INVARIANT AMPLITUDES FOR VIRTUAL COMPTON SCATTERING OFF POLARIZED NUCLEONS FREE FROM KINEMATICAL SINGULARITIES, ZEROS AND CONSTRAINTS

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A B S T R A C T

A complete set of independent gauge invariant and Lorentz covariant tensors which carry the maximal amount of kinematics and which are either even or odd under the discrete symmetries and crossing is given for virtual photon scattering off polarized nucleons. The remaining kinematics is explicitly shown in the amplitudes. A great variety of different processes of physical interest may be described with this set by taking the adequate limits. The possible connections among these different limits are now open.

It is our purpose to undertake and, hopefully, solve a problem which is by now "classical" and which has been many times a "preliminary exercise" to the dispersive treatment of Compton scattering on polarized nucleons [see, e.g., Prange [1], Hearn and Leader [2], Gourdin [3], Meyer [4]]. This approach requires analyticity of the amplitudes in certain regions of the complex plane of the variable for which the relation is written. But analyticity is an assumption on the dynamics, the amplitudes for which it is postulated should therefore be free from kinematical zeros, constraints and singularities. Our interest points into a different direction, but it also requires a clean separation of kinematics and dynamics : it is related to the connections which exist between real photon processes (Compton scattering) and virtual forward photon processes (whose absorptive parts are given by deep inelastic electron scattering) on nucleons due to the fact that both are limits of the same general process of virtual Compton scattering on polarized nucleons and that they have both again a common limit [for spin averaged nucleons, see 5]. The commutativity of these limits depends essentially on the analytic structure of the amplitudes and we arrive thus to the same requirement of knowing how to write the tensor for virtual Compton scattering on polarized nucleons in terms of a basis which exhibits all the kinematics. Furthermore, different limits of this tensor are responsible for a great variety of interactions involving leptons, photons and nucleons, as the two-photon contribution to lepton-nucleon scattering, the same but for the bound system [which is by now the most important missing contribution to the hyperfine splitting of the hydrogene atom, see, e.g., 6], the Bethe-Heitler scattering, etc.

To our knowledge, the closest approach to this problem has been given by Perrottet [7],[8],[*]. He gives a basis of gauge-invariant tensors, but which exhibit poles in the masses of the two photons, introduced precisely by the gauge projectors. Due to these unphysical poles, the amplitudes have kinematical zeros and constraints and as a consequence the real photon limit cannot be taken without explicitly finding these kinematical zeros and constraints. It is our aim to solve this problem by directly constructing a basis of gauge-invariant tensors, free from poles and minimal. The consequence of not having poles is that the amplitudes are free from kinematical zeros and constraints. That the basis is minimal means that any other pole

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free basis may be obtained from this one in such a way that none of the elements of the matrix which transforms the minimal basis into the other one has a pole in any of the invariants. This ensures that the amplitudes are free from kinematical singularities.

Let us denote by k and p (k' and p') the four-momenta of the incoming (outgoing) photon and nucleon and by π and η (π' and η') the corresponding spin indices, and let us define

$$P = \frac{p + p'}{2}$$
$$K = \frac{k + k'}{2} \tag{1}$$

and

$$\epsilon^{*\mu}(k',\pi')\, \bar{u}(p',\eta')\; T_{\nu\mu}(k,k',P)\, u(p,\eta)\, \epsilon^{\nu}(k,\pi)\,(2\pi)^4\, \delta^{(4)}(p+k-p'-k')$$

$$= \langle N(p',\eta')\, \gamma(k',\pi') \mid T \mid \gamma(k,\pi)\, N(p,\eta) \rangle \tag{2}$$

where $\epsilon^{\nu}(k,\pi)$ $(\epsilon^{*\mu}(k',\pi'))$ is the polarization vector of the incoming (outgoing) photon and $u(p,\eta)$ $(\bar{u}(p',\eta'))$ is the four-spinor of the incoming (outgoing) nucleon. Our aim is to expand $T^{\nu\mu}(k,k',P)$ in terms of a complete set of independent tensors, $\tau_i^{\nu\mu}(k,k',P)$,

$$T^{\nu\mu}(k,k',P) = \sum_i \tau_i^{\nu\mu}(k,k',P)\, A_i(k^2, k'^2, k\cdot k', P\cdot K) \tag{3}$$

in such a way that each one is Lorentz covariant and gauge invariant by itself, even or odd under the discrete symmetries and crossing and carries all the kinematics (which means that the basis is minimal and pole free). As a consequence the invariant amplitudes $A_i(k^2, k'^2, k\cdot k', P\cdot K)$ are not related to each other by the discrete symmetries and crossing but satisfy certain symmetry conditions in terms of the four independent invariants k^2, k'^2, $k\cdot k'$, $P\cdot K$ and are free from all kinematical zeros, constraints and singularities.

Let us first state the discrete symmetries and the crossing relations. Parity conservation implies

$$T^{\nu\mu}(k, k', \ell) = \gamma_0 \, T_{\nu\mu}(k^T, k'^T, \ell^T) \gamma_0 \tag{4}$$

where $k_\mu^T = k^\mu$, $\gamma_0 \gamma^\mu \gamma_0 = \gamma^{\mu+}$ and $+$ means complex adjoint [*]. Time reversal plus parity gives

$$T^{\nu\mu}(k, k', \ell) = \mathcal{Z}^+ \, T^{\mu\nu T}(k', k, \ell) \, \mathcal{Z} \tag{5}$$

where $\mathcal{Z}^+ \gamma^\mu \mathcal{Z} = \gamma^{\mu T}$ and T means transposed. Photon crossing gives

$$T^{\nu\mu}(k, k', \ell) = \mathcal{Z}^+ \gamma_0 \, T^{\mu\nu *}(-k', -k, \ell) \, \gamma_0 \, \mathcal{Z} \tag{6}$$

and finally nucleon crossing plus charge conjugation implies

$$T^{\nu\mu}(k, k', \ell) = \gamma^0 \, C \, T^{\mu\nu *}(k', k, -\ell) \, C^+ \gamma^0 \tag{7}$$

where $C\gamma^\mu C^+ = -\gamma^{\mu T}$. The hermiticity of the electromagnetic current has been used in order to obtain this last condition, which incidentally (although expected on physical grounds) does not give anything which had not already been given by the other three. There is still another kind of restrictions, due to positivity, for the absorptive part of the tensor. These have been extensively studied by De Rujula and de Rafael [9].

We start by writing down all the tensors which satisfy (4), are either even or odd under (5) and (6), form a system of generators [**], are pole free and minimal in the sense described above. This, one may do with four independent vectors, e.g., k^μ, k'^μ, P^μ and γ^μ (but not with $\epsilon^{\mu\alpha\beta\delta} k_\alpha k'_\beta P_\delta$ instead of one of those) giving (indices $\nu\mu$) will be understood always for all tensors from now on).

$$T_1 = g_{\nu\mu}$$
$$T_2 = k_\nu k'_\mu$$
$$T_3 = k'_\nu k_\mu$$
$$T_4 = k_\nu k_\mu + k'_\nu k'_\mu$$
$$T_5 = k_\nu k_\mu - k'_\nu k'_\mu$$
$$T_6 = \ell_\nu \ell_\mu$$
$$T_7 = \ell_\nu k_\mu + \ell_\mu k'_\nu$$
$$T_8 = \ell_\nu k_\mu - \ell_\mu k'_\nu$$
$$T_9 = \ell_\nu k'_\mu + \ell_\mu k_\nu$$
$$T_{10} = \ell_\nu k'_\mu - \ell_\mu k_\nu$$
$$T_{11} = g_{\nu\mu}\,\not{K}$$
$$T_{12} = k_\nu k'_\mu\,\not{K}$$
$$T_{13} = k'_\nu k_\mu\,\not{K}$$
$$T_{14} = (k_\nu k_\mu + k'_\nu k'_\mu)\,\not{K}$$
$$T_{15} = (k_\nu k_\mu - k'_\nu k'_\mu)\,\not{K}$$
$$T_{16} = \ell_\nu \ell_\mu\,\not{K}$$
$$T_{17} = (\ell_\nu k_\mu + \ell_\mu k'_\nu)\,\not{K}$$

$$T_{18} = (\ell_\nu k_\mu - \ell_\mu k'_\nu)\,\not{K}$$
$$T_{19} = (\ell_\nu k'_\mu + \ell_\mu k_\nu)\,\not{K}$$
$$T_{20} = (\ell_\nu k'_\mu - \ell_\mu k_\nu)\,\not{K}$$
$$T_{21} = \ell_\nu \gamma_\mu + \ell_\mu \gamma_\nu$$
$$T_{22} = \ell_\nu \gamma_\mu - \ell_\mu \gamma_\nu$$
$$T_{23} = k_\nu \gamma_\mu + k'_\mu \gamma_\nu$$
$$T_{24} = k_\nu \gamma_\mu - k'_\mu \gamma_\nu$$
$$T_{25} = k'_\nu \gamma_\mu + k_\mu \gamma_\nu$$
$$T_{26} = k'_\nu \gamma_\mu - k_\mu \gamma_\nu$$
$$T_{27} = (\ell_\nu \gamma_\mu + \ell_\mu \gamma_\nu)\,\not{K} - \not{K}\,(\ell_\nu \gamma_\mu + \ell_\mu \gamma_\nu)$$
$$T_{28} = (\ell_\nu \gamma_\mu - \ell_\mu \gamma_\nu)\,\not{K} - \not{K}\,(\ell_\nu \gamma_\mu - \ell_\mu \gamma_\nu)$$
$$T_{29} = (k_\nu \gamma_\mu + k'_\mu \gamma_\nu)\,\not{K} - \not{K}\,(k_\nu \gamma_\mu + k'_\mu \gamma_\nu)$$
$$T_{30} = (k_\nu \gamma_\mu - k'_\mu \gamma_\nu)\,\not{K} - \not{K}\,(k_\nu \gamma_\mu - k'_\mu \gamma_\nu)$$
$$T_{31} = (k'_\nu \gamma_\mu + k_\mu \gamma_\nu)\,\not{K} - \not{K}\,(k'_\nu \gamma_\mu + k_\mu \gamma_\nu)$$
$$T_{32} = (k'_\nu \gamma_\mu - k_\mu \gamma_\nu)\,\not{K} - \not{K}\,(k'_\nu \gamma_\mu - k_\mu \gamma_\nu)$$
$$T_{33} = \gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu$$
$$T_{34} = (\gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu)\,\not{K} + \not{K}\,(\gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu)$$

(8)

where $\not{K} = K^\mu \gamma_\mu$ and use has been made of the Dirac equation in eliminating $\not{p}$ and $\not{p}'$ as the tensors have to be understood always between the four spinors of the nucleons. It is clear that two of these 34 tensors have to be dependent on the other ones, as with four vectors and one scalar matrix one can construct only 32 independent tensors, the superfluous two coming from the non-commutativity of the γ matrices. The choice of the two tensors to be eliminated in a way that it transforms the system of generators into a minimal basis has created certain difficulties for a time [see Perrottet [7),8)] and references therein for details and the solution]. The relations between the tensors are given in these last references and has been checked by the author [in view of discrepancies between different papers, see again Perrottet [7),8)]]. They will be reproduced here for completeness in terms of our tensors and our invariants

$$2 \left(T_{17} - T_{19} \right) - \left(k^2 - k'^2 \right) T_{22} + 2 \ell \cdot K \left(T_{23} - T_{25} \right) - 2 M T_{28}$$
$$- 2 M \ell \cdot K T_{33} + \left(M^2 + \frac{k \cdot k'}{2} - \frac{k^2 + k'^2}{4} \right) T_{34} = 0 \qquad (9)$$

and

$$\ell \cdot K \left(T_2 - T_3 \right) + \frac{1}{4} \left(k^2 + k'^2 + 2 k \cdot k' \right) \left(T_7 - T_9 \right) - \frac{k^2 - k'^2}{4} \left(T_8 + T_{10} \right)$$
$$- M \left(T_{12} - T_{13} \right) + \frac{M}{4} \left(k^2 + k'^2 + 2 k \cdot k' \right) \left(T_{23} - T_{25} \right) - M \frac{k^2 - k'^2}{4} \left(T_{24} + T_{26} \right)$$
$$- \ell \cdot K T_{28} + \frac{k^2 - k'^2}{8} \left(T_{29} - T_{31} \right) - \frac{1}{8} \left(k^2 + k'^2 - 2 k \cdot k' \right) \left(T_{30} + T_{32} \right) \qquad (10)$$
$$- \left[\left(\ell \cdot K \right)^2 + \frac{1}{4} \left(k^2 k'^2 - \left(k \cdot k' \right)^2 \right) \right] T_{33} + \frac{M}{2} \ell \cdot K T_{34} = 0$$

which show immediately that only T_{17}, T_{19} or T_{28} can be eliminated due to (9) and T_{12} or T_{13} due to (10). There is one convenient choice, the reasons will be given later, and this is to eliminate T_{13} and T_{28}. This ensures freedom from kinematics up to this stage. There is one symmetry condition left : gauge invariance. It is well known that gauge invariance, i.e., $k^\nu T_{\nu\mu} = k'^\mu T_{\nu\mu} = 0$, gives constraints on the amplitudes. The only way to avoid this is to have a basis of gauge invariant tensors. The method of how to construct this gauge invariant basis, pole free and minimal, or equivalently, how to introduce gauge invariance without spoiling the freedom from kinematical singularities, zeros and constraints of the invariant amplitudes was given by Bardeen and Tung [10] for real photon processes. It may be extended to virtual photon processes which we are interested in and the essential steps are the following ones.

a) Act with a gauge projector on both indices of each of the 32 tensors which form a pole free, minimal basis,

$$\widetilde{T}_i^{\nu\mu} = \left(g^{\nu\nu'} - \frac{k''^\nu k'^{\nu'}}{k \cdot k'} \right) \left(g^{\mu\mu'} - \frac{k'^\mu k'^{\mu'}}{k \cdot k'} \right) T_{\nu'\mu'}; \qquad i = 1 \cdots 32 \quad (11)$$

leaving thus 18 independent, gauge invariant tensors which exhibit single and double poles in $k \cdot k'$.

b) Eliminate as many single and double poles as possible by simply adding to each tensor $\widetilde{T}_i^{\nu\mu}$ linear combinations of the other ones with coefficients which should not have poles. This leaves us with three pole free tensors, 10 with only single poles and 5 with both single and double poles.

c) Multiply the 5 tensors with essential but unphysical double poles in
k·k' by k·k' and apply again the procedure of b). 10 tensors with
essential single poles in k·k' are left.

d) Multiply these 10 tensors by k·k'.

The so-obtained basis of gauge invariant tensors is expected to be
minimal and pole free by construction and is the following one :

$$\mathcal{T}_1 = k \cdot k' \, T_1 - T_3$$

$$\mathcal{T}_2 = k^2 k'^2 \, T_1 + k \cdot k' \, T_2 - \frac{k^2 + k'^2}{2} T_4 + \frac{k^2 - k'^2}{2} T_5$$

$$\mathcal{T}_3 = (\ell \cdot K)^2 \, T_1 + k \cdot k' \, T_6 - \ell \cdot K \, T_7$$

$$\mathcal{T}_4 = \ell \cdot K \, (k^2 + k'^2) \, T_1 - \ell \cdot K \, T_4 - \frac{k^2 + k'^2}{2} T_7 + \frac{k^2 - k'^2}{2} T_8 + k \cdot k' \, T_9$$

$$\mathcal{T}_5 = - \ell \cdot K \, (k^2 - k'^2) \, T_1 + \ell \cdot K \, T_5 + \frac{k^2 - k'^2}{2} T_7 - \frac{k^2 + k'^2}{2} T_8 + k \cdot k' \, T_{10}$$

$$\mathcal{T}_6 = \ell \cdot K \, T_2 - \frac{k^2 + k'^2}{2} T_9 - \frac{k^2 - k'^2}{2} T_{10} - M \, T_{12} + M \frac{k^2 + k'^2}{4} T_{23} - M \frac{k^2 - k'^2}{4} T_{24}$$
$$+ \frac{k^2 - k'^2}{8} T_{29} - \frac{k^2 + k'^2}{8} T_{30} - \frac{k^2 k'^2}{4} T_{33}$$

$$\mathcal{T}_7 = 8 \, T_{16} - 4 \, \ell \cdot K \, T_{21} + \ell \cdot K \, T_{34}$$

$$\mathcal{T}_8 = T_{19} + \frac{k^2 - k'^2}{2} T_{22} - \ell \cdot K \, T_{23} + \frac{k^2 + k'^2}{8} T_{34}$$

$$\mathcal{T}_9 = T_{20} - \frac{k^2 + k'^2}{2} T_{22} + \ell \cdot K \, T_{24} - \frac{k^2 - k'^2}{8} T_{34}$$

$$\mathcal{T}_{10} = - 8 \, k \cdot k' \, T_6 + 4 \, \ell \cdot K \, T_7 + 4 M \, k \cdot k' \, T_{21} - 4 M \, \ell \cdot K \, T_{25} - 2 \, \ell \cdot K \, T_{32} - 2 k \cdot k' \, \ell \cdot K \, T_{33} + M \, k \cdot k' \, T_{34}$$

$$\mathcal{T}_{11} = T_{18} - k \cdot k' \, T_{22} + \ell \cdot K \, T_{26}$$

$$\mathcal{T}_{12} = \ell \cdot K \, T_4 - \frac{k^2 - k'^2}{2} T_8 - k \cdot k' \, T_9 - M \, T_{14} + M \, k \cdot k' \, T_{23} - M \frac{k^2 - k'^2}{2} T_{26} - \frac{k^2 + k'^2}{4} T_{32} - k \cdot k' \frac{k^2 + k'^2}{4} T_{33}$$

$$\mathcal{T}_{13} = \ell \cdot K \, T_5 - \frac{k^2 + k'^2}{2} T_8 + k \cdot k' \, T_{10} - M \, T_{15} + M \, k \cdot k' \, T_{24} - M \frac{k^2 + k'^2}{2} T_{26} - \frac{k^2 - k'^2}{4} T_{32} - k \cdot k' \frac{k^2 - k'^2}{4} T_{33}$$

$$\mathcal{T}_{14} = 2 \, \ell \cdot K \, T_8 - 2 M \, k \cdot k' \, T_{22} + 2 M \, \ell \cdot K \, T_{26} - k \cdot k' \, T_{27} + \ell \cdot K \, T_{31}$$

$$\mathcal{T}_{15} = - (k^2 - k'^2) \, T_7 + (k^2 + k'^2) \, T_8 - 2 k \cdot k' \, T_{10} - 2 M \, k \cdot k' \, T_{24} + M (k^2 - k'^2) \, T_{25} + M (k^2 + k'^2) \, T_{26}$$
$$- k \cdot k' \, T_{29} + \frac{k^2 + k'^2}{2} T_{31} + \frac{k^2 - k'^2}{2} T_{32}$$

$$\mathcal{T}_{16} = - (k^2 + k'^2) \, T_7 + (k^2 - k'^2) \, T_8 + 2 k \cdot k' \, T_9 - 2 M \, k \cdot k' \, T_{23} + M (k^2 + k'^2) \, T_{25} + M (k^2 - k'^2) \, T_{26}$$
$$- k \cdot k' \, T_{30} + \frac{k^2 - k'^2}{2} T_{31} + \frac{k^2 + k'^2}{2} T_{32}$$

$$\mathcal{T}_{17} = - 4 \, \ell \cdot K \, T_1 + 2 \, T_7 + 4 M \, T_{11} - 2 M \, T_{25} + T_{32} + k \cdot k' \, T_{33}$$

$$\mathcal{T}_{18} = 4 \, T_{17} - 4 \, \ell \cdot K \, T_{25} + k \cdot k' \, T_{34}$$

(12)

A fundamental question arises now. We have assumed, up to now, that this minimal basis exists, but this is a fact which has to be proved [*]. As no general proof of this is known to us and as in any case the most subtle point of the procedure we have used to construct the evidently gauge invariant and pole free basis (12) is that it is also minimal, we have to check this explicitly. Let us only sketch the method. We would like to prove that given any transformation matrix to another gauge invariant and pole free basis, none of its elements can have a pole. As an example, let us show it for all the elements of the first column. These elements multiply τ_1, but τ_1 is the only element of the basis which carries T_3, so that a pole in the first column would appear necessarily as such attached to T_3 in the new basis, which would then not be pole free, contrary to our assumption. An extension of this method gives the following result: the basis (12) is <u>not</u> minimal, there exists essentially three gauge invariant, pole free tensors which can be obtained from our basis only with factors which carry a single pole in $k \cdot k'$:

$$
\begin{aligned}
\mathcal{T}_{19} &= \frac{1}{k \cdot k'} \left[2 (\ell \cdot K)^2 T_2 + 2 k^2 k'^2 T_3 - \ell \cdot K (k^2 + k'^2) T_4 - \ell \cdot K (k^2 - k'^2) T_5 \right] = \\
&= 2 (\ell \cdot K)^2 T_2 + 2 k^2 k'^2 T_6 - \ell \cdot K (k^2 + k'^2) T_9 - \ell \cdot K (k^2 - k'^2) T_{10}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{20} &= \frac{1}{4 k \cdot k'} \left[(k^2 - k'^2) T_{10} - 2 (k^2 + k'^2) T_{14} + 4 \ell \cdot K T_{15} \right] = \\
&= -2 (k^2 - k'^2) T_6 - 2 \ell \cdot K T_{10} + M (k^2 - k'^2) T_{21} + M (k^2 + k'^2) T_{22} - 2M \ell \cdot K T_{24} \\
&\quad + \frac{k^2 + k'^2}{2} T_{27} - \ell \cdot K T_{29} - \ell \cdot K \frac{k^2 - k'^2}{2} T_{33} + M \frac{k^2 - k'^2}{4} T_{34}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{21} &= \frac{1}{4 k \cdot k'} \left[(k^2 + k'^2) T_{10} - 2 (k^2 - k'^2) T_{14} + 4 \ell \cdot K T_{16} \right] = \\
&= -2 (k^2 + k'^2) T_6 + 2 \ell \cdot K T_9 + M (k^2 + k'^2) T_{21} + M (k^2 - k'^2) T_{22} - 2M \ell \cdot K T_{23} \\
&\quad + \frac{k^2 - k'^2}{2} T_{27} - \ell \cdot K T_{30} - \ell \cdot K \frac{k^2 + k'^2}{2} T_{33} + M \frac{k^2 + k'^2}{4} T_{34}
\end{aligned}
$$

(13)

The meaning of these three relations is the following : as long as we do not go to the point $k \cdot k' = 0$ the set of gauge invariant tensors (12) forms a basis, although not a minimal one as there are three gauge invariant tensors (13) whose amplitudes, once written in terms of the basis (12), show,

although explicitly and in a known manner, singularities in $k \cdot k'$. At the point $k \cdot k' = 0$, (12) is no longer a basis, as there are now three relations (13) between the tensors of this basis but the tensors τ_{19}, τ_{20} and τ_{21} are then independent, so that a new basis may easily be constructed. In cases of physical interest, when $k \cdot k' = 0$ is reached through forward $(k = k')$ real $(k^2 = k'^2 = 0)$ scattering, this problem does not arise, as we will see later on.

We are now compelled to ask a final question. Could it be that a minimal basis exists but that Bardeen and Tung's procedure for obtaining it does not work in this case for unclear reasons ? It is not so and let us explain it for the spin-independent tensors τ_1 to τ_5 and τ_{19} (which correspond to virtual pion Compton scattering). A minimal basis will have, from gauge invariance, one tensor with energy dimensions 2 (which has to be τ_1) and four with energy dimensions 4, which therefore have to be related to τ_2 to τ_5 through numerical coefficients, so that if one basis is minimal so would be the other, contrary to the facts. We have to conclude that <u>for the general case of virtual Compton scattering on polarized nucleons, there does not exist a basis of gauge invariant tensors which carry all the kinematics</u>.

We have succeeded in showing explicitly all the kinematics, which is contained in the 21 tensors of (12) and (13), we have failed, because of it being impossible, to put all the kinematics in a tensorial basis, having thus amplitudes free from kinematics. We know, nevertheless, which is the analytic structure of the amplitudes coming from the kinematical requirements, as it is contained completely in (13). We therefore propose (12) as a basis which carries the maximal amount of kinematics and show explicitly the analytic structure of the amplitudes which still carry kinematics and which correspond to the tensors $\tau_{2,3,4,5,10,14,15,16}$

$$A_2' = A_2 + \frac{2(P \cdot K)^2}{k \cdot k'} A_{19}$$

$$A_3' = A_3 + 2 \frac{k^2 k'^2}{k \cdot k'} A_{19}$$

$$A_4' = A_4 - P \cdot K \frac{k'^2 + k'^2}{k \cdot k'} A_{19}$$

$$A_5' = A_5 - P \cdot K \frac{k^2 - k'^2}{k \cdot k'} A_{19}$$

$$A_{10}' = A_{10} + \frac{k^2 - k'^2}{4 k \cdot k'} A_{20} + \frac{k^2 + k'^2}{4 k \cdot k'} A_{21}$$

$$A_{14}' = A_{14} - \frac{k^2 + k'^2}{2 k \cdot k'} A_{20} - \frac{k^2 - k'^2}{2 k \cdot k'} A_{21}$$

$$A_{15}' = A_{15} + \frac{P \cdot K}{k \cdot k'} A_{20}$$

$$A_{16}' = A_{16} + \frac{P \cdot K}{k \cdot k'} A_{21}$$

(14)

The amplitudes A_i, $i = 1,\dots,21$ are now free from kinematics, conjectures on dynamics should be formulated on them, any kind of limits of physical interest may be taken in a straightforward way in (12), (13) and (14). Before studying these limits let us state the symmetry structure of the invariant amplitudes and their Born contributions.

Time reversal and crossing give, respectively,

$$A_i(k^2, k'^2, k\cdot k', \ell\cdot K) = A_i(k'^2, k^2, k\cdot k', \ell\cdot K) = A_i^*(k^2, k'^2, k\cdot k', -\ell\cdot K)$$

$$A_j(k^2, k'^2, k\cdot k', \ell\cdot K) = A_j(k'^2, k^2, k\cdot k', \ell\cdot K) = -A_j^*(k^2, k'^2, k\cdot k', -\ell\cdot K)$$

$$A_\ell(k^2, k'^2, k\cdot k', \ell\cdot K) = -A_\ell(k'^2, k^2, k\cdot k', \ell\cdot K) = A_\ell^*(k^2, k'^2, k\cdot k', -\ell\cdot K)$$

$$A_m(k^2, k'^2, k\cdot k', \ell\cdot K) = -A_m(k'^2, k^2, k\cdot k', \ell\cdot K) = -A_m^*(k^2, k'^2, k\cdot k', -\ell\cdot K)$$

$$(15)$$

for $i = 1, 2, 3, 8, 10, 18, 19, 21$; $j = 4, 6, 7, 12, 16, 17$; $\ell = 9, 11, 14, 20$; $m = 5, 13, 15$.

It is very useful to have the Born contributions to the amplitudes in our basis. These are originated by the nucleon intermediate states in the s and u channel and by the pion (and similarly the η meson) intermediate state in the t channel. The residues at the poles may then immediately be obtained. With the following definition for the Born terms

$$T_B^{\nu\mu}(k, k', \ell) =$$

$$\left[F_1(k'^2)\gamma^\mu + \frac{\mu}{4M} F_2(k'^2)(\gamma^\mu\slashed{k}' - \slashed{k}'\gamma^\mu) \right] \frac{\slashed{\ell} + \slashed{k} + M}{k\cdot k' + 2\ell\cdot K} \left[F_1(k^2)\gamma^\nu - \frac{\mu}{4M} F_2(k^2)(\gamma^\nu\slashed{K} - \slashed{K}\gamma^\nu) \right]$$

$$+ \left[F_1(k^2)\gamma^\nu - \frac{\mu}{4M} F_2(k^2)(\gamma^\nu\slashed{K} - \slashed{K}\gamma^\nu) \right] \frac{\slashed{\ell} - \slashed{k} + M}{k\cdot k' - 2\ell\cdot K} \left[F_1(k'^2)\gamma^\mu + \frac{\mu}{4M} F_2(k'^2)(\gamma^\mu\slashed{K} - \slashed{K}\gamma^\mu) \right]$$

$$+ i\, \frac{g_{\pi N}\, g_{\pi\gamma\gamma}(k^2, k'^2)}{m_\pi}\, \frac{1}{k^2 + k'^2 - 2k\cdot k' - m_\pi^2}\, \epsilon^{\nu\mu\alpha\beta} k_\alpha k'_\beta \gamma_5 \qquad (16)$$

where μ is the anomalous magnetic moment of the nucleon and

$$F_1(0) = F_2(0) = 1 \qquad g_{\pi N} \simeq 13.5 \qquad \text{for protons}$$

$$F_1(0) = 0 \quad F_2(0) = 1 \qquad g_{\pi N} \simeq -13.5 \qquad \text{for neutrons}$$

(17)

$$g_{\pi \gamma \gamma}(0,0) \simeq -0.037$$

The result is

$$T_B^{\nu\mu}(k,k',\ell) = \frac{1}{2M\left[(k\cdot k')^2 - 4(\ell\cdot K)^2\right]} \left\{ F_1(k^2)F_1(k'^2)\left[8\,T_3 + T_{10}\right] \right.$$

$$+ \frac{\mu}{2}\left[\left(F_1(k'^2)F_2(k^2) + F_1(k^2)F_2(k'^2)\right)T_{10} + 2\left(F_1(k'^2)F_2(k^2) - F_1(k^2)F_2(k'^2)\right)T_{11}\right]\right\}$$

$$+ \frac{1}{4M}\left[\frac{1}{k\cdot k' - 2\ell\cdot K} - \frac{1}{k\cdot k' + 2\ell\cdot K}\right]\left\{\left[F_1(k^2)F_1(k'^2) + \mu\left(F_1(k'^2)F_2(k^2) + F_1(k^2)F_2(k'^2)\right)\right]T_{17}\right.$$

$$\left. - \left(\frac{\mu}{2M}\right)^2 F_2(k^2)F_2(k'^2)\left[2\,T_4 - 4\,T_6 + 2M\,T_7 + T_{16} - 4M^2\,T_{17}\right]\right\}$$

$$+ \frac{\mu}{16M^2}\left[\frac{1}{k\cdot k' - 2\ell\cdot K} + \frac{1}{k\cdot k' + 2\ell\cdot K}\right]\left\{\left(F_1(k'^2)F_2(k^2) + F_1(k^2)F_2(k'^2)\right)\left(8M\,T_1 + 4\,T_8 - T_{18}\right)\right.$$

$$\left. - \frac{\mu}{M}F_2(k^2)F_2(k'^2)\left[-8M^2\,T_1 + 8\,T_3 - 8M\,T_8 + T_{10}\right]\right\}$$

$$- \frac{1}{16M^2}\left(\frac{\mu}{2M}\right)^2 F_2(k^2)F_2(k'^2)\left[2 - \frac{k\cdot k' + 2\ell\cdot K}{k\cdot k' - 2\ell\cdot K} - \frac{k\cdot k' - 2\ell\cdot K}{k\cdot k' + 2\ell\cdot K}\right]\left(8M\,T_1 + 4\,T_8 - T_{18}\right)$$

$$- \frac{g_{\pi N}\,g_{\pi\gamma\gamma}(k^2,k'^2)}{4\,m_\pi\,M}\,\frac{1}{k^2 + k'^2 - 2k\cdot k' - m_\pi^2}\,\left(4\,T_8 - T_{18}\right) \tag{18}$$

where use has been made of (9) and (10), showing again the correctness of
these two relations. It is clear from (18) that the amplitudes A_i,
i = 2,5,9,11,12,13,15,19,20,21 do not have a contribution coming from the
Born terms [one has to look also at (13) to ensure this for the last three
amplitudes] and the remaining ones show, apart from the typical Born factors,
coefficients which do not depend on the invariants (only on the nucleon mass
M) which is an additional check for their alleged freedom from kinematics.

Let us stress here that the procedure which led us to our basis (12) is not unique. One could have taken different gauge projectors in a) and different linear combinations in b) giving thus a different basis but which, as is now clear, could be written as linear combinations of our 21 tensors τ_i with pole free factors.

Let us now comment on some particular cases of interest, whose corresponding tensors are easily obtained taking the adequate limit in (12), (13) and (14). It is important to know how many tensors actually contribute to the physical processes, which means after contracting with the polarization vector of the two (real or virtual) photons. Due to the orthogonality of a polarization vector to the corresponding four-momentum vector, this limit corresponds just to eliminate from our original system of generators (8) the elements T_i, $i = 2,4,5,9,10,12,14,15,19,20,23,24,29,30$. It turns out that after doing this the 18 final gauge invariant tensors τ_i, $i = 1,...,18$, of (12) are still all independent. We will call this the physical limit. At this point the reason for not having chosen either T_{12} or T_{19} as dependent tensors becomes clear. They do not contribute to the physical process and in writing the physical tensor we would have had to use again relations (9) and (10) (now only for the physical tensors) to ensure independence, losing thus the beautiful feature of (12) that it gives all limits taking into account only (13) and (14) but no longer (9) and (10).

Another case of interest is $k^2 = k'^2$. This eliminates, due to the symmetry requirements (15), the amplitudes A_i, $i = 5,9,11,13,14,15,20$, leaving in general 12 independent tensors which remain independent in the physical limit. Taking now $k^2 = k'^2 = 0$ there are still 12 general independent tensors, but only six of them are physical [*], as it is well known. It is easy to see, using again (9) and (10), that they are equivalent to Bardeen and Tung's [10]. Incidentally this last case (physical limit of $k^2 = k'^2 = 0$) is the most general for which the remaining amplitudes are completely free from kinematics (only A_2' and A_{16}' still carry kinematics, but their corresponding tensors are non-physical).

----

[*] This does not agree with the results of Ref. 8). When one or two photons are real our counting of physical amplitudes agrees with his, although

$$3 \, \mathcal{A}_1(q, k_2; p_3, p_4) = \mathcal{A}_2(q, k_2; p_3, p_4)$$

$$+ 8 g_s^2 g^2 e_Q^2 \left[\frac{(p_1 k_2)}{(p_1 p_2)} \right]^2 \left\{ \frac{(p_2 p_4)^2 + (p_2 p_3)^2}{(t - M^2)(u - M^2)} - \frac{2M^2 + q^2}{k_2^2} \left[\frac{(p_2 p_3)}{t - M^2} - \frac{(p_2 p_4)}{u - M^2} \right]^2 \right\} ,$$

(A.12)

where $t = (q - p_4)^2$, $u = (q - p_3)^2$ are the usual Mandelstam invariants. We refer to Appendix A of Ref. [7] for the high-energy approximation of the phase space (A.8).

The factorization formula (A.7) and the explicit results (A.9), (A.11), (A.12) can be used for detailed theoretical (e.g. along the lines of Ref. [7]) and phenomenological analyses as well as implemented in Monte Carlo event generators [15].

In this paper we limit ourselves to present a simple application, namely the analytic evaluation of the perturbative contributions to the lepto-production cross section. Inserting into eq. (A.7) the perturbative expansion of the gluon structure function $\mathcal{F}$ and defining

$$\sigma_i(\rho, Q^2/M^2; M^2/Q_0^2) \equiv \sigma_{i,\text{Born}}(\rho, Q^2/M^2) + \sigma_{i,1-\text{loop}}(\rho, Q^2/M^2; M^2/Q_0^2) + O(\alpha_s^3) , \quad \text{(A.13)}$$

the Born level and 1-loop contribution are respectively given by

$$4M^2 \sigma_{i,\text{Born}}(\rho, Q^2/M^2) = \hat{\sigma}_i(\rho, \mathbf{q}/M, \mathbf{0}) , \tag{A.14}$$

$$4M^2 \sigma_{i,1-\text{loop}} \left(\rho, \frac{Q^2}{M^2}; \frac{M^2}{Q_0^2} \right) = C_A \frac{\alpha_s}{\pi} \int_\rho^1 \frac{dz}{z} \left\{ \hat{\sigma}_i \left(z, \frac{\mathbf{q}}{M}, \mathbf{0} \right) \ln \frac{M^2}{Q_0^2} \right.$$

$$\left. + \int \frac{d^2 \mathbf{k}_2}{\pi \mathbf{k}_2^2} \left[\hat{\sigma}_i \left(z, \frac{\mathbf{q}}{M}, \frac{\mathbf{k}_2}{M} \right) - \hat{\sigma}_i \left(z, \frac{\mathbf{q}}{M}, \mathbf{0} \right) \right] \right\} \tag{A.15}$$

$$\equiv \alpha \, \alpha_s^2 \left[\bar{c}_i(\rho, Q^2/4M^2) \ln M^2/Q_0^2 + c_i(\rho, Q^2/4M^2) \right] .$$

By using the matrix elements (A.11), (A.12) and performing the $Q\bar{Q}$ phase space integration in eq. (A.9), we find the explicit form

$$4M^2 \sigma_{2,\text{Born}}(\rho, Q^2/M^2) = \hat{\sigma}_2(\rho, \mathbf{q}/M, \mathbf{0})$$

$$= 2\pi \, \alpha \, e_Q^2 \, \alpha_s \, \Theta\left(\frac{1}{\rho} - 1 - \frac{Q^2}{4M^2} \right) \rho \beta' \left\{ \left[(1 + \rho - \frac{1}{2}\rho^2) \mathcal{L}(\beta') - 1 - \rho \right] \right.$$

$$\left. + [8 + \rho - (2 + 3\rho)\mathcal{L}(\beta')] \frac{\rho Q^2}{4M^2} + [-8 + 2\mathcal{L}(\beta')] \left(\frac{\rho Q^2}{4M^2} \right)^2 \right\} , \tag{A.16}$$

$$4M^2[\sigma_{1,\text{Born}}(\rho, Q^2/M^2) - \sigma_{2,\text{Born}}(\rho, Q^2/M^2)] =$$

$$= \pi \, \alpha \, e_Q^2 \, \alpha_s \, \Theta\left(\frac{1}{\rho} - 1 - \frac{Q^2}{4M^2}\right) \rho^2 \beta' \frac{Q^2}{M^2} \left[\rho \mathcal{L}(\beta') - 2 + \frac{\rho Q^2}{2M^2}\right] \quad , \tag{A.17}$$

where

$$\beta' = \sqrt{1 - \rho \left(1 - \frac{\rho Q^2}{4M^2}\right)^{-1}} \quad , \tag{A.18}$$

and $\mathcal{L}(\beta)$ is the bremsstrahlung function in eq. (2.4). The result in eqs. (A.16), (A.17) agrees with previous calculations [5].

We should recall that the formula (A.15) does not give the exact one loop result. It contains only the one loop contribution increasing with the energy and, eventually, giving a constant factor at high energies. Other one loop effects, like Coulomb and bremsstrahlung contributions relevant near threshold (and, so far, computed only for the photo- and hadro-production cases [1-3,16]), are not included. We have analytically evaluated the constant term at high energy ($\rho \to 0$) and we find

$$\bar{c}_2(0, a) = \frac{4}{3} C_A e_Q^2 \left[\frac{1}{2a} + \left(1 - \frac{1}{4a}\right) J(a)\right] \quad ,$$

$$c_2(0, a) = \frac{4}{3} C_A e_Q^2 \left[\frac{5}{6a} + \left(\frac{13}{6} - \frac{5}{12a}\right) J(a) + \left(1 - \frac{1}{4a}\right) I(a)\right] \quad , \tag{A.19}$$

$$\bar{c}_1(0, a) = \bar{c}_2(0, a) - \frac{4}{3} C_A e_Q^2 \frac{1}{1+a} \left[\frac{3}{2a} + 1 - \left(1 + \frac{3}{4a}\right) J(a)\right] \quad ,$$

$$c_1(0, a) = c_2(0, a) - \frac{4}{3} C_A e_Q^2 \frac{1}{1+a} \left[\frac{1}{a} - \frac{1}{3} + \left(\frac{1}{3} + a - \frac{1}{2a}\right) J(a) - \left(1 + \frac{3}{4a}\right) I(a)\right] \quad , \tag{A.20}$$

where

$$J(a) = \frac{1}{\sqrt{a(1+a)}} \ln \frac{\sqrt{1+a} + \sqrt{a}}{\sqrt{1+a} - \sqrt{a}} \quad ,$$

$$I(a) = \frac{1}{\sqrt{a(1+a)}} \left[-\frac{\pi^2}{6} - \frac{1}{2} \ln^2 \frac{\sqrt{1+a} + \sqrt{a}}{\sqrt{1+a} - \sqrt{a}} \right. \tag{A.21}$$

$$\left. + \ln^2 \frac{\sqrt{1+a} - \sqrt{a}}{2\sqrt{1+a}} + 2 Li_2\left(\frac{\sqrt{1+a} - \sqrt{a}}{2\sqrt{1+a}}\right)\right] \quad ,$$

and Li_2 is the dilogarithm function

$$Li_2(x) = -\int_0^x \frac{dz}{z} \ln(1 - z) \quad . \tag{A.22}$$

Eqs. (A.19) and (A.20) generalize the known results for photoproduction [1] taking into account the photon offshellness. In particular we have

$$
\bar{c}_2(0, Q^2/4M^2) \simeq \begin{cases} \frac{28}{9} C_A e_Q^2 \,, & (Q^2 \ll M^2) \\ \frac{16}{3} C_A e_Q^2 \frac{M^2}{Q^2} \ln \frac{Q^2}{M^2} \,, & (Q^2 \gg M^2) \end{cases}
$$

$$
c_2(0, Q^2/4M^2) \simeq \begin{cases} \frac{164}{27} C_A e_Q^2 \,, & (Q^2 \ll M^2) \\ \frac{8}{3} C_A e_Q^2 \frac{M^2}{Q^2} \ln^2 \frac{Q^2}{M^2} \,, & (Q^2 \gg M^2) \end{cases}
\tag{A.23}
$$

$$
\bar{c}_1(0, Q^2/4M^2) - \bar{c}_2(0, Q^2/4M^2) \simeq \begin{cases} -\frac{8}{45} C_A e_Q^2 \frac{Q^2}{M^2} \,, & (Q^2 \ll M^2) \\ -\frac{16}{3} C_A e_Q^2 \frac{M^2}{Q^2} \,, & (Q^2 \gg M^2) \end{cases}
$$

$$
c_1(0, Q^2/4M^2) - c_2(0, Q^2/4M^2) \simeq \begin{cases} -\frac{8}{27} C_A e_Q^2 \frac{Q^2}{M^2} \,, & (Q^2 \ll M^2) \\ -\frac{16}{3} C_A e_Q^2 \frac{M^2}{Q^2} \ln \frac{Q^2}{M^2} \,, & (Q^2 \gg M^2) \end{cases}
\tag{A.24}
$$

Let us make a final comment on a point which has been discussed at length in this Workshop. We refer to the question of the choice of the evolution scale for the gluon structure function in heavy flavour production. Obviously, this problem cannot properly be addressed at the Born level, i.e. before higher order corrections have been evaluated. The $k_\perp$-factorization formula (A.7) contains most of the higher order contributions. It turns out that the hard cross sections $\hat{\sigma}_i$ in eq. (A.7) have a k_2-shape which is almost constant for $k_2^2 \lesssim \max(Q^2, M^2)$ and then drops off rapidly (see Ref. [7] for a more detailed discussion). It follows that the k_2-integration of the unintegrated parton distribution $\mathcal{F}(z_2, k_2; Q_0^2)$ in eq. (A.7) will lead to the gluon structure function $G(z_2, \mu^2; Q_0^2)$ evaluated at the scale $\mu^2 \simeq Q^2 + M^2$. A different scale choice, like for instance $\mu^2 \sim \hat{s}$ ($\hat{s} = (q + k_2)^2$ being the centre of mass energy of the hard subprocess) overestimates scaling violations in the gluon density.

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Figure Captions

Figure 1:   The heavy flavour photo-production process.

Figure 2:   Heavy flavour photo-production at high-energy: (a) factorized structure of the cross section in the Regge limit and (b) hard vertex cross section contribution.

Figure 3:   The hard vertex function $h_N(\gamma)$ for small values of N.

Figure 4:   The $k_\perp$-shape of the (normalized) $N = 0$-moment $\hat{\sigma}_{N=0}(\mathbf{k}^2/M^2)/\hat{\sigma}_{N=0}(\mathbf{0})$ of the heavy flavour hard vertex.

Figure 5:   The one-loop coefficient function for bottom quark production ($M = 5$ GeV, $\Lambda = 260$ MeV).

Figure 6:   Resummed ($\tilde{C}$) and one-loop ($C^{(1)}$) coefficient functions for bottom quark production in the small-ρ region ($M = 5$ GeV, $\Lambda = 260$ MeV).

Figure 7:   Resummed coefficient function $\tilde{C}^{(1)}$ for bottom quark production ($M = 5$ GeV, $\Lambda = 260$ MeV) after performing the matching with the one-loop result $C^{(1)}$.

Figure 8:   The neutral current γg contribution to heavy flavour lepto-production.

Figure 9:   Born diagrams with off-shell incoming lines for hevy flavour lepto-production.

Fig. 1
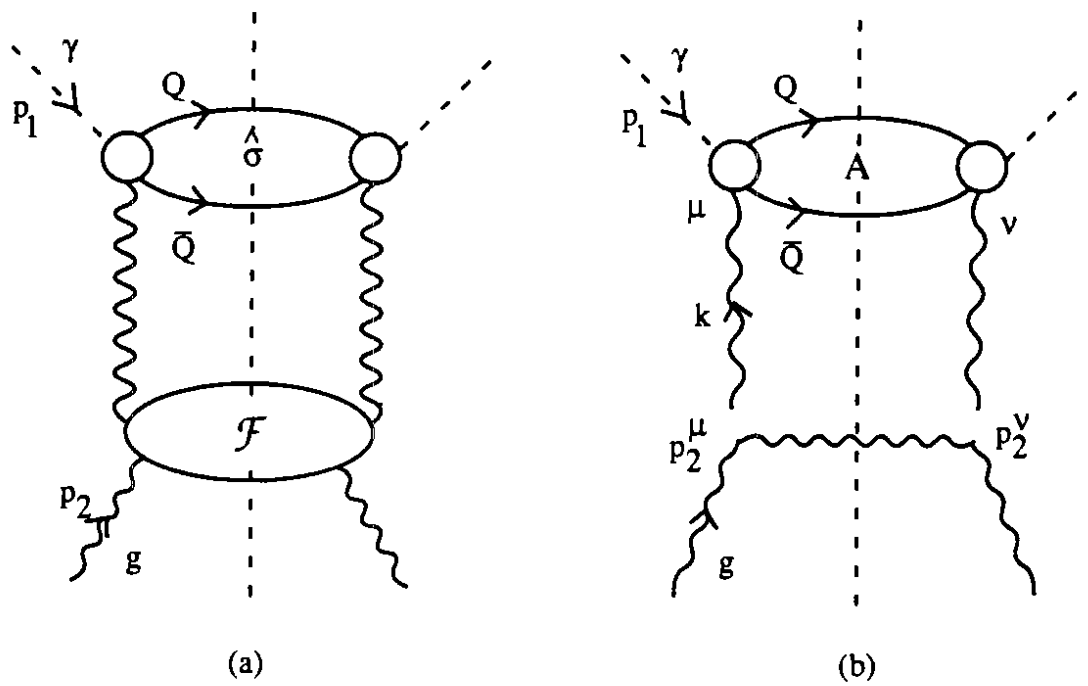

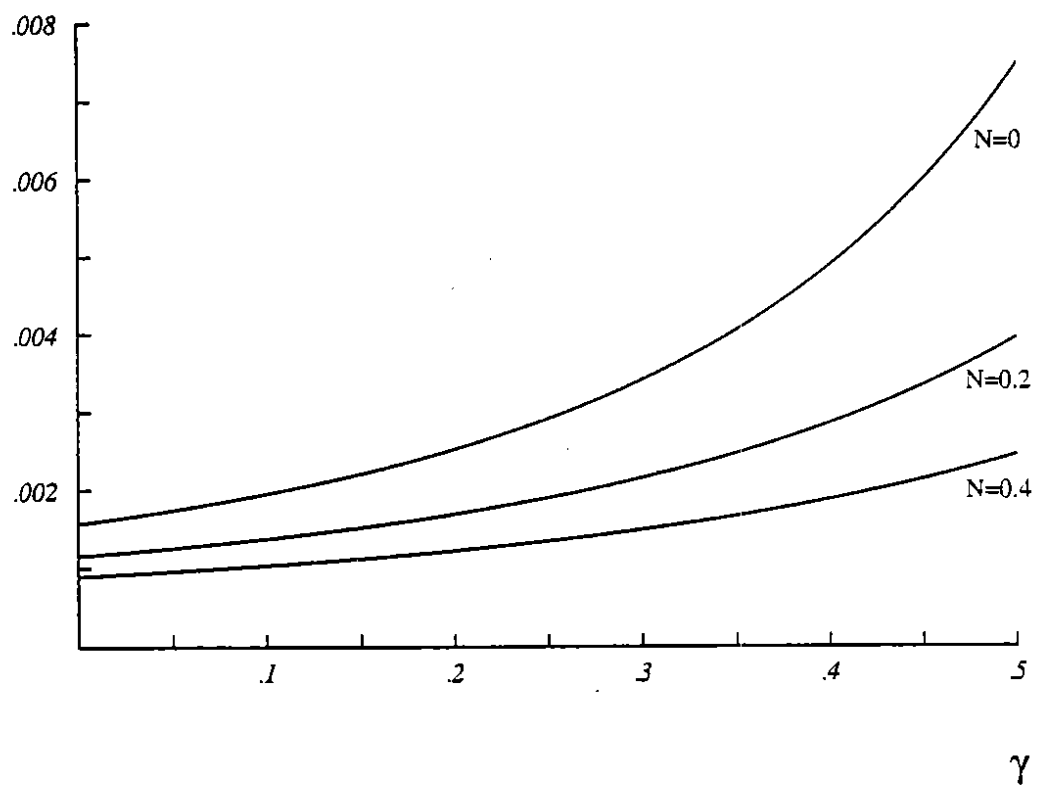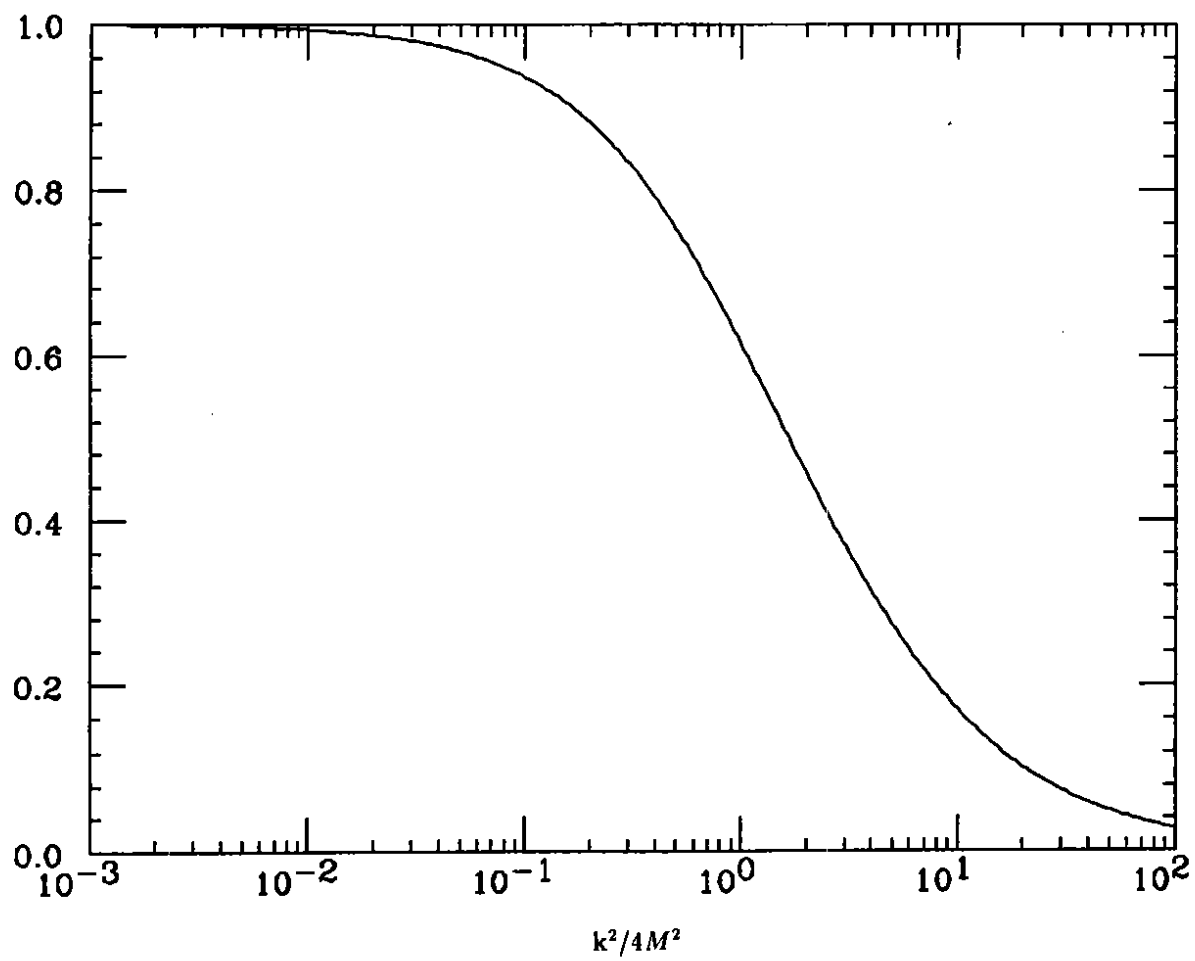(a)   (b)

Fig. 2

$h_N(\gamma)$

.008

.006

.004

.002

N=0

N=0.2

N=0.4

.1   .2   .3   .4   .5

γ

Fig.3

1.0

0.8

0.6

0.4

0.2

0.0

10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2

$k^2/4M^2$

Fig. 4

$C^{(1)}*10^3$

Fig.5

$*10^{-3}$

$\log_{10}(1/\rho)$

Fig.6

Fig.7

Fig.8

Fig.9