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Intuitionistic fuzzy time series functions approach for time series forecasting

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Abstract

Fuzzy inference systems have been commonly used for time series forecasting in the literature. Adaptive network fuzzy inference system, fuzzy time series approaches and fuzzy regression functions approaches are popular among fuzzy inference systems. In recent years, intuitionistic fuzzy sets have been preferred in the fuzzy modeling and new fuzzy inference systems have been proposed based on intuitionistic fuzzy sets. In this paper, a new intuitionistic fuzzy regression functions approach is proposed based on intuitionistic fuzzy sets for forecasting purpose. This new inference system is called an intuitionistic fuzzy time series functions approach. The contribution of the paper is proposing a new intuitionistic fuzzy inference system. To evaluate the performance of intuitionistic fuzzy time series functions, twenty-three real-world time series data sets are analyzed. The results obtained from the intuitionistic fuzzy time series functions approach are compared with some other methods according to a root mean square error and mean absolute percentage error criteria. The proposed method has superior forecasting performance among all methods.

Keywords Intuitionistic fuzzy sets · Fuzzy inference · Forecasting · Fuzzy functions approach

1 Introduction

Forecasting is very important for future planning in many technological areas. Forecasting techniques are attracted by managers and other decision-makers. Forecasting techniques can be based on probability theory, fuzzy set theory or computational techniques. Many of forecasting techniques use fuzzy sets in their algorithms. Fuzzy sets were proposed by Zadeh (1965). Chen (1996a, b) proposed a fuzzy reasoning approach. Chen (1998) proposed a fuzzy system for group decision-making. Bai and Chen

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(2008a, b) and proposed a method for creating automatically membership functions of fuzzy rules. Bai and Chen (2008a, b) proposed adaptive fuzzy system-based automatically determined concept maps. Fuzzy inference systems and fuzzy time series methods can be used for forecasting. Takagi and Sugeno (1985) system, adaptive network fuzzy inference system proposed by Jang (1993) and a fuzzy function approach proposed by Turksen (2008) are well-known fuzzy inference systems in the forecasting literature. Fuzzy time series methods are also popular methods in the forecasting literature. Song and Chissom (1993) were firstly defined fuzzy time series concept, and they proposed a fuzzy time series forecasting method. Chen and Wang (2010), Chen et al. (2012, 2013), Chen and Chen (2015), Chen and Phuong (2017) and Chen and Jian (2017) proposed forecasting methods based on fuzzy sets.

Recent years, many applications of classical fuzzy systems have been made in the literature. Zarandi et al. (2013) proposed a new fuzzy functions model tuned by hybridizing imperialist competitive algorithm and simulated annealing. Bezdek (2013) used fuzzy objective function algorithms for pattern recognition. Baykasoğlu and Maral (2014) proposed fuzzy functions approach via genetic

programming. Baser and Apaydin (2015) proposed a hybrid fuzzy support vector regression analysis. Barak and Sadegh (2016) used ensemble ARIMA-ANFIS hybrid algorithm for forecasting of energy consumption. Goudarzi et al. (2016) proposed an interactively recurrent fuzzy function with multi-objective learning. Aladag et al. (2016) proposed a type 1 fuzzy time series function method based on binary particle swarm optimization. Tan et al. (2017) proposed a new adaptive network-based fuzzy inference system for forecasting. Yang et al. (2017) used linear fuzzy information granules and fuzzy inference system for long-term forecasting of time series. Son et al. (2017) proposed a new neuro-fuzzy inference system for insurance forecasting. Ranganayaki and Deepa (2017) proposed a support vector machine-based neuro-fuzzy model for short-term wing power forecasting. Tak et al. (2018) proposed a recurrent fuzzy function approach for forecasting. Pelka and Dudek (2018) proposed a neuro-fuzzy system for forecasting. Vanhoenshoven et al. (2018) proposed a fuzzy cognitive map employing ARIMA components for time series forecasting. Moreover, there are many fuzzy time series forecasting methods. The fuzzy time series concept was introduced by Song and Chissom (1993). Chen (1996a, b) proposed a fuzzy time series method based on fuzzy relation tables, and it constituted a base for many methods. In recent studies, Chen and Chang (2010), Chen and Chen (2011), Chen et al. (2012), Garg and Garg (2016), Singh (2016), Cagcag Yolcu et al. (2016), Kumar and Gangwar (2016), Kocak (2017), Bose and Mali (2018) and Chang and Yu (2019) proposed fuzzy time series methods. Wang (2018) used a fuzzy time series forecasting method for big data analysis. Bisht and Kumar (2019) used hesitant fuzzy sets based on the computational method for financial time series forecasting. Egrioglu et al. (2019) proposed a forecasting method for single-variable high-order intuitionistic fuzzy time series forecasting model. Gupta and Kumar (2019a) proposed a novel high-order fuzzy time series forecasting method based on probabilistic fuzzy sets. Gupta and Kumar (2019b) proposed a hesitant probabilistic fuzzy set-based time series forecasting method.

Recent years, intuitionistic (hesitant) fuzzy sets have been commonly used in fuzzy techniques. In a fuzzy set, there are membership values for each member of the universal set. Non-membership values can be obtained from membership values by using a simple subtract operation. Atanassov (1983) introduced an intuitionistic fuzzy set. In an intuitionistic fuzzy set, non-membership values have different information than membership values have. Besides, hesitation degrees are obtained from the simple mathematical operation of membership and non-membership values. Atanassov (1986) and Atanassov (1999) gave the details of the theory and some applications for intuitionistic fuzzy sets. Bustince et al. (1995), Cornelis and Deschrijver (2001), Szmidt and Kacprzyk (2001), Marinov and Atanassov (2005), Own (2009) and Davarzani and Khorheh (2013) applied intuitionistic fuzzy sets on different implementations. Moreover, Zheng et al. (2013), Kumar and Gangwar (2016), Wang et al. (2016), Bisht and Kumar (2016) and Fan et al. (2017) proposed intuitionistic fuzzy time series method in their studies. Chen and Chang (2016), Chen et al. (2016a, b) and Liu et al. (2017) applied intuitionistic fuzzy sets in their proposed methods.

Castillo et al. (2007) proposed an intuitionistic fuzzy system for time series analysis. Olej and Hájek (2010a) proposed an intuitionistic fuzzy inference system design for prediction of ozone time series. Olej and Hájek (2010b) showed the possibilities of air quality modeling based on intuitionistic fuzzy set theory. Olej and Hájek (2011) compared of fuzzy operators for intuitionistic fuzzy inference system of Takagi–Sugeno type. Hájek and Olej (2012) used adaptive intuitionistic fuzzy inference system of Takagi–Sugeno type for regression problems. The parameters of the intuitionistic fuzzy inference system are determined by using particle swarm optimization in Angelov (2012), Maciel et al. (2012) and Henzgen et al. (2014). Bas et al. (2019) proposed a type 1 fuzzy function method based on ridge regression for forecasting. Kizilaslan et al. (2019) and Cagcag Yolcu et al. (2019) proposed intuitionistic fuzzy function approaches. Egrioglu et al.(2020) proposed picture fuzzy regression functions method based on picture fuzzy clustering.

The motivation of this paper is explained in the following sentences. Fuzzy inference systems are efficient tools for forecasting purposes. It is possible to create new fuzzy inference systems for obtaining more accurate forecasts. Especially, the intuitionistic fuzzy inference system is needed to improve by using different updated techniques. Because intuitionistic fuzzy inference systems employee non-membership values, they can give more accurate forecast results than classical fuzzy inference systems.

The main contribution of this paper can be expressed as proposing a new intuitionistic fuzzy inference system. In this new system, membership values and non-membership values in intuitionistic fuzzy sets and their nonlinear transformations are used as inputs. Thus, the dimension of the input matrix in type 1 fuzzy function approach is augmented by using non-membership values in intuitionistic fuzzy sets. In the new approach, the membership and non-membership values are obtained from intuitionistic fuzzy c-means as in Chaira (2011). The proposed intuitionistic systems do not need to determine the combination parameter of a dual system which are separately designed according to membership and non-membership. In the second section, the proposed method is summarized. The applications for real data sets are given in the third

section. In the last section, conclusions and discussions are given.

2 Intuitionistic fuzzy time series functions approach

In the literature, many of fuzzy inference methods have been proposed. The fuzzy functions approach proposed by Turksen (2008) is fairly different from others because it does not have a rule base and it can use directly linear regression models. Although the fuzzy functions approach uses just fuzzy sets, it does not use intuitionistic fuzzy sets. In the fuzzy functions approach, Turksen (2008) showed that the augmentation of the input matrix's elements by using nonlinear transformations of membership values can drastically improve prediction performance. In this paper, an intuitionistic fuzzy time series functions approach is proposed. In this approach, the input matrix contains nonlinear transformations of non-membership values as well as membership values. The proposed method is based on the ordinary least square estimation instead of ridge regression like in Kizilaslan et al. (2019). The proposed methods use membership and membership values in the same input matrices apart from Cagcag Yolcu et al. (2019). The proposed approach has the following advantages:

- The proposed approach employs intuitionistic fuzzy c-means clustering. Creation of intuitionistic fuzzy sets is more realistic than creation fuzzy sets because of using hesitation margin.
- The input matrix has a higher dimension in the proposed approach so that it uses more information compared with other fuzzy functions approaches.
- The proposed approach has superior forecasting performance in many real-world time series applications.

- The proposed intuitionistic systems do not need to determine the combination parameter of a dual system which are separately designed according to membership and non-membership.

The proposes step-by-step algorithm for intuitionistic fuzzy time series functions algorithm is shown as follows, where its flowchart is given in Fig. 1.

Algorithm 1. Intuitionistic Fuzzy Time Series Functions (IFTSF) Algorithm

Step 1 Parameters of the method are determined.
Parameters are the number of intuitionistic fuzzy clusters (cn), and inputs of the system are the number of lagged variables (p), hesitation margin (π), alpha cut ($\alpha - cut$) and the length of the test set ($ntest$).
Step 2 Clustering the data.
The input and targets are constituted IO matrix. Intuitionistic fuzzy c-means clustering algorithm proposed by Chaira (2011) is used to obtain memberships and non-memberships.

$$
IO = \begin{bmatrix}
x_1 & x_2 & \cdots & x_p & x_{p+1} \\
x_2 & x_3 & \cdots & x_{p+1} & x_{p+2} \\
\cdot & \cdot & \cdots & \cdot & \cdot \\
\cdot & \cdot & \cdots & \cdot & \cdot \\
\cdot & \cdot & \cdots & \cdot & \cdot \\
x_{n-p} & x_{n-p+1} & \cdots & x_{n-1} & x_n
\end{bmatrix} \tag{1}
$$

The last element of cluster centers is excluded, and reduced cluster centers are obtained. Intuitionistic membership values ($\mu_A(x)$) and non-membership values ($\vartheta_A(x)$) are calculated according to reduced cluster centers. If the $\mu_A(x) < \alpha - cut$, then $\mu_A(x) = 0$. Similarly, if the $\vartheta_A(x) < \alpha - cut$ then $\vartheta_A(x) = 0$. After applying $\alpha - cut$ operation, normalization applied to membership and non-membership values is shown u_{ij} and μ_{ij}.
**Step 3** Fuzzy regression functions are obtained by using the least square method. The parameters of linear functions are estimated. Let n be the length of training time series data.

$$
Input^{(i)} = \begin{bmatrix}
1 & u_{i1} & u_{i1}^2 & \exp(u_{i1}) & \mu_{i1} & \mu_{i1}^2 & \exp(\mu_{i1}^2) & x_1 & x_2 & \cdots & x_p \\
1 & u_{i2} & u_{i2}^2 & \exp(u_{i2}) & \mu_{i2} & \mu_{i2}^2 & \exp(\mu_{i2}^2) & x_2 & x_3 & \cdots & x_{p+1} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
1 & u_{in} & u_{in}^2 & \exp(u_{in}) & \mu_{in} & \mu_{in}^2 & \exp(\mu_{in}^2) & x_{n-p} & x_{n-p+1} & \cdots & x_{n-1}
\end{bmatrix} \tag{2}
$$

$$
Target^{(i)\prime} = \begin{bmatrix} x_{p+1} & x_{p+2} & \cdots & x_n \end{bmatrix} \tag{3}
$$

Fig. 1 Flowchart for the IFTSF approach
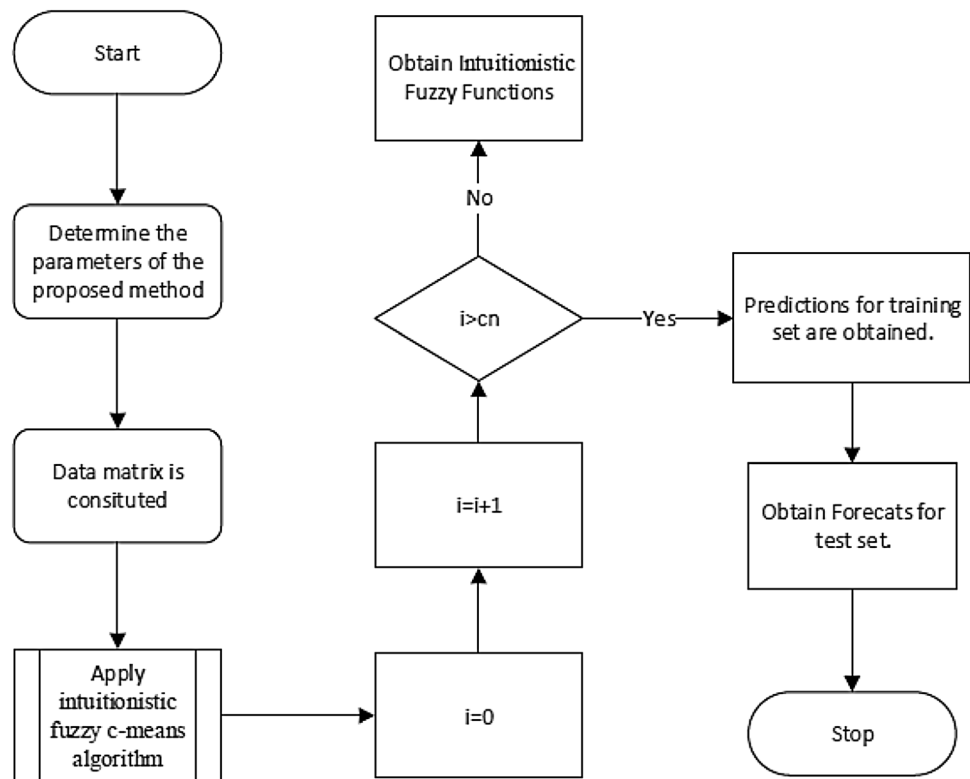
$$\hat{\theta}^{(i)} = (Input^{(i)\prime} Input^{(i)})^{-1} Input^{(i)\prime} Target^{(i)} \qquad (4)$$

Step 4 Predictions are obtained for training data. Predictions for the training set are obtained by using Eq. (7) by combining outputs of linear functions.

$$\widehat{Output}_j^{(1)} = \frac{\sum_{i=1}^{c} u_{ij} Input^{(i)} \hat{\theta}^{(i)}}{\sum_{i=1}^{c} u_{ij}} \quad j = 1, 2, \ldots, n \qquad (5)$$

$$\widehat{Output}_j^{\text{IFTSF}} = \pi \widehat{Output}_j^{(1)} + (1 - \pi) \widehat{Output}_j^{(2)} \qquad (7)$$

$\widehat{Output}_j^{\text{IFTSF}}$ is the prediction of intuitionistic fuzzy time series function (IFTSF) method for jth observation.

Step 5 Forecasts are obtained for test sets.

The design matrix ($Itest^{(i)}$) is constituted for each intuitionistic fuzzy cluster and test set. The test set forecasts ($\widehat{Yt}^{(i)}$) of each intuitionistic fuzzy regression functions are computed as follows.

$$Itest^{(i)} = \begin{bmatrix} 1 & u_{i,n+1} & u_{i,n+1}^2 & \exp(u_{i,n+1}) & \mu_{i,n+1} & \mu_{i,n+1}^2 & \exp(\mu_{i,n+1}^2) & x_{n-p+1} & x_{n-p+2} & \cdots & x_n \\ 1 & u_{i,n+2} & u_{i,n+1}^2 & \exp(u_{i,n+2}) & \mu_{i,n+2} & \mu_{i,n+2}^2 & \exp(\mu_{i,n+2}^2) & x_{n-p+2} & x_{n-p+3} & \cdots & x_{n+1} \\ . & . & . & . & . & . & . & . & . & \cdots & . \\ . & . & . & . & . & . & . & . & . & \cdots & . \\ . & . & . & . & . & . & . & . & . & \cdots & . \\ 1 & u_{i,n+ntest} & u_{i,n+ntest}^2 & \exp(u_{i,n+ntest}) & \mu_{i,n+ntest} & \mu_{i,n+ntest}^2 & \exp(\mu_{i,n+ntest}^2) & x_{n-p+ntest} & x_{n-p+ntest+1} & \cdots & x_{n+ntest-1} \end{bmatrix}$$

$$(8)$$

$$\widehat{Output}_j^{(2)} = \frac{\sum_{i=1}^{c} \mu_{ij} Input^{(i)} \hat{\theta}^{(i)}}{\sum_{i=1}^{c} \mu_{ij}} \quad j = 1, 2, \ldots, n \qquad (6)$$

Table 1 Names and features of time series and parameter values for the proposed method

The number of series	Series/year	Number of observation	Number of lag (p)	Number of clusters (cn)	Length of test set ($ntest$)
1	BIST100/2009	103	1:5	3:10	7; 15
2	BIST100/2010	104	1:5	3:10	7; 15
3	BIST100/2011	106	1:5	3:10	7; 15
4	BIST100/2012	106	1:5	3:10	7; 15
5	BIST100/2013	106	1:5	3:10	7; 15
6	TAIEX/1999	266	1:5	3:10	45
7	TAIEX/2000	271	1:5	3:10	47
8	TAIEX/2001	244	1:5	3:10	43
9	TAIEX/2002	248	1:5	3:10	43
10	TAIEX/2003	249	1:5	3:10	43
11	TAIEX/2004	250	1:5	3:10	45
12	Dow-Jones/2010	252	1:5	3:10	10
13	Dow-Jones/2011	251	1:5	3:10	10
14	Dow-Jones/2012	250	1:5	3:10	10
15	Dow-Jones/2013	252	1:5	3:10	10
16	Dow-Jones/2014	252	1:5	3:10	10
17	TEC	144	2:16	3:10	12

$$\widehat{Otest}_j^{(1)} = \frac{\sum_{i=1}^c u_{ij} Itest^{(i)} \hat{\beta}^{(i)}}{\sum_{i=1}^c u_{ij}} \tag{9}$$
$$j = n+1, n+2, \ldots, n+ntest$$

$$\widehat{Otest}_j^{(2)} = \frac{\sum_{i=1}^c \mu_{ij} Itest^{(i)} \hat{\beta}^{(i)}}{\sum_{i=1}^c \mu_{ij}} \tag{10}$$
$$j = n+1, n+2, \ldots, n+ntest$$

$$\widehat{Output}_j^{\text{IFTSF}} = \pi \widehat{Otest}_j^{(1)} + (1-\pi)\widehat{Otest}_j^{(2)} \tag{11}$$
$$j = n+1, n+2, \ldots, n+ntest$$

where u_{ij} and μ_{ij} membership and non-membership values are computed by using reduced cluster centers which are obtained in Step 2.

3 Applications

The forecasting performance of the proposed method is investigated by using some real-world time series data sets. The list of time series and their features are given in Table 1. The first data set is daily BIST 100 (Borsa Istanbul 100) index computed for Istanbul Stock Exchange between the years 2009 and 2013 as totally five data sets. The time series were taken from the Turkish Central Bank official Web site. The second data set is the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) data observed daily between the years 1999 and 2004. The

TAIEX data sets were taken from Sarica et al. (2018). The third data set is daily Dow-Jones Industrial Average index between the years 2010 and 2014 as totally five time series. The first three data sets are stock exchange data sets. The last data are Turkey Electricity Consumption (TEC) data observed monthly between the first month of 2002 and last month of 2013. TEC data set was taken from Turkey Energy Ministry.

The parameters of the proposed method (p, cn and $ntest$) are used like in Table 1 for the analysis of all data sets. Firstly, BIST100 data set is analyzed by using ARIMA (Box and Jenkins 1976), ANFIS (Jang 1993) and modified ANFIS (MANFIS) proposed by Egrioglu et al. (2014), fuzzy time series method (SC) proposed by Song and Chissom (1993), AR-ANFIS proposed by Sarica et al. (2018), type 1 fuzzy function (T1FF) proposed by Turksen (2008) and the proposed method in this paper (IFTSF). The root of mean square error (RMSE) and mean absolute percentage error (MAPE) values for test sets of BIST100 are given in Tables 2 and 3, respectively.

$$\text{RMSE} = \sqrt{\frac{1}{ntest} \sum_{t=1}^{ntest} (y_t - \hat{y}_t)^2} \tag{12}$$

$$\text{MAPE} = \frac{1}{ntest} \sum_{t=1}^{ntest} \left| \frac{y_t - \hat{y}_t}{y_t} \right| \tag{13}$$

In Eqs. 12 and 13, y_t and $\hat{y}_t$ are real observations and predicted values, respectively.

Table 2 RMSE values for test sets for BIST100 data set

Time series	Length of the test sets	Box–Jenkins (1976)	Jang (1993)	Egrioglu et al. (2014)	Song and Chissom (1993)	Sarıca et al. (2018)	Turksen (2008)	IFTSF
BIST100/ 2009	7	344	405	261	1402	240	446	323
BIST100/ 2009	15	540	647	503	1754	467	534	483
BIST100/ 2010	7	1221	1141	1144	1128	1136	1180	1010
BIST100/ 2010	15	1612	2033	1303	1742	1451	1852	1539
BIST100/ 2011	7	1057	1007	960	1396	987	1083	862
BIST100/ 2011	15	1129	1134	1009	1360	999	1146	996
BIST100/ 2012	7	651	634	634	1292	631	1034	669
BIST100/ 2012	15	621	938	629	1047	619	1038	698
BIST100/ 2013	7	1361	1477	1418	1450	1362	1512	1006
BIST100/ 2013	15	1268	1413	1264	1931	1256	1279	1194
Mean		980.4	1082.9	912.5	1450.2	914.8	1110.4	878

Table 3 MAPE values for test sets for BIST100 data set

Time series	Length of the test sets	Box–Jenkins (1976)	Jang (1993)	Egrioglu et al. (2014)	Song and Chissom (1993)	Sarica et al. (2018)	Turksen (2008)	IFTSF
BIST100/ 2009	7	0.0087	0.0097	0.0059	0.0396	0.0052	0.0101	0.0077
BIST100/ 2009	15	0.0120	0.0157	0.0112	0.0438	0.0102	0.0122	0.0105
BIST100/ 2010	7	0.0183	0.0153	0.0187	0.0182	0.0191	0.0179	0.0148
BIST100/ 2010	15	0.0219	0.0277	0.0193	0.0265	0.0230	0.0264	0.0196
BIST100/ 2011	7	0.0144	0.0141	0.0141	0.0200	0.0134	0.0153	0.0123
BIST100/ 2011	15	0.0150	0.0153	0.0136	0.0189	0.0130	0.0156	0.0130
BIST100/ 2012	7	0.0084	0.0092	0.0082	0.0183	0.0087	0.0162	0.0089
BIST100/ 2012	15	0.0088	0.0139	0.0094	0.0153	0.0086	0.0161	0.0104
BIST100/ 2013	7	0.0116	0.0131	0.0134	0.0108	0.0118	0.0131	0.0096
BIST100/ 2013	15	0.0109	0.0125	0.0109	0.0176	0.0108	0.0108	0.0102
Mean		0.0130	0.0147	0.0125	0.0229	0.0124	0.0154	0.0117

Table 4 RMSE values for test sets for TAIEX data

	Chen (1996a, b)	Chen and Chang (2010)	Chen and Chen (2011)	Chen et al. (2012)	Jang (1993)	Egrioglu et al. (2014)	Sarica et al. (2018)	IFTSF
TAIEX/ 1999	120	101.97	112.47	99.87	101.16	101.94	98.37	95.09
TAIEX/ 2000	176.32	129.42	123.62	119.98	137.02	124.92	122.81	119.91
TAIEX/ 2001	147.84	113.33	115.33	114.47	114.72	112.47	111.49	111.42
TAIEX/ 2002	101.18	66.82	71.01	67.17	65.99	62.57	65.86	63.86
TAIEX/ 2003	74.46	53.51	58.06	52.49	57.04	52.33	51.83	51.64
TAIEX/ 2004	84.28	60.48	57.73	52.27	61.36	53.66	53.63	51.86
Mean	117.34	87.58	89.70	84.37	89.54	84.64	84.00	82.29

Table 5 RMSE values for test sets of Dow Jones data set

Time series	Jang (1993) ANFISgrid	Jang (1993) ANFISsub	Egrioglu et al. (2014)	Turksen (2008)	IFTSF
Dow-Jones/2010	39.7287	30.1699	21.7519	20.5995	20.4857
Dow-Jones/2011	132.3766	130.1214	123.5594	123.9386	116.015
Dow-Jones/2012	93.4754	102.3491	100.5147	97.7848	98.3116
Dow-Jones/2013	197.8908	176.4183	95.2613	92.0896	86.9267
Dow-Jones/2014	177.7626	202.5687	160.9774	178.1782	169.548
Mean	128.2468	128.3255	100.4129	102.5181	98.2574

Table 6 MAPE values for test sets of Dow Jones data set

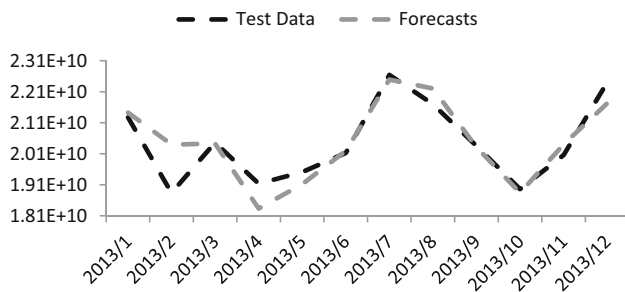| Time series	Jang (1993) ANFISgrid	Jang (1993) ANFISsub	Egrioglu et al. (2014)	Turksen (2008)	IFTSF
Dow-Jones/2010	0.0031	0.0023	0.0015	0.0015	0.0014
Dow-Jones/2011	0.0088	0.0080	0.0079	0.0073	0.0070
Dow-Jones/2012	0.0065	0.0066	0.0069	0.0062	0.0064
Dow-Jones/2013	0.0103	0.0102	0.0036	0.0033	0.0034
Dow-Jones/2014	0.0086	0.0082	0.0070	0.0070	0.0053
Mean	0.0074	0.0071	0.0054	0.0051	0.0047

The proposed method produces the best forecasts for 50% of all results in BIST100 data set applications according to RMSE value. The similar results are obtained according to the MAPE criterion. Moreover, the proposed method produces the minimum mean of RMSE and MAPE values in Tables 2 and 3 when the proposed method is compared with other methods. When the length of the test set is 7, the success rate of the proposed method is 60%. For the length of the test set 15, the success rate of the proposed method is 40%. As a result of the analysis, IFTSF

method can produce better forecasts for a small test set of BIST100 data set. The proposed method produces the competitive results for long test set lengths.

The analyzed results of the TAIEX data set are given in Table 4. In Table 4, it is seen that the proposed method has 83.33% succeeds of all results in TAIEX data set according to RMSE value. Besides this superior succeed performance, the proposed method is the best method for mean statistics of RMSE values among all other analytical methods for TAIEX data set.

Table 7 RMSE, MAPE values and forecasts for TEC data

Test data	Rumelhart et al. (1986)	Box–Jenkins (1976)	Yadav et al. (2007)	Yolcu et al. (2013)	Turksen (2008)	IFTSF
21275408487	21504260392	21690314907	21929976335	22005935580	21666285710	21428154349
18841712637	21407932217	19879748318	19674336447	21273737442	20685602131	20398939503
20463933683	19934741061	20679491118	20759353666	20386359964	19996267563	20429969260
19139248871	17339157313	18616749529	18177421217	19068902087	17987700997	18336252114
19511728912	19132107582	18166751109	19502618441	19337714416	19394021620	19123035466
20132602347	20521392811	19476718063	19961628854	20493224715	20584842565	20197056647
22648523194	21559624073	22996373189	21466603203	22493619106	22702026988	22496003285
21698207982	21608712088	22602364226	20970484983	22642925619	21714016063	22207009728
20358717408	20450417618	19372692274	20819409088	20387962879	19965826638	20317736156
18964661109	19608790983	19145368564	18842139533	19384311943	18624614607	18840613768
20061232838	20798229864	18281990717	20341486661	20668793095	20140094911	20406199920
22405662577	21396071971	21319220705	20523920152	22329629382	22138069735	21728867097
RMSE	1065870606	917321409.3	813259007.7	820978567.5	687192130.1	586354861.9
MAPE	0.039841988	0.038842	0.030114538	0.025444349	0.023662007	0.020225362

Fig. 2 Graph of real observations and proposed method forecasts for TEC data

Table 8 Conditions for the best results of IFTSF

Time series	Number of clusters	Number of lag	Ntest
BIST100/2009	7	6	7
BIST100/2009	4	6	15
BIST100/2010	6	3	7
BIST100/2010	4	1	15
BIST100/2011	7	2	7
BIST100/2011	3	3	15
BIST100/2012	7	1	7
BIST100/2012	7	2	15
BIST100/2013	7	4	7
BIST100/2013	7	2	15
TAIEX/1999	4	3	45
TAIEX/2000	5	6	47
TAIEX/2001	3	2	43
TAIEX/2002	5	3	43
TAIEX/2003	6	2	43
TAIEX/2004	6	5	45
Dow-Jones/2010	4	2	10
Dow-Jones/2011	9	2	10
Dow-Jones/2012	9	2	10
Dow-Jones/2013	10	4	10
Dow-Jones/2014	5	5	10
TEC	10	16	10

Dow Jones data set is analyzed by ANFIS with the grid partition method (ANFISgrid), ANFIS with subtractive clustering (ANFISsub), MANFIS, T1FF and IFTSF. The results of the analysis for Dow Jones data set are given in Tables 5 and 6. The success rate of the IFTSF is 80% according to RMSE criterion, and it is 60% for MAPE criterion. For mean statistics, the proposed method is the best one according to both RMSE and MAPE criteria.

In the final application, the analyze results of TEC data are given in Table 7. TEC data are analyzed by multilayer perceptron artificial neural network (MLP-ANN), seasonal autoregressive integrated moving average model (SARIMA; Box–Jenkins 1976), single multiplicative neuron model artificial neural network (SMNM-ANN) proposed by Yadav et al. (2007), linear and nonlinear neural network introduced by Yolcu et al. (2013) and TIFF and IFTSF methods.

It is clearly seen that the proposed method has superior forecasting performance for both RMSE and MAPE

criteria. Besides, the graph of real observations and forecasted values is given in Fig. 2.

The best parameter sets in IFTSF method for all analyzed time series are given in Table 8. An important remark in Table 8 is that while the number of clusters is generally

obtained as 7, there is no a certain value for the number of lags that means the best results obtained in the different numbers of lags for the BIST 100. Moreover, there are no certain parameter values in other time series.

4 Conclusion

The main contribution of this paper can be stated that an intuitionistic fuzzy time series functions approach is proposed. The proposed method inspired by T1FF is an improved modification of T1FF with the use of hesitation margins and non-membership values. Non-memberships have different information to determine time series relations from membership values, unlike T1FF approach. The new inference system takes into consideration hesitation values, and it has an additional approach to uncertainty like type 2 fuzzy systems. Because of using second-order uncertainty, the system can define better relations between lagged variables of time series. In IFTSF, input matrix contains non-memberships and their transformations as well as lagged variables, memberships and transformations of memberships. Augmentation of the input matrix provides extra information to the inference mechanism.

The proposed method is compared with some well-known fuzzy inference, fuzzy time series methods, artificial neural networks and classical time series methods. According to analyzing results, it is clearly seen that IFTSF can produce better forecasting results than others for almost all of the time series. Moreover, it can be generally said that the proposed IFTSF has better forecasting performance for short-term or small test sets.

In future studies, the proposed method can be adapted to be working in a dual structure for memberships and non-memberships like other intuitionistic fuzzy inference systems. Input selection can be made by using artificial intelligence optimization techniques. Moreover, the fuzzy functions can be obtained by using artificial neural networks instead of the linear model.

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