

INTUITIONISTIC H -FUZZY RELATIONS

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We introduce the category $\mathbf{IRel}(H)$ consisting of intuitionistic fuzzy relational spaces on sets and we study structures of the category $\mathbf{IRel}(H)$ in the viewpoint of the topological universe introduced by Nel. Thus we show that $\mathbf{IRel}(H)$ satisfies all the conditions of a topological universe over \mathbf{Set} except the terminal separator property and $\mathbf{IRel}(H)$ is cartesian closed over \mathbf{Set} .

1. Introduction

In 1965, Zadeh [30] introduced a concept of a fuzzy set as the generalization of a crisp set. Also, in 1971, he introduced a fuzzy relation naturally, as a generalization of a crisp relation in [31].

Nel [27] introduced the notion of a topological universe which implies concrete quasitopos [1]. Every topological universe satisfies all the properties of a topos except one condition on the subobject classifier. The notion of a topological universe has already been put to effective use in several areas of mathematics in [24, 25, 28]. In 1980, Cerruti [8] introduced the category of L -fuzzy relations and investigated some of its properties. After that time, Hur [14] introduced the category $\mathbf{Rel}(H)$ of the fuzzy relational spaces with a complete Heyting algebra H as a codomain and he studied the category $\mathbf{Rel}(H)$ in the sense of a topological universe.

In 1983, Atanassov [2] introduced the concept of an intuitionistic fuzzy set as the generalization of fuzzy sets and he also investigated many properties of intuitionistic fuzzy sets (cf. [3]). After that time, Banerjee and Basnet [4], Biswas [6], and Hur and his colleagues [15, 16, 17, 20] applied the concept of intuitionistic fuzzy sets to algebra. Also, Çoker [9], Hur and his colleagues [21], and S. J. Lee and E. P. Lee [26] applied one to topology. In particular, Hur and his colleagues [18] applied the notion of intuitionistic fuzzy sets to topological group.

In this paper, we introduce the category $\mathbf{IRel}(H)$ of intuitionistic H -fuzzy relational spaces and study the category $\mathbf{IRel}(H)$ in a topological universe viewpoint. In particular, we show that $\mathbf{IRel}(H)$ satisfies all the conditions of a topological universe over \mathbf{Set} except

the terminal separator property. Also $\mathbf{IRel}(H)$ is shown to be cartesian closed over \mathbf{Set} . For general categorical background, we refer to Herrlich and Strecker [12].

2. Preliminaries

In this section, we will introduce some basic definitions and well-known results which are needed in the next sections.

Let X be a set, let $(X_i)_{i \in I}$ be a family of sets indexed by a class I , and let f_i be a mapping with domain X for each $i \in I$. Then a pair $(X, (f_i)_I)$ (simply, $(f_i)_I$) is called a *source of mappings*. A *sink of mappings* is the dual notion of a source of mappings.

Definition 2.1 [12]. Let \mathbf{A} be a concrete category and let I be a class.

(1) A *source in \mathbf{A}* is a pair $(X, (f_i)_I)$ (simply, (X, f_i) or $(f_i)_I$), where X is an \mathbf{A} -object and $(f_i : X \rightarrow X_i)_I$ is a family of \mathbf{A} -morphisms each with domain X . In this case, X is called the *domain of the source* and the family $(X_i)_I$ is called the *codomain of the source*.

(2) A source (X, f_i) is called a *monosource* provided that the f_i can be simultaneously canceled from the left; that is, provided that for any pair $Y \begin{smallmatrix} r \\ \rightrightarrows \\ s \end{smallmatrix} X$ of morphisms such that $f_i \circ r = f_i \circ s$ for each $i \in I$, it follows that $r = s$.

Dual notions: *sink in \mathbf{A}* and *episink*.

Definition 2.2 [23]. Let \mathbf{A} be a concrete category and let $((Y_i, \xi_i))_I$ be a family of objects in \mathbf{A} indexed by a class I . For any set X , let $(f_i : X \rightarrow Y_i)_I$ be a source of mappings indexed by I . An \mathbf{A} -structure ξ on X is said to be *initial with respect to $(X, (f_i), ((Y_i, \xi_i)))$* provided that the following conditions hold.

(1) For each $i \in I$, $f_i : (X, \xi) \rightarrow (Y_i, \xi_i)$ is an \mathbf{A} -morphism.

(2) If (Z, ρ) is an \mathbf{A} -object and $g : Z \rightarrow X$ is mapping such that for each $i \in I$, the mapping $f_i \circ g : (Z, \rho) \rightarrow (Y_i, \xi_i)$ is an \mathbf{A} -morphism, then $g : (Z, \rho) \rightarrow (X, \xi)$ is an \mathbf{A} -morphism. In this case, $(f_i : (X, \xi) \rightarrow (Y_i, \xi_i))_I$ is called an *initial source in \mathbf{A}* .

Dual notions: *final structure* and *final sink*.

Definition 2.3 [23]. A concrete category \mathbf{A} is said to be *topological over \mathbf{Set}* provided that for each set X , for any family $((Y_i, \xi_i))_I$ of \mathbf{A} -objects, and for any source $(f_i : X \rightarrow Y_i)_I$ of mappings, there exists a unique \mathbf{A} -structure ξ on X which is initial with respect to $(X, (f_i), ((Y_i, \xi_i)))$.

Dual notions: *cotopological category*.

RESULT 2.4 [23, Theorem 1.5]. *A concrete category \mathbf{A} is topological if and only if \mathbf{A} is cotopological.*

RESULT 2.5 [23, Theorem 1.6]. *Let \mathbf{A} be a topological category over \mathbf{Set} . Then \mathbf{A} is complete and cocomplete.*

Definition 2.6 [11]. A category \mathbf{A} is called *cartesian closed* provided that the following conditions hold.

(1) For any \mathbf{A} -objects A and B , there exists a product $A \times B$ in \mathbf{A} .

(2) Exponential exists in \mathbf{A} , that is, for any \mathbf{A} -object A , the functor $A \times - : \mathbf{A} \rightarrow \mathbf{A}$ has a right adjoint, that is, for any \mathbf{A} -object B , there exists an \mathbf{A} -object B^A and an \mathbf{A} -morphism $e_{A,B} : A \times B^A \rightarrow B$ (called the *evaluation*) such that for any \mathbf{A} -object C

and any \mathbf{A} -morphism $f : A \times C \rightarrow B$, there exists a unique \mathbf{A} -morphism $\bar{f} : C \rightarrow B^A$ such that the diagram

$$\begin{array}{ccc}
 A \times B^A & \xrightarrow{e_{A,B}} & B \\
 \swarrow \exists!_{A \times \bar{f}} & & \nearrow f \\
 & A \times C &
 \end{array}
 \tag{2.1}$$

commutes.

Definition 2.7 [23]. Let \mathbf{A} be a concrete category.

- (1) The \mathbf{A} -fiber of a set X is the class of all \mathbf{A} -structures on X .
- (2) \mathbf{A} is called *properly fibered over Set* provided that the following conditions hold.
 - (i) *Fiber-smallness*. For each set X , the \mathbf{A} -fiber of X is a set.
 - (ii) *Terminal separator property*. For each singleton set X , the \mathbf{A} -fiber of X has precisely one element.
 - (iii) If ξ and η are \mathbf{A} -structures on a set X such that $1_X : (X, \xi) \rightarrow (X, \eta)$ and $1_X : (X, \eta) \rightarrow (X, \xi)$ are \mathbf{A} -morphisms, then $\xi = \eta$.

Definition 2.8 [27]. A category \mathbf{A} is called a *topological universe over Set* provided that the following conditions hold.

- (1) \mathbf{A} is well structured over \mathbf{Set} , that is, (i) \mathbf{A} is a concrete category; (ii) \mathbf{A} has the fiber-smallness condition; (iii) \mathbf{A} has the terminal separator property.
- (2) \mathbf{A} is cotopological over \mathbf{Set} .
- (3) Final episinks in \mathbf{A} are preserved by pullbacks, that is, for any final episink $(g_\lambda : X \rightarrow Y)_\Lambda$ and any \mathbf{A} -morphism $f : W \rightarrow Y$, the family $(e_\lambda : U_\lambda \rightarrow W)_\Lambda$, obtained by taking the pullback of f and g_λ for each λ , is again a final episink.

Definition 2.9 [29]. A category \mathbf{A} is called a *topos* provided that the following conditions hold.

- (1) There is a terminal object U in \mathbf{A} , that is, for each \mathbf{A} -object A , there exists one and only one \mathbf{A} -morphism from A to U .
- (2) \mathbf{A} has equalizers, that is, for any \mathbf{A} -objects A and B and \mathbf{A} -morphisms

$$A \begin{array}{c} \xrightarrow{f} \\ \xrightarrow{g} \end{array} B
 \tag{2.2}$$

there exist an \mathbf{A} -object C and an \mathbf{A} -morphism $h : C \rightarrow A$ such that

- (a) $f \circ h = g \circ h$,
- (b) for each \mathbf{A} -object C' and \mathbf{A} -morphism $h' : C' \rightarrow A$ with $f \circ h' = g \circ h'$, there exists a unique \mathbf{A} -morphism $\bar{h}' : C' \rightarrow C$ such that $h' = h \circ \bar{h}'$, that is, the diagram

$$\begin{array}{ccc}
 C & \xrightarrow{h} & A \begin{array}{c} \xrightarrow{f} \\ \xrightarrow{g} \end{array} B \\
 \uparrow \exists \bar{h}' & \nearrow h' & \\
 C' & &
 \end{array}
 \tag{2.3}$$

commutes;

- (3) \mathbf{A} is cartesian closed;
- (4) there is a subobject classifier in \mathbf{A} , that is, there is an \mathbf{A} -object Ω and \mathbf{A} -morphism $\nu : U \rightarrow \Omega$ such that for each \mathbf{A} -monomorphism $m : A' \rightarrow A$, there exists a unique \mathbf{A} -morphism $\phi_m : A \rightarrow \Omega$ such that the following diagram is a pullback:

$$\begin{array}{ccc}
 A' & \longrightarrow & U \\
 m \downarrow & & \downarrow \nu \\
 A & \xrightarrow{\phi_m} & \Omega
 \end{array} \tag{2.4}$$

Remark 2.10. Let \mathbf{A} be any category with a subobject classifier. If f is any bimorphism in \mathbf{A} , then f is an isomorphism in \mathbf{A} (cf. [7]).

3. The category $\mathbf{IRel}(H)$

First we will list some concepts and one result which are needed in this section and the next section. Next, we introduce the category $\mathbf{IRel}(H)$ of intuitionistic H -fuzzy relational spaces and show that it has similar structures as those of $\mathbf{ISet}(H)$.

Definition 3.1 [5, 22]. A lattice H is called a *complete Heyting algebra* if H satisfies the following conditions:

- (1) H is a complete lattice;
- (2) for any $a, b \in H$, the set $\{x \in H : x \wedge a \leq b\}$ has a greatest element denoted by $a \rightarrow b$ (called *pseudocomplement of a and b*), that is, $x \wedge a \leq b$ if and only if $x \leq (a \rightarrow b)$.

In particular, for each $a \in H$, $N(a) = a \rightarrow 0$ is called the *negation* or the *pseudocomplement of a* .

RESULT 3.2 [5, Example 6, page 46]. *Let H be a complete Heyting algebra and let $a, b \in H$. Then*

- (1) if $a \leq b$, then $N(b) \leq N(a)$, that is, $N : H \rightarrow H$ is an involutive order-reversing operation in (H, \leq) ;
- (2) $a \leq NN(a)$;
- (3) $N(a) = NNN(a)$;
- (4) $N(a \vee b) = N(a) \wedge N(b)$ and $N(a \wedge b) = N(a) \vee N(b)$.

Throughout this paper, we use H as a complete Heyting algebra.

Definition 3.3 [19]. Let X be a set. A triple (X, μ, ν) is called an *intuitionistic H -fuzzy set* (in short, *IHFS*) on X if the following conditions holds:

- (i) $\mu, \nu \in H^X$, that is, μ and ν are H -fuzzy sets;
- (ii) $\mu \leq N(\nu)$, that is, $\mu(x) \leq N(\nu(x))$ for each $x \in X$, where $N : H \rightarrow H$ is an involutive order-reversing operation in (H, \leq) .

Definition 3.4 [19]. Let (X, μ_X, ν_X) and (Y, μ_Y, ν_Y) be IHFSs. A mapping $f : X \rightarrow Y$ is called a *morphism* if $\mu_X \leq \mu_Y \circ f$ and $\nu_X \geq \nu_Y \circ f$.

From Definitions 3.3 and 3.4, we can form a concrete category $\mathbf{ISet}(H)$ consisting of all IHFSs and morphisms between them. In this case, each $\mathbf{ISet}(H)$ -morphism will be called an *$\mathbf{ISet}(H)$ -mapping*.

It is clear that if $f : (X, \mu_X, \nu_X) \rightarrow (Y, \mu_Y, \nu_Y)$ is an **ISet**(H)-mapping, then $f : (X, \mu_X) \rightarrow (Y, \mu_Y)$ is a **Set**(H)-mapping (cf. [13]).

Definition 3.5 [14]. (1) Let X be a set. R is called an H -fuzzy relation (or simply, a fuzzy relation) on X if $\mu_R : X \times X \rightarrow H$ is a mapping. In this case, (X, R) is called an H -fuzzy relational space (or simply, a fuzzy relational space).

(2) Let (X, R_X) and (Y, R_Y) be any fuzzy relational spaces. A map $f : X \rightarrow Y$ is called a relation-preserving map provided that $\mu_R \leq \mu_R \circ f^2$, where $f^2 = f \times f$.

From Definition 3.5, we can form a concrete category **Rel**(H) consisting of all relational spaces and relation preserving mappings between them. Every **Rel**(H)-morphism will be called a **Rel**(H)-mapping.

Definition 3.6. Let X be a set. A pair $R = (\mu_R, \nu_R)$ is called an intuitionistic H -fuzzy relation (in short, *IHFR*) on X if it satisfies the following conditions:

- (i) $\mu_R : X \times X \rightarrow H$ and $\nu_R : X \times X \rightarrow H$ are mappings, where μ_R and ν_R denote the degree of membership (namely, $\mu_R(x, y)$) and the degree of nonmembership (namely, $\nu_R(x, y)$) of each $(x, y) \in X \times X$ to R ;
- (ii) $\mu_R \leq N(\nu_R)$, that is, $\mu_R(x, y) \leq N(\nu_R(x, y))$ for each $(x, y) \in X \times X$.

In this case, (X, R) or (X, μ_R, ν_R) is called an intuitionistic H -fuzzy relational space (in short, *IHFRS*).

Definition 3.7. Let (X, R_X) and (Y, R_Y) be an *IHFRS*s. A mapping $f : X \rightarrow Y$ is called a relation-preserving mapping if $\mu_{R_X} \leq \mu_{R_Y} \circ f^2$ and $\nu_{R_X} \geq \nu_{R_Y} \circ f^2$, where $f^2 = f \times f$.

The following is the immediate result of Definition 3.7.

PROPOSITION 3.8. Let (X, R_X) , (Y, R_Y) , and (Z, R_Z) be *IHFRS*s.

- (1) $1_X : (X, R_X) \rightarrow (X, R_X)$ is a relation-preserving mapping.
- (2) If $f : (X, R_X) \rightarrow (Y, R_Y)$ and $g : (Y, R_Y) \rightarrow (Z, R_Z)$ are relation-preserving mappings, then $g \circ f : (X, R_X) \rightarrow (Z, R_Z)$ is a relation-preserving mapping.

From Definitions 3.6 and 3.7, and Proposition 3.8, we can form a concrete category **IRel**(H) consisting of all *IHFRS*s and relation-preserving mappings between them. Every **IRel**(H)-morphism will be called an **IRel**(H)-mapping. Moreover, it is clear that if $f : (X, R_X) \rightarrow (Y, R_Y)$ is an **IRel**(H)-mapping, then $f : (X, \mu_{R_X}) \rightarrow (Y, \mu_{R_Y})$ is a **Rel**(H)-mapping.

THEOREM 3.9. **IRel**(H) is topological over **Set**.

Proof. Let X be any set and let $((X_\alpha, R_\alpha))_\Gamma$ be any family of *IHFRS*s indexed by a class Γ . Let $(f_\alpha : X \rightarrow X_\alpha)_\Gamma$ be any source of mappings. We define two mappings $\mu_R : X \times X \rightarrow H$ and $\nu_R : X \times X \rightarrow H$ by $\mu_R = \bigwedge_\Gamma \mu_{R_\alpha} \circ f_\alpha^2$ and $\nu_R = \bigvee_\Gamma \nu_{R_\alpha} \circ f_\alpha^2$. Then, by the definition of $R = (\mu_R, \nu_R)$, $\mu_R \leq N(\nu_R)$. Thus $(X, R) \in \mathbf{IRel}(H)$. Moreover, $f_\alpha : (X, R) \rightarrow (X_\alpha, R_\alpha)$ is an **IRel**(H)-mapping for each $\alpha \in \Gamma$.

For any $(Y, R_Y) \in \mathbf{IRel}(H)$, let $g : Y \rightarrow X$ be any mapping for which $f_\alpha \circ g : (Y, R_Y) \rightarrow (X_\alpha, R_\alpha)$ is an **IRel**(H)-mapping for each $\alpha \in \Gamma$. Then we can easily check that $g : (Y, R_Y) \rightarrow (X, R)$ is an **IRel**(H)-mapping. Hence $R = (\mu_R, \nu_R)$ is the initial structure on X with respect to $(X, (f_\alpha), ((X_\alpha, R_\alpha)))$. This completes the proof. □

Example 3.10. (1) *Inverse image of an IHFR.* Let X be a set, let (Y, R_Y) be an IHFRS, and let $f : X \rightarrow Y$ be any mapping. Then there exists the initial IHFR R on X for which $f : (X, R) \rightarrow (Y, R_Y)$ is an **IRel**(H)-mapping. In this case, R is called the *inverse image* of R_Y under f . In particular, if $X \subset Y$ and $f : X \rightarrow Y$ is the canonical mapping, then (X, R) is called an *intuitionistic H -fuzzy relational subspace* of (Y, R_Y) , where $R = (\mu_R, \nu_R)$ is the inverse image of R_Y under f . In fact, $\mu_R = \mu_{R_Y}|_{X \times X}$ and $\nu_R = \nu_{R_Y}|_{X \times X}$.

(2) *Intuitionistic fuzzy product structure.* Let $((X_\alpha, R_\alpha))_\Gamma$ be any family of IHFRSs and let $X = \prod X_\alpha$ be the product set of $(X_\alpha)_\Gamma$. Then there exists the initial IHFR R on X for which each projection $\pi_\alpha : (X, R) \rightarrow (X_\alpha, R_\alpha)$ is an **IRel**(H)-mapping. In this case, R is called the *product* of $(R_\alpha)_\Gamma$ and is denoted by $R = \prod R_\alpha$ and $(\prod X_\alpha, \prod R_\alpha)$ is called the *intuitionistic H -fuzzy product relational space* of $((X_\alpha, R_\alpha))_\Gamma$. In fact, $\mu_{\text{IIR}} = \bigwedge_\Gamma \mu_{R_\alpha} \circ \pi_\alpha^2$ and $\nu_{\text{IIR}} = \bigvee_\Gamma \nu_{R_\alpha} \circ \pi_\alpha^2$.

In particular, if $H = \{1, 2\}$, then $\mu_{R_1 \times R_2}((x_1, y_1), (x_2, y_2)) = \mu_{R_1}(x_1, x_2) \wedge \mu_{R_2}(y_1, y_2)$ and $\nu_{R_1 \times R_2}((x_1, y_1), (x_2, y_2)) = \nu_{R_1}(x_1, x_2) \vee \nu_{R_2}(y_1, y_2)$ for any $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

COROLLARY 3.11. **IRel**(H) is complete and cocomplete. Moreover, by definition, it is easy to show that **IRel**(H) is well powered and co-well-powered.

From Result 2.4 and Theorem 3.9, it is clear that **IRel**(H) is cotopological. However, we show directly that **IRel**(H) is cotopological.

THEOREM 3.12. **IRel**(H) is cotopological over **Set**.

Proof. Let X be any set and let $((X_\alpha, R_\alpha))_\Gamma$ be any family of IHFRSs indexed by a class Γ . Let $(f_\alpha : X_\alpha \rightarrow X)_\Gamma$ be any sink of mappings. We define two mappings $\mu_R : X \times X \rightarrow H$ and $\nu_R : X \times X \rightarrow H$ by, for each $(x, y) \in X \times X$,

$$\begin{aligned} \mu_R(x, y) &= \bigvee_{\Gamma} \bigwedge_{(x_\alpha, y_\alpha) \in f_\alpha^{-1}(x, y)} \mu_{R_\alpha}(x_\alpha, y_\alpha), \\ \nu_R(x, y) &= \bigwedge_{\Gamma} \bigvee_{(x_\alpha, y_\alpha) \in f_\alpha^{-1}(x, y)} \nu_{R_\alpha}(x_\alpha, y_\alpha), \end{aligned} \tag{3.1}$$

where $f_\alpha^{-1} = f_\alpha^{-1} \times f_\alpha^{-1}$. Then clearly $(X, R) \in \mathbf{IRel}(H)$. Moreover, $f_\alpha : (X_\alpha, R_\alpha) \rightarrow (X, R)$ is an **IRel**(H)-mapping for each $\alpha \in \Gamma$.

For any $(Y, R_Y) \in \mathbf{IRel}(H)$, let $g : X \rightarrow Y$ be any mapping for which $g \circ f_\alpha : (X_\alpha, R_\alpha) \rightarrow (Y, R_Y)$ is an **IRel**(H)-mapping for each $\alpha \in \Gamma$. Then we can easily check that $g : (X, R) \rightarrow (Y, R_Y)$ is an **IRel**(H)-mapping. Hence $R = (\mu_R, \nu_R)$ is the final structure on X with respect to $((X_\alpha, R_\alpha), (f_\alpha, X))$. This completes the proof. \square

Example 3.13. (1) *Intuitionistic H -fuzzy quotient relation.* Let $(X, R) \in \mathbf{IRel}(H)$, let \sim be an equivalence relation on X , and let $\varphi : X \rightarrow X/\sim$ the canonical mapping. Then there exists the final intuitionistic H -fuzzy relation $(\mu_{X/\sim}, \nu_{X/\sim})$ on X/\sim for which $\varphi : (X, R) \rightarrow (X/\sim, \mu_{X/\sim}, \nu_{X/\sim})$ is an **IRel**(H)-mapping. In this case, $(\mu_{X/\sim}, \nu_{X/\sim})$ is called the *intuitionistic H -fuzzy quotient relation* of X by R .

(2) *Sum of intuitionistic H -fuzzy relations.* Let $((X_\alpha, R_\alpha))_\Gamma$ be a family of IHFRSs, let X be the sum of $(X_\alpha)_\Gamma$ and let $j_\alpha : X_\alpha \rightarrow X$ be the canonical (injection) mapping for

each $\alpha \in \Gamma$. Then there exists the final IHFR R on X . In fact, for each $((x_\alpha, \alpha), (y_\beta, \beta)) \in X \times X$, $\mu_R((x_\alpha, \alpha), (y_\beta, \beta)) = \bigvee_\Gamma \mu_{R_\alpha}(x, y)$ and $\nu_R((x_\alpha, \alpha), (y_\beta, \beta)) = \bigwedge_\Gamma \nu_{R_\alpha}(x, y)$. In this case, R is called the *sum* of $(R_\alpha)_\Gamma$ and (X, R) is called the *sum* of $((X_\alpha, R_\alpha))_\Gamma$.

THEOREM 3.14. *Final episinks in $\mathbf{IRel}(H)$ are preserved by pullbacks.*

Proof. Let $(g_\alpha : (X_\alpha, R_\alpha) \rightarrow (Y, R_Y))_\Gamma$ be any final episink in $\mathbf{IRel}(H)$ and let $f : (W, R_W) \rightarrow (Y, R_Y)$ be any $\mathbf{IRel}(H)$ -mapping. For each $\alpha \in \Gamma$, let $U_\alpha = \{(w, x_\alpha) \in W \times X_\alpha : f(w) = g_\alpha(x_\alpha)\}$ and let us define two mappings $\mu_{R_{U_\alpha}} : U_\alpha \times U_\alpha \rightarrow H$ and $\nu_{R_{U_\alpha}} : U_\alpha \times U_\alpha \rightarrow H$ for each $((w, x_\alpha), (w', x'_\alpha)) \in U_\alpha \times U_\alpha$,

$$\begin{aligned} \mu_{R_{U_\alpha}}((w, x_\alpha), (w', x'_\alpha)) &= \mu_{R_W}(w, w') \wedge \mu_{R_\alpha}(x_\alpha, x'_\alpha), \\ \nu_{R_{U_\alpha}}((w, x_\alpha), (w', x'_\alpha)) &= \nu_{R_W}(w, w') \vee \nu_{R_\alpha}(x_\alpha, x'_\alpha). \end{aligned} \tag{3.2}$$

Let $e_\alpha : U_\alpha \rightarrow W$ and $p_\alpha : U_\alpha \rightarrow X_\alpha$ denote the usual projections of U_α . Then clearly $(U_\alpha, R_{U_\alpha}) \in \mathbf{IRel}(H)$ for each $\alpha \in \Gamma$. Moreover, $e_\alpha : (U_\alpha, R_{U_\alpha}) \rightarrow (W, R_W)$ and $p_\alpha : (U_\alpha, R_{U_\alpha}) \rightarrow (X_\alpha, R_\alpha)$ are $\mathbf{IRel}(H)$ -mappings for each $\alpha \in \Gamma$. And the following diagram is a pullback square in $\mathbf{IRel}(H)$:

$$\begin{array}{ccc} (U_\alpha, R_{U_\alpha}) & \xrightarrow{p_\alpha} & (X_\alpha, R_\alpha) \\ e_\alpha \downarrow & & \downarrow g_\alpha \\ (W, R_W) & \xrightarrow{f} & (Y, R_Y) \end{array} \tag{3.3}$$

We will show that $(e_\alpha : (U_\alpha, R_{U_\alpha}) \rightarrow (W, R_W))_\Gamma$ is a final episink in $\mathbf{IRel}(H)$. By the process of the proof of [14, Theorem 2.5], $(e_\alpha)_\Gamma$ is an episink in $\mathbf{IRel}(H)$. Suppose $R = (\mu_R, \nu_R)$ is another final IHFR on W with respect to $(e_\alpha)_\Gamma$. By the process of the proof of [14, Theorem 2.5], $\mu_R = \mu_{R_W}$. Thus it is sufficient to show that $\nu_R = \nu_{R_W}$. Let $(w, w') \in W \times W$. Then

$$\begin{aligned} \nu_{R_W}(w, w') &= \nu_{R_W}(w, w') \vee \nu_{R_W}(w, w') \\ &\geq \nu_{R_W}(w, w') \vee [\nu_{R_Y} \circ f^2(w, w')] \\ &\quad (\text{since } f : (W, R_W) \rightarrow (Y, R_Y) \text{ is an } \mathbf{IRel}(H)\text{-mapping}) \\ &= \nu_{R_W}(w, w') \vee \nu_{R_Y}(f(w), f(w')) \\ &= \nu_{R_W}(w, w') \vee \left[\bigwedge_\Gamma \bigwedge_{(x_\alpha, x'_\alpha) \in g_\alpha^{-1}(f(w), f(w'))} \nu_{R_\alpha}(x_\alpha, x'_\alpha) \right] \\ &\quad (\text{since } (g_\alpha)_\Gamma \text{ is final}) \\ &= \bigwedge_\Gamma \bigwedge_{(x_\alpha, x'_\alpha) \in g_\alpha^{-1}(f(w), f(w'))} [\nu_{R_W}(w, w') \vee \nu_{R_\alpha}(x_\alpha, x'_\alpha)] \\ &= \bigwedge_\Gamma \bigwedge_{((w, x_\alpha), (w', x'_\alpha)) \in e_\alpha^{-1}(w, w')} \nu_{R_{U_\alpha}}((w, x_\alpha), (w', x'_\alpha)). \end{aligned} \tag{3.4}$$

Thus $\nu_{R_W}(w, w') \geq \nu_R(w, w')$ for each $(w, w') \in W \times W$. So $\nu_{R_W} \geq \nu_R$. On the other hand, since $(e_\alpha : (U_\alpha, R_{U_\alpha}) \rightarrow (W, R))_\Gamma$ is final, $1_W : (W, R) \rightarrow (W, R_W)$ is an $\mathbf{IRel}(H)$ -mapping. Thus $\nu_R \geq \nu_{R_W}$. So $\nu_R = \nu_{R_W}$. Hence $R = R_W$. This completes the proof. \square

For any singleton set $\{a\}$, since the IHFR R on $\{a\}$ is not unique, the category $\mathbf{IRel}(H)$ is not properly fibered over \mathbf{Set} . Hence, by Theorems 3.12 and 3.14, we obtain the following result.

THEOREM 3.15. $\mathbf{IRel}(H)$ satisfies all the conditions of a topological universe over \mathbf{Set} except the terminal separator property.

THEOREM 3.16. $\mathbf{IRel}(H)$ is cartesian closed over \mathbf{Set} .

Proof. It is clear that $\mathbf{IRel}(H)$ has products by Corollary 3.11. We will show that $\mathbf{IRel}(H)$ has exponential objects.

For any IHFRSs $\mathbf{X} = (X, R_X)$ and $\mathbf{Y} = (Y, R_Y)$, let Y^X be the set of all mappings from X into Y . We define two mappings $\mu_R : Y^X \times Y^X \rightarrow H$ and $\nu_R : Y^X \times Y^X \rightarrow H$ as follows: for each $(f, g) \in Y^X \times Y^X$,

$$\begin{aligned} \mu_R(f, g) &= \bigwedge \{h \in H : \mu_{R_X}(x, y) \wedge h \leq \mu_{R_Y}(f(x), g(y)) \text{ for each } (x, y) \in X \times X\}, \\ \nu_R(f, g) &= \bigvee \{h \in H : \nu_{R_X}(x, y) \vee h \geq \nu_{R_Y}(f(x), g(y)) \text{ for each } (x, y) \in X \times X\}. \end{aligned} \tag{3.5}$$

Then clearly $(Y^X, R) \in \mathbf{IRel}(H)$. Let $\mathbf{Y}^{\mathbf{X}} = (Y^X, R)$. Then, by the definition of R ,

$$\begin{aligned} \mu_{R_X}(x, y) \wedge \mu_R(f, g) &\leq \mu_{R_Y}(f(x), g(y)), \\ \nu_{R_X}(x, y) \vee \nu_R(f, g) &\geq \nu_{R_Y}(f(x), g(y)) \end{aligned} \tag{3.6}$$

for each $(f, g) \in Y^X$ and $(x, y) \in X \times X$.

Define $e_{X,Y} : X \times Y^X \rightarrow Y$ by $e_{X,Y}(x, f) = f(x)$ for each $(x, f) \in X \times Y^X$. Let $((x, f), (y, g)) \in (X \times Y^X) \times (X \times Y^X)$. Then, by the process of the proof of [14, Theorem 2.7], $\mu_{R_X \times R}((x, f), (y, g)) \leq \mu_{R_Y} \circ e_{X,Y}^2((x, f), (y, g))$. So $\mu_{R_X \times R} \leq \mu_{R_Y} \circ e_{X,Y}^2$. On the other hand,

$$\begin{aligned} \nu_{R_X \times R}((x, f), (y, g)) &= \nu_{R_X}(x, y) \vee \nu_R(f, g) \\ &\geq \nu_{R_Y}(f(x), g(y)) \\ &= \nu_{R_Y}(e_{X,Y}(x, f), e_{X,Y}(y, g)) \\ &= \nu_{R_Y} \circ e_{X,Y}^2((x, f), (y, g)). \end{aligned} \tag{3.7}$$

Thus $\nu_{R_X \times R} \geq \nu_{R_Y} \circ e_{X,Y}^2$. Hence $e_{X,Y} : \mathbf{X} \times \mathbf{Y}^{\mathbf{X}} \rightarrow \mathbf{Y}$ is an $\mathbf{IRel}(H)$ -mapping.

For any $\mathbf{Z} = (Z, R_Z) \in \mathbf{IRel}(H)$, let $h : \mathbf{X} \times \mathbf{Z} \rightarrow \mathbf{Y}$ be an $\mathbf{IRel}(H)$ -mapping. We define $\bar{h} : Z \rightarrow Y^X$ by $[\bar{h}(z)](x) = h(x, z)$ for each $z \in Z$ and each $x \in X$. Let $z, z' \in Z$ and

let $x, x' \in X$. Then, by the process of the proof of [14, Theorem 2.7], $\mu_{R_Z}(z, z') \leq \mu_R \circ \bar{h}^2(z, z')$. So $\mu_{R_Z} \leq \mu_R \circ \bar{h}^2$. On the other hand,

$$\begin{aligned} \nu_{R_X \times R_Z}((x, z), (x', z')) &= \nu_{R_X}(x, x') \vee \nu_{R_Z}(z, z') \\ &\geq \nu_{R_Y} \circ h^2((x, z), (x', z')) \\ &\quad (\text{since } h : \mathbf{X} \times \mathbf{Z} \longrightarrow \mathbf{Y} \text{ is an } \mathbf{IRel}(H)\text{-mapping}) \quad (3.8) \\ &= \nu_{R_Y}(h(x, z), h(x', z')) \\ &= \nu_{R_Y}([\bar{h}(z)](x), [\bar{h}(z')](x')). \end{aligned}$$

Thus, by the definition of R , $\nu_{R_Z}(z, z') \geq \nu_R(\bar{h}(z), \bar{h}(z')) = \nu_{R_Y} \circ \bar{h}^2(z, z')$. So $\nu_{R_Z} \geq \nu_R \circ \bar{h}^2$. Hence $\bar{h} : \mathbf{Z} \rightarrow \mathbf{Y}^X$ is an $\mathbf{IRel}(H)$ -mapping. Moreover, \bar{h} is the unique $\mathbf{IRel}(H)$ -mapping such that $e_{X, Y} \circ (1_X \times \bar{h}) = h$. This completes the proof. \square

Remark 3.17. $\mathbf{IRel}(H)$ has no subobject classifier. Hence $\mathbf{IRel}(H)$ is not topos.

Example 3.18. Let $H = \{0, 1\}$ be the two points chain and let $X = \{a\}$. Let R_1 and R_2 be the IHFRs on X given by $\mu_{R_1}(a, a) = 0, \nu_{R_1}(a, a) = 1$ and $\mu_{R_2}(a, a) = 1, \nu_{R_2}(a, a) = 0$. Let $1_X : (X, R_1) \rightarrow (X, R_2)$ be the identity mapping. Then clearly, 1_X is both a monomorphism and an epimorphism in $\mathbf{IRel}(H)$. But, 1_X is not an isomorphism in $\mathbf{IRel}(H)$. Hence $\mathbf{IRel}(H)$ has no subobject classifier (see [7]).

4. The relations between $\mathbf{IRel}(H)$ and $\mathbf{Rel}(H)$

LEMMA 4.1. Define $G_1, G_2 : \mathbf{IRel}(H) \rightarrow \mathbf{Rel}(H)$ by

$$\begin{aligned} G_1(X, \mu_R, \nu_R) &= (X, \mu_R), \\ G_2(X, \mu_R, \nu_R) &= (X, N(\nu_R)), \\ G_1(f) &= G_2(f) = f. \end{aligned} \quad (4.1)$$

Then G_1 and G_2 are functors.

Proof. Clearly $G_1(X, \mu_{R_X}, \nu_{R_X}) = (X, \mu_{R_X}) \in \mathbf{Rel}(H)$ for each $(X, \mu_R, \nu_R) \in \mathbf{IRel}(H)$. Let $(X, \mu_{R_X}, \nu_{R_X}), (Y, \mu_{R_Y}, \nu_{R_Y}) \in \mathbf{IRel}(H)$ and let $f : (X, \mu_{R_X}, \nu_{R_X}) \rightarrow (Y, \mu_{R_Y}, \nu_{R_Y})$ be an $\mathbf{IRel}(H)$ -mapping. Then $\mu_{R_X} \leq \mu_{R_Y} \circ f^2$. Thus $G_1(f) = f : (X, \mu_{R_X}) \rightarrow (Y, \mu_{R_Y})$ is a $\mathbf{Rel}(H)$ -mapping. Hence $G_1 : \mathbf{IRel}(H) \rightarrow \mathbf{Rel}(H)$ is a functor. Also $G_2(X, \mu_{R_X}, \nu_{R_X}) = (X, N(\nu_{R_X})) \in \mathbf{Rel}(H)$ for each $(X, \mu_{R_X}, \nu_{R_X}) \in \mathbf{IRel}(H)$. Now let $(X, \mu_{R_X}, \nu_{R_X}), (Y, \mu_{R_Y}, \nu_{R_Y}) \in \mathbf{IRel}(H)$ and let $f : (X, \mu_{R_X}, \nu_{R_X}) \rightarrow (Y, \mu_{R_Y}, \nu_{R_Y})$ be an $\mathbf{IRel}(H)$ -mapping. Then $\nu_{R_X} \geq \nu_{R_Y} \circ f^2$. Thus $N(\nu_{R_X}) \leq N(\nu_{R_Y}) \circ f^2$. So $G_2(f) = f : (X, N(\nu_{R_X})) \rightarrow (Y, N(\nu_{R_Y}))$ is a $\mathbf{Rel}(H)$ -mapping. Hence $G_2 : \mathbf{IRel}(H) \rightarrow \mathbf{Rel}(H)$ is a functor. \square

LEMMA 4.2. Define $F_1 : \mathbf{Rel}(H) \rightarrow \mathbf{IRel}(H)$ by $F_1(X, \mu_R) = (X, \mu_R, N(\mu_R))$ and $F_1(f) = f$. Then F_1 is a functor.

Proof. For each $(X, \mu_{R_X}) \in \mathbf{Rel}(H)$, $\mu \leq NN(\mu_{R_X})$. Thus $F_1(X, \mu_{R_X}) = (X, \mu_{R_X}, N(\mu_{R_X})) \in \mathbf{IRel}(H)$. Let $(X, \mu_{R_X}), (Y, \mu_{R_Y}) \in \mathbf{Rel}(H)$ and let $f : (X, \mu_{R_X}) \rightarrow (Y, \mu_{R_Y})$ be an $\mathbf{Rel}(H)$ -mapping. Then $\mu_{R_X} \leq \mu_{R_Y} \circ f$. Consider the mapping $F_1(f) = f : (X, \mu_{R_X}, N(\mu_{R_X})) \rightarrow (Y, \mu_{R_Y}, N(\mu_{R_Y}))$. Since $\mu_{R_X} \leq \mu_{R_Y} \circ f$, $N(\mu_{R_X}) \geq N(\mu_{R_Y}) \circ f$. So $f : (X, \mu_{R_X}, N(\mu_{R_X})) \rightarrow (Y, \mu_{R_Y}, N(\mu_{R_Y}))$ is an $\mathbf{IRel}(H)$ -mapping. Hence F_1 is a functor. \square

LEMMA 4.3. Define $F_2 : \mathbf{Rel}(H) \rightarrow \mathbf{IRel}(H)$ by $F_2(X, \mu_R) = (X, NN(\mu_R), N(\mu_R))$ and $F_2(f) = f$. Then F_2 is a functor.

Proof. It is clear that $F_2(X, \mu_{R_X}) \in \mathbf{IRel}(H)$ for each $(X, \mu_{R_X}) \in \mathbf{Rel}(H)$. Let $(X, \mu_{R_X}), (Y, \mu_{R_Y}) \in \mathbf{Rel}(H)$ and let $f : (X, \mu_{R_X}) \rightarrow (Y, \mu_{R_Y})$ be an $\mathbf{Rel}(H)$ -mapping. Consider the mapping $F_2(f) = f : F_2(X, \mu_{R_X}) \rightarrow (Y, NN(\mu_{R_Y}), N(\mu_{R_Y}))$, where $F_2(X, \mu_{R_X}) = (X, NN(\mu_{R_X}), N(\mu_{R_X}))$ and $F_2(Y, \mu_{R_Y}) = (Y, NN(\mu_{R_Y}), N(\mu_{R_Y}))$. Since $f : (X, \mu_{R_X}) \rightarrow (Y, \mu_{R_Y})$ is a $\mathbf{Rel}(H)$ -mapping, $\mu_{R_X} \leq \mu_{R_Y} \circ f^2$. Thus $NN(\mu_{R_X}) \leq NN(\mu_{R_Y}) \circ f^2$. Moreover $N(\mu_{R_X}) \geq N(\mu_{R_Y}) \circ f^2$. So $F_2(f) = f : F_2(X, \mu_{R_X}) \rightarrow F_2(Y, \mu_{R_Y})$ is an $\mathbf{IRel}(H)$ -mapping. Hence F_2 is a functor. \square

THEOREM 4.4. The functor $F_1 : \mathbf{Rel}(H) \rightarrow \mathbf{IRel}(H)$ is a left adjoint of the functor $G_1 : \mathbf{IRel}(H) \rightarrow \mathbf{Rel}(H)$.

Proof. For each $(X, \mu_R) \in \mathbf{Rel}(H)$, $1_X : (X, \mu_R) \rightarrow G_1 F_1(X, \mu_R) = (X, \mu_R)$ is a $\mathbf{Rel}(H)$ -mapping. Let $(Y, \mu_{R_Y}, \nu_{R_Y}) \in \mathbf{IRel}(H)$ and let $f : (X, \mu_R) \rightarrow G_1(Y, \mu_{R_Y}, \nu_{R_Y})$ be an $\mathbf{IRel}(H)$ -mapping. We will show that $f : F_1(X, \mu_R) = (X, \mu_R, N(\mu_R)) \rightarrow (Y, \mu_{R_Y}, \nu_{R_Y})$ is an $\mathbf{IRel}(H)$ -mapping. Since $f : (X, \mu_R) = G_1(Y, \mu_{R_Y}, \nu_{R_Y}) \rightarrow (Y, \mu_{R_Y})$ is a $\mathbf{Rel}(H)$ -mapping, $\mu_R \leq \mu_{R_Y} \circ f^2$. Then $N(\mu_R) \geq N(\mu_{R_Y}) \circ f^2$. Since $\mu_{R_Y} \leq N(\nu_{R_Y})$, $\nu_{R_Y} \leq NN(\nu_{R_Y}) \leq N(\mu_{R_Y})$. Thus $N(\mu_R) \geq \nu_{R_Y} \circ f^2$. So $f : F_1(X, \mu_R) = (X, \mu_R, N(\mu_R)) \rightarrow (Y, \mu_{R_Y}, \nu_{R_Y})$ is an $\mathbf{IRel}(H)$ -mapping. Hence 1_X is a G_1 -universal map for (X, μ_R) in $\mathbf{Rel}(H)$. This completes the proof. \square

For each $(X, \mu_R) \in \mathbf{Rel}(H)$, $F_1(X, \mu_R) = (X, \mu_R, N(\mu_R))$ is called an *intuitionistic H -fuzzy set in X induced by (X, μ_R)* . Let us denote the category of all induced intuitionistic H -fuzzy sets and $\mathbf{IRel}(H)$ -mappings as $\mathbf{IRel}^*(H)$. Then it is clear $\mathbf{IRel}^*(H)$ is a full subcategory of $\mathbf{IRel}(H)$.

THEOREM 4.5. Two categories $\mathbf{Rel}(H)$ and $\mathbf{IRel}^*(H)$ are isomorphic.

Proof. It is clear that $F_1 : \mathbf{Rel}(H) \rightarrow \mathbf{IRel}^*(H)$ is a functor by Lemma 4.2. Consider the restriction $G_1 : \mathbf{IRel}^*(H) \rightarrow \mathbf{Rel}(H)$ of the functor G_1 in Lemma 4.1. Let $(X, \mu_R) \in \mathbf{Rel}(H)$. Then, by Lemma 4.2, $F_1(X, \mu_R) = (X, \mu_R, N(\mu_R))$. Thus $G_1 F_1(X, \mu_R) = G_1(X, \mu_R, N(\mu_R)) = (X, \mu_R)$. So $G_1 \circ F_1 = 1_{\mathbf{Rel}(H)}$. Now let $(X, \mu_R, N(\mu_R)) \in \mathbf{IRel}^*(H)$. Then, by Lemma 4.1, $G_1(X, \mu_R, N(\mu_R)) = (X, \mu_R)$. Thus $F_1 G_1(X, \mu_R, N(\mu_R)) = (X, \mu_R, N(\mu_R))$. So $F_1 \circ G_1 = 1_{\mathbf{IRel}^*(H)}$. Hence $F_1 : \mathbf{Rel}(H) \rightarrow \mathbf{IRel}^*(H)$ is an isomorphism. This completes the proof. \square

Remark 4.6. We are going to investigate “intuitionistic H -fuzzy reflexive relations,” “some subcategories of the category $\mathbf{IRel}(H)$,” and “intuitionistic H -fuzzy relations on intuitionistic H -fuzzy sets” in the viewpoint of topological universe.

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