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Intuitionistic Trapezoidal Fuzzy based Aggregation Operator: Applications in Medical Diagnosis

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Abstract: *Since the inception of fuzzy sets given by Zadeh, uncertainty arises due to partial information or imprecise information has been measured. The generalized version of fuzzy sets has been introduced by Atanassov, known as intuitionistic fuzzy sets (IFSs), which have wide applications in decision making processes and consider both membership and non-membership functions. A generalized fuzzy based decision making method with aggregation operator has been discussed with application in medical diagnosis. In this paper, IF based method have been discussed for the application of the diagnosis of the type of child cancer.*

Keywords: *Fuzzy sets, Medical decision making, Fuzzy information theory, Childhood cancer*

I. INTRODUCTION

Population across the globe is increasing exponentially and there is a shortage of basic healthcare facilities in most countries. Many life-threatening diseases such as cancer are spreading like anything and the diagnostic facilities are not enough to diagnose the diseases at the initial level. The available diagnostic facilities are very costly and beyond the reach of the common man. Also, doctors in the medical health care centers are not sufficient to treat the diseases. Due to the evolution of many innovative techniques, the diagnosis of diseases is easy and cost-effective, which not only helps the patients but also enhances the efficiency of the doctors. With the help of computational techniques, medical diagnosis becomes transparent and becomes a decision-making tool for doctors to execute the right treatment among the available treatments. Among these techniques, soft computing techniques are useful in decision-making under uncertainty. Atanassov[1986] introduced IFS as a generalization of fuzzy sets, which are effective in describing imprecise or uncertain decision information and can be applied to the problems of decision-making. This theory can be proved as a useful tool for the selection of the best treatment for diseases. In this paper, the diagnosis of the type of childhood cancer has been discussed on the basis of the given information to patients.

According to (Steliarova et al. 2017), every year approximately 3 lakh children aged between 0 to 19 years are diagnosed with childhood cancer which includes brain cancer, leukemia, solid tumors, etc. Howard et al. (2018), the recovery rate of the disease is more in high-income countries, which is more than 80% as compared to low and middle-income countries, which is about 20%. In childhood cancer, most types of cancer can be cured with medicines but some forms can only be tackled with surgery and radiotherapy. In low and middle-income countries, most deaths result from lack of diagnosis, delay in diagnosis, lapses in care, etc. Therefore, in such countries, computational techniques have been useful for the purpose of diagnosis. Hwang & Yoon (1981), MADM problems contribute to decision-making for evaluation, prioritization, and selection over the available alternatives which are conflicting and multiple in nature

II. INFORMATION THEORETIC MEASURES

For a random variable, entropy is a quantitative measure, used to calculate uncertainty. Shannon (1948), coined the term entropy, which gave birth to a separate discipline namely Information theory (Ash

1965). He defined the concepts of information in mathematical insight and proved many general results and their consequences. Since then, entropy has been extensively used in different domains including quantum information theory, communication theory, image recognition, finance, decision-making, etc. Zadeh (1969) anticipated that fuzzy theory can handle the problems of medical diagnosis very well to deal with imprecise and uncertain information gathered by humans. The primary characteristic of a fuzzy theory is that it accepts the information in both qualitative and quantitative ways and can develop an ecosystem, which describes it by simple human-friendly rules. In any diagnostic process, a general medical knowledge base is imprecise and uncertain, which is a relationship between the symptoms and the disease. Information about patients is gathered from past history, clinical examination, pathological test results, and other available investigative procedures. Many researchers have defined the IF entropy from different viewpoints, such as probabilistic viewpoint, non-probabilistic viewpoint, and geometric viewpoint. Over the past few years, many researchers have proposed different entropy measures, including (Verma & Sharma 2013), (Mishra 2016), (Wei 2012), (Wang & Zhang 2009a), (Liu & Ren 2014). They have applied these entropies to different real-world problem.

III. DECISION MAKING

In decision-making, one alternative is selected among the available alternatives. In the deterministic process, decision-making can be an optimization problem but in non-deterministic problems, making decisions is challenging because of the uncertainty involved in it. Therefore, the criteria for evaluation of alternatives are difficult for decision-makers due to ambiguous information. In such a situation, fuzzy set theory can model and handle vague information and situation very well. The medical information-based computational techniques optimize decisions in each step to bridge the gaps which occur during the treatment. These techniques take care of all types of complexities and navigate the patients and doctors by offering other candidate alternatives. Any form of illness present in the human body is deadly and the trajectory of the disease from diagnosis to treatment passes through certain challenging decisions. Information-based management systems could be the key factor in understanding the diagnosis of diseases, making decisions, and if possible, reconfiguring the treatment. Many decision-making models focus on one criterion and have limited applications; therefore, they fail in most the real-life situations.

IV. MEDICAL DIAGNOSIS

Globally, thousands of people die every year due to errors in the diagnosis of diseases. Like other domains, the medical domain is characterized by an exponential evolution of knowledge. There are many computational tools related to medical diagnosis, which can reduce the risk of errors and have many advantages. Medical diagnosis begins when a patient consults a doctor. The doctor evaluates the whole situation of the patient and prepares a knowledge base to prescribe the suitable treatment. If required, the whole process might be iterated and reconfigured, refined, or even rejected to diagnose the diseases.

V. BASIC PRELIMINARIES

Zadeh (1965) introduced the concept of fuzzy sets as an extension of the classical set. Atanassov (1986) introduced intuitionistic fuzzy sets as a generalization of fuzzy sets. Dubios & Prade (1980) introduced fuzzy numbers and (Burillo, Bustince & Mohedano 1994) introduced IFN. Wang (2008) applied the concept of ITrFN.

A. Aggregation

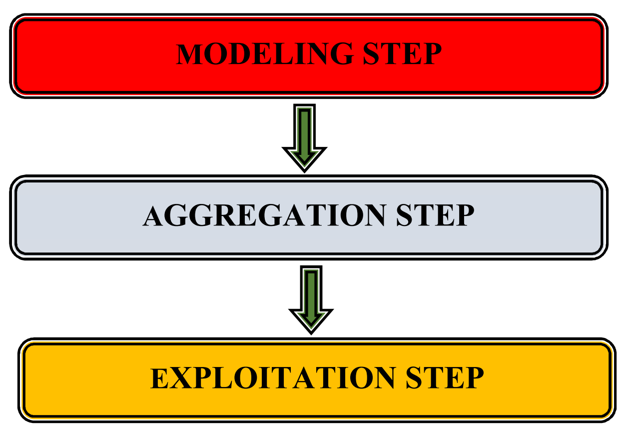

Figure 1: Steps in MCDM Problems

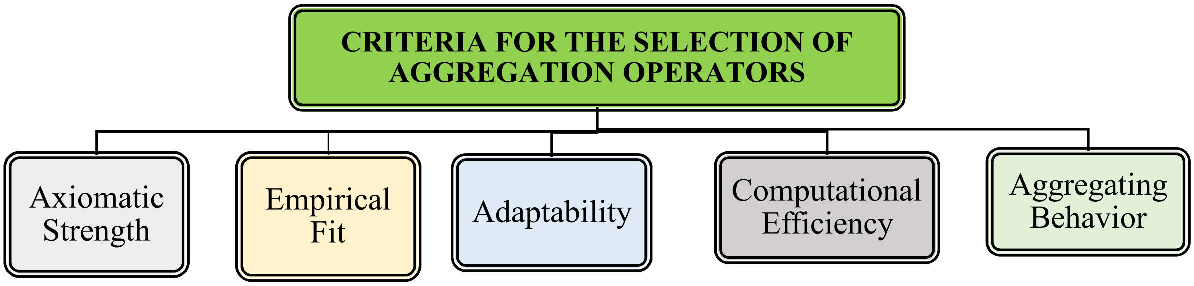

Figure 2: Criteria for the Selection of Aggregation Operator

The process of representing a collection of data with one representative value is called aggregation, the representative value could be some average, maximum, or minimum value. Over the last many years, many MCDM techniques have been proposed by using various types of aggregation operators under a fuzzy environment with the assumption that both the criteria and the decision-makers are at the same level of priority. Aggregation operators in a fuzzy environment are used in combining the finite set of numerical values into a single numerical value and are the essential components in solving MCDM problems. Marichal (1998), every MCDM problem consists of the three main steps given in Figure 1. Various generalized fuzzy aggregation operators are available for the purpose of decision-making. It is very difficult to choose a suitable aggregation operator. The criteria for the selection of operators are given in Figure 2.

B. Dynamic Intuitionistic Fuzzy Weighted Geometric (DIFWG) Aggregation Operator

Wei (2009), IF decision matrix $R(t_k) = (r_{ij}(t_k))_{m\times n}$ is aggregated into a complex IF decision matrix $R = (r_{ij})_{n\times m}$, where $r_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$

DIFWG operator is defined as

$$r_{ij} = DIFWG_{\alpha(t)}\left(r_{ij}(t_1), r_{ij}(t_2), \ldots, r_{ij}(t_p)\right)$$

VI. LITERATURE REVIEW

The problems of MADM are interdisciplinary, which can help in the process of decision making. When it is used with generalized fuzzy sets, it gives more strength to the concept. The domains of IFSs and FSs are discrete for dealing with imperfect and imprecise information. Wang & Zhang (2008) explicitly defined the expected values of ITrFN and explained the MCDM with imprecise and incomplete information. Wang & Zhang (2009b) and (Guorong 2011) presented some aggregation operators of ITrFWAA over the expected values of ITrFN for better decision making. They investigated that the ITrF information measure approach was better to deal with MADM problems. Wan & Dong (2010) defined the expected score of ITrFNs across the geometric point and presented the ITrFOWA operator and its hybridization. Yager (2004, 2008, 2009) proposed prioritized operators to streamline decision-making. Wei (2012) discussed the generalized concept of prioritized aggregation operators proposed by (Yager 2004, 2009) in a hesitant fuzzy environment and introduced some operators of hesitant fuzzy prioritized aggregation. Yu & Xu (2013) described the prioritization relationship of attributes over MADM under the IF information environment and present some prioritized operators of IF aggregation for the sake of MADM. But these operators have certain limitations that these operators cannot be used with ITrFNs and difficulty in implementation in MADM problems.

where, $$r_{ij} = \left(1 - \prod_{k=1}^{k=p}\left(1 - \mu_{r_{ij}(t_k)}\right)^{\alpha(t_k)}, \prod_{k=1}^{k=p} \nu_{r_{ij}(t_k)}^{\alpha(t_k)}, 1 - \prod_{k=1}^{k=p}\left(1 - \mu_{r_{ij}(t_k)}\right)^{\alpha(t_k)} - \prod_{k=1}^{k=p} \nu_{r_{ij}(t_k)}^{\alpha(t_k)}\right)$$

thus, $\mu_{ij} = 1 - \prod_{k=1}^{k=p}\left(1 - \mu_{r_{ij}(t_k)}\right)^{\alpha(t_k)}$; $\nu_{ij} = \prod_{k=1}^{k=p} \nu_{r_{ij}(t_k)}^{\alpha(t_k)}$ and $\pi_{ij} = 1 - \prod_{k=1}^{k=p}\left(1 - \mu_{r_{ij}(t_k)}\right)^{\alpha(t_k)} - \prod_{k=1}^{k=p} \nu_{r_{ij}(t_k)}^{\alpha(t_k)}$

To overcome these shortcomings, (Zhang 2014), proposed some prioritized operators that work well for ITrF information. Also, they are considering prioritization among the input arguments. Gani, Sritharan & Kumar (2011) proposed the WAR method for decision-making problems using ITrFHA operators.

An algorithmic approach based on the DIFWG operator has been used for the selection of the type of Childhood Cancer over certain attributes with the IFNs information. The aim behind the proposal is to establish a decision support system, which not only helps the doctors but also provides transparent support to the patient regarding the treatment. The support system gathers the initial information from three domain experts and finally, with the help of an algorithm, the system ranks the diseases. The results obtained from the findings can help doctors to select the type of disease.

For this purpose, a hypothetical case study has been considered to explain the algorithm. The expert doctors provide their views in the form of linguistic terms, which are then converted to IFNs. Aggregation operators are used to aggregating the information, as information is received through multiple sources. The ranking of alternatives is done using the closeness coefficient, and then best alternative is selected out of the available alternatives

VII. TYPES OF CHILDHOOD CANCER

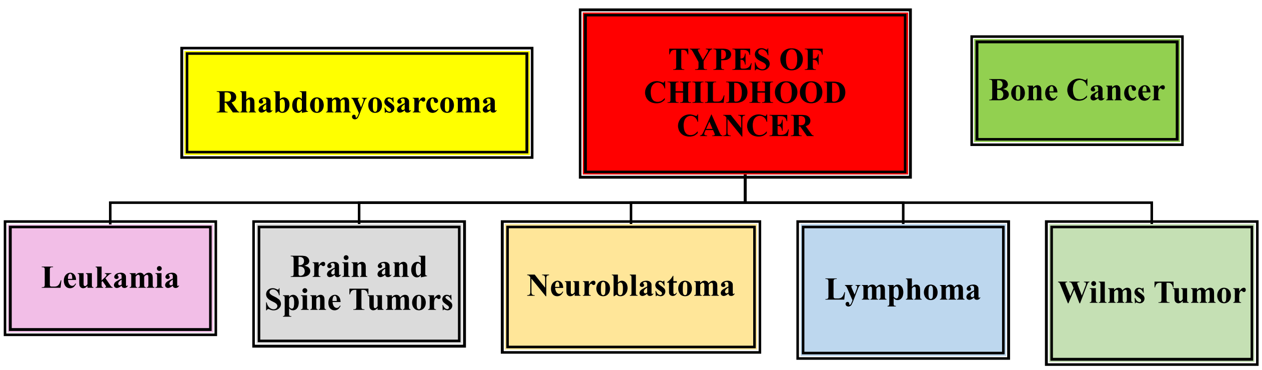

Figure 3. Types of Childhood Cancer

VIII. RELATION BETWEEN SYMPTOMS AND TYPE OF THE CHILDHOOD CANCER

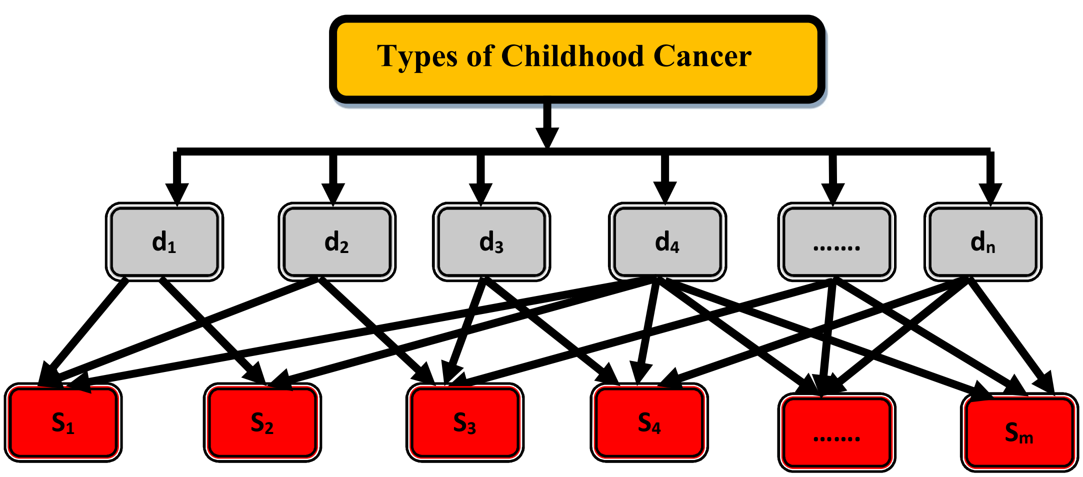

Figure 4. Relationship between the Type of Diseases and their Respective Symptoms for Childhood Cancer

If the correct diagnosis for the disease has been done, then the blueprint for the treatment can be framed. There are certain life-threatening diseases whose diagnosis cannot be easily found because of a variety of reasons. Childhood cancer is one such disease, whose diagnosis is not a cup of tea for the decision-makers, as there are many symptoms, which are common among certain illnesses. The nature of the treatment depends on the diagnosis of the disease and moreover, the seriousness of the disease.

Therefore, there is a very close tradeoff between the diseases and the symptoms. In any case, the type of disease provides the roadmap to curing the disease.

Let $D = \{ d_1, d_2, \ldots, d_n \}$ and $S = \{ s_1, s_2, \ldots, s_m \}$ be the finite set of the type of diseases and symptoms respectively. The relationship between the type of diseases and symptoms for Childhood Cancer are given in figure 2

IX. CASE STUDY: SELECTION OF THE TYPE OF CHILDHOOD CANCER

Childhood cancer is difficult to detect as compared to cancer in adults. Approximately 80% of death occurs from childhood cancer. Dynamic intuitionistic fuzzy weighted geometric aggregation (DIFWG) operator has been used for the selection of the type of childhood cancer and ranks them according to the closeness coefficient.

The algorithm based on the DIFWG operator for selection from the alternatives $x_i\ (i=1,2,\ldots,7)$ is given as:

Step I: Obtain IF decision matrices from three decision makers in different period of time.

Step II: Utilize DIFWG operator to aggregate result of obtained from three decision makers.

Step III: Define Intuitionistic fuzzy positive ideal solution (IFPIS) $\lambda^+ = (\lambda_1^+, \lambda_2^+, \ldots, \lambda_m^+)$ and Intuitionistic fuzzy negative ideal solution (IFNIS) $\lambda^- = (\lambda_1^-, \lambda_2^-, \ldots, \lambda_m^-)$.

Step IV: Calculate relative closeness to the ideal solutions i.e. closeness coefficient as:

$$C(x_i) = \frac{d_i^-}{d_i^+ + d_i^-}; i = 1, 2, \ldots, n$$

where

$$d_i^+ = \sqrt{\Sigma(x_i - \lambda^+)}; \quad d_i^- = \sqrt{\Sigma(x_i - \lambda^-)}$$

Step V: Rank all the alternatives $x_i\ (i=1,2,\ldots,7)$ according to closeness coefficient.

Step VI: Select the best option.

With the help of three experts, a knowledge base has been formed based on symptoms and the available set of types of childhood cancer among patients. Finally, the prominent type of disease has been selected among the available set of diseases by using the said algorithm:

Let $x_i\ (i=1,2,..,7)$ be the set of common type of diseases for childhood cancer, which could be Leukemia, Brain and spine tumors, Neuroblastoma, Wilms tumor, Lymphoma, Rhabdomyosarcoma, Bone cancer respectively. Let $G_i\ (i=1,2,3,4)$ be the set of symptoms present in the patients, which could be fever/ headache, body pain and joint pain / swelling, cough / vomiting, weakness / weight loss respectively.

Let $k(t) = \left(\frac{1}{6}, \frac{2}{6}, \frac{3}{6}\right)^T$ be the weight vector of the decision maker's $t_k\left(k = 1, 2, 3\right)$, at different period and $w = \left(0.1, 0.2, 0.3, 0.4\right)^T$ be the weight vector of the attributes $G_j\left(j = 1, 2, 3, 4\right)$.

The information given by three decision makers are present in Tables 1,2 and 3. The collective information provided by the three decision makers using DIFWG operator is given in Table 4.

Table 1: IF Decision matrix given by decision maker $R(t_1)$

	G_1	G_2	G_3	G_4
x_1	(0,8,0.1,0.1)	(0.9,0.1,0.0)	(0.7,0.2,0.1)	(0.7,0.2,0.1)
x_2	(0.7,0.3,0.0)	(0.6,0.2,0.2)	(0.6,0.3,0.1)	(0.5,0.2,0.3)
x_3	(0.5,0.4,0.1)	(0.7,0.3,0.0)	(0.6,0.1,0.3)	(0.4,0.6,0.2)
x_4	(0.9,0.1,0.0)	(0.7,0.1,0.2)	(0.8,0.2,0.0)	(0.7,0.1,0.2)
x_5	(0.6,0.1,0.3)	(0.8,0.2,0.0)	(0.5,0.1,0.4)	(0.2,0.4,0.4)
x_6	(0.3,0.6,0.1)	(0.5,0.4,0.1)	(0.4,0.5,0.1)	(0.2,0.7,0.1)
x_7	(0.5,0.2,0.3)	(0.4,0.6,0.0)	(0.5,0.5,0.0)	(0.1,0.8,0.1)

Table 2: IF Decision matrix given by decision maker $R(t_2)$

	G_1	G_2	G_3	G_4
x_1	(0.9,0.1,0.0)	(0.8,0.2,0.0)	(0.8,0.1,0.1)	(0.6,0.3,0.1)
x_2	(0.8,0.2,0.0)	(0.5,0.1,0.4)	(0.7,0.2,0.1)	(0.4,0.3,0.3)
x_3	(0.5,0.5,0.0)	(0.7,0.2,0.1)	(0.8,0.2,0.0)	(0.7,0.1,0.2)
x_4	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.7,0.3,0.0)	(0.3,0.5,0.2)
x_5	(0.5,0.2,0.3)	(0.6,0.3,0.1)	(0.6,0.2,0.2)	(0.6,0.1,0.3)
x_6	(0.4,0.6,0.0)	(0.3,0.4,0.3)	(0.5,0.5,0.0)	(0.2,0.3,0.5)
x_7	(0.3,0.5,0.2)	(0.5,0.3,0.2)	(0.6,0.4,0.0)	(0.1,0.5,0.4)

Table 3: IF Decision matrix given by decision maker $R(t_3)$

	G_1	G_2	G_3	G_4
x_1	(0.7,0.1,0.2)	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.6,0.1,0.3)
x_2	(0.9,0.1,0.0)	(0.6,0.2,0.2)	(0.5,0.2,0.3)	(0.5,0.2,0.3)
x_3	(0.4,0.5,0.1)	(0.8,0.1,0.1)	(0.7,0.1,0.2)	(0.3,0.3,0.4)
x_4	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.1,0.0)	(0.4,0.4,0.2)
x_5	(0.6,0.3,0.1)	(0.8,0.2,0.0)	(0.7,0.2,0.1)	(0.5,0.5,0.0)
x_6	(0.2,0.7,0.1)	(0.5,0.1,0.4)	(0.3,0.1,0.6)	(0.1,0.4,0.5)
x_7	(0.4,0.6,0.0)	(0.7,0.3,0.0)	(0.5,0.5,0.0)	(0.2,0.3,0.5)

Table 4: Collective IF Decision matrix R

	G_1	G_2	G_3	G_4
x_1	(0.806,0.100,0.094)	(0.874,0.126,0.000)	(0.849,0.112,0.039)	(0.619,0.162,0.219)
x_2	(0.849,0.151,0.000)	(0.569,0.159,0.272)	(0.594,0.214,0.192)	(0.469,0.229,0.302)
x_3	(0.452,0.482,0.066)	(0.755,0.015,0.094)	(0.725,0.126,0.149)	(0.486,0.233,0.281)
x_4	(0.859,0.100,0.041)	(0.792,0.141,0.067)	(0.838,0.162,0.000)	(0.437,0.342,0.221)
x_5	(0.569,0.218,0.213)	(0.748.0.229,0.023)	(0.640,0.178,0.182)	(0.510,0.282,0.208)
x_6	(0.289,0.648,0.063)	(0.441,0.200,0.359)	(0.390,0.224,0.383)	(0.151,0.399,0.449)
x_7	(0.387,0.470,0.143)	(0.601,0.337,0.062)	(0.536,0.464,0.000)	(0.151,0.512,0.337)

The (IFPIS) λ^+ and (IFNIS) λ^- for each alternative $x_i\ (i=1,2,...,7)$ are given as:

$$\lambda^+ = \left((1,0,0),(1,0,0),(1,00)\right)^T ;\ \lambda^- = \left((0,1,0),\ (0,1,0),\ (0,1,0)\right)^T$$

$$x_1 = \left((0.806,0.100,0.094),(0.874,0.126,0.000),(0.849,0.112,0.039),(0.619,0.162,0.219)\right)^T$$
$$x_2 = \left((0.849,0.151,0.000),(0.569,0.159,0.272),(0.594,0.214,0.192),(0.469,0.229,0.302)\right)^T$$
$$x_3 = \left((0.452,0.482,0.066),(0.755,0.015,0.094),(0.725,0.126,0.149),(0.486,0.233,0.281)\right)^T$$
$$x_4 = \left((0.859,0.100,0.041),(0.792,0.141,0.067),(0.838,0.162,0.000),(0.437,0.342,0.221)\right)^T$$
$$x_5 = \left((0.569,0.218,0.213),(0.748.0.229,0.023),(0.640,0.178,0.182),(0.510,0.282,0.208)\right)^T$$
$$x_6 = \left((0.289,0.648,0.063),(0.441,0.200,0.359),(0.390,0.224,0.383),(0.151,0.399,0.449)\right)^T$$
$$x_7 = \left((0.387,0.470,0.143),(0.601,0.337,0.062),(0.536,0.464,0.000),(0.151,0.512,0.337)\right)^T$$

Table 5: Closeness Coefficient $C(x_i)$

$C(x_1)$	$C(x_2)$	$C(x_3)$	$C(x_4)$	$C(x_5)$	$C(x_6)$	$C(x_7)$
0.781456	0.646739	0.691586	0.702053	0.658417	0.486959	0.466294

Figure 5: Closeness Coefficient of the Type of Childhood Cancer

The ranking of the alternatives as per the closeness coefficient is given as:

$$x_1 > x_4 > x_3 > x_5 > x_2 > x_6 > x_7.$$

The closeness coefficient for each alternative is given in Table 5 and depicted in Figure 5.

X. RESULTS AND DISCUSSION

Based on the closeness coefficient given in Figure 5, it is concluded that the child is suffering from Leukemia. Thus, the use of aggregation operators and ranking methods provide the information to the decision-makers to take a suitable decision. The algorithm used can be applied to other decision-making problems.

XI. CONCLUSION

The multi-criteria techniques are useful in the problems of decision-making with uncertainty associated with it. The use of aggregation operators handle the uncertain situation effectively and propose better decision and rank it as per the available information. In this paper, the diagnosis of the type of childhood cancer has been identified using the proposed algorithm. From the available information, it is concluded that the general the type of cancer in children is Leukemia.

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