Invariant cone-excitation ratios may predict transparency

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Received May 3, 1999; revised manuscript received September 16, 1999; accepted October 18, 1999

Cone-excitation ratios for pairs of surfaces are almost invariant under changes in illumination and offer a possible basis for color constancy [Proc. R. Soc. London Ser. B **257**, 115 (1994)]. We extend this idea to the perception of transparency on the basis of the close analogy between the changes in color signals that occur for surfaces when the illumination changes and the changes in color signals when the surfaces are covered by a filter. This study presents measurements and simulations to investigate the conditions under which cone-excitation ratios are statistically invariant for physically transparent systems. The invariance breaks down when the spectral transmission of the filters is low at some or all wavelengths. We suggest that cone-excitation ratios might be useful to define the stimulus conditions necessary for the perception of transparency.

OCIS codes: 330.1690, 330.1720, 330.7310, 330.1710.

1. INTRODUCTION

The perception of transparency[1–5] requires that the visual system correctly interprets changes in color signals in terms of physical properties of surfaces.[5] Thus a surface that is seen both directly and through a transparent overlay or filter can be identified as a single surface despite the variation in color signals. The term color signal is used to refer to the product of the surface reflectance R and illuminant energy E distributions of wavelength λ at a given spatial position.[6] What conditions on color signals are required for the perception of transparency? If we consider the color signals alone for two-dimensional stationary stimuli (and thus ignore any other potential cues such as depth or motion), then two or more surfaces must be seen both directly and under a filter for the perception of transparency.[1] Furthermore, changes in the color signals at the four-way intersections or x-junctions must be constrained such as to lead the visual system to infer that the surface regions seen directly correspond to the surface regions seen under the filter.[5] This paper concerns the nature of these constraints.

The visual system processes spatiochromatic information from visual scenes to provide knowledge about the position, color, and identity of surfaces in the world. In this respect, the approximate invariance of color perception is an important property of our visual systems. However, this invariance seems surprising when it is considered that the color signals reaching the eye from a given surface change markedly depending on the nature of the illumination under which the surface is viewed. This phenomenon of color constancy has been studied extensively.[7–16] How does the visual system accomplish this approximate invariance despite large changes in color signals when the illumination changes? One possibility is that the visual system might be able to recover the spectral reflectance of surfaces from the cone excitations that occur when those surfaces are viewed under a given light source. It has been established that this would be possible only if the spectral properties of the surfaces in the world are highly constrained.[7] Although the reflectance of natural surfaces and the energy distributions of natural light sources do indeed vary only slowly with wavelength,[6] it is unlikely that they are so highly constrained as to allow the robust recovery of spectral reflectance functions from cone excitations.[17] Recently, an alternative approach has shown that cone-excitation ratios for pairs of surfaces are almost invariant under changes in illumination and that these ratios offer a possible, although not necessarily unique, basis for perceptual color constancy.[14]

We argue that the notion of invariant cone-excitation ratios might also be useful to define the stimulus conditions necessary for the perception of transparency. This study tests, by empirical measurements and numerical Monte Carlo simulation, an invariant cone-excitation-ratio model of transparency and investigates the conditions under which the invariance is true. The simplicity of this approach is appealing, especially since it suggests that similar neural mechanisms might account for both color constancy and transparency.

2. HUMAN CONE-EXCITATION RATIOS AND TRANSPARENCY

The human cone-excitation ratios for two surfaces have been shown to be almost statistically invariant for surfaces under changes in illumination for a large class of surfaces and illuminants.[14] We draw an analogy between the changes in the color signals that take place for a pair of surfaces when the illumination changes and the changes when a pair of surfaces are overlayed by a transparent filter and viewed under fixed illumination. This analogy is most obvious if a change in illumination is imagined to be implemented by the placement of a transparent filter between a light source and a surface.

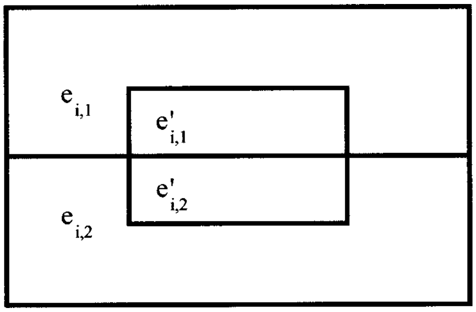

Fig. 1. Invariant cone-excitation-ratio model. The two surfaces ($j = 1, 2$) are partially covered by a transparent layer. The ratio of cone excitations $e_{i,1}/e_{i,2}$ for the surfaces seen directly is equal to the ratio $e'_{i,1}/e'_{i,2}$ when the surfaces are seen through a transparent layer.

Let e_{ij} be the cone excitation in cone class i for stimulus j, where $i \in \{L, M, S\}$ denoting long-, medium-, and short-wavelength-sensitive cone classes and j is a color signal given by the scalar product of the reflectance R of the surface and the energy distribution E of the light source. According to our hypothesis, the condition for perceptual transparency is given when the cone-excitation ratios for two surfaces, R_1 and R_2, viewed directly is equal to the ratios for the same two surfaces (now with effective reflectance R'_1 and R'_2) viewed through a transparent filter (Fig. 1). Thus

$$e_{i,E\cdot R_1}/e_{i,E\cdot R_2} = e_{i,E\cdot R'_1}/e_{i,E\cdot R'_2}. \tag{1}$$

If we assume that the color signal for the filtered surfaces is given by $T^2(\lambda)R(\lambda)E(\lambda)$, where $T(\lambda)$ is the spectral transmittance of the filter, and if we denote the cone sensitivity functions by ϕ_i, then we can expand Eq. (1) thus:

$$\int R_1(\lambda)E(\lambda)\phi_i(\lambda)\delta\lambda \Big/ \int R_2(\lambda)E(\lambda)\phi_i(\lambda)\delta\lambda = \int R_1(\lambda)T^2(\lambda)E(\lambda)\phi_i(\lambda)\delta\lambda \Big/ \int R_2(\lambda)T^2(\lambda)E(\lambda)\phi_i(\lambda)\delta\lambda. \tag{2}$$

It is clear that Eq. (2) will be true when the filter transmittance T is a scalar that is independent of wavelength (this could be the case for certain achromatic or neutral filters), since under these conditions the terms $T^2(\lambda)$ on the right-hand side of Eq. (2) simply cancel. It is not so clear, however, that Eq. (2) will hold if we do not constrain T in this way. Furthermore, we note that Eq. (1) can be expanded to Eq. (2) only if the color signal for the filtered areas can be given by $T^2(\lambda)R(\lambda)E(\lambda)$, which is likely to be true only approximately. Note that Eq. (2) ignores surface reflections (both internal and external) that will occur in real systems because of changes in refractive index as light passes into and out of a filter. The invariance that is investigated in this work is defined strictly by Eq. (1).

3. EMPIRICAL MEASUREMENTS CONCERNING CONE-EXCITATION RATIOS AND PHYSICAL TRANSPARENCY

A simple pilot experiment was conducted with reflectance measurements of real surfaces considered to be viewed both directly and through real filters. Section 5 describes a more comprehensive and controlled experiment, but one that relies on optical models that allow the color signals to be computed for surfaces covered by filters with known optical properties. This pilot experiment did not rely on any such approximations.

A. Methods

Six (nominally white, gray, red, green, blue, and yellow) opaque samples of cards were obtained, and their spectral reflectance values were measured with an Ihara S900 reflectance spectrophotometer.[18] The spectrophotometer was calibrated for dark current by use of a black trap and for gain by use of a white ceramic tile, according to the instructions of the manufacturer. The measurement area of the instrument was a circular aperture (diameter 10 mm) with an optical geometry that approximated to CIE 0/45. Spectral reflectance data (corrected for bandpass dependence) were obtained between 400 and 700 nm at intervals of 10 nm (Appendix A). Measurements were also obtained for the same surfaces covered in turn with each of three neutral-density Wratten gelatin filters[19] with optical densities of 0.1, 0.3, and 0.6 log units and four colored filters. Two of the colored filters, referred to as blue and green, had bandpass spectral transmittance properties (half-width half-height ~10 nm) with maximum transmission of ~20% at 440 and 520 nm, respectively; the other two filters, referred to as yellow and red, had long-pass spectral transmittance properties (maximum transmittance ~90%) with cutoffs at 500 and 580 nm, respectively. Spectral reflectance data were converted to cone excitations with the Smith–Pokorny cone fundamentals.[20]

B. Results

Figure 2 shows scatterplots of ratios of cone excitations pooled for each of the three cone classes for the neutral-density (N.D.) filters. Each point represents a pair of ratios of excitation in cone class i produced by light from two surfaces illuminated by D65 and viewed either directly yielding $e_{i,E\cdot R_1}/e_{i,E\cdot R_2}$ or through a filter yielding $e_{i,E\cdot R'_1}/e_{i,E\cdot R'_2}$. The data are in Figs. 2(a), 2(b), and 2(c) for filters with N.D. 0.1, 0.3 and 0.6, respectively. There is a striking linearity (coefficient of determination R^2—not to be confused with other uses of this symbol elsewhere in this paper—greater than 0.980 for all three cone classes and for all three filters) in the plots of Fig. 2.

For invariance of cone-excitation ratios, however, the plots of Fig. 2 should be linear with gradient 1 exactly. Table 1 shows the coefficient of determination and the gradient of a least-squares linear regression fit to each of the three plots for each of the three cone classes separately. The physical relevance of the residual scatter in the data is not immediate from the R^2 values alone, and a more direct quantifier[14] of the invariance of cone ratios is perhaps given by the proportions of pairs of ratios falling

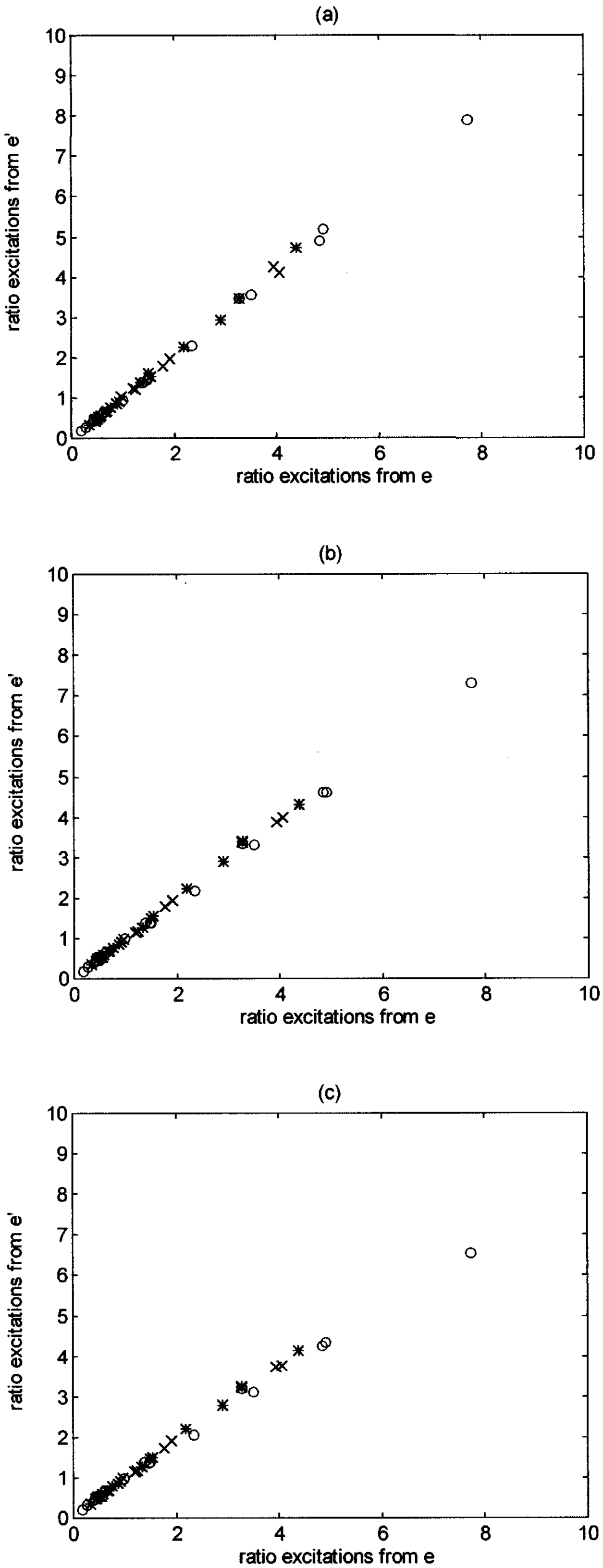

Fig. 2. Scatterplot of ratios of cone excitations for each of the three cone classes (circles, short-wavelength; crosses, medium wavelength; stars, long-wavelength) for (a) N.D. 0.1 filter, (b) N.D. 0.3 filter, and (c) N.D. 0.6 filter. Each point represents a pair of ratios produced by light (under simulated natural daylight at 6500 K) from two surfaces viewed directly e and through a transparent filter e'.

within a certain range of each other, say 10%. For the N.D. 0.1 filter all the pairs of excitation ratios fall within 10% of each other. For the N.D. 0.3 filter the proportions that fall within 10% of each other are 0.467, 1.000, and 1.000 for short-, medium-, and long-wavelength-sensitive cone classes, respectively. For the N.D. 0.6 filter the proportions are 0.133, 0.533, and 0.666 for the three cone classes, respectively. We have no formal justification for the use of the 10% limit as an indicator of invariance, but we note that 10% was used for similar work on the invariance of cone ratios for color constancy[14] based on the same previous psychophysical analyses.[10] In subsequent analyses we have simply computed R^2 and the gradient of the least-squares linear regression.

Equation (2) predicts that the invariance should be perfect for filters whose transmittance values do not vary with wavelength. How can we therefore explain the data in Table 1? First, although the transmittance of N.D. filters will be broadly independent of wavelength, they are not perfect in this sense. Second, and probably more significantly, Eq. (2) is a valid expansion of Eq. (1) given only certain assumptions. For example, Eq. (2) ignores the fact that there will be reflections at the air–filter and filter–air interfaces that are caused by a difference in the refractive index between air and the filter material. The amount of light reflected at these interfaces will further depend on the angle of incidence at which the light strikes the interface. The real measured data of Table 1 show cone-excitation-ratio invariance that is not dependent on assumptions made by a model of filter behavior but that decreases as $T \rightarrow 0$. Without these data it could be argued that the invariance might not hold at all for real systems.

Nevertheless, it seems clear that the invariance weakens as the transmittance of the filter decreases. Clearly at the limit when $T \rightarrow 0$ the cone excitations for the surfaces viewed through the filter will tend toward zero and the ratio toward unity. Hence a plot analogous to Fig. 2 would show a regression line of zero gradient, indicating total breakdown of the invariance when $T \rightarrow 0$. We note that it might reasonably be predicted that the transparency percept would also become weaker as $T \rightarrow 0$ and of course must disappear altogether when $T = 0$.

Table 1 shows similar data for the four colored filters. We note that the correlations between $e_{i,E \cdot R_1}/e_{i,E \cdot R_2}$ and $e_{i,E \cdot R'_1}/e_{i,E \cdot R'_2}$ are less strong than for the N.D. filters. Furthermore, colored filters give the strongest correlation for cone classes whose peak sensitivity is at wavelengths close to the wavelength of maximum transmittance for the filter. Thus the blue filter gives the strongest correlations for the short-wavelength class of cones, while the yellow, green, and red filters give relatively stronger correlations for the medium- and long-wavelength cones. The fact that the correlation is weaker with the colored filters might be expected on the basis of Eq. (2). If the data for the four colored filters are pooled, then the proportions that fall within 10% of each other are 0.150, 0.183, and 0.100 for short-, medium-, and long-wavelength-sensitive cone classes, respectively. However, these results must be treated with great caution because they are based on a small and arbitrary set of surfaces and filters. We note that the blue- and green-

Table 1. Gradient of Least-Squares Linear Fit for Plots of Ratios for Surfaces Viewed through a Filter versus the Same Surfaces Viewed Directly for Three N.D. Filters and Four Color Filters

Filter Type	Short-Wavelength[a]	Medium-Wavelength[a]	Long-Wavelength[a]
0.1 N.D. log unit	1.0301 (0.9992)	1.0575 (0.9980)	1.0760 (0.9986)
0.3 N.D. log unit	0.8489 (0.9975)	0.9352 (0.9982)	0.9526 (0.9987)
0.6 N.D. log unit	0.5733 (0.9801)	0.7813 (0.9971)	0.8012 (0.9907)
Blue	1.0112 (0.8288)	0.5857 (0.6583)	0.5271 (0.5695)
Green	0.5352 (0.8254)	0.9882 (0.7870)	0.8155 (0.5158)
Yellow	0.3756 (0.4612)	0.9786 (0.7671)	1.2089 (0.8499)
Red	0.0567 (0.2205)	0.4638 (0.2535)	0.9367 (0.4745)

[a] Coefficient of determination is given in parentheses.

colored filters in particular had low transmittance even at the wavelengths of maximum transmittance and close to zero transmittance at the longer wavelengths. Interestingly, informal visual observations of these filters yielded quite weak percepts of transparency.

This empirical evidence supports the notion that cone-excitation ratios can be approximately invariant for transformations, owing to the overlay of some transparent filters. The invariance is seen to weaken, however, when the transmittance of N.D. filters becomes small. Also, for colored filters the relative invariance of the separate cone classes seems to depend on the color of the filter and, more generally, presumably depends on the spectral transmittance of the filter. This empirical evidence is interesting, not least because the results do not depend upon any model of physical transparency, but a more systematic study of the dependence of the invariance is required. In this study we investigate, using Monte Carlo simulations, cone-excitation-ratio invariance for physically transparent systems. Reasonably accurate physical models are required for implementing such simulations.

4. PHYSICAL MODELS OF TRANSPARENCY

There is a necessary distinction between physical transparency and perceptual transparency, since it has been demonstrated that physical transparency is not always accompanied by perceptual transparency.[1] Furthermore, visual stimuli can be generated that are perceptually transparent but are physically unrealizable.[5] If we consider the set of systems that are physically transparent, only a subset of these systems would be perceived to be perceptually transparent. We note that this subset is not even an inclusive set of perceptually transparent systems. Nevertheless, in this study we investigate the conditions for physically transparent systems under which cone-excitation ratios are invariant, and we conjecture that the invariance may be a cue for perceptual transparency.

Several models of physical transparency based on both additive and subtractive color mixing have been reported. For example, it has been shown that additive mixtures of light can lead to strong impressions of transparency,[1,4] yielding models of transparency that are in accordance with Talbot's law of color fusion[3] and the operation of an episcotister. In an episcotister the color fusion is temporal. In transparency models that are based on an episcotister, the transparent layer is assumed to consist of opaque areas and holes; the fusion of the two color signals (from the opaque areas and the holes) is spatial.

It has also been proposed that transparency can result from a subtractive mixture that takes place whenever light is affected by selectively absorbing materials.[2,3] In the simplest case the effective reflectance $R'(\lambda)$ of a surface with reflectance $R(\lambda)$ covered by a filter with transmission $T(\lambda)$ is given by

$$R'(\lambda) = R(\lambda)T^2(\lambda). \tag{3}$$

Unless the surface and the filter have identical refractive indices (and are in optical contact), a more realistic model [Eq. (4)] needs to take into account the internal reflectance r of the filter and multiple reflections.[3,21,22] [We note, however, that Eq. (4) does not take into account the external reflectance of the filter. The external reflectance depends on the refractive index of the filter material and the angle of incidence at which the light strikes the filter surface. Equation (4) also ignores the multiple reflection between the filter and the opaque surface.] Thus

$$R'(\lambda) = R(\lambda)[T(\lambda)(1 - r)^2]^2. \tag{4}$$

The subtractive model [Eq. (4)—note that Eq. (3) is a special case of Eq. (4) when $r = 0$] was used in a numerical simulation in which pairs of different opaque surfaces were overlayed by a transparent filter such that the surfaces could be viewed both directly and through the transparent filter. The color signals were calculated for CIE D65 illuminant and the excitation of the three classes of human cones computed.

5. METHODS

The spectral reflectances $R(\lambda)$ for opaque surfaces were taken from 1269 samples[23,24] from the *Munsell Book of Color*.[25] Although the Munsell surfaces are not natural (in the sense that they are man made), extensive studies have confirmed that their reflectance spectra are smoothly varying functions of wavelength with a Fourier band limit of 0.01–0.02 cycles/nm that is also typical of sets of reflectance spectra measured from natural surfaces.[6]

Achromatic filters were defined by $T(\lambda) = c$, where c was selected within the range of $0 \leqslant c \leqslant 1$. Chromatic filters were defined by $1 - A(\lambda)$ with the absorption spectrum of the filter defined by a Gaussian distribution

$A(\lambda) = A_0 \exp[-(\lambda - \lambda_m)^2/2\sigma^2]$, where $400\ \text{nm} \leq \lambda_m \leq 700\ \text{nm}$, $5\ \text{nm} \leq \sigma \leq 200\ \text{nm}$, and A_0 is a scaling factor. The color of natural dyes is produced by absorptions that result in transitions between molecular orbitals of the dye molecules, and therefore it is reasonable to assume (given additional rotational and vibrational energy levels) that absorption spectra produced by such processes would be approximately Gaussian in spectral shape.[26] The value of the internal reflectance of the filter was varied in the range $0 \leq r \leq 0.5$. In addition, numerical spectral transmittances were obtained for a set of 105 Wratten filters and were used to compare two sets of simulation results: those obtained with use of idealized filters and similar results obtained with use of realistic filters.

In all simulations 1000 pairs of surfaces from the full set of 1269 were randomly selected. Smith–Pokorny cone excitations[20] were computed (with 10-nm intervals for the numerical integration) for the surfaces viewed directly and for predictions of color signals from the surfaces viewed through transparent layers. For each simulation the cone-excitation ratios were computed for the two surfaces simulated as seen through the filters and were regressed against the cone-excitation ratios for the surfaces themselves for each cone class.

6. RESULTS

Results from a typical example of the simulation are presented in Fig. 3. Figure 3(a) shows the reflectance spectra for two of the Munsell surfaces selected at random; Fig. 3(b) shows the spectral transmittance for a typical filter ($\lambda_{max} = 670\ \text{nm}$, $\sigma = 100\ \text{nm}$) that was used in the simulation; and Fig. 3(c) shows the reflectance spectra of the two surfaces in Fig. 3(a) computed with Eq. (4) ($r = 0$) when covered by the filter in Fig. 3(b).

The cone excitations can be computed for the surfaces viewed directly [Fig. 3(a)] and through the filter [Fig. 3(c)]. Figure 4 shows a scatterplot of ratios of cone excitations computed for 1000 randomly selected pairs of Munsell surfaces both with and without a randomly selected filter ($\sigma = 50\ \text{nm}$ and $r = 0$, but λ_{max} is random).

Table 2 shows a summary of results obtained for achromatic and chromatic filters with no internal reflectance ($r = 0$). An analysis of cone-excitation ratios for 1000 pairs of unrelated surfaces (control condition) selected from the Munsell set showed coefficients of determination of less than 0.002 for all three cone classes respectively (the proportion of cone-excitation ratios within 10% of each other was 0.046, 0.058, and 0.053 for short-, medium-, and long-wavelength-sensitive cone classes, respectively). This illustrates that the high coefficients of determination recorded in Table 2 for other conditions really represents a strong correspondence between the cone-excitation ratios for surfaces viewed directly and for the same surfaces viewed through a transparent filter.

For idealized achromatic filters the invariance of cone-excitation ratios is perfect for all cone classes irrespective of the value of r. This is expected, since the cone excitations are the integrals of the product of the color signal and the cone-class sensitivity; if $T(\lambda)$ in Eq. (4) is represented by a constant, then it simply cancels if ratios of cone excitations are considered. For chromatic filters the invariance of cone-excitation ratios steadily decreases as

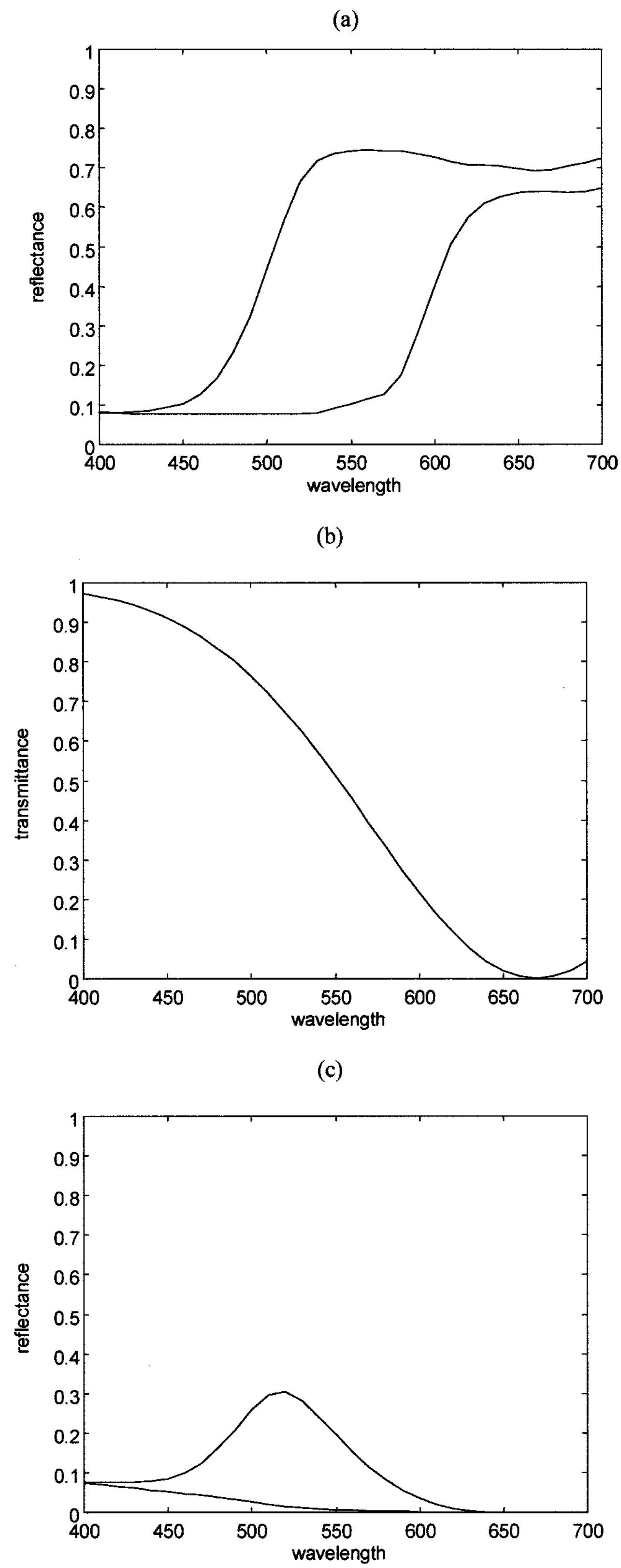

Fig. 3. (a) Spectral reflectance data for two typical surfaces from the Munsell set; (b) typical filter with $\lambda_{max} = 670\ \text{nm}$, $\sigma = 100\ \text{nm}$, and $r = 0$; (c) spectral reflectance data for the two surfaces in (a) covered by the filter in (b).

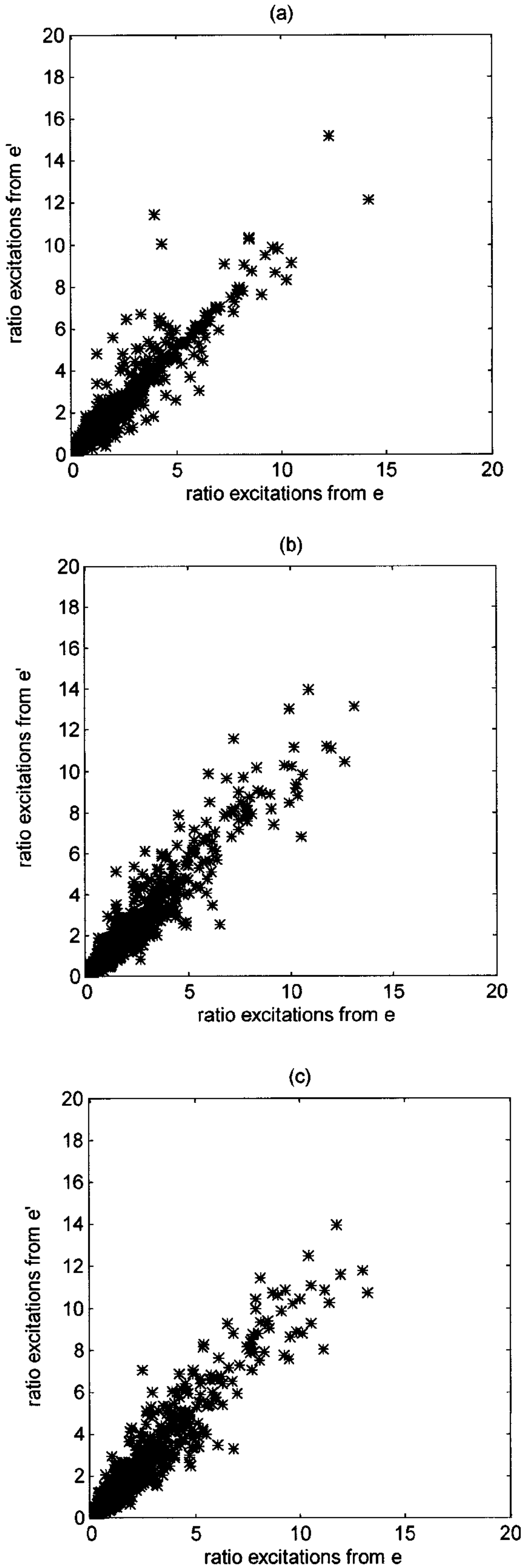

Fig. 4. Scatterplot of ratios of cone excitations for (a) short-, (b) medium-, and long-wavelength-sensitive cone classes for a filter with $\sigma = 50\,\text{nm}$, and $r = 0$. Each point represents a pair of ratios produced by light (under simulated natural daylight at 6500 K) from two surfaces viewed directly e and through a transparent filter e'. Based on 1000 iterations.

the bandwidth (controlled by σ) of the absorption spectrum of the filter increases. However, the coefficients of determination are greater than 0.7 even when $\sigma = 200\,\text{nm}$ ($r = 0$). The color signals from areas viewed through transparent layers are intuitively less related to the color signals from the same areas viewed directly as the transmission properties of the transparent layers become increasingly narrowband.

Table 3 shows results for a fixed chromatic filter ($\sigma = 50\,\text{nm}$, but λ_m variable) with varying internal reflectance r. Estimates[22] of r for physical systems have been reported between 0 and 0.4. The parameter r had little effect on the cone-excitation-ratio invariance.

The data in Table 2 show that the invariance becomes weaker as the bandwidth (controlled by σ) of the filter's absorption spectrum increases. It must be noted, however, that these absorption spectra were not controlled to have the same area under their curve. The maximum absorption at λ_m nm was always 1.0 (consequently, the minimum transmission was 0.0), and hence the general transmission of the filter also decreased with σ. To investigate whether the general level of transmission (in terms of the optical strength of the filter) affected the cone-excitation-ratio invariance for colored filters, the parameter A_0 was reduced for a fixed value of σ (see Table 4).

The data in Table 4 show that as the optical strength of the filter decreases the cone-excitation-ratio invariance increases. If $A_0 = 0$, with the consequence that the filter was 100% transparent at all wavelengths, then by definition the invariance would be perfect.

To what extent do these results depend on the assumption that the filters have a single absorption peak with a Gaussian absorption profile? The data in Table 5 show that increasing the number n of absorption peaks actually increases the strength of the invariance. These data were obtained by allowing the filter to have a variable number of absorption peaks by summing n absorption profiles (each with an independent peak, but each with $A_0 = 1/n$).

Finally, Fig. 5 shows a scatterplot of ratios of cone excitations for 1000 randomly selected pairs of surfaces viewed both directly and through a filter [Eq. (4)], where $T(\lambda)$ was selected from a set of 105 Wratten filters. The gradient (coefficient of determination in parentheses) of the scatterplots was 0.9872 (0.7556), 0.9929 (0.8336), and 0.9838 (0.8361) for short-, medium-, and long-wavelength-sensitive cone classes, respectively.

The results from the simulation of the achromatic filters show that the invariance is perfect given that $T > 0$. How can these be reconciled with the actual measurements presented in Section 3 that show invariance becoming progressively weaker as $T \rightarrow 0$? There are two possible explanations. First, the invariance measured for a real system might degrade with decreasing T because of a deteriorating signal-to-noise ratio. Second, the omission of the external reflectance from our models of the filter [Eq. (4)], and indeed our opaque surfaces, might become important when the signal from the filtered area becomes small. For the colored filters, data obtained from the final simulation with 105 Wratten filters are not inconsistent with the data in Table 2 that show a

Table 2. Gradient of Least-Squares Linear Fit for Plots of Ratios for Surfaces Viewed through a Filter (with $r = 0$ and $A_0 = 1$) versus the Same Surfaces Viewed Directly for Achromatic and Chromatic Filters[a]

Model Parameter	Short-Wavelength[b]	Medium-Wavelength[b]	Long-Wavelength[b]
Control (unrelated surfaces)	0.0195 (0.0003)	0.0433 (0.0018)	0.0421 (0.0019)
Achromatic ($r = 0$)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
Chromatic ($\sigma = 10$ nm; $r = 0$)	0.9988 (0.9968)	0.9964 (0.9951)	0.9996 (0.9955)
Chromatic ($\sigma = 20$ nm; $r = 0$)	0.9971 (0.9897)	0.9851 (0.9806)	0.9797 (0.9814)
Chromatic ($\sigma = 50$ nm; $r = 0$)	0.9978 (0.9231)	1.0037 (0.8666)	0.9788 (0.8599)
Chromatic ($\sigma = 100$ nm; $r = 0$)	0.9765 (0.8807)	0.9578 (0.8129)	0.9658 (0.8082)
Chromatic ($\sigma = 200$ nm; $r = 0$)	0.9231 (0.8032)	0.8885 (0.7154)	0.9144 (0.7284)

[a] Each entry is based on 1000 random samples of pairs of surfaces.
[b] Coefficient of determination is given in parentheses.

Table 3. Gradient of Least-Squares Linear Fit for Plots of Ratios for Surfaces Viewed through a Chromatic Filter ($\sigma = 50$ nm and $A_0 = 1$) versus the Same Surfaces Viewed Directly for Different Values of r[a]

Model Parameter	Short-Wavelength[b]	Medium-Wavelength[b]	Long-Wavelength[b]
Chromatic ($\sigma = 50$ nm; $r = 0.0$)	0.9846 (0.9403)	0.9792 (0.9112)	0.9832 (0.9063)
Chromatic ($\sigma = 50$ nm; $r = 0.1$)	0.9800 (0.9270)	0.9671 (0.9226)	0.9702 (0.9240)
Chromatic ($\sigma = 50$ nm; $r = 0.2$)	1.0069 (0.9413)	0.9997 (0.8947)	1.0002 (0.8968)
Chromatic ($\sigma = 50$ nm; $r = 0.3$)	1.0231 (0.9351)	1.0102 (0.9035)	1.0137 (0.8946)
Chromatic ($\sigma = 50$ nm; $r = 0.4$)	0.9761 (0.9320)	0.9777 (0.8886)	0.9764 (0.8919)
Chromatic ($\sigma = 50$ nm; $r = 0.5$)	0.9832 (0.9417)	0.9840 (0.8949)	0.9983 (0.8959)

[a] Each entry is based on 1000 random samples of pairs of surfaces.
[b] Coefficient of determination is given in parentheses.

Table 4. Gradient of Least-Squares Linear Fit for Plots of Ratios for Surfaces Viewed through a Chromatic Filter ($\sigma = 50$ nm) versus the Same Surfaces Viewed Directly for Different Values of A_0[a]

Model Parameter	Short-Wavelength[b]	Medium-Wavelength[b]	Long-Wavelength[b]
Chromatic ($\sigma = 50$ nm; $A_0 = 1.0$)	0.9789 (0.9328)	1.0007 (0.9200)	0.9901 (0.9077)
Chromatic ($\sigma = 50$ nm; $A_0 = 0.8$)	1.0109 (0.9767)	1.0268 (0.9387)	1.0251 (0.9312)
Chromatic ($\sigma = 50$ nm; $A_0 = 0.6$)	0.9886 (0.9774)	1.0111 (0.9705)	1.0153 (0.9690)
Chromatic ($\sigma = 50$ nm; $A_0 = 0.4$)	0.9916 (0.9936)	1.0018 (0.9866)	1.0075 (0.9858)
Chromatic ($\sigma = 50$ nm; $A_0 = 0.2$)	1.0018 (0.9987)	0.9998 (0.9974)	1.0005 (0.9970)

[a] Each entry is based on 1000 random samples of pairs of surfaces.
[b] Coefficient of determination is given in parentheses.

Table 5. Gradient of Least-Squares Linear Fit for Plots of Ratios for Surfaces Viewed through a Chromatic Filter ($\sigma = 50$ nm) versus the Same Surfaces Viewed Directly with Increased Number n of Absorption Peaks in the Filter[a]

Model Parameter	Short-Wavelength[b]	Medium-Wavelength[b]	Long-Wavelength[b]
Chromatic ($\sigma = 50$ nm; $n = 1$)	0.9901 (0.9184)	0.9708 (0.8924)	0.9820 (0.8895)
Chromatic ($\sigma = 50$ nm; $n = 2$)	1.0017 (0.9732)	0.9901 (0.9454)	0.9957 (0.9365)
Chromatic ($\sigma = 50$ nm; $n = 3$)	0.9926 (0.9822)	0.9932 (0.9661)	0.9926 (0.9622)
Chromatic ($\sigma = 50$ nm; $n = 4$)	0.9874 (0.9875)	1.0125 (0.9622)	1.0244 (0.9571)
Chromatic ($\sigma = 50$ nm; $n = 5$)	0.9808 (0.9880)	0.9747 (0.9676)	0.9791 (0.9660)

[a] Each entry is based on 1000 random samples of pairs of surfaces.
[b] Coefficient of determination is given in parentheses.

deterioration in invariance when the absorption bandwidth of the filter increases. Our results show that cone-excitation ratios can be approximately invariant for a range of achromatic and chromatic filter systems but that the invariance weakens (in the limit disappears altogether) when the transmittance of the filters becomes small at some or all wavelengths. Since we do not present any psychophysical experiments of transparency perception, we can conclude nothing directly about transparency perception. However, we postulate that the conditions under which the cone-excitation ratios are approximately invariant are the same conditions under which the perception of transparency might reasonably be expected to be strong (given the necessary figural con-

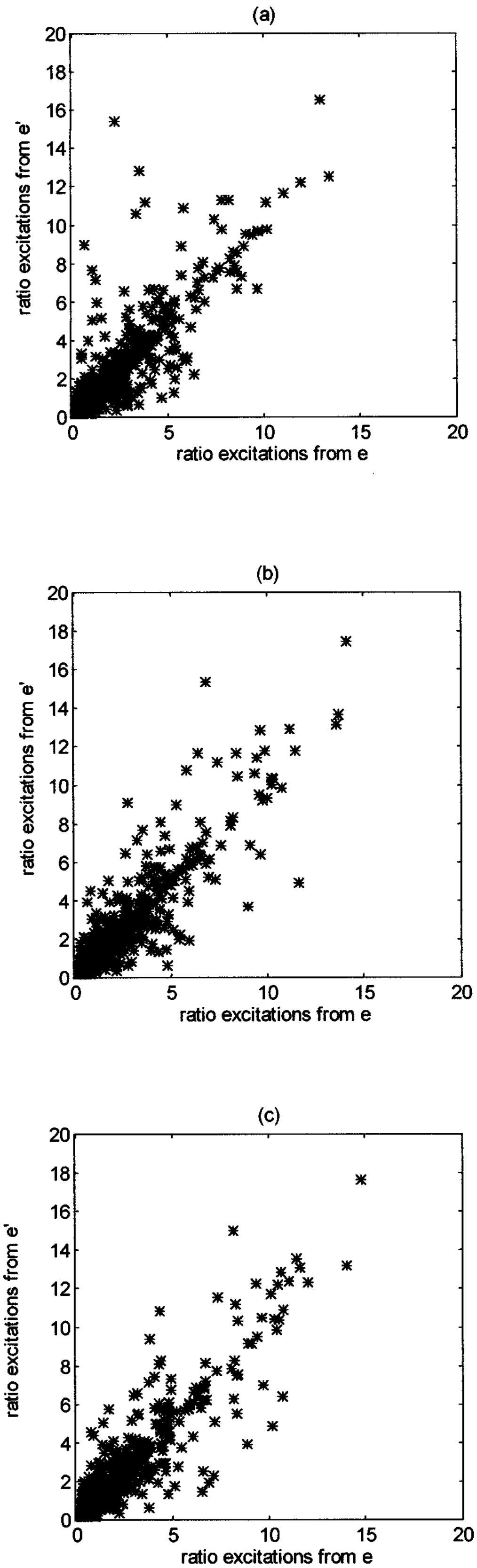

Fig. 5. Scatterplot of ratios of cone excitations for (a) short-, (b) medium-, and long-wavelength-sensitive cone classes randomly selected Wratten filters with $r = 0$. Each point represents a pair of ratios produced by light (under simulated natural daylight at 6500 K) from two surfaces viewed directly e and through a transparent filter e'. Based on 1000 iterations.

straints). Conversely, the invariance appears to break down for filters with low transmittance at some or all wavelengths, and these filters would be expected to yield weaker percepts of perceptual transparency.

7. DISCUSSION

We have argued that the notion of the invariance of cone-excitation ratios that has been developed by Foster and Nascimento[14] as a possible basis for perceptual color constancy may also be used to define the stimulus conditions necessary for the perception of transparency. The encoding of cone-excitation ratios by the visual system may be one way of preserving spatial color relations for pairs of surfaces either when the illuminant changes or when the surfaces are overlaid with a transparent layer. We have shown that cone-excitation ratios are almost invariant in some cases from a set of physically transparent systems and for a wide range of simulated systems. It is important to note that we do not propose the invariance of cone-excitation ratios as a constraint for physical transparency. We do find, however, that the invariance is weakest when the transmittance of the filter is low at some or all wavelengths and note that in the limit (when the spectral transmittance of filters approaches zero) cone-excitation ratios cannot be invariant. Thus we find that the invariance is weaker for the colored filters (which allowed little transmittance at certain wavelengths) in Section 3 than for the achromatic filters. Similarly, the Monte Carlo simulations show that the invariance is strongest for filters that allow substantial transmittance in most of the visible spectrum (these filters have low values of σ). The physical systems that would be expected to give rise to strongest perceptions of transparency yield cone-excitation ratios that are nearly invariant, whereas those systems that would be expected to give rise to only weak (if at all) perceptions of transparency yield less invariant cone-excitation ratios. This study does not report any psychophysical data; however, some preliminary psychophysical experiments[27] support the hypothesis that the strength of the invariance of cone-excitation ratios is correlated with the strength of the transparency percept for simulations of physically transparent systems. The invariance of cone-excitation ratios may also be a useful constraint for perceptually transparent systems that are not physically transparent.

The use of cone-excitation ratios is one possible way of expressing the color relations between spatially adjacent surfaces, but it is by no means unique. For example, D'Zmura *et al.*[5] also considered the color shifts that are necessary for the perception of transparency and reported that these shifts [referring to Fig. 1 and Eq. (1), $e_{i,1}$ to $e'_{i,1}$ and $e_{i,2}$ to $e'_{i,2}$] can be expressed as translation or convergence in color space. If cone-excitation ratios are invariant, it is mathematically equivalent to saying that the shifts $e_{i,1}$ to $e'_{i,1}$ and $e_{i,2}$ to $e'_{i,2}$ are invariant. It is possible that cone-excitation invariance may be a special case of a more general set of conditions proposed by D'Zmura *et al.*[5] for transparency perception. However, although the perfect simultaneous invariance of all three cone classes corresponds with the shifts postulated by D'Zmura *et al.*, it may be subsequently shown that the simultaneous invariance of only one or two cone classes might correspond to the more general conditions that D'Zmura *et al.* showed to predict perceptual transparency. Psychophysical studies are required to ascertain whether transparency perception is correlated with the simultaneous invariance of all three cone classes or whether some cone classes are more important than others. Nascimento and Foster showed that for color constancy, observers are most sensitive to violations in invariance of long-wavelength-sensitive cones and least sensitive for short-wavelength-sensitive cones.[28]

The expression of the spatial relation in terms of the ratios of cone responses for spatially adjacent surfaces explicitly relates the perception of transparency to the phenomenon of color constancy. Furthermore, it might indicate that these two tasks are accomplished by common mechanisms and possibly by common neural structures. However, we note that color constancy and transparency can share mechanisms independent of what those mechanisms are; that is, the validity of this point is independent of whether the basis is invariant cone-excitation ratios. Even if cone-excitation ratios are used for the perception of transparency, they would undoubtedly form only one of a set of cues that could be available to the visual system. For example, the invariance of cone-excitation ratios might be combined with depth and specular cues and, in certain circumstances, cognitive cues including observer expectations.

Color constancy allows us to correctly interpret changes in color signals as changes in the physical world, and it is possible to construct arguments with evolutionary driving forces for the development of color constancy in humans. Although the perception of transparency is often useful in the man-made world, it is not so easy to construct arguments that might explain the development of transparency perception. Why would humans and other animals develop special visual processes for the perception of transparency? If transparency shares mechanisms and possibly structures with color constancy, then this would represent an extremely efficient biological design and might support the notion that transparency perception is a bonus ("something for nothing") for a visual system that develops color constancy.

Our computational analyses have shown that cone-excitation ratios can be broadly invariant for a wide range of filters properties and for Munsell surfaces. In these analyses both the surfaces and the filters were defined by smooth functions of wavelength. There is also an implicit assumption that the illuminant is broadband. However, these assumptions are not inconsistent with some estimates of natural scenes[6] that reveal that spectral properties of both are slowly varying functions of wavelength and with measurements of natural daylight.[21] Whether the constraints on the transmission properties and internal reflectances of transparent layers that are necessary for cone-excitation-ratio invariance are consistent with objects in the natural or man-made world is an empirical problem yet to be solved. Furthermore, psychophysical work is required to investigate whether the invariance of cone-excitation ratios is a potential cue for perceptual transparency for a wide range of stimuli including those that would not be physically transparent.

APPENDIX A: TABLE 6

Table 6. Spectral Reflectance Factors for Surfaces Used to Generate Data Shown in Table 1

nm	White	Gray	Red	Green	Blue	Yellow
400	0.3647	0.2253	0.1496	0.0907	0.4535	0.1106
410	0.6243	0.2505	0.1404	0.0900	0.5075	0.1082
420	0.8121	0.2666	0.1315	0.0909	0.5593	0.1054
430	0.9221	0.2715	0.1231	0.0921	0.6103	0.1004
440	0.9720	0.2714	0.1150	0.0977	0.6561	0.0985
450	0.9667	0.2727	0.1071	0.1101	0.6926	0.1045
460	0.9392	0.2735	0.0999	0.1327	0.7199	0.1169
470	0.9295	0.2741	0.0940	0.1704	0.7377	0.1358
480	0.9215	0.2742	0.0892	0.2209	0.7432	0.1608
490	0.9118	0.2740	0.0848	0.2921	0.7323	0.1899
500	0.9020	0.2732	0.0829	0.3580	0.7084	0.2277
510	0.8920	0.2713	0.0872	0.3944	0.6737	0.2794
520	0.8838	0.2697	0.0918	0.4031	0.6277	0.3421
530	0.8794	0.2702	0.0888	0.3697	0.5690	0.4184
540	0.8773	0.2713	0.0889	0.3178	0.5039	0.4981
550	0.8758	0.2718	0.0974	0.2660	0.4382	0.5710
560	0.8765	0.2726	0.1222	0.2153	0.3732	0.6402
570	0.8810	0.2741	0.1741	0.1724	0.3095	0.7084
580	0.8876	0.2756	0.2502	0.1385	0.2551	0.7666
590	0.8959	0.2766	0.3593	0.1182	0.2210	0.8037
600	0.9032	0.2769	0.4785	0.1068	0.1994	0.8292
610	0.9056	0.2761	0.5907	0.1010	0.1862	0.8482
620	0.9065	0.2748	0.6879	0.1006	0.1811	0.8616
630	0.9080	0.2738	0.7532	0.1048	0.1842	0.8721
640	0.9103	0.2727	0.7971	0.1109	0.1903	0.8797
650	0.9158	0.2718	0.8219	0.1156	0.1936	0.8852
660	0.9211	0.2708	0.8354	0.1196	0.1959	0.8890
670	0.9227	0.2696	0.8466	0.1205	0.1944	0.8921
680	0.9226	0.2681	0.8540	0.1250	0.1984	0.8940
690	0.9215	0.2661	0.8593	0.1398	0.2174	0.8946
700	0.9192	0.2637	0.8620	0.1625	0.2482	0.8939

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