes his program written by him in FORTRAN language corresponding to a given engineering problem. But for this, it does not require that the programmer must be aware or knowledgeable about machine language programming!. The analogous fact is true for a fuzzy decision

maker too, who estimates $\mu(x)$ using the domain of his fuzzy knowledge whereas at the lowest level inside his cognition system the exact execution happens under intuitionistic fuzzy systems only. This is the core philosophy of the Theory of CIFS established in [15].

2.1 Two Facts of CIFS

The following two hypothesis are hidden facts in fuzzy computing or in any soft computing process [15]**:-**
Fact-1 :
A decision maker (intelligent agent) can never use or apply 'fuzzy theory' or any soft-computing set theory without intuitionistic fuzzy system.
Fact-2 :
The Fact-1 does not necessarily require that a fuzzy decision maker (or a crisp ordinary decision maker or a decision maker with any other soft theory models or a decision maker like animal/bird which has brain, etc.) must be aware or knowledgeable about IFS Theory!

Theory of CIFS is unique and common to the brains of all the living things (human or animal or bird etc.) irrespective of any soft-computing set theory (viz. fuzzy set theory, intuitionistic fuzzy set theory, i-v fuzzy set theory, i-v intuitionistic fuzzy set theory, L-fuzzy set theory, type-2 fuzzy set theory, flou set theory, L-set theory and also rough set theory, soft set theory, etc.) or any unknown theory/logic used by the concerned decision maker by his own choice. By the word 'unknown' here we mean that it is unknown to the human beings. For a detail study of the Theory of CIFS one could see [15]. However we recollect below some of the basic elements of the Theory of CIFS from [15] which are required as preliminaries for the progress of our actual work in this paper .

It is obvious that the judgment process of a decision maker (say, a human being named by Mr. Sen) by which the element x of the universe U is given the membership value $\mu(x) = \omega$ (say) in a fuzzy set A of U can not be finished in zero amount of processing time by the brain of the decision maker Mr. Sen. It may take an infinitesimally small amount of time Δt or it could be in few nano-seconds or in few micro-seconds or in few seconds/minutes or in hours or in days, etc. Suppose that for the element x the complete processing time taken by the decision maker to come to his final judgment that "$\mu(x) = \omega$" is T (>0) unit of time. This amount of time being required by the cognition system of the decision maker Mr. Sen for processing to evaluate the membership value $\mu(x)$ is called **"Atanassov Processing Time"** (APT) for the element x and is denoted by $APT_{Sen}(x) = T$ or in short by the notation $APT(x) = T$. Thus, the value of $\mu(x)$ proposed by the decision maker Mr. Sen is ω for which the time-cost is T (>0), and hence by fuzzy theory one can compute $\upsilon(x) = (1 - \omega)$ by doing an arithmetic just, without any further cost of time towards decision process, without any requirement of further decision process. In 'Fuzzy Theory', decision process

is required just to propose $\mu(x)$, but the value of $\upsilon(x)$ is not proposed. The value of $\upsilon(x)$ is crisp-computed using a mathematical formula, and so there is no decision process involved in it. The brain (CPU) or the cognition system of the decision maker Mr. Sen was continuously busy during the every instant of time in the interval [0,T] in blossoming the appropriate value of of $\mu(x)$ assuming that the business commenced at time $t = 0$. It is absolutely sure that exactly at the starting time $t = 0$, evaluation of $\mu(x)$ was not complete and can not be complete, and hence its value can not be equal to ω at time $t = 0$. It is also sure that at any instant of time $t < T$, i.e. in the right-open interval [0, T), the complete evaluation of $\mu(x)$ was not over and hence the "under-process value of $\mu(x)$" $\leq \omega$ in the semi-closed time interval [0, T) because of the fact that it is a pre-mature stage of the cognition processor to output the final value of $\mu(x)$; and finally at time $t = T$, the mercury of "under-process $\mu(x)$" stops permanently at ω inside the cognition system of Mr. Sen. Since value of "under-process $\mu(x)$" is sometimes $\leq$ ω and then finally equal to ω, it is thus fact that "under-process $\mu(x)$" is a continuous non-decreasing function of 'time' t during the period of evaluation in the cognition system, until it becomes fully-matured and frozen at a constant value ω at time T. In the Theory of CIFS, this continuous non-decreasing function is denoted by $m_x(t)$ or by $m(x,t)$ which is a function of time t and whose Decision Process Period domain is the closed interval [0,T] of time t; i.e. the function $m(x,t)$ starts growing from time $t = 0$ and continues to grow (at least it does not diminish) till the time $t = T$ when it achieves the final functional value ω. This final value ω is what we call the 'membership value' $\mu(x)$ in the fuzzy set A (or any other soft-computing set) of the universe X proposed by the concerned decision maker by his best possible judgment, for the element x of X. Thus $0 \leq m(x,t) \leq \mu(x)$, where $\mu(x)$ is a constant value (here it is ω) but $m(x,t)$ updates itself with the continuous progress of time starting from $t = 0$ till $t = T$.

2.2 Trio Functions

Let R^* be the set of all non-negative real numbers. Consider a fuzzy decision maker. For any given element x of the set X to belong to the fuzzy set A of X, the membership value $\mu(x)$ is the final output of a hidden "cognitive intuitionistic fuzzy system" in the brain of the fuzzy decision maker where the following three functions are co-active :-

(i)   **h(x,t)** called by 'Hesitation Function' whose domain is $X \times R^*$ and range is [0,1]. For a fixed element x of the set X, $h(x, t)$ is a non-increasing continuous function of time t.

(ii)   **m(x,t)** called by 'Membership Function' whose domain is $X \times R^*$ and range is [0,1]. For a fixed element x of the set X, $m(x, t)$ is a non-decreasing continuous function of time t.

(iii)   **n(x,t)** called by 'Non-membership Function' whose domain is $X \times R^*$ and range is [0,1]. For a fixed element x of the set X, $n(x, t)$ is a non-decreasing continuous function of time t.

2.3 Atanassov Constraint

The AT functions are subject to the following constraint called by Atanassov Constraint:

$$h(x, t) + m(x, t) + n(x, t) = 1 \quad \text{at every time t.}$$

The value $\mu(x)$ in the fuzzy set A comes at the instant $t = T$ from the function $m(x,t)$ because $m(x,t)$ finally converges at the value $\mu(x)$ after a course of sufficient growth. The functions $m(x,t)$ and $n(x, t)$ get feeding from $h(x,t)$ in a continuous manner starting from time $t = 0$ till time $t = T$. None else feeds $m(x,t)$ and $n(x, t)$.

2.4 Atanassov Initialization

At time $t = 0$ i.e. at the starting instant of time for evaluating the membership value $\mu(x)$, any decision process in the cognition system starts with AT functions with the following initial values :-

$$\textbf{h(x,0) = 1,} \quad \textbf{with} \quad \textbf{m(x,0) = 0} \quad \textbf{and} \quad \textbf{n(x,0) = 0.}$$

The clock starts from time $t = 0$ and the whistle blows from this initialization only. This initialization <0, 0, 1> for <$\mu(x)$, $\upsilon(x)$, $\pi(x)$> is called by 'Atanassov Initialization'.

It is important to understand that Atanassov Initialization is not initialized by any choice of the decision maker or by any decision of the decision maker or by any prior information from the kernel of the cognition system to the outer-sense of the decision maker. It is never initialized by the decision maker himself by his own possessed knowledge, but it gets automatically initialized at the kernel of the brain (CPU) during the execution of any decision making process (see [15] for details). By decision maker, we shall mean here a human or an animal or a bird or any living thing which has a physical brain (ignoring the robots or machines or software which have artificial intelligence).

2.5 Impossible Types of Initialization

A decision maker, be him highly intelligent or not, can never conclude the value of $\mu(x)$ at time $t = 0$ i.e. at no cost of time. He must need some amount of time t > 0, which could be infinitesimal small Δt or moderately small or a large amount. Imagine a case of any soft computing set model (for example: Fuzzy Set Theory) where the dimension $h(x,t)$ does not exist in the mathematical model of its notion proposed by Prof. Zadeh. In such a case too, it is quite obvious that **none** of the following three types of initializations can anytime happen (can be anytime possible) in the cognition system of the decision maker while evaluating the membership value $\mu(x)$ **:**

Type-(i) :   m(x, 0) = 0 and n(x, 0) = 1   at time t = 0,   but
finally converging at m(x, T) = μ(x) and n(x, T) = 1- μ(x)
after time t = T (>0).

Type-(ii) :   m(x, 0) = 1 and n(x, 0) = 0   at time t = 0,   but
finally converging at m(x, T) = μ(x) and n(x, T) = 1- μ(x)
after time t = T (>0).

Type-(iii) :   m(x, 0) = k and n(x, 0) = 1- k,   at time t = 0   where k is some
initial scalar constant, and finally converging at m(x, T) = μ(x) and n(x, T) = 1-
μ(x) after time t = T (>0).

It is because of the fact that whatever be the soft computing set theory under
consideration of the decision maker (or no standard theory under consideration of
the decision maker), the initialization is always the Atanassov Initialization only,
can not be any alternative. Although μ(x) in fuzzy set theory does not have any
scope of link with the hesitation part h(x,t) which is also called by 'undecided
part', but the software of the cognition system or brain has the in-built function
h(x,t) without which no decision process can initiate. This was a fact during the
stone age of the earth too, and will continue to remain as a fact for ever.

2.6 Trio Bags

These are three imaginary bags. During the progress of decision making process
with respect to the variable 'time' in the brain while evaluating the membership
value μ(x), imagine that the values of AT functions are stored and updated
continuously, with respect to time t, in the three bags : h-bag, m-bag and n-bag,
but always replacing their previous values. These three bags are called by
Atanassov Trio Bags or Trio Bags (see Figure 1).

It is obvious that at time t = 0 each of the Atanassov Trio Bags contains the value
corresponding to Atanassov Initialization, not else.
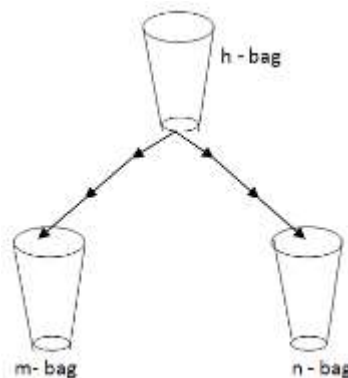
Fig. 1.   Atanassov Trio Bags

Immediately after that, the m-bag and n-bag start getting credited with zero or more amount of values continuously from the h-bag, subject to fulfillment of Atanassov Constraint at every instant of time t assuming that the transaction time from h-bag to any bag is always nil.
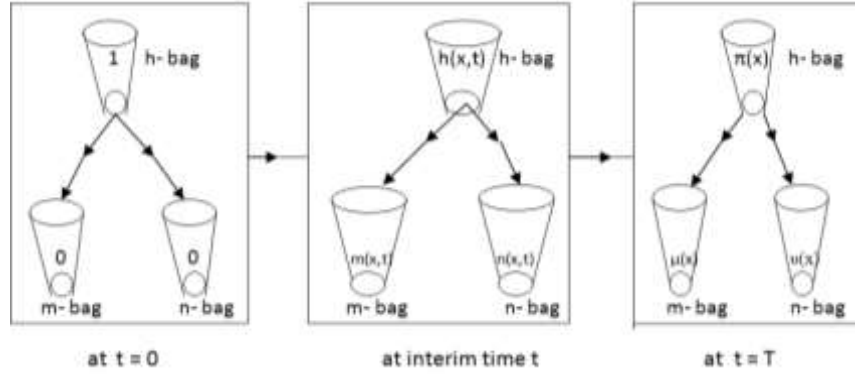
Fig. 2.   Evaluation of μ(x) starting from 'Atanassov Initialization'

But there never happens a reverse flow, i.e. the h-bag does never get credited from any or both of m-bag and n-bag. The feeding continues till time t = T as shown in Figure 2.

2.7 Atanassov Processing Time

While evaluating the membership value μ(x) of an element x, the Atanassov Initialization happens at time t = 0 at the human cognition system (or at the cognition system of the animal or bird whoever be the decision maker). At the very next instant of time, i.e. from time t>0, the following <u>actions</u> happens simultaneously to the AT functions subject to fulfillment of Atanassov constraint (assuming that the transaction time from h-bag to any bag is always nil):

     (i) h(x, t) starts reducing   (at least non-increasing),   and
     (ii) m(x, t) as well as n(x, t)   start increasing (non-decreasing).

After certain amount of time, say after t = T (>0) the processing of the decision making process stops (converges) at the following state, say :

$$h(x, T) = \pi(x), \quad \text{with } m(x, T) = \mu(x) \quad \text{and} \quad n(x, T) = \nu(x)$$

where $\pi(x) + \mu(x) + \nu(x) = 1$, and after which there is no further updation happens to the values of AT functions in the cognition system. Then we say that for the complete evaluation of the membership value μ(x) by the concerned decision maker corresponding to the element x, the Atanassov Processing Time (APT) is T unit of time.

The final values of the AT functions <m(x,t), n(x,t), h(x,t)> corresponding to the element x are given by :

$$m(x,T), \ n(x,T), \ h(x,T) ; \quad \text{where } APT(x) = T.$$

We call these final matured values m(x,T) by $\mu(x)$, n(x, T) by $\nu(x)$, and h(x,T) by $\pi(x)$.   They are respectively the 'membership value $\mu(x)$ ', the 'non-membership value $\nu(x)$ ' and the 'hesitation value $\pi(x)$ ' of the element x by the best possible judgement of the concerned decision maker in the Theory of IFS of Atanassov.

It is fact that the cognition system of a decision maker (fuzzy decision maker or intuitionistic fuzzy decision maker or crisp decision maker) can not evaluate the membership value $\mu(x)$ of an element x without initiating from the Atanassov Initialization "h(x,0) = 1 with m(x,0) = 0 and n(x,0) = 0" by default, irrespective of his awareness/knowledge of IFS Theory. It is because of the fact that this intuitionistic fuzzy processing happens at the kernel of the brain (CPU) of the decision maker, analogous to the case of execution of FORTRAN codes in CPU, irrespective of the awareness/knowledge of the concept of Machine Language by the concerned 'higher level language programmer'. Here the decision maker may be a fuzzy decision maker or any kind of decision maker (who may be a layman of IFS theory or of Fuzzy theory, or who could be even an animal or a living thing having brain).

The following important proposition is established in [15, 16].

Proposition 1.
For any decision maker, be it a human or an animal or any living thing which has brain, it is impossible that his brain (kernel of his cognitive system) does always have the indeterministic component (i.e. the hesitation component) h(x,t) to be nil for the element x of the universe X, while going to propose the corresponding membership value $\mu(x)$.

3 Achieving 'Reality' by 'Imagination'

Usually in an ill-defined decision problem a fuzzy set is exercised by concepts or properties described by a set of words or by a phrase(s) denoted by Λ , in particular by adjectives   (with or without adverb) like : YOUNG, TALL, SLIGHTLY MORE THAN, HEAVY, VERY BEAUTIFUL, etc. Let us call the phrase by **"Zadeh phrase"** in memory of the great philosopher Prof. L. A. Zadeh. Thus a Zadeh phrase describes a fuzzy set in a universe of discourse. However, corresponding to a given Zadeh phrase in a given universe of discourse, there will be different fuzzy sets proposed by different decision makers.
Few Examples of 'Zadeh phrase':
(1)   corresponding to the fuzzy set 'Collection of all YOUNG boys' in a school, the Zadeh phrases we may take as $\Lambda \equiv$ "YOUNG boys";
(2)   corresponding to the fuzzy set 'Collection of all beautiful cities in a country', the Zadeh phrases we may take as $\Lambda \equiv$ "beautiful cities";

(3)   corresponding to the fuzzy set 'Collection of all real numbers which are slightly greater than 47', the Zadeh phrases we may take as $\Lambda \equiv$ "slightly greater than 47"; etc.

Before proceeding for a rigorous analysis about the main issues and objectives, let us consider one new terminology called by "Identical Imaginary Environment" but with philosophical eyes. We propose its definition under some imaginary conditions. This new term "Identical Imaginary Environment" is used in our discussion here to propose two membership values using two different (or, may be same) soft-computing set theories respectively, but corresponding to a given common element x of the universe of discourse U to represent a common ill-defined collection of the elements of U under certain interesting conditions. The most important part of this section is that although we will use few imaginary conditions but the final objectives and goals are to achieve reality without any contradiction on common logic and reasoning.

3.1 Two imaginary brains

Before explaining the term "Identical Imaginary Environment", we shall imagine that the actual physical brain of the decision maker is replaced by two imaginary brains each of which is exactly same as the actual real brain of him (see Figure 3) in all biological respects. These two imaginary brains are called by **brain-1** and **brain-2**. Thus each of brain-1 and brain-2 are 100% identical to the actual real physical brain of the decision maker in terms of all its elements (i.e. in terms of biological constitutes, physical constitutes, scientific constitutes, and in terms of all type of constitutes).
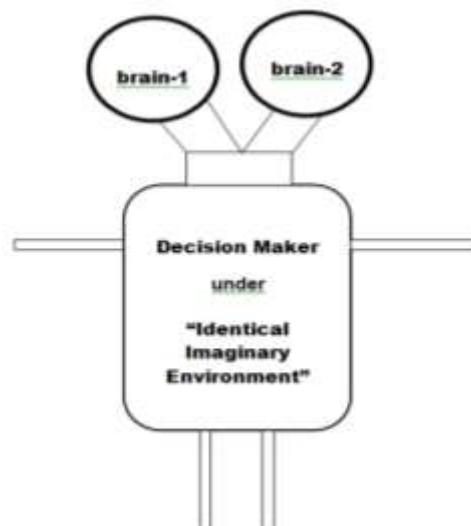
Fig. 3.   An imagination that the actual brain of the decision maker is replaced by two independent but identical imaginary brains

However, with no loss of anything of our interest, let us also imagine that for the decision maker the two imaginary brains brain-1 and brain-2 are completely

independent.   None even knows the existence of the other. Each of them acts like the actual physical(biological) brain thinking that it itself is the actual physical(biological) brain.

<u>Fulfillment of "Non-contradictory" Condition :-</u>
It is quite obvious and very important to notice that that brain-1 and brain-2 can not be contradictory to each other as both are imagined to be the same brain in all respect. The brain-1 can not give a statement which contradicts brain-2 and vice-versa. All results delivered by them must be 100% consistent.

For instance, it is impossible that the brain-1 says 'it is red' and the brain-2 says 'it is blue'; similarly it is impossible that the brain-1 says $\mu(x) = 0.8$ and the brain-2 says $\mu(x) = 0.1$, corresponding the same element x to propose a fuzzy set A of the universe U.
Thus, it is very important to note that brain-1 does not know what amount brain-2 proposes and brain-2 does not know what amount brain-1 proposes as a membership value. It is so because of the fact that both brain-1 and brain-2 are same but independent as per their construction. In fact brain-1 even does not know the existence of brain-2 and vice-versa. Each of them assumes that it is the actual brain of the corresponding decision maker. Thus brain-1 and brain-2 can never happen to contradict each other. All results delivered by them must be fully consistent.

It is to be assumed that the actual physical brain is kept at 'sleeping mode', where the brain-1 and brain-2 are in 'action mode' but concurrently (i.e. in parallel).

3.2 Identical Imaginary Environment

The most popular soft computing set theories ([1-8], [22-27], [30-43]) to the world scientists are : fuzzy set theory, intuitionistic fuzzy set theory (vague sets are nothing but intuitionistic fuzzy sets, as justified and reported by many authors), i-v fuzzy set theory, i-v intuitionistic fuzzy set theory, L-fuzzy set theory, type-2 fuzzy set theory, flou set theory, L-set theory and also rough set theory, soft set theory, etc. Which soft-computing set theory (theories) is to be used for solving a decision problem is the personal choice of the concerned decision maker. In such theories, the value of $\mu(x)$ for every x of the universe U is proposed by the concerned decision maker by his best possible judgment.

Let U be a given universe. Suppose that we are interested in a particular soft-defined collection **C** of objects of U. Also suppose that for some interest, we need to represent the collection **C** using two different soft-computing sets <u>independently</u> and in <u>parallel</u>, analogous to the situation of two compatible questions or events [19].

Under such a requirement, two soft-sets of a given universe U are said to be

proposed at an **"Identical Imaginary Environment"** if the following three conditions i(i), i(ii) and i(iii) are fulfilled :-

i(i) :   both the soft-computing sets be proposed by the same decision maker,
       (but one is proposed by brain-1 and the other is proposed by brain-2).

  i(ii) :   both be proposed initiating at the same instant of time on the same day,
          and

  i(iii) :   both be proposed at the same place of geographical location.

A decision maker considered under "Identical Imaginary Environment" should not be confused to have any link with the real cases of child births of type 'a person with two real physical brains' (as such type of births happen sometimes in the world in reality, but it is an extremely rare and rare case).

The concept of "Identical Imaginary Environment", although imaginary, but very useful in our logical discussion here as we will be in the process of "Achieving **Reality** by **Imagination**". It will be realized at the end here that although it is called an imaginary environment but truly speaking it is not so at any loss of reality, rather it is in quest of hidden reality. The "Identical Imaginary Environment" by definition does not contradict or oppose any element of our interest here in quest of reality. It need not be confused with the Law of Non-Contradiction of Aristotle too.

3.3 Ignoring 'Hesitation Part' in Fuzzy Theory

In this section we discuss few prominent consequences of ignoring 'Hesitation Part' in Fuzzy Theory. Fuzzy Set Theory was formalized by the great philosopher Professor Zadeh in 1965. The development of Fuzzy Set Theory from conventional bivalent set theory is no doubt a paradigm shift. As per the philosophy of Zadeh, some of the essential characteristics of fuzzy logic are :

(i)   exact reasoning is viewed as a limiting case of approximate reasoning.

(ii)   everything is a matter of degree.

(iii)   knowledge is interpreted a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.

(iv)   inference is viewed as a process of propagation of elastic constraints.

(v)   any logical system can be fuzzified.

Let A be a fuzzy set of the universe U with the membership function μ_A. The complement of the fuzzy set A is denoted by A^c for which the membership function $\mu_A{}^c$ (or ν_A) given by

$$\nu_A(x) \text{ i.e. } \mu_A{}^c(x) = 1 - \mu_A(x)$$

using the standard negation function (standard involution) $N : I \rightarrow I$ given by $N(z) = 1 - z$.

In our discussion throughout here, we consider the above classical complement of a fuzzy set proposed by Zadeh in his pioneering work (Zadeh, 1965), which is also the Sugeno's complement for $\lambda = 0$ and Yager's complement for $\omega = 1$,

having the equilibrium at the point 0.5**.** For this fuzzy complement of Zadeh, the Kosko's theorem hold good. We do not consider here the <u>generalized</u> axiomatic/involutive complement or round complement or the Sugeno's complement or Yager's complement. Most of the decision makers apply the classical notion of complement of a fuzzy set unless a highly specialized domain is under consideration. This classical notion of complement is most popular to the fuzzy theorists in the world, and also extensively used by Zadeh himself in case of dealing with linguistic hedges [28, 29, 34, 35, 38-41]. In Fuzzy Set theory the value $v(x)$ or $\mu_A^c(x)$ is interpreted as the degree to which x belongs to the fuzzy set A^c, i.e. as the degree to which the element x does not belong to the fuzzy set A. The significance of complement in a crisp set is like: who does not belong to the set. But in a fuzzy set it is : how much does an element not belong to the set? To the decision makers during soft-computing, one non-supportive element of the notion of fuzzy complement A^c is that it is not cutworthy, i.e. the α-cut of the fuzzy set A^c is not equal to the crisp complement of the α-cut of the fuzzy set A.

In [15,16] it is established that in some cases fuzzy set theory is weak (in fact, not an appropriate model) to deal with ill-defined <u>large size</u> decision problems. In this section here we trace the weakness of fuzzy theory in a further dimension, identifying the root-cause of the weakness. It is further exercised here that the fuzzy set theory sometimes is inappropriate for soft-computing not only for the large size decision problems but also for any decision problem, irrespective of its size, be it of large size or small size. However, there are also a number of real life imprecise cases where only the best/excellent decision makers (in most of such cases being pre-selected or pre-chosen) are allowed to take decisions who are capable to do the decision job with a guaranteed excellence and are supposed to do the decision job 'without any hesitation' on every issue of the problem under consideration. In such type of particular cases the outcome '$\pi(x) = 0$' is to happen to be a 'true' everywhere across all the elements of all the universes during the execution of the decision process. The method of CFE presented in [14] is a very important and revolutionary proposal to FIFA (IFAB) which is a very ideal example to understand this type of situation where fuzzy theory is the most appropriate tool than intuitionistic fuzzy theory or any other soft-computing theory, in some special cases. In the CFE method it is shown where fuzzy set theory is rated to be the most appropriate tool for finding out solution, but in general it is not so for an arbitrarily chosen problem or for a randomly chosen problem. Similarly, the fuzzy theory is assumed to be appropriate for application made in Election System [13] and in an Engineering Model for Higher Education Management [17], for examples.

In this section we do also unearth the following types of weakness of Fuzzy Theory which is just owing to the ignoring the existence of 'Hesitation Part' in its theoretical birth model, and consequently it is observed to be too costly as justified in this work in length :-
(i)   An element of inconsistency in 'Fuzzy Theory'

(ii) An intelligent Expert 'Fuzzy Theorist' may decide Not to apply 'Fuzzy Theory', while facing a fuzzy problem which he has to mandatorily solve!

(iii) An Example showing that 'Fuzzy Theory' contradicts itself!

All these three cases are expected to be of serious matter to the fuzzy experts who want to apply fuzzy theory in solving ill-defined problems! In the next section we do also show by an example that there is an occurrence of upto 220% Error on applying 'Fuzzy Theory' by a Fuzzy Expert in a small size decision problem which is a problem of very common and frequent nature in our daily life environment.

3.3.1 Inconsistency in 'Fuzzy Theory'

In this subsection it is unearthed that there is a hidden element of inconsistency in Fuzzy Theory. To justify it, we begin with a hypothetical but real life example of very simple and common nature.

Example-1

Consider a decision maker Mr. Bose who is an expert in both 'Fuzzy Set theory' and 'Intuitionistic Fuzzy Set theory'. Suppose that there are 50 students in Class-V in Calcutta St. Paul's School. Let U be the set of all these 50 students, say U = $\{ x_1, x_2, x_3, \ldots, x_{50} \}$.

Consider the Identical Imaginary Environment entitled by **'E1'** as prescribed below by the three conditions as defined earlier:

> *i(i):   The decision maker is Mr. Bose.*
> *i(ii) : Time is now 4.30 PM   on 5th September'2018,   and*
> *i(iii) : at the office of the Principal of Calcutta St. Paul's School.*

(Throughout in this paper , whenever we use identical environment **E1**, we shall mean the above environment only constituted by the above three conditions).

We suppose that for execution of a statistical project, this decision maker (Mr. Bose) wants to consider a soft-collection of objects of U which is the "collection of all the TALL students of U". For representing this soft-collection by his "best possible judgment" at the Identical Imaginary Environment **E1**, suppose that the decision maker Mr. Bose proposes two soft-computing sets independently:

(i)   the fuzzy set A_1 by his brain-1, and

(ii)   the IFS A_2 by his brain-2,

of the universe of discourse U as follows:

$A_1 = \{ (x_1,0.2), (x_2, 0.85), (x_3, 0.35), \ldots , (x_{50}, 0.1) \}$.   .... (1)

$A_2 = \{((x_1, (0.2,0.3)), ((x_2, (0.85,0.05)), ((x_3, (0.35,0.1)), \ldots , ((x_{50}, (0.1,0.4)) \}$.

.... (2)

It may be noted that in the above two soft sets A_1 and A_2, nothing has gone out of logic/reasoning contradicting any crisp or soft theory.

About the fulfillment of "Non-contradictory" Condition

It can be easily understood that neither brain-1 by proposing the soft-set A_1 does contradicts brain-2 nor brain-2 by proposing the soft-set A_2 does contradicts brain-1. Thus the "Non-contradictory" Condition is fulfilled.

With no loss of generality, consider the student x_1 and its membership values $\mu(x_1)$ in both A_1 and A_2 which is 0.2 here, given by Mr. Bose by his "best possible judgment". For proposing "collection of all the TALL students of U", the decision maker is same (Mr. Bose) for both the soft-computing sets A_1 and A_2. The decision maker Mr. Bose evaluated $\mu(x_1)$ for both the soft-computing sets A_1 and A_2 for the common element x_1. Since the decision maker is common (Mr. Bose) and both the soft-sets A_1 and A_2 as represented above by (1) and (2) are proposed at the identical environment **E1**, it is obvious that the value of $\mu(x_1)$ will be same for both A_1 and A_2, which is 0.2 here. It is because of the fact that both brain-1 and brain-2 have independently proposed here some amount of numerical grade on a common evaluation parameter which is the 'membership value' of a common element 'x_1', corresponding to a common soft collection which is "collection of all the TALL students of U" at the Identical Imaginary Environment **E1**.

Since the decision maker is common (the person Mr. Bose) who evaluated at the Identical Imaginary Environment **E1,** highest amount (100%) of consistency in the common evaluation parameter (here it is "membership value of x_1" for A_1 and A_2 both) must be there, and this consistency is absolutely logical in our soft-computing.

It is to be recollected that while proposing a membership value the psychological sequence of steps to a fuzzy theorist can not be other than the following order:
No.1 :   Total amount of $\mu(x)$ and $\nu(x)$ is 1, which is the ceiling amount and which is absolutely prefixed in the fuzzy set theory. This is one of the core hypothesis in fuzzy set theory; and then with this philosophy in mind,
No.2 :   the value $\mu(x)$ is proposed by the decision maker by his best
         possible judgment, and then
No.3 :   anyone can easily calculate the value of $\nu(x)$ whenever required,
using a crisp mathematical formula $\nu(x) = 1 - \mu(x)$.

The above sequence of steps is to be kept in mind because of the reason that No.2 is guided by No.1, and No.3 is guided by No.2 in fuzzy set theory. It is assumed in fuzzy set theory that other than the amount of $\mu(x)$, the rest amount fully goes to non-membership value by default. It is earlier mentioned that in our discussion throughout here we consider the classical complement of a fuzzy set defined by Zadeh [37], which is also used extensively used by Zadeh in all his pioneering

works [37-41] while dealing with linguistic hedges. Being motivated, most of the decision makers use this classical complement of a fuzzy set unless a problem of highly specialized domain is under consideration.

In the above example (Example-1), one must agree that the decision maker Mr. Bose at the same identical environment **E1** has surely delivered correct decisions by his best possible judgment for a common soft-defined set "the set of all TALL students". Nevertheless the two values of $v(x)$ for this common soft-defined set "the set of all TALL students" coming to be highly deviated as reproduced below :
(i) $v(x) = 0.3$ in "the set of all TALL students" A_2,   and
(ii) $v(x) = 0.8$ in "the set of all TALL students" A_1.

Due to cognition homogeneity (brain-1 $\equiv$ brain-2 $\equiv$ 'actual physical brain' of the decision maker) throughout the identical environment **E1**, there is no deviation in the value of $\mu(x)$ in A_1 and A_2.   But a serious amount of inconsistency (deviation) happening in the two values of $v(x)$ which is obviously not acceptable to the world scientists!

An interesting but very funny situation arises as a consequence, which is explained below.
Suppose that Mr. Bose is asked to submit to his boss the data/information about the value of $v(x)$.   For a moment now let consider the following three (3) questions (cases):-
Case(i) :   If Mr. Bose be a fuzzy theorist, not having any knowledge of intuitionistic fuzzy theory, then what will be his answer?
Case(ii) :   If Mr. Bose be an intuitionistic fuzzy theorist, not having any knowledge of fuzzy theory, then what will be his answer?
Case(iii) :   If Mr. Bose be an expert in both fuzzy theory and intuitionistic fuzzy
              theory, then what will be his answer?

Any of the above three cases could be the reality. But the soft information i.e. the value of $v(x)$, which is to be submitted to the boss, should be in a reasonable proximity among internally whatever be the case out of the above three. The value of $v(x)$ by any of the above three cases can not and should not differ by huge amount from case to case among these three cases!

Let us now analyze his possible answers corresponding to the above three cases:

For Case(i) :
His answer will be 0.8 and by giving this report to his boss he is fully satisfied as a fuzzy theorist (not having any knowledge of Intuitionistic Fuzzy Set theory) because he has submitted this report by his best possible judgment and he is having no self-contradiction in his sense too.

For Case(ii) :
His answer will be 0.3 and by giving this report to his boss he is fully satisfied as an intuitionistic fuzzy theorist (not having any knowledge of Fuzzy Set theory) because he has submitted this report by his best possible judgment and he is having no self-contradiction in his sense too.

For Case(iii) :
But what will be his answer if this Case(iii) is in real action? Which value of $\nu(x)$ he will report to his boss? Will he report the value 0.8? or Will he report the value 0.3? The decision maker Mr. Bose in this case is in a peculiar trouble although he is an Expert, highly educated, very intelligent and highly knowledgeable.

<u>Consequently an important question arises :</u>
What is the source of a seriously high amount of non-match in these two values of $\nu(x)$ which are proposed by a common decision maker (Mr. Bose)?

The deviation amount (i.e. the non-matching amount) is huge as it is compared to the measure of the unit interval [0,1]. The situation is <u>alarming</u> because of the reason that this kind of hidden inconsistency is not only to happen for only one element x but for many elements, in fact for all the elements, of the universe U!. For an hypothetical instance, if U has 100 elements than for all the 100 elements possibility is there to have huge non-match scenario. Quite obviously the immediate question arises : What is the source of such amount of error happening here? Undoubtedly, it is a major weakness of Fuzzy Set Theory in Soft-computing.

Surely, out of the two values of $\nu(x)$, the value $\nu(x) = 0.8$ obtained by fuzzy theory can not produce good result in the final conclusion of the decision problem. Because it carries a high amount of error. But the value $\nu(x) = 0.3$ can produce much better result in the final conclusion of the decision problem.

The error in the Fuzzy Set theory does not come from the part of $\mu(x)$, but comes from the part while calculating $\nu(x)$ i.e. from other side of the coin. In Intuitionistic Fuzzy Set theory $\nu(x)$ is evaluated independently as a part of decision making process (as an element of soft-computing). In Fuzzy Set theory $\nu(x)$ is evaluated not by any additional cost of decision, but just by using a simple crisp mathematical formula. The two steps to be followed are :
(i)   use the proposed value of $\mu(x)$, and
(ii)   use the crisp mathematical formula $\nu(x) = 1- \mu(x)$ to crisp-compute the value of $\nu(x)$ without applying any further decision process.

Quite obviously, the source of this error is :-
In Fuzzy Set Theory it is <u>blindly</u> assumed that other than the amount of $\mu(x)$, the rest amount fully goes to non-membership value by default.

The Questions are:
What is the guarantee that the rest amount fully goes to non-membership value?
By what logic or reasoning?

Another example is presented in the next subsection to unearth the hidden reality**.**

3.3.2 Fuzzy Theorist decides Not to apply Fuzzy Theory : Any such situation is possible in reality?

In this section we by an interesting hypothetical example that an expert 'Fuzzy Theorist' Dr. Sen decides not to apply Fuzzy Theory to solve his fuzzy problem! To realize the harsh impact of the hidden error residing inside fuzzy set theory in our real life situation, consider next another very common type of decision making problem as explained below.

Example-2

Consider an excellent oncologist Dr. Sen in a cancer research institute at Calcutta who is not only a famous oncologist but also a top Expert in 'Fuzzy Set Theory' too. The problem like : "whether the patient has fever?" or "whether it is a case of loose motion", etc. are crisp issues to a doctor. These are very precise cases of patients. But the problem of cancer diagnosis is no doubt an ill defined problem to the oncologists, although with the pursuance of rigorous amount of research work carried out around the world the subject now has been coming to a certain amount of control and grasp. The oncologist Dr. Sen, to the best of his knowledge and by his best possible judgment, may say that : "there is 20% chance that this patient x is a cancer patient". But in an immediate statement to the patient party, he strongly denies to give the statement that: "there is 80% chance that this patient x is not a cancer patient".

Although it is an imprecise problem and although Dr. Sen is a top Expert in fuzzy theory, nevertheless he denies to use fuzzy theory to give the above statement about 'Not a Cancer Patient'!. Can somebody guess the reason behind his denial? Actually while he denies to give this statement, his internal sense and awareness of his cognition system for a moment enforced him to ignore his knowledge and expertise in 'fuzzy set theory' or enforced him not to use 'fuzzy set theory'. Because some amount of chance is still residing in his knowledge-pocket which is presently an 'hesitation amount' or 'undecided amount' to him. This amount may happen to be alarming to him! And this is the reality which neither can be ignored nor can be altered, by any acceptable logic. The oncologist has no way at all to ignore his hesitation amount, even though he is one of the best oncologists in the populous city Calcutta and one excellent fuzzy theorist too. Consequently, for not having confidence and faith upon 'fuzzy theory', the oncologist (i.e. his cognition system) has decided for not applying his fuzzy set theory for doing any kind of soft-computing to process this medical diagnosis case even though it is fact that

(i)   it is an imprecise/ill-defined problem,
(ii)   he is a top expert in fuzzy set theory and
(iii)   he is an expert oncologist too.

Since the issue of diagnosis here is not a crisp issue, the oncologist is bound to use some kind of necessary soft-computing way for yielding an accurate and excellent decision. A simple mistake in his soft-computing may lead to a worst situation to the patient!

In that case it is not the fault of the oncologist Dr. Sen by virtue of his medical knowledge, but of the soft-computing theory being used by him. He understands quite surely that Fuzzy Theory should not be applied in this case. But at the same time, the problem is undoubtedly an imprecise problem and according to the existing literature of fuzzy set theory the problem is a fuzzy decision problem. Consequently the general question then arises: Is 'Fuzzy Theory' Always Good for Soft-Computing?

We present below another case which is also interesting in quest of hidden reality, and this case has been possible for presentation and explanation by the imaginary model of brain-1 and brain-2 introduced in Section earlier.

3.3.3 'Fuzzy Theory' contradicts itself!

We present an interesting example below (Example-3) which shows that 'Fuzzy Theory' contradicts itself (!) in some cases.

We remind here that we are in the process of "**achieving hidden reality by imagination**"; but our imagination will not make any element of contradiction with the common sense of human being, logic, common reasoning, science, and mathematics.

Example-3

Let us carefully refer to the statement of Example-1 presented earlier. But imagine, for the time being, that the decision maker Mr. Bose never proposed any soft-set representing the "collection of all the TALL students of the universe U", i.e. he never proposed the soft sets A_1 and A_2 representing the "collection of all the TALL students of the universe U".   It is even better to imagine for the time being that the decision maker Mr. Bose never even happened earlier in his life to deal with the "collection of all the TALL students of the universe U" so far.
Instead of it, imagine now that there was a demand of information from the same decision maker Mr. Bose for a different issue. Let us reproduce some portion of the statement of the Example-1 first of all.

Consider a decision maker Mr. Bose who is an expert in both 'Fuzzy Set theory'

and 'Intuitionistic Fuzzy Set theory'. Suppose that there are 50 students in Class-V in Calcutta St. Paul's School. Let U be the set of all these 50 students, say $U = \{ x_1, x_2, x_3, \ldots, x_{50} \}$.

Consider the Identical Imaginary Environment entitled by **'E1'** as prescribed below (by the three conditions as defined earlier):

> *i(i):   The decision maker is Mr. Bose.*
> *i(ii) : Time is now 4.30 PM on 5th September'2018, and*
> *i(iii) : at the office of the Principal of Calcutta St. Paul's School.*

(Throughout in this paper, whenever we use identical environment **E1**, we shall mean only the above environment constituted by the above three conditions).

We suppose that for execution of a statistical project, this decision maker (Mr. Bose) wants to consider a soft-collection of objects of U which is the "the collection of all '<u>Not TALL</u>' students". For representing this soft-collection by his "best possible judgment" at the Identical Imaginary Environment **E1**, suppose that the decision maker Mr. Bose proposes three soft-computing sets independently:
(i)   the fuzzy set B_1 (by his brain-1 and brain-2 independently), and
(ii)   the IFS B_2 by his brain-2,
of the universe of discourse U.

Suppose that the soft-set B_2 proposed by Mr. Bose (by brain-2) is given by :
$B_2 = \{ ((x_1, (0.3,.2)), ((x_2, (.05,.85)), ((x_3, (.1, .35)), \ldots, ((x_{50}, (.4, .1)) \}$.

Mr. Bose has proposed "the collection of all '<u>Not TALL</u>' students" <u>directly</u> by his best possible judgment, not by calculating/computing from any other soft-computing set. In the eyes of the readers of this paper, it is quite logical and quite certain that the soft-sets B_1 and B_2 proposed by Mr. Bose can not be inconsistent with the soft-sets A_1 and A_2 of Example-1 (where A_1 and A_2 are also fully inter-consistent between themselves mutually).   It is not contradictory to any truth by any reasoning, not contradictory to common sense of human being, not contradictory to any logic, science or mathematics.

Now consider the soft-set B_1 proposed by brain-1 and brain-2 independently. For the sake of our analysis, let us denote the soft-set B_1 proposed by brain-1 by the notation $B_{1.1}$ and the soft-set B_1 proposed by brain-2 by the notation $B_{1.2}$.

Suppose that most appropriately and most correctly brain-1 has proposed the soft-set B_1 in the form of a fuzzy set as below by its best possible judgment:
$B_{1.1} = \{ (x_1, 0.8), (x_2, .15), (x_3, .65), \ldots, (x_{50}, .9) \}$,
whereas most appropriately and most correctly brain-2 has proposed the soft-set B_1 in the form of a fuzzy set as below by its best possible judgment:

$B_{1.2}$ = { $(x_1, 0.3)$, $(x_2, .05)$, $(x_3, .1)$, ……, $(x_{50}, .4)$ }.

The words "most appropriately and most correctly" is used here in the sense that brain-1/brain-2 does not have in their cognition system any more possibility or any more capability for a further accurate or better judgment by any further amount. To the best of his (brain-1/brain-2) knowledge and by his best possible judgment the fuzzy set ($B_{1.1}$ / $B_{1.2}$) is proposed "most appropriately and most correctly".
However at the background the readers of this article (not surely Mr. Bose) can correlate $B_{1.1}$ and $B_{1.2}$ with the Example-1.

But am alarming situation has arisen now automatically, almost in a hidden way as mentioned and explained below.

Alarming Situation :-
The brain-1 and brain-2 happen to be mutually Contradictory! Why?

Now we will justify that while proposing the soft-set B_1 in the form of a fuzzy set ($B_{1.1}$ and $B_{1.2}$), the two brains brain-1 and brain-2 are not fulfilling the "Non-contradictory" Condition as prescribed in earlier subsection although they proposed most appropriately and most correctly by their best possible judgment ! This is a serious matter, an alarming situation, and needs to be analyzed to unearth the root cause behind so. We already mentioned our philosophy that we will achieve the hidden reality by imagination. To explore the root-cause, we present the proceedings of an Interview Board below. The exchange of dialogues presented here is purely hypothetical story-based.

Interview Board

With no loss of generality and with no loss of any amount of sense of this article, consider below the proceedings of an 'Interview Board' of which the **Chairman** is the Actual Physical (biological) Brain of Mr. Bose with few expert-logicians as Members of the Board. There are only two candidates to be interviewed today independently who are : brain-1 and brain-2. But none of brain-1 and brain-2 is aware of other's presence or existence for this important interview.

First of all the candidate brain-1 is called inside for interview. The brain-1 is then sitting before the Board Members facing the Chairman of the Interview Board.

(1) Proceedings of the session of Interviewing the candidate 'brain-1'

Chairman : Tell me Mr. brain-1, how you have proposed the fuzzy set $B_{1.1}$

during the Identical Imaginary Environment **E1**?

Candidate (brain-1) **:** Sir, I have proposed the fuzzy set $B_{1.1}$ by my best possible judgment upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" over the universe U. For example: corresponding to the element x_1 here I have proposed the 'membership value' equal to 0.8 for x_1 upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by my best possible judgment; similarly, corresponding to the element x_2 here I have proposed the 'membership value' equal to 0.15 for x_2 upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by my best possible judgment, and so on.

Chairman : OK. Thank you. Your interview is over.
    (*the candidate brain-1 left the Board room then*).

Confidential Discussion of Board Members :
The Chairman then initiated a confidential discussion among the Board members about the performance, about the statements delivered by the candidate brain-1 in today's interview. The Chairman announced that from some 'highly reliable source' he is having the information about the soft-sets A_1 and A_2 which are excellent and very rich asset today for everybody here of the esteemed members. This prior information disclosed by the Chairman to the members are that :
$A_1 = \{ (x_1, 0.2), (x_2, 0.85), (x_3, 0.35), \ldots, (x_{50}, 0.1) \}$,   and
$A_2 = \{ ((x_1, (0.2, 0.3)), ((x_2, (0.85, 0.05)), ((x_3, (0.35, 0.1)), \ldots, ((x_{50}, (0.1, 0.4)) \}$.

The Chairman also pointed out by saying that : Dear Members, please note it very carefully that A_1 and A_2 are fully consistent, fully non-contradictory with each other. (*However at the background the readers of this article (not surely Mr. Bose) can correlate with the Example-1*).

The members of the Board noted the above information very carefully as announced by the Chairman that these are from a 'highly reliable source' and are excellent information.
The Chairman says : Dear hon'ble members, you may please correlate that the fuzzy set $B_{1.1}$ proposed by this candidate brain-1 is <u>fully consistent</u> with the above information of A_1 and A_2 both by using fuzzy set theory. All the Members correlated the data, and finally agreed that the candidate brain-1 has very appropriately proposed the fuzzy set $B_{1.1}$ (which is consequently to be regarded as the fuzzy set B_1 proposed by brain-1).

Conclusion of the Board:
It is an excellent and highly acceptable fuzzy set proposed by Mr. Bose (brain-1) upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by his best possible judgment. Let us award him full marks (10/10), ten out of ten.

Next *the candidate brain-2 is called inside for interview.* *The brain-2 is then sitting before the Board Members facing the Chairman of the Interview Board.*

(2) Proceedings of the session of Interviewing the candidate 'brain-2'

Chairman :   Tell me Mr. brain-2, how you have proposed the fuzzy set $B_{1.2}$ during the Identical Imaginary Environment **E1**?

Candidate (brain-2) **:** Sir, I have proposed the fuzzy set $B_{1.2}$ by my best possible intuitionistic judgment upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" over the universe U. As the demand here is to propose a fuzzy set, I have done so. I have done so by my best possible intuitionistic judgment upon the Zadeh Phrase Λ without proposing any intuitionistic fuzzy set. For example: corresponding to the element x_1 here I have proposed the 'membership value' equal to 0.3 for x_1 upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by my best possible judgment, but I have not mentioned the 'non-membership value' upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" because it is not required as per demand; similarly, corresponding to the element x_2 here I have proposed the 'membership value' equal to 0.05 for x_2 upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by my best possible judgment, and so on.

Chairman : OK. Thank you. Your interview is over.
              (*the candidate brain-2 left the Board room then*).

Confidential Discussion of Board Members :
The Chairman then initiated a confidential discussion among the Board members about the performance, about the statements delivered by the candidate brain-2 in today's interview.

The Chairman says : Dear hon'ble members, you may please correlate that the fuzzy set $B_{1.2}$ proposed by this candidate brain-2 is fully consistent with the information of A_2. All the Members correlated the data, and finally agreed that the candidate brain-2 has very appropriately proposed the fuzzy set $B_{1.2}$ (which is consequently to be regarded as the fuzzy set B_1 but proposed by brain-2).

Conclusion of the Board :
It is an excellent and highly acceptable fuzzy set proposed by Mr. Bose (brain-2) upon the Zadeh Phrase $\Lambda \equiv$ "Not Tall" by his best possible judgment. Let us award him full marks (10/10), ten out of ten.

The interview session is over. The Chairman has submitted the following 'evaluation report' to the appropriate higher authority:-

Evaluation Report

Serial No.	Candidates Identity	Marks awarded (out of Total 10 marks)	Comment (about the performance)
1	brain-1	10	Excellent
2	brain-2	10	Excellent

Let us do now an overall analysis of the results, including those of the interview performance Report. And then we arrive at an interesting conclusion below.

An Analysis :
To begin the analysis, consider the following three outcomes :

No.1
In Example-1 the brain-1 proposed the fuzzy set A_1 and the brain-2 proposed the IFS A_2, where both were A_1 and A_2 are mutually consistent.

No.2
In the above Example-3 the brain-2 proposed B_2 which is consistent with A_2.

No.3
The brain-1 proposed $B_{1.1}$ consistent with A_1 and A_2 both.
The brain-2 proposed $B_{1.2}$ consistent with A_2.

Then quite naturally the following questions now arise :-
Q(i) : Why the two fuzzy sets $B_{1.1}$ and $B_{1.2}$ are mutually inconsistent by a so huge amount?
Q(ii) : Then, what the "actual physical brain" of Mr. Bose will propose as B_1? Will Mr.Bose propose $B_{1.1}$ as B_1 or $B_{1.2}$ as B_1?

Q(iii) : In fact both $B_{1.1}$ or $B_{1.2}$ are supposed to be acceptable to Mr.Bose for proposing B_1 as both are decisions of the same brain under the Identical Imaginary Environment E1. Then why this major amount of <u>contradiction</u> has occurred? Will the "actual physical brain" of Mr. Bose hang up and stop to propose the fuzzy set B_1?

Conclusion :
Thus the fuzzy set $B_{1.1}$, because of the reason that it has been developed using the 'fuzzy set theory', happens here to lead to self-contradiction causing a serious amount of inconsistency to the in-built data!
This example (Example-3) clearly shows that 'Fuzzy Set Theory' may contradict even itself in some cases in a very hidden way!

3.3.4 Occurrence of upto 220% Error

In this section we present an example showing an 'Occurrence of upto 220% Error' which has happened on applying Fuzzy Theory in a small size Decision Problem.
In the recent literatures in [15,16] it is justified in length that Fuzzy Set theory is not appropriate for large size decision problems. We present here an example showing an occurrence of upto 220% error on applying fuzzy theory by a fuzzy expert, even in a small size decision problem.

For this, let us think about the hypothetical cases presented in Example-4 below for an interesting analysis.

Example-4

Consider an Identical Imaginary Environment **E2** at the cancer hospital at Calcutta of the famous oncologist Dr. Sen. Suppose that the oncologist Dr. Sen is a top Expert in both 'Fuzzy Set theory' and 'Intuitionistic Fuzzy Set theory' too. Consider the universe of discourse U which is a given collection of 100 medically ill people in his hospital. Dr. Sen here considers the collection **C** of all suspected cancer patients in U (under investigation).

Consider any patient x of U.

Suppose that by his "best possible judgment", Dr. Sen at the Identical Imaginary Environment **E2** considers the patient x as below in two independently proposed cases:

Case(i):   there is 20% chance that this patient is a cancer patient, and consequently Dr. Sen (brain-1) proposes that $\mu(x) = 0.20$ to represent the soft collection **C**, (fuzzy set theory is used by Dr. Sen).

Case(ii):   Dr. Sen (brain-2) proposes the degree (0.20, 0.25) using intuitionistic fuzzy theory, i.e. Dr. Sen proposes $\mu(x) = 0.20$, and $v(x) = 0.25$ to represent the soft collection **C**.

It is very important to note that brain-1 does not know what amount brain-2 proposes and brain-2 does not know what amount brain-1 proposes as a membership value. It is so because of the fact that both brain-1 and brain-2 are same but independent as per their construction philosophy in the earlier Section. In fact brain-1 even does not know the existence of brain-2 and vice-versa. Thus they can never happen to contradict each other. All results delivered by them must be 100% consistent.

Since the decision makers for both the above cases are an identical brain at the Identical Imaginary Environment **E2,** consistency in the common evaluation parameter (here it is "membership value") must be there. In both the cases here the value of $\mu(x)$ is 0.20, the consistency which is highly obvious. But see that at the specified instant of time corresponding to this Identical Imaginary Environment **E2**, the amount of mistake happened in case(i) above due to Fuzzy theory could be upto as high as **220%**! Because the fuzzy evaluation grade about "Not Cancer" for this patient x is 0.80 in case (i).

Consequently the deviation amount of 0.80-0.25 = 0.55 has led to a possible limit of 220% error(!), where .80 is the non-membership value in the fuzzy set and 0.25 is the non-membership value in the intuitionistic fuzzy set corresponding to the soft collection **C**. The directly but independently proposed value 0.25 of $v(x)$ by the best possible judgement is surely the best available and most reliable value of it, assuming that

(i)   The decision maker Dr. Sen is an excellent oncologist, and
(ii)   Dr. Sen is an expert in Fuzzy Theory, and
(iii)   Dr. Sen is an expert in Intuitionistic Fuzzy Theory.

In case Dr. Sen does not know intuitionistic fuzzy theory, and applies his fuzzy theory for initiating treatment for this patient as in case(i) above, it will yield a disaster situation to the treatment of this patient!. It is not the fault of the oncologist Dr. Sen, but it is sourced from the weakness in the constructive model of fuzzy set theory.

Any soft-computing tool is supposed to be a supporting and complementary tool to the decision maker, not a competitive tool. At worst it could be a non-applicable tool to a decision maker for the particular problem under consideration. But, being well regarded as an applicable tool, it is surely not expected to inject upto 220% error in a hidden way to all the soft-computing algorithms developed by the innocent decision maker for solving his fuzzy problem!. So much error can not be acceptable to any expert or scientist.

Out of the two non-membership values 0.80 and 0.25, the later value 0.25 is acceptable to be appropriate as it is directly but independently proposed by the decision maker by his best possible judgment. The other value 0.80 is a crisp-computed value using the crisp mathematical function $N(\mu(x))$ for which the input argument $\mu(x)$ is reliable because of the fact that it is directly proposed by the decision maker by his best possible judgment, but we comment that the function 'N' is a doubtful model to us for the purpose it is adopted in fuzzy set theory. It is to be noted that $\mu(x)$ and $v(x)$ are two sides of the coin, both the sides being extensively used in soft-computing applications and results during the last five decades, including Zadeh (see [28, 29, 34, 35, 38-41]).

Mathematically the maximum possible error amount in this Example-4 is equal to the deviation amount of $\{1- \mu(x)\}$ from $v(x)$, which is thus $[\{1- \mu(x)\} - v(x)] = 0.55$ where $v(x)$ is the directly proposed non-membership value. This amount 0.55 does also happen to be equal to the value of the hesitation amount $\pi(x)$ of Atanassov theory. Consequently, the percentage of this kind of error in fuzzy theory is equal to

$$\left[\frac{\pi(x)}{\vartheta(x)} \times 100\right] \%, \quad \text{where } \vartheta(x) \neq 0.$$

These errors evolve due to NO fault of the decision makers, due to NO fault of the nature of the fuzzy problem under consideration, but due to some fault in some part of the theory model itself. Consequently, for the case of fuzzy set theory let us call this type of error by the term **'soft error'** instead of calling it 'error'. For a given decision maker, the soft error is in general different for different elements x. Similarly, for a given element x, the soft error is in general different for different decision makers.

But the extremely alarming issue is that this decision problem of medical science in Example-4 is not just a particular case or an exceptional case, or a rare situation!. Almost all the decision problems of each and every fields are of this type. It is the ground reality in real life situation in each and every ill-defined decision making problem being faced everyday by human beings on this earth. It is the ground reality in all the decision making problems of our daily life!. None can ignore this fact, even if he be a top expert in fuzzy set theory. The hypothesis : "hesitation can not be ignored", is a mandatory and a quite logical situation to any decision maker whenever he uses any kind of soft-computing set theory for the sake of much better treatment of the fuzziness existing in the concerned ill-defined problem. However, in some cases the hesitation may be very low, and could be nil for one or few or many objects. One can suggest a different nomenclature or name for this 'hesitation' part like **:** residue part, undecided part, remaining part, non-deterministic part, etc., but **can not ignore** whatever be the soft-computing set model used for solving any ill-defined decision problem in real life.

The soft error (which could be upto 220% for this element x here in Example-4) gets <u>multiplied</u> in Fuzzy Set theory because of the fact that there are n elements x_1, x_2, x_3, … , x_n of the universe U. The situation gets further worsened and manifolded if the decision problem involves many universes U_1, U_2, U_3, ..., U_r. Consequently the final results of soft-computing of the decision problem, if done using the fuzzy set theory, will remain far away in a hidden way from the unknown destination of the reality. The most unfortunate and shocking fact is that in such cases the decision makers remain completely in dark, rather remain in full confidence being fuzzy experts.

This type of error in the fuzzy set theory will propagate to all the next generation fuzzy sets like modified fuzzy sets [38-41] as hereditary carried germs. For example, if a decision maker wants to soft-compute modified fuzzy sets like CON or DIL of a fuzzy set in his soft-computing steps or algorithms (consider the case of Example-1, where the decision maker may want to soft-compute "Collection of 'Very TALL' students", "Collection of 'Very Very TALL' students", etc.), then by virtue of hereditary propagation of this type of error the final decision results will be extremely affected.

4 Special Cases of Intuitionistic Fuzzy Set

To understand about the occurrences of error during the soft-computing exercised by the decision maker (because of no fault of himself), let us attempt to view the crisp sets and fuzzy sets as special cases of intuitionistic fuzzy sets (see Figure-5, 6, 7).

4.1 Viewing a Crisp Set

In crisp set theory there is no scope of such error at all. If we view a crisp set as a special case of intuitionistic fuzzy set then the following Figures 5 reveal the fact :
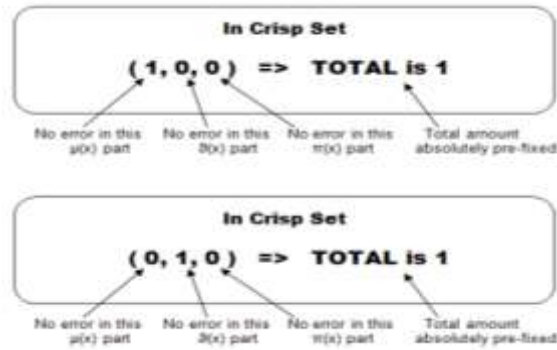

Fig. 5. a crisp set viewed as an intuitionistic fuzzy set

In case of a crisp set, the complete amount (100%) goes genuinely to either $\mu(x)$ or $\nu(x)$ for each and every element of the universe without any soft-decision but by a straightforward crisp decision. As 100% goes genuinely to either $\mu(x)$ or $\nu(x)$, there is no scope of error to be committed. Because the rest amount is absolutely 0% for each and every element x across the complete duration, and any unknown or hidden percentage of this 0 amount is mathematically equal to 0 again.
Since $\pi(x) = 0$, one could use our proposed calculator

$$\left[\frac{\pi(x)}{\vartheta(x)} \times 100 \right] \%, \quad \text{where } \vartheta(x) \neq 0,$$

to calculate the error percentage in a crisp set.

There is no scope of any alternative values for $\mu(x)$ or $\nu(x)$ even if the decision makers are different and even if the identical environment is followed (or not followed). Every element in a crisp set is equivalent to $(x, (1,0))$ or $(x, (0,1))$ if its grade value is viewed as a particular case of intuitionistic fuzzy philosophy of Atanassov. But these grades are not available freely, neither even at the cost of zero amount of time (as discussed in [15]). These grades are always the output of the execution of the CIFS processor at the kernel of the brain [15] of the decision maker.

4.2 Viewing a Fuzzy Set

It is most logical to agree that as far as the amount of $\mu(x)$ is concerned, there can not happen any inconsistency in the soft sets A_1 and A_2 of Example-1. Similarly as far as $\mu(x)$ is concerned there can not be any inconsistency in the soft sets B_2

and $B_{1.2}$ too in Example-3. But for the fuzzy set $B_{1.1}$ in Example-3 the consistency with the soft set B_2 can never be expected because of the fact that the soft set $B_{1.1}$ is proposed by soft-computing but applying the mathematical theory of 'Fuzzy Set'. The source of this kind of error is hidden in the theory itself, neither in the decision making process of the decision maker nor in the intellectual capability of the decision maker, but keeping the decision maker in a complete dark.
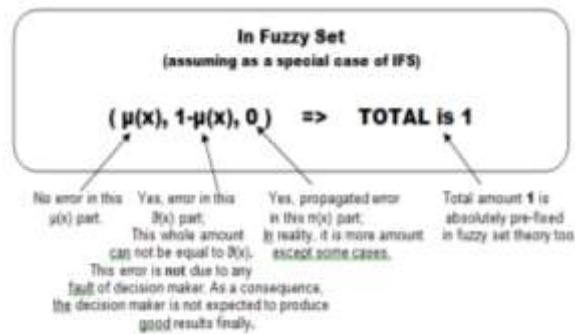

Fig. 6.   a fuzzy set viewed as an intuitionistic fuzzy set
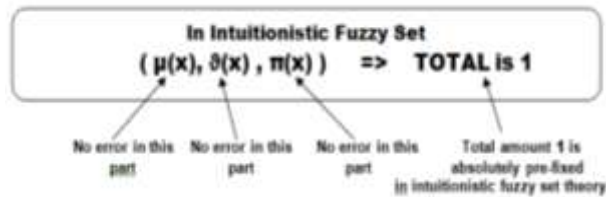
Fig. 7.   an intuitionistic fuzzy set

4.3 Impossible Types of Initialization

In the Theory of CIFS established in [15], it is explained that the following are impossible types of initialization:

Type-(i) : $m(x, 0) = 0$ and $n(x, 0) = 1$ at time $t = 0$,   but
                   finally converging at $m(x, T) = \mu(x)$ and $n(x, T) = 1 - \mu(x)$
   after time $t = T$ (>0).
Type-(ii) : $m(x, 0) = 1$ and $n(x, 0) = 0$ at time $t = 0$,   but
                   finally converging at $m(x, T) = \mu(x)$ and $n(x, T) = 1 - \mu(x)$
                   after time $t = T$ (>0).
Type-(iii) : $m(x, 0) = k$ and $n(x, 0) = 1 - k$,   at time $t = 0$ where k is some initial scalar constant, and finally converging at $m(x, T) = \mu(x)$ and $n(x, T) = 1 - \mu(x)$ after time $t = T$ (>0).

We shall now analyze the following fact.

Initialization (of the assignments at time t = 0 as introduced in [15]) is unique and absolutely unique for all the decision makers irrespective of any Soft-computing Set Theory or crisp theory used by him by his own choice.

For this, let us plot both the curves m = m(x,t) and n = n(x,t) on the same plane as shown in Figure-8 and Figure-9. Let the curve OPM is m = m(x,t) and the curve OQN is n = n(x,t). For an arbitrary element x of the universe U, these two curves will be either as shown in Figure-8 or as shown in Figure 9, exceptional case is if M and N are coincident points. But the case shown in Figure-10 is am impossible case in the Theory of CIFS, as it can never happen in the brain in reality under any circumstances.
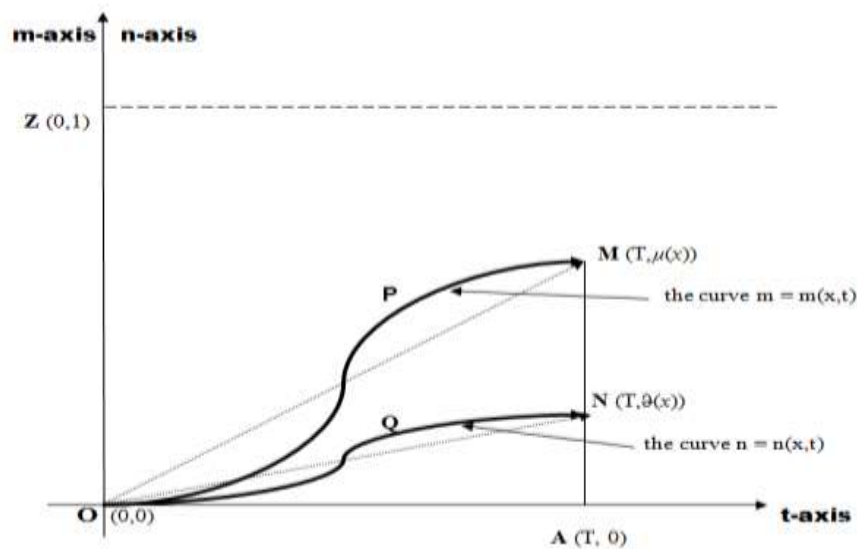
Fig. 8.   m-curve and n-curve on a common plane (m dominates n)
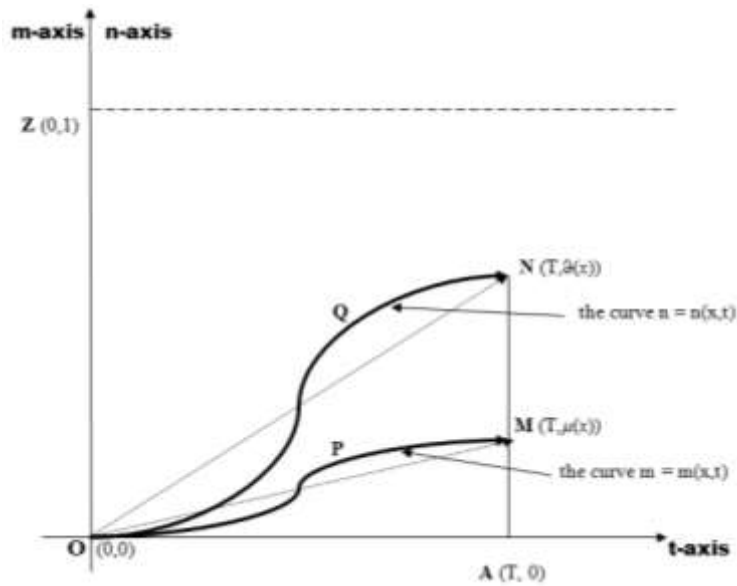
Fig. 9. m-curve and n-curve on a common plane (n dominates m)
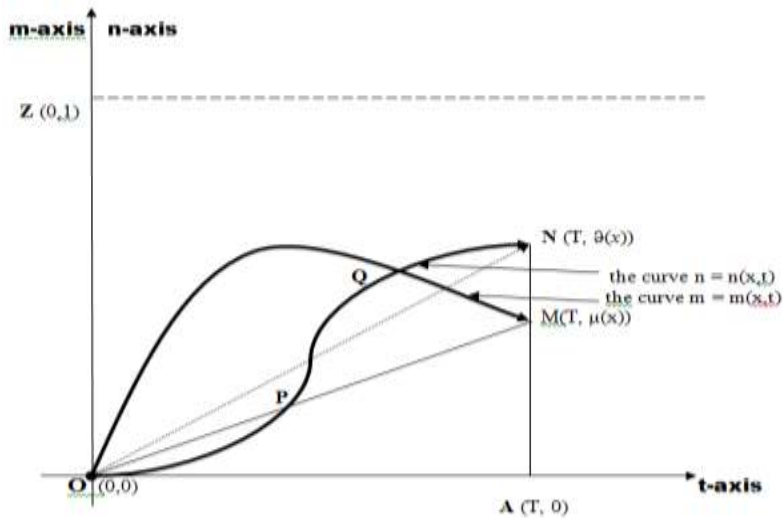
Fig. 10. An impossible case of m-curve and n-curve (none dominates the other)

Let S1 denoted the length of the curve OPM (from O to M), and S2 denotes the length of the curve OQN (from O to N). It is obvious that for an arbitrary x and for an arbitrary decision maker, be it the case of Figure-8 or Figure-9, in general we have

$$S1 \neq S2 \quad \text{.. (4)}$$

ignoring the cases 'S1 = S2' which is an exceptional case, not a general case.

Now let us see what kind of contradiction happens if we suppose that the above

initializations Type-(i), Type-(ii) and Type-(iii) are possible. If possible, it means that the function h(x,t) is a zero function for all these three Types.
Suppose that APT(x) = T.
Atanassov constraint for a given element x at any variable time t is

$$m(x,t) \;+\; n(x,t) \;+\; h(x,t) \;=\; 1.$$

Differentiating w.r.t time t, we see that

$$\frac{d}{dt}\,m(x,t) \;+\; \frac{d}{dt}\,n(x,t) \;=\; 0 \quad \text{as h(x,t) is a zero function in all the three Types.}$$

i.e.
$$\frac{dm}{dt} + \frac{dn}{dt} \;=\; 0$$

or,
$$1 + \left(\frac{dm}{dt}\right)^2 \;=\; 1 + \left(\frac{dn}{dt}\right)^2$$

or,
$$\int_0^T \left[1+\left(\frac{dm}{dt}\right)^2\right]^{\frac{1}{2}} dt \;=\; \int_0^T \left[1+\left(\frac{dn}{dt}\right)^2\right]^{\frac{1}{2}} dt$$

or, S1 = S2, which contradicts the result (4).

Hence it is now established that our initial supposition : "the above initializations Type-(i), Type-(ii) and Type-(iii) are possible" is wrong.

However, at any particular instant, say at t = 0, the Atanassov Constraint m(x,0) + n(x,0) + 0 = 1 itself contradicts, and thus justifies the result directly.

Hence it is justified that whatever be the decision problem under consideration, whatever be the soft computing set theory under consideration of the decision maker by his own choice out of several options and alternatives, the initialization at the kernel of the cognitive system is always unique which is the Atanassov Initialization only, can not be any alternative as discussed in details in [15].

Thus, the initialization in CIFS is unique and absolutely unique for all the decision makers by default, irrespective of any Soft-computing Set Theory or crisp theory used by him by his own choice. This was a fact in the stone age of the earth too, and will continue to remain as a fact forever.

4.4 Necessary Conditions For Using 'Fuzzy Theory' in Soft-computing

The two parameters "$\mu(x)$ and $\nu(x)$" are "Head and Tail" of the philosophy-coin of fuzzy set theory [28, 29, 34-41]. During last five decades the fuzzy decision

makers have been fluently using both of them during the course of soft-computing while solving fuzzy problems in various fields. But it is now unearthed that **at least one** of the following three necessary conditions is to be mandatorily satisfied by every fuzzy decision maker as an eligibility condition for using 'Fuzzy Set Theory' in his soft-computing.

Necessary Eligibility Conditions (at least one) :

Condition-1 : The decision maker must be self-confident that he will not need to use $v(x)$ during his soft-computing process till end, for each and every element x of all the universes involved in the concerned decision problem.

Condition-2 : The decision maker must be self-confident that for each and every element x of all the universes involved in the concerned decision problem, the hesitation amount $\pi(x)$ is NIL while proposing the membership value.

Condition-3 : The decision maker must be self-confident that he will not need to use $v(x)$ during his soft-computing process till end for each and every element x of all the universes involved in the concerned decision problem, except for few elements $x_k, x_i, x_j, x_l, \ldots\ldots$ for each of which he is self-confident that the hesitation amount $\pi(x)$ is NIL.

If at least one of these three necessary conditions is not fulfilled, **the decision maker should not use 'Fuzzy Set Theory' in his soft-computing process to solve any fuzzy problem**, be it a large size or a small size problem. Not fulfilling the condition even for a single element may lead to a 'serious amount of errors' in the final results, the errors which will occur silently in a hidden way keeping the decision maker in dark. This is a hidden truth, so far <u>ignored</u> by the researchers.

However, it may be noted that for any decision maker using classical set theory, the Condition-2 is always fulfilled by default. Consequently, for using crisp set theory to solve a well precise problem, the above proposed necessary eligible conditions need not be checked.


5 Conclusion

Soft Computing (SC) represents a significant paradigm shift in the aims of computing, which reflects the fact that the human mind, unlike present day computers, possesses a remarkable ability to store and process information which is pervasively imprecise, uncertain and lacking in categoricity. Soft-computing mimics the remarkable human ability to make rational decisions in an environment of uncertainty and impreciseness. The 'Fuzzy Set Theory' is presently being regarded as one among the most powerful existing soft-computing theories to the decision makers. But the rigorous analysis made in this paper with several examples justifies that in many cases if Fuzzy Theory gives sometimes excellent results, then it is by chance only. Instead, if the same decision maker

applies the IFS theory then he will obviously get at least the equally excellent results corresponding to any decision making problems, not by any chance but by its model strength (assuming that the decision maker is an expert in Intuitionistic Fuzzy Set Theory too). Consequently, as justified in length in this paper, the IFS theory is free of that kind of risk as far as soft-computing is concerned, unlike Fuzzy Theory.

In [15,16] it is observed that Fuzzy Theory is not an appropriate tool for any decision maker to solve large size problems. By a 'decision maker' we mean a human being or a living animal or a bird or any living thing which has physical brain (ignoring the machines, robots, or software which have artificial intelligence). In the Theory of CIFS in [15] it is observed that in ground reality, for a decision maker by the best possible processing in his cognition system, the data "$\pi(x) = 0$" can not be true in general for all and across all the elements x of the universe X while proposing any model of soft set A of the universe X (as justified further in Proposition 1 and 2). Even if it be true for one or few or many elements, it is illogical to believe that it is true for all and across all the elements of the universe X while proposing an IFS A. The situation is further worsened if there are many universes in a decision making problem, which is in fact a frequent and very common phenomenon.

It is fact that any real life soft-computing problem on this earth usually occurs involving more than one universe. There could be r number of universes viz. X_1, X_2, ……, X_r in a given problem under consideration by a decision maker for soft-computing. And in that case it is extremely illogical to believe that '$\pi(x) = 0$' is true for all the elements of all the r universes.   Neither any real logical system(s) nor the Nature can force a decision maker (human being or animal or bird or any living object having a physical brain) either to stand strictly at the decision: "$\pi(x) = 0$ for every x of every X", or "to abandon his decision process otherwise".

It is a very rare case that for a fuzzy set proposed by a fuzzy expert the CIFS at the kernel of the cognition system, while converges finally, will output at the outermost layer of the cognition system the result $\pi(x) = 0$ for every x of X. A fuzzy expert has no alternative way but to impose $\pi(x) = 0$ for each and every x for each and every universe of the corresponding decision problem. And retaining this constraint he proposes the value of $\mu(x)$ which is the true output of the CIFS execution (see Figure 11).
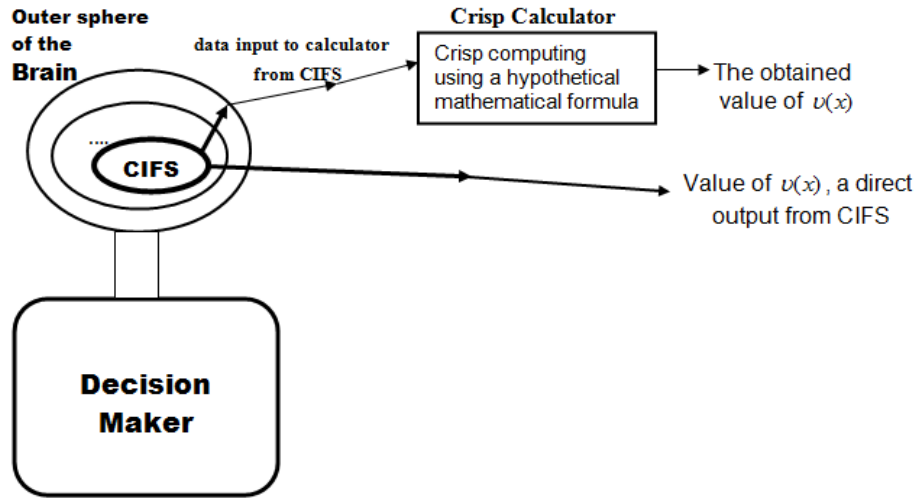
Fig. 11   The birth origin of $\vartheta(x)$ in two different soft-computing set theories :
'Fuzzy Set Theory' and 'Intuitionistic Fuzzy Set Theory'.

The decision maker is a fuzzy decision maker, who is using his fuzzy set theory by his own choice. But the execution of the CIFS at the innermost kernel is not by any choice or by virtue of any knowledge of the decision maker be it a human being or a living animal or a bird or any living thing which has physical brain. The CIFS processor has no link with the quality or quantity of knowledge of the decision maker, or with the areas covering the acquired knowledge of the decision maker   or with the intellectual capability of the decision maker. It can never be a feasible case that a fuzzy decision maker can ignore $\pi(x)$ for all the elements x because of the fact that the CIFS can not sponsor this result from his brain (excluding crisp cases or few special cases).

The Theory of CIFS in [15] says that corresponding to every element x, any decision process for deciding the membership value $\mu(x)$ for any soft set theory starts with Atanassov's initialization $< 0, 0, 1 >$ and then after certain amount of time T (called by Atanassov Processing Time) it converges to the trio $<\mu(x), v(x), \pi(x)>$ without any further updation of the AT functions. In general in most of the cases, $\pi(x) \neq$ NIL. Even if $\pi(x) =$ NIL for one element x, it is a very rare situation that $\pi(x)$ will be Nil for all the elements x of X in the soft set proposed by the decision maker. The situation is further faint if there are more than one universe in the concerned ill-defined decision problem under consideration. It is a very very special case, and the following justification will support further to this hypothesis :-

Suppose that, to solve an ill-defined problem we have to consider 10 number of universes, while in each universe there is at least 100 elements. Suppose that, to solve this problem an intelligent decision maker (say, a fuzzy human expert)

needs to consider more than 10 fuzzy sets of each universe. Clearly, he has to propose membership values by his best possible judgment for more than 10,000 elements. For deciding the membership value for each of these 10,000 elements, the cognition system of the decision maker by default begins with Atanassov's initialization $< 0, 0, 1 >$ and after certain amount of time converges to the decision about $\mu(x)$ for the element. Can we presume that for each of these 10,000 elements, his convergence process starting from the Atanassov's initialization trio $< 0, 0, 1 >$ will stop at the trio $<\mu(x), \nu(x), 0>$ with $\pi(x) = 0$ for each and every x**?** Can we presume that there is not a single element x out of 10,000 elements for which the convergence process ends with some non-zero amount of $\pi(x)$? In [15,16] it is justified that it may not be appropriate to use fuzzy theory if the problem under study involves the estimation of membership values for <u>large</u> number of elements of one or more universes. For instance, the populations in Big Data Statistics [18], which are in maximum cases region [11,12] based Algebraic Statistics (be it R-Statistics or NR-Statistics [18]) are all about big data expanding in 4Vs very fast; and decision analysis in many such cases involve the application of various soft-computing tools where it is most important to have excellent results only. In our everyday life, every human being (or animal or bird etc.) plays the role of a decision maker at every moment of time (ignoring his sleeping period at night). He is compelled to decide every day for large number of imprecise problems of various nature. But one can not be expected to be always an excellent and outstanding decision maker for all the unknown (or known), unpredictable, homogeneous/heterogeneous, precise/ imprecise real time decision problems he faces every day without 'any element of hesitation' at all. Consequently, it is well justified in [15,16] that for a large or moderate size soft computing problem, it may not be appropriate to use the tool 'Fuzzy Theory' in order to get excellent results. In this paper the author locates the weakness of fuzzy theory in a further dimension, which is not caused due to the part $\mu(x)$ of its theoretic model but due to the other part $\nu(x)$ of the coin. Note that this type of hesitation element and hence the issue of 'soft error' is not a topic in the Theory of Probability. If one throws a biased or unbiased dice, and and if an 'even integer' does not appear then certainly a NOT(even integer) will appear! In this case NOT(even integer) is the full complement of 'even integer' without any element of hesitation, irrespective of the person who does the tossing, irrespective of the decision maker who does his soft-computing using Theory of Probability. If one tosses a biased or unbiased coin, and if 'Head' does not appear then certainly NOT(Head) will appear! In this case NOT(Head) is the full complement of 'Head' without any element of hesitation.

The literatures of last five decades including a lot of pioneering works of Prof. Zadeh show that the application of $\vartheta(x)$ (i.e. $\mu^c(x)$) is very fluent in almost all application areas of fuzzy theory. The huge volume of existing literature says that in decision and organization sciences, fuzzy sets had been so far proposed to have a great impact in preference modelling and multicriteria evaluation, and has helped bringing optimization techniques closer to the users needs in the last 50 years!

A directly proposed value of $\vartheta(x)$ by a decision maker (if he can do) by his best possible judgment is surely much more reliable than the crisp computed value of $\vartheta(x)$ using the model of fuzzy set theory, while in the quest of better results for better final conclusion. This is justified in the first half of this paper by several examples of real life environment. Consequently, considering the theories developed in [15,16] and considering the rigorous analysis made in this present paper , the eyebrows raise with a question **:** Is 'Fuzzy Theory' really good for Soft-Computing? It is rigorously exercised here that in many cases the fuzzy set theory is inappropriate not only for large size decision problems but also for any decision problem, irrespective of its size, large or small.

It is known to us that the 'law of excluded middle' and 'law of non-contradiction' are not valid in fuzzy set theory, and it can be easily observed that these results remains to be absolutely true irrespective of the soft-error part in fuzzy set theory. But owing to the 'nature of weakness' identified in fuzzy set theory in this paper , the questions do arise : How far correctly we have studied so far the 'law of excluded middle' and 'law of non-contradiction' in fuzzy theory? How far correctly we have studied so far the 'Fuzzy Algebra', 'De Morgan Algebra', 'Kleene Algebra', etc. in fuzzy theory? How far correctly we have studied so far the important areas like : Fuzzy Statistics, Fuzzy event, Fuzzy Probability theory, Fuzzy Possibility theory, Fuzzy measure theory, Fuzzy evidence theory, Fuzzy theory of belief and plausibility, Fuzzy Optimization, Fuzzy Game theory, Fuzzy Algebra (fuzzy group, fuzzy ring, fuzzy fields, fuzzy De'Morgan Algebra, etc.), Fuzzy proposition, Fuzzy Expert System, Fuzzy Medical Expert System, Fuzzy RDBMS, Fuzzy Electronics, Fuzzy rough theory and Rough fuzzy theory, Fuzzy soft set theory and Soft fuzzy set theory, Fuzzy neural network, Fuzzy Graph theory & Fuzzy Multigraph theory, Fuzzy bag theory, etc. to list a few only out of many more?.

It is now obvious that all the fuzzy problems in this world whichever can be attempted by using 'Fuzzy Set Theory' in quest of a solution, can be better solved using 'Intuitionistic Fuzzy Set Theory' for better results, at least with the guarantee of no chance of poorer results. However, the example of Continuous Fuzzy Evaluation Method for Football Matches presented in [14] is a very important and revolutionary proposal to FIFA (IFAB) and is a very ideal example to understand a very particular situation where fuzzy theory is the most appropriate tool than intuitionistic fuzzy theory or any other soft-computing theory in some special cases (the proposal is presently under consideration of IFAB, Zurich). This proposal is a complete proposal for a major improvement of the existing evaluation method of football matches in World Cup, Euro Cup, etc. as per FIFA(IFAB) rules. Otherwise the 'Theory of Fuzzy Sets' may not be an appropriate theory for soft-computing in many cases. Even in many cases it may lead to large amount of errors in the final results in a hidden way keeping the fuzzy decision maker unaware and uninformed. For application of fuzzy set theory, an eligibility criteria has been proposed. It is not the issue here to note :

how much intellectual is the decision maker as a fuzzy expert. This kind of hidden risk factor will not be faced if the same problems be solved using intuitionistic fuzzy set theory. To eradicate the source of error in the 'Theory of Fuzzy Sets', one could attempt to modify or to improve the existing notion and properties of fuzzy sets. In that case, i.e. if he is in the quest of improvement and corrections of Fuzzy Set Theory by his all possible research thoughts and endeavor, he will not arrive at any new model of soft-computing sets but at the common destination which is the existing 'Theory of Intuitionistic Fuzzy Sets' or at some 'higher order intuitionistic fuzzy sets'. It is just because of the reason that none has any control over the theory and methods of CIFS being executed by the CIFS processor at the innermost kernel, except that the logic or theory or knowledge which is the personal matter of the decision maker working at the outermost annulus sphere [15]. Although fuzzy sets are being regarded as a special case of intuitionistic fuzzy sets, but the existing concept that 'the intuitionistic fuzzy sets are higher order fuzzy sets" is very much incorrect (a similar incorrect concept can be imagined if one says that 'fuzzy sets can be viewed as higher order crisp sets'). It is fact that Fuzzy Set Theory and Intuitionistic Fuzzy Set Theory are not competing hypotheses although both are theories for soft-computing. Consequently, one should not think about Occam's razor principle to view these two theories, because the issue is subject to another principle known as 'Principle of Optimization'. It is not important how complex is the theory, but it is important how much excellence in the results can be finally obtained; and accordingly the dimension needs to be optimized (i.e neither should be lower nor to be higher than requirements). The 'Intuitionistic Fuzzy Set Theory' took birth almost two decades after the inception of 'Fuzzy Set Theory', probably being encouraged by the innovative philosophy of Prof. Zadeh [37]. But it is fact that the intuitionistic fuzzy sets are the most appropriate **"optimal dimensional model"** for translation of imprecise objects, while the fuzzy sets are 'lower order' or 'lower dimensional' intuitionistic fuzzy sets. And that is the reason why CIFS in the brain of a decision maker is based upon the platform of Atanassov philosophy of intuitionistic architecture. One can think about the fact that a dog is given four legs, not three or five or else in number, because of the reason that it is the most appropriate "optimal dimensional model" for his translation for survival on this earth. Any variation from the number four (4) will lead to uncomfort to his daily life which the almighty Creator of the cosmological universe is well aware of. Similarly, one can think about the fact that a human being is given two legs, not three or one or else in number, because of the reason that it is the most appropriate "optimal dimensional model" for translation. Any variation from the number two (2) will lead to uncomfort to his daily life which the almighty Creator of the cosmological universe is well aware of. The analogous situation is followed in the CIFS too, where an "optimal dimensional model" for translation of imprecise objects is followed. While modeling the cognition system mathematically as a set like object with grades like μ value, ϑ value, etc., optimization of the dimension (number of grades) is very important. Because if there is a shortage in the dimension then it could be a kind of 'physically handicapped' situation and if

there is an excess in the dimension then it could be a kind of 'over burden' situation. The Atanassov model of intuitionistic fuzzy set does not suffer from any of such kind of 'physically handicapped' situation or 'over burden' situation. But there is no existing parent model or method which can authenticate the world scientists the correct optimal dimension, except being convinced by case studies and human logic only, an analysis of which has been done here with examples, and achieved **the reality by imagination**.

It is also justified that 'Fuzzy Set Theory" can only be applied if it is known in advance that the concerned fuzzy decision maker is highly knowledgeable in the field of the subject on which the phrase Λ describes the fuzzy set, if it is known in advance that the concerned decision maker is highly intelligent, highly brilliant, having no confusion (and no hesitation i.e. the amount $\pi(x)$ is NIL), to deal with the grading of each element by an amount in its 'TRUTH' part so that by default the rest amount goes to its 'FALSE' part leading to no amount of soft-error corresponding to any element x of U. Subject to fulfillment of this necessary condition, if 'Fuzzy Theory' be applicable for solving a decision problem, then surely 'Fuzzy Theory' will be the most appropriate theory for this type of particular case compared to any other soft-computing set theory. A very unique example showing application of fuzzy set theory as the dominant soft-computing set theory compared to any other existing soft-computing set theories is presented in [14] while introducing CFE Method for an excellent evaluation of football matches (proposed to FIFA/IFAB, and presently under consideration of IFAB, Zurich). Although all the above prescribed three necessary conditions for applying 'Fuzzy Set Theory' in soft-computing seem to be very **tightly constrained** to any fuzzy decision maker, but we can not ignore the fact.

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