

# Invasion percolation on regular trees

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**Abstract:** We consider invasion percolation on a rooted regular tree. For the infinite cluster invaded from the root, we identify the scaling behaviour of its  $r$ -point function for any  $r \geq 2$  and of its volume both at a given height and below a given height. In addition, we derive scaling estimates for simple random walk on the cluster starting from the root. We find that while the power laws of the scaling are the same as for the incipient infinite cluster for ordinary percolation, the scaling functions differ. Thus, somewhat surprisingly, the two clusters behave differently. We show that the invasion percolation cluster is stochastically dominated by the incipient infinite cluster. Far above the root, the two clusters have the same law locally, but not globally. A key ingredient in the proofs is an analysis of the forward maximal weights along the backbone of the invasion percolation cluster. These weights decay towards the critical value for ordinary percolation, but only slowly, and this slow decay causes an anomalous scaling behaviour

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