Review of Economic Studies (1982) XLIX, 567-582 0034-6527/82/00440567$00.50


# Invention and Innovation Under Alternative Market Structures: The Case of Natural Resources

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This paper examines the interactions between market structure and resource allocation over time when there is endogenous technical progress. The structures considered are a planned economy, pure monopoly, and competition with patent rights. In an efficient allocation the date of invention coincides with the date of innovation (the date at which technology is used). This is also true with a pure monopoly, but monopoly retards technical progress relative to the efficient level.

Competition for patents rights to a new technology results in excessively rapid technical progress if the resource endowment of the economy is sufficiently large. Also, competition may lead to "sleeping patents", where invention strictly precedes the date of innovation.

## 1. INTRODUCTION

The potential for economic growth and transition from the use of exhaustible resources depends on the rate of development of new technology. The effect of market structures on the timing of invention and innovation has long been a subject of controversy. Arrow (1962) argued that the private incentive for research and development is less than its social value and that monopoly is less progressive than competitive markets. Others (e.g. Barzel (1968), Stiglitz (1971) and Loury (1979)) suggested there may be excessive research expenditures in competitive markets.

This paper develops a model of resource depletion which allows a precise comparison between the timing of new technology substitution in different markets. The structures considered are the polar examples of a planned economy, pure monopoly, and competition with patent rights. The restriction is in part a consequence of limited space and in part a deliberate attempt to analyse the allocation of R & D in extreme market structures.

We distinguish the date of invention (the date at which a new technology becomes commercially practicable) from the date of innovation (the date at which the new technology is used). Section 2 examines the socially optimal investment in R & D. With no uncertainty invention and innovation coincide, and with constant marginal costs in the production of the substitute technology price falls discontinuously after invention. Section 3 shows that invention and innovation also coincide in a pure monopoly, but monopoly invests less than the socially optimal amount in research and development.

Section 4 introduces patent rights into an otherwise perfectly competitive model of resource depletion. For small values of the resource stock competitors may spend less than the optimal level on R & D; but, subject to a restriction on the invention technology, competition always results in excessively rapid technical change when the resource stock is sufficiently large. In addition, competition in the patent race may lead to "sleeping patents". A firm may patent the new technology before it is actually used. Since patenting is costly by assumption, the existence of "sleeping patents" is inefficient. They occur because any attempt to delay invention (and reduce the inefficiency) allows a competitor to obtain the patent. Also, any attempt to accelerate innovation causes a price response which lowers profits to the point where the inventor makes a net loss.

The characterization of optimal expenditures on research and development parallels work by Dasgupta, Heal and Majumdar (1976) and Kamien and Schwartz (1978), which explore optimal growth with endogenous technical change. The focus here is the effects of industry organization on research and development, and although the particular problem is the development of a substitute for a natural resource, the framework of analysis can be applied to any situation in which commodities are produced by durable capital goods. Essentially, the questions of what rate the resource should be depleted before and after invention are equivalent to what stock of machines to maintain before invention of a new technology and what rate to depreciate the now obsolescent machines after invention.

## 2. THE PLANNED ECONOMY

This section examines the efficient allocation of effort to invention and innovation with exhaustible resources and extends the analysis of exogenous technical change in Dasgupta and Stiglitz (1981*b*). By assumption, a perfect substitute for a natural resource can be developed at an investment cost that increases (in present value terms) if the delay until the date of invention is reduced. Let $T$ be the date of invention; the R & D cost function is[1]

$$x = x(T), \quad x'(T) \leqq 0.$$

The demand for the commodity is given by

$$Q = D(P)$$

where $Q$ is the amount consumed and $P$ is the price. For simplicity the interest rate is fixed at $r$.

Since the solution trajectories derived in the following sections are not, in general, continuous functions of time, we economize on notation and adopt the convention that

$$f(t) = \limsup_{\tau \to t} f(\tau).$$

In the planned economy, the government must choose consumption, production, research and extraction plans to maximize social welfare. We take as the objective of the government the maximization of consumer plus producer surplus. Define

$$u(Q) = \int_0^Q P(Q)dQ \tag{1}$$

where

$$P(Q) = D^{-1}(Q), \quad \text{the inverse demand curve.}$$

By definition,

$$u'(Q) = P(Q). \tag{2}$$

The problem of the government is to maximize

$$\int_0^\infty u(Q(t))e^{-rt}dt - \int_T^\infty y(t)\bar{P}e^{-rt}dt - x(T) \tag{3}$$

subject to

$$Q(t) \leqq z(t) + y(t) \tag{3a}$$

and

$$\int_0^\infty z(t)dt \leqq S \tag{3b}$$

where $y(t)$ is production of the substitute at time $t$, $z(t)$ is extraction of the natural resource at time $t$, and $\bar{P}$ is the cost of production using the new technology.

The optimal policy is characterized by

**Proposition 1**

(a) $z(t) = 0$ *for* $t > T$; *production of the substitute begins immediately after invention (invention and innovation coincide).*

(b) *For* $t < T$, $\dot{P}/P = r$; *prior to invention, price rises at the rate of interest.*

(c) For $t > T$, $P = \bar{P}$; *after the invention, price is equal to the marginal cost of producing the substitute.*

(d) *Price is discontinuous at the date of invention for* $T < \infty$, *provided the marginal cost of invention,* $dx/dT$, *is non-zero and demand is not perfectly elastic.*

Propositions 1(a), (b), and (c) are immediate (see Dasgupta and Stiglitz (1981*a*)) and represent standard results on production and resource extraction. For proof of 1(d), use results 1(a) and (c) to rewrite the maximization problem as

$$\max_T \left\{ V(S, T) + e^{-rT} \frac{[u(\bar{Q}) - \bar{P}\bar{Q}]}{r} - x(T) \right\} \tag{4}$$

where $\bar{Q} = D(\bar{P})$ and

$$V(S, T) = \max_{Q(t)} \int_0^T u(Q(t))e^{-rt}dt \tag{5}$$

subject to

$$\int_0^T Q(t)dt \leqq S.$$

At the optimal invention date the marginal benefit from delaying the extraction programme,

$$\frac{dV(S, T)}{dT}$$

equals the discounted marginal cost,

$$e^{-rT}[u(\bar{Q}) - \bar{P}\bar{Q}] + x'(T) \quad \text{for } T \in (0, \infty).$$

Appendix I shows that the marginal benefit from lengthening the extraction programme is

$$\frac{dV(S, T)}{dT} = e^{-rT}[u(Q(T)) - u'(Q(T))Q(T)] = e^{-rT}N(Q(T)), \tag{6}$$

where $N(Q(T))$ is the surplus from consumption of $Q(T)$ evaluated at the shadow price $P(T) = u'(Q(T))$.

Writing $N(\bar{Q}) = u(\bar{Q}) - \bar{P}\bar{Q}$, then if the marginal cost of advancing the invention date is positive,

$$e^{-rT}[N(\bar{Q}) - N(Q(T))] = -x'(T) > 0 \tag{7}$$

for $T \in (0, \infty)$. Since $dN(Q)/dQ = -Q\,dP/dQ$ and this is strictly positive if demand is not perfectly elastic, it follows that $\bar{Q} > Q(T)$ and price falls discretely at the invention date.

**Proposition 2.** *The date of invention is an increasing function of the stock of the resource.*

When the optimal invention date is $T \in (0, \infty)$,

$$\frac{dV(S,T)}{dT} - e^{-rT}N(\bar{Q}) - x'(T) = 0. \tag{8}$$

Differentiating (8) with respect to $S$ gives

$$\frac{dT}{dS} = \frac{-\frac{\partial}{\partial S}\left[\frac{dV(S,T)}{dT}\right]}{\frac{d^2V(S,T)}{dT^2} + re^{-rT}N(\bar{Q}) - \frac{d^2X}{dT^2}} \tag{9}$$

Since

$$\frac{dV(S,T)}{dT} = e^{-rT}N(Q(T)),$$

$$\frac{\partial}{\partial S}\left[\frac{dV(S,T)}{dT}\right] = -e^{-rT}Q(T)\frac{dP}{dQ}\frac{\partial Q(T)}{\partial S}$$

which is positive since $Q(T)$ is an increasing function of the initial resource stock. (This follows because extraction paths for different resource stocks do not cross.) The second-order necessary condition for the optimal invention date requires that the denominator in (9) must be negative, and therefore

$$dT/dS > 0.$$

This assumes a finite invention date. For sufficiently large values of the resource stock, the marginal benefit from delaying invention may exceed the marginal cost and the optimal R & D programme may involve no research at all.

## 3. THE PURE MONOPOLIST

The problem of the monopolist is identical to that of the social planner with revenues, $R(Q(t))$, replacing social benefits, $u(Q(t))$. The monopolist maximizes

$$\int_0^\infty R(Q(t))e^{-rt}dt - \int_T^\infty y(t)\bar{P}e^{-rt}dt - x(T). \tag{10}$$

The optimal policy is characterized by:[2]

**Proposition 3**

(a) *$z(t) = 0$ for $t > T$; invention and innovation coincide;*
(b) *for $t < T$, $\dot{m}/m = r$, (where $m(Q(t)) = R'(Q(t))$, marginal revenue) marginal revenue rises at the rate of interest;*
(c) *for $t > T$, $m = \bar{P}$; after the invention, marginal revenue is equal to the marginal cost of producing the substitute;*
(d) *Price is discontinuous at the date of invention, provided $x'$ and $P'(Q)$ are non-zero.*

The proof is isomorphic to the planning problem in Proposition 1 with social benefits replaced by monopoly profits.

A comparison of the socially-optimal and the monopoly allocation of investment in the development of the substitute source of supply can be obtained if both social benefits and monopoly profits are concave in the date of innovation, $T$. Sufficient conditions for concavity are downward sloping demand for social benefits and downward sloping marginal revenue for monopoly profits, along with the stronger assumption that marginal development costs, compounded to the date of innovation, are a decreasing function of $T$. If these conditions are satisfied, we may compare the monopoly and socially-optimal allocation of investment by determining the sign, evaluated at the socially-optimal invention date $T^*$, of

$$\frac{d\Pi^m}{dT} = e^{-rT}[M(Q(T)) - M(Q^m) - x'(T)e^{rT}] \tag{11}$$

where $M(Q) = R(Q) - R'(Q)Q$ and $Q^m$ is defined by $R'(Q^m) = \bar{P}$. If the sign is positive, monopoly profits are increased by choosing a later innovation date, and conversely if the sign is negative.

The determination of the sign of the marginal profit function proceeds by establishing a contradiction, and because the proof is somewhat lengthy, it is reserved for the Appendix. The result is summarized in the following proposition.

**Proposition 4.** *If social benefits and monopoly profits are concave functions of the invention date, a monopolist introduces a substitute source of supply at a date later than the socially optimal invention date. The monopolist also spends less on invention than is socially optimal, provided the marginal cost of invention is not zero.*

This result accords with the intuition that the monopolist, because he is able to capture only a fraction of the consumer surplus associated with the invention, has too little incentive for doing research. The matter, however, is more subtle than that, because there are three distinct effects from pursuing a faster R & D policy:

(a) The costs of R & D increase;
(b) The earlier invention decreases the value of the existing stock of natural resources, to both society and to the monopolist; the magnitude of the decrease, for a given date of invention, is actually larger for society than for the monopolist.
(c) The earlier invention increases the present discounted value of social and private returns accruing directly from the invention; again, because the monopolist has a higher price and because he appropriates only a fraction of social profits, the private profits are smaller than the social profits.

The net return from more R & D is found by adding these three effects; what we have managed to establish is that effect (c) always dominates effect (b) i.e. the net return for a monopolist is less than for society, and hence he engages in too little research.

We can establish, in a manner similar to that used for the socially optimal date of invention that, if profits are concave in output,

**Proposition 5.** $dT^m/dS > 0$, *the date of invention is an increasing function of the stock of the natural resource.*

## 4. COMPETITIVE RESOURCE SECTOR AND PATENTS

This section introduces patent rights into an otherwise perfectly competitive model of resource exhaustion. The firm that first discovers the substitute receives a monopoly position in the use of the new technology. Prior to the invention there is competition (free entry) for the right to make the discovery and obtain the patent. After the invention the monopolist of the substitute will be forced to compete with owners of the natural resource, if they have not exhausted their stocks by the invention date.

The results of this section contrast with the classical observations of the economics of the patent system (e.g. Arrow (1962)). The competitive investment in R & D may be greater than or less than the socially optimal level. If the marginal cost of invention, compounded to the invention date, decreases as the invention date increases, the competitive investment in R & D is always excessive for a sufficiently large resource stock. Furthermore, there may be "sleeping patents". Let $T_1$ be the date of invention and $T_2$ the date of innovation (the earliest date at which $y(t) > 0$). In the previous two cases invention and innovation coincided, but we show this is not necessarily true with competition for patents: invention may precede innovation and give rise to "sleeping patents".

Dasgupta and Stiglitz (1981*a*, *b*) show that unexploited technological capacity can occur in a stochastic economy by virtue of unexpectedly rapid discovery of substitute sources of supply. The point here is that uncertainty is not a necessary condition for this result. Sleeping patents may occur because competition for the monopoly profits from a substitute technology dissipates those profits by advancing the date of invention prior to the date at which the technology will be exploited. The analysis allows for binding contracts between resource suppliers and the inventor of a substitute, although this institutional assumption is not central to the sleeping patent results. Maskin and Newbery (1978) suggest an approach to the relaxation of the binding contract assumption that would apply here, but at a cost of considerable additional complexity.

In general the equilibrium which emerges in this market structure is considerably different from either of the two cases (pure monopoly and the socially planned economy) examined so far. One might expect that competition for patents would generate equilibria that are "in between" those of pure monopoly and the planned economy, but this is not so. The specific results are summarized in Propositions 6 and 7 below. Proposition 6 describes the dynamics of the price path and Proposition 7 compares the investment in R & D with the investment for the planned economy and pure monopoly. In what follows it is useful to define the date $T_3$ as the earliest date, after invention of the substitute, at which the spot price $P(t)$ equals the long-run monopoly price $P^m$. Recall that $T_1$ is the date of invention and $T_2$ the date of innovation.

**Proposition 6.** *There exist two critical values of the initial resource stock*: $S'$ *and* $S''$ ($S'' > S'$).

(a) For $S \leqq S'$,

(i) $T_1 = T_2 = \hat{T}$; *the dates of invention and innovation coincide and are independent of the size of the stock.*

(ii) *For* $t < \hat{T}$, $\dot{P}/P = r$; *price rises at the rate of interest. For* $t > \hat{T}$, $P = P^m$; *price equals the long-run monopoly price.*

(iii) $\text{Lim}_{t \to \hat{T}} P(t) \geqq P^m$; *price just prior to invention/innovation equals or exceeds the monopoly price.*

(b) *For* $S \geqq S''$,

(i) $T_1 < T_2 < T_3$; *invention precedes innovation (there are "sleeping patents") and the resource is exhausted at a price* $P(T_2) < P^m$.

(ii) $dT_1/dS > 0$; *the date of invention is an increasing function of the resource stock.*

(iii) *The price-production pattern has three phases: For* $t < T_3$, $\dot{P}/P = r$. *For* $t \geqq T_3$, $P = P^m$, *where* $T_3 > T_2$. *Price is a continuous function of time. For* $T_1 < t < T_2$ *there is no production of the substitute (even though it is feasible). The resource is exhausted at* $t = T_2$.

(c) *For* $S' < S \leqq S''$

(i) *Either* $T_1 \leqq T_2 < T_3$; *invention and innovation may coincide, but, if so,* $P(T_1) = P(T_2) < P^m$. *Or* $T_1 < T_2 < T_3$. *This pattern is similar to* (b), *but invention and innovation may coincide for some S in the interval* $(S', S'')$.

(ii) *Same as* (b)(ii).

(iii) *Same as* (b)(iii).

The pricing-production properties of Proposition 6 follow immediately from the analysis of Dasgupta and Stiglitz (1978). The properties relating to the timing of innovation are, however, more difficult to establish. We now turn to these.

### 4.1. *Invention when the stock of the resource is small* ($S \leqq S'$)

The competitive date of invention must be independent of the resource stock if, just prior to invention, the resource stock is exhausted and the price equals or exceeds the monopoly price, $P^m$. These conditions ensure that the inventor can earn monopoly profits from the date of invention equal to

$$\pi^m = (P^m - \bar{P})Q^m.$$

Assuming free entry and zero profits from R & D, the invention date, $T^c$, is the smallest solution to

$$e^{-r\hat{T}^c} \frac{\pi^m}{r} = x(\hat{T}^c). \tag{12}$$

Given $\hat{T}^c$, which depends on $\pi^m$, the rate of interest, and the invention technology, there is a maximum stock for which exhaustion will occur when $P(\hat{T}^c) \geqq P^m$ as assumed. This is determined by

$$S' = \int_0^{\hat{T}^c} Q(P^m e^{-r(\hat{T}^c - t)})dt. \tag{13}$$

Now consider what happens as $S$ is increased above $S'$. If $T^c = \hat{T}^c$, there is an interval after invention when $P(t) < P^m$. Hence profits are decreased and the inventor must make a loss; i.e. the date of invention must increase for $S > S'$.

### 4.2. *Invention and innovation when* $S > S'$

For $S > S'$ the equilibrium price rises continuously to the long-run monopoly price, $P^m$. Profit maximization by competitive resource suppliers and by the inventor with the substitute technology may entail the negotiation of binding contracts with the following notable features: (a) the inventor is obligated to withhold production of the substitute even though production is profitable at the market price; and (b) the resource is exhausted when the price is less than the long-run monopoly price of the substitute. It is clear that this equilibrium presumes the existence of binding contracts, since *ex post* incentives are in conflict with *ex ante* agreements. The equilibrium also presumes perfect certainty (or complete state-contingent markets), since only then could the prospective inventor enter into the necessary agreements. While these are important objections, the binding contract equilibrium is the result of intertemporal tradeoffs between the inventor and resource owners which are of sufficient interest to justify a close examination.

The reason for the structure of this equilibrium closely follows the argument in Gilbert (1978). There a cartel faced with a competitive fringe agrees to lower the present price in order to hasten the date at which it can fully exploit its monopoly power.

Similarly, the inventor may want to lower today's resource price (up to a point) in order to advance the date of innovation and thereby increase profits. The peculiar twist in this problem is that the date of *invention*, which responds directly to the inventor's profits, may precede the agreed upon date of *innovation*. If the inventor commences production before the agreed date, the competitive response will cause the current price to fall further and the net effect will lower total profits to the inventor. Since the patent is awarded to the first inventor, and profits must cover the cost of research and development, an attempt to advance innovation closer to the invention date will ultimately delay invention by lowering profits. This means another firm could win the patent by committing itself to the later innovation date. All this, of course, assumes that a prospective inventor cannot fool competitive suppliers into accepting an innovation date which is ultimately advanced.

Consider what happens as $S$ increases above $S'$. When $S = S'$, the market price is $P^m$ at the invention date and the resource is exhausted. Furthermore, the inventor makes zero profits. If $S > S'$ and the invention date is unchanged at $\hat{T}$, the inventor must make a loss because profits from the date $\hat{T}$ are decreased relative to profits when $S = S'$ while the cost of invention is unchanged. Invention must be delayed.

The first inventor wins the patent, and in this world of certainty the firm that can earn the largest profit can afford to invent first. The inventor maximizes profits by announcing a price path with the property

$$P(t) = P_0 e^{rt} \quad \text{for } P(t) \leqq P^m. \tag{14}$$

This condition is a necessary consequence of competition in the supply of the exhaustible resource. The price path also must satisfy

$$\int_0^{T_2} Q(P(t))dt = S \tag{15}$$

with $P(T_2) \leqq P^m$.

The resource can be exhausted at date $T_2$ when $P(T_2) < P^m$, provided price continues to rise at the rate of interest to the long-run monopoly price $P^m$. By choosing a price path with $P(T_2) < P^m$, the inventor can accelerate the date of innovation, although he must accept in return a lower price at the innovation date. Production of the substitute must satisfy

$$\begin{aligned} y(t) &= 0 \quad \text{for } t < T_2 \\ y(t) &> 0 \quad \text{for } t \geqq T_2. \end{aligned} \tag{16}$$

Given the equilibrium price trajectory, production of the substitute is feasible for $T < T_2$. It is not profit-maximizing because given the equilibrium price trajectory the total production of the substitute in the interval $[0, T_3]$ is fixed, and since price rises at the rate of interest, the inventor increases profits by delaying production until the resource is exhausted.[3]

The inventor's profits are given by

$$V(P_0, S) = \int_{T_2}^{T_3} Q(P_0 e^{rt})[P_0 e^{rt} - \bar{P}] e^{-rt} dt + e^{-rT_3} \frac{\pi^m}{r}. \tag{17}$$

The date $T_2$ is defined by equation (15), conditional on $P_0$ and $S$, and $T_3$ is given by

$$P_0 e^{rT_3} = P^m.$$

Differentiating (17) with respect to $P_0$ and making use of (15) gives

$$\frac{\partial V}{\partial P_0}=\int_{T_2}^{T_3}\left[Q(P(t))+\frac{dQ(P(t))}{dP(t)}(P_0e^{rt}-\bar{P})\right]dt$$
$$+(P(T_2)-\bar{P})e^{-rT_2}\int_0^{T_2}\frac{dQ(P(t))}{dP(t)}e^{rt}dt. \tag{18}$$

For $P_0$ sufficiently high that $T_2=T_3$ and $P(T_2)=P^m$, then

$$\frac{\partial V}{\partial P_0}=(P^m-\bar{P})e^{-rT_2}\int_0^{T_2}Q'e^{rt}dt$$

which is negative if $Q'<0$ for $P(t)\in[P_0, P^m]$.

Profits from the substitute technology are a maximum when the resource is exhausted before the market price reaches the long-run monopoly price. Over the interval $[T_2, T_3]$ price rises at the rate of interest. This is inconsistent with *ex post* profit-maximization, but it is a necessary outcome of *ex ante* profit maximization with binding contracts. Of course implicit in the result $T_2<T_3$ is the feasibility condition $T_1\leqq T_2$. For $S\leqq S'$, any attempt to make $T_2<T_3$ must advance the date of invention to the point where profits fall short of R & D expenditures. Note that whenever $T_1<T_2$ advancing the date of innovation is feasible and, as equation (18) shows, profits are maximized with $T_2<T_3$.

We now have the results necessary to prove Proposition 6. First note that if $T_1=T_2=T_3$, then invention is independent of the resource stock. Since this is true only for $S\leqq S'$, for $S>S'$ either $T_1<T_2$ or $T_2<T_3$. In the first case there are "sleeping patents", while in the second case invention and innovation occur when $P(T_2)<P^m$. For $S>S'$, the inventor may be viewed as choosing $T_2$ to maximize

$$\Pi(T_2, S)=V(T_2(P_0, S), S)$$

subject to the constraints

$$\Pi(T_2, S)=x(T_1)\quad\text{(free entry)}$$

and

$$T_2\geqq T_1\quad\text{(innovation cannot precede invention).}$$

Figure 1 illustrates the two possible sequences of invention and innovation when $S>S'$. The values of $\Pi(T_2, S)$ are represented in the figure by the (locally) concave curves. The curves are drawn for two values of the initial stock, with $S_2>S_1$. Since the gross profits to the inventor fall with larger values of $S$, $\Pi(T_2, S_1)$ is drawn higher than $\Pi(T_2, S_2)$. The convex curve is the cost of invention, $x(T_1)$.

The function $\Pi(T_2, S)$ is defined with the implicit assumption that $T_2\geqq T_1$ (i.e. invention must precede or coincide with innovation). Given $T_2\geqq T_1$, there is an optimal innovation date for each value of the initial stock. This is shown as $T_2^*$ for $S_1$ and $T_2^{**}$ for $S_2$. Figure 6 illustrates the case where at $S_1$, $x(T_2^*)>\Pi(T_2^*, S)$. In other words, at the innovation date which maximizes the gross profits of the inventor, the cost of invention exceeds the return from new technology. Therefore in equilibrium innovation must occur at a date $T_2'>T_2^*$, and invention and innovation coincide. At $S_2$ the gross profits from the substitute are a maximum at $T_2^{**}$ and $\Pi(T_2^{**}, S_2)>x(T_2^{**})$. Free entry dissipates the net profits from R & D by pushing forward the date of invention, $T_1''$, until $x(T_1'')=\Pi(T_2^{**}, S_2)$. Thus at $S_2$ we have invention strictly preceding innovation. Indeed, sleeping patents occur whenever the present value of profits earned from the date of innovation, $\Pi(T_2, S)$, exceed $x(T_2)$.

Figure 1 shows the conditions for $T_1=T_2$ and $T_1<T_2$. We know that if $T_1=T_2$ and $S>S'$, then $T_1=T_2<T_3$. If $T_1<T_2$, then from equation (18), $T_2<T_3$ for $S>S'$.

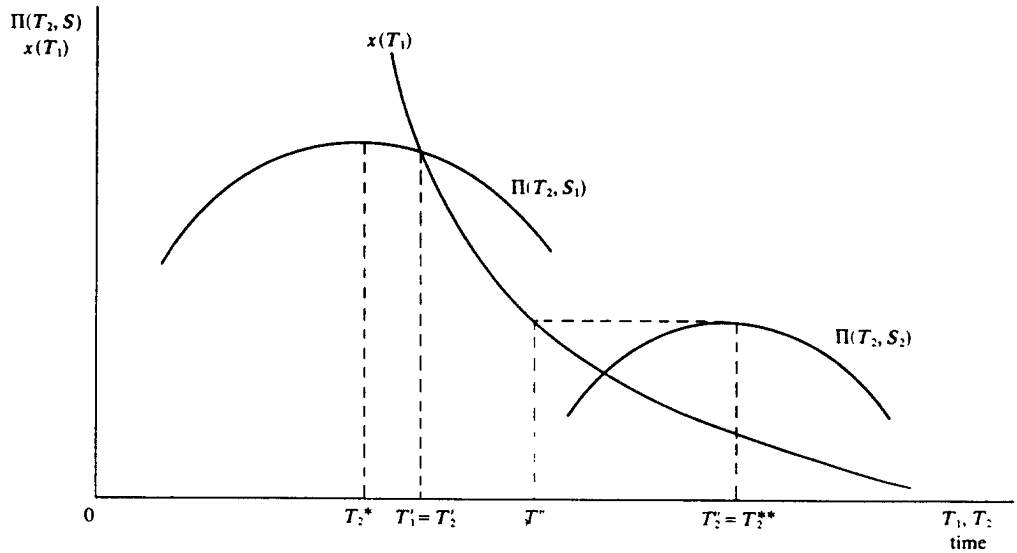


FIGURE 1
Patent values and cost of invention

To complete the proof of Proposition 6, we need only show that there exists an $S''$ such that $T_1(S) < T_2(S) < T_3(S)$ for all $S \geqq S''$. Note that the gross return to R & D is bounded below by

$$e^{-rT_3}\frac{\pi^m}{r}.$$

Let $T_3 = T_1 + \alpha(S)$. The free entry condition implies

$$e^{-rT_1}\left[e^{-r\alpha(S)}\frac{\pi^m}{r}\right] \leqq x(T_1).$$

Now suppose invention and innovation coincide. If so, the price at the date of invention must be at least $\bar{P}$. Since $\alpha(S) = T_3 - T_1$, $P(T_3) = P^m$, and $P(T_1) \geqq \bar{P}$, $e^{-r\alpha(S)}$ is bounded below by $\bar{P}/P^m$, and

$$x(T_1)e^{rT_1} \geqq \frac{\bar{P}}{P^m}\frac{\pi^m}{r}.$$

If invention and innovation coincide, the latest date at which invention can occur is the largest $T_1$ that satisfies the above inequality. Call this date $\hat{T}_1$. Now as $S$ increases, the earliest date at which innovation is feasible is determined by $P(\hat{T}_2) \geqq \bar{P}$, and this date increases monotonically with $S$. Hence there is an $S''$ for which $\hat{T}_2 > \hat{T}_1$, so invention must precede innovation (and this implies $T_2 < T_3$).

The date $T_3$, at which the post-invention price equals the monopoly price, increases with $S$ for $S > S'$. Over this range of resource stocks $T_3 - T_2$ is positive, while $T_2 > T_1$ or $T_2 = T_1$ is possible. An increase in $S$ lowers the return from invention for $S > S'$ and hence increases the date of invention, $T_1$; the exact amount depends on properties of the R & D cost function $x(T)$. The length of time during which the patent sleeps, $T_2 - T_1$, need not be a monotonic function of $S$, but given the assumed convexity of the invention possibility function, sleeping patents always occur in the limiting case of an arbitrarily large resource stock.

4.3. *Comparison with monopoly and the planned economy*

Proposition 6 describes the price trajectories under competition; we now use this information to compare the dates of invention (and, equivalently, the expenditures on research and development) for the three market structures under consideration. Since the socially optimal and pure monopoly investment in R & D depends on the marginal return to research expenditures but the total competitive investment in R & D with free entry depends on the average return, a general comparison of research intensity is not possible without specifying the shape of the invention-possibility function. Let $\gamma(T)$ be the elasticity of the invention function,

$$\gamma(T) = -\frac{d \ln x(T)}{d \ln T}.$$

**Proposition 7.** *Assume* $dx/dT < 0$,

$$\frac{d}{dT}\left(-\frac{dx}{dT} e^{rT}\right) < 0 \quad \textit{(decreasing marginal invention cost)},$$

*and* $T_1(S) = T^*(S)$ *for some* $S = S_0$ *(competitive and optimal invention dates coincide at* $S_0$*).*

(a) *If* $S_0 \leqq S'$ *(where* $S'$ *is defined by equation* (13)*), then*
  (i) *for* $0 \leqq S \leqq S'$, $T_1(S) \gtreqless T^*(S)$ *as* $S \gtreqless S_0$.
  (ii) *If, in addition,* $d(\gamma(T)/T)/dT \geqq 0$, *then* $T_1(S) \gtreqless T^*(S)$ *as* $S \gtreqless S_0$ *for all* $S$.

(b) *For any* $S_0$, *if* $\lim_{T\to\infty} \gamma(T)/T > 0$, *then*
  (i) $\lim_{S\to\infty} T_1(S) < \lim_{S\to\infty} T^*(S)$.
  (ii) *If, in addition,* $d(\gamma(T)/T)/dT \geqq 0$, *then there exists a* $\tilde{S}$ *such that* $T_1(S) < T^*(S)$ *for* $S \geqq \tilde{S}$.

Proposition 7 orders the competitive and optimal invention dates conditional on the behaviour of $\gamma(T)/T$. If $\gamma(T)/T$ is non-decreasing, then for a sufficiently large resource stock, the competitive investment in R & D always *exceeds* the socially optimal investment. The proof of Proposition 7 shows that for very large resource stocks, the value of the patent always exceeds the marginal value of the invention to society, so that the incentive to patent is excessive. The restriction on $\gamma(T)/T$ assures that the marginal cost of invention relative to the average cost does not fall fast enough to offset the excessive incentives for patenting.

Stronger conclusions are possible if the competitive and optimal invention dates coincide when $S_0 \leqq S'$, so that innovation always occurs without delay. In this case competitive R & D is excessive for $S' > S > S_0$ and for any $S > S_0$ if $\gamma(T)/T$ is non-decreasing. Since the monopoly expenditure on R & D is always less than the socially optimal level, whenever $T_1(S) < T^*(S)$, *a fortiori* $T_1(S) < T^m(S)$.

*Proof of Proposition* 7. If $S \leqq S'$, we know from Proposition 6 that $T_1(S) = \hat{T}$, a constant. The optimal invention date is determined by

$$-x'(T^*) = e^{-rT^*}[N(\bar{Q}) - N(Q(S, T^*))],$$

and from equation (9), $dT^*/dS > 0$. Thus if $T^*(S_0) = \hat{T}$, then

$$T^*(S) \gtreqless \hat{T} = T_1(S) \quad \text{as } S \gtreqless S_0,$$

provided $S \leqq S'$.

For $S > S'$, the value of the patent is

$$\Pi(T_2, S) = \int_{T_2}^{T_3} Q(P_0 e^{rt})[P_0 e^{rt} - \bar{P}]e^{-rt}dt + e^{-rT_3}\frac{\pi^m}{r} \geqq e^{-rT_3}\frac{\pi^m}{r}.$$

Let $\alpha(S) = T_3 - T_1$. If $T_1(S) \geqq T^*(S)$, then $P_0^* \leqq P_0^c$. This implies

$$T_3(S) - T^*(S) = \alpha(S) + (T_1(S) - T^*(S)) \leqq \frac{1}{r}\ln\frac{P^m}{P^*(T^*)}. \tag{19}$$

Since

$$x(T_1) > e^{-rT_1}\frac{\pi^m}{r}e^{-r\alpha}$$

and because $P_0^* \leqq P_0^c$,

$$\frac{P^*(T^*)}{P^m} \leqq e^{-r(T_3 - T^*)} = e^{-r\alpha}e^{-r(T_1 - T^*)},$$

we have

$$\frac{P^*(T^*)}{P^m} < \frac{x(T_1)e^{rT^*}}{\pi^m/r}. \tag{20}$$

Now suppose $T_1(S) \geqq T^*(S)$. Then

$$x(T_1) \leqq x(T^*) = -\frac{x'(T^*)}{\gamma(T^*)/T^*}. \tag{21}$$

At $T^*(S)$,

$$-x'(T^*) = e^{-rT^*}[N(\bar{Q}) - N(Q(S, T^*))] = e^{-rT^*}\pi^*(S, T^*). \tag{22}$$

Substituting (21) and (22) in (20) gives

$$\frac{P^*(T^*)}{P^m} < \frac{\pi^*(S, T^*)}{\pi^m/r}\frac{1}{\gamma(T^*)/T^*}. \tag{23}$$

As $S \to \infty$, $Q(S, T^*) \to \bar{Q}$ and $\pi^*(S, T^*) \to 0$. Therefore, if $T_1 \geqq T^*$ and if

$$\lim_{T\to\infty}\frac{\gamma(T)}{T} > 0,$$

then in the limit

$$\frac{P^*(T^*)}{P^m} < 0.$$

But $P^*(T^*) \geqq \bar{P}$, so that assuming $T_1(S) \geqq T^*(S)$ yields a contradiction as $S \to \infty$. We conclude

$$\lim_{S\to\infty}[T_1(S) < T^*(S)].$$

For a sufficiently large resource stock, competition for a patent on a substitute technology results in excessive R & D if $\lim_{T\to\infty}\gamma(T)/T > 0$. Also, if $\gamma(T)/T$ is non-decreasing, then since $\pi^*$ is non-increasing in $S$ and $P^*(T^*) \geqq \bar{P}$, there is an $\tilde{S}$ such that for all $S > \tilde{S}$ the inequality (23) is a contradiction. Hence $T_1(S) < T^*(S)$ for $S \geqq \tilde{S}$. ||

A somewhat stronger result is possible if $T_1(S_0) = T^*(S_0)$ for $S_0 \leqq S'$. If $\gamma(T)/T$ is non-decreasing, then $T_1(S) \gtreqless T^*(S)$ as $S \lesseqgtr S_0$ for all $S$. The proof requires additional computation with no new insights, so it is presented in the Appendix.

## 5. CONCLUDING REMARKS

This paper has attempted to examine the dynamics of technical change by studying a particular intertemporal model. The model of exhaustible resource depletion is a convenient paradigm for this problem because the timing of substitute development is an obvious decision variable, but similar results could be obtained using other partial equilibrium models with the services of producer durables replacing the flow of the natural resource and technical change appearing as the development of improved machines.

The explicitly dynamic formulation permits a number of observations which either do not emerge from static models or have been overlooked in the past. For example Arrow (1962) concluded that the incentive to invest in a competitive economy with patents is less than optimal, yet the results of this paper show that competitors may spend more, as well as less, than the efficient amount on research and development. This arises in part because we assume that free entry dissipates the profits from a new technology while Arrow assumed profits from R & D are maximized.[4] A stronger result is that, subject to restrictions on the invention technology, a competitive economy always spends an excessive amount on R & D when the stock of natural resources is very large. This suggests a troublesome bias in competitive economies with patent rights toward too much R & D when the pay-off from new technologies is relatively low and perhaps too little R & D when the pay-off is relatively high.

A possible consequence of the dissipation of profits from R & D is the appearance of "sleeping patents". These are new technologies or products which are not introduced in commercial applications even though they are profitable at current prices. We have shown that with binding futures contracts, equilibrium in a competitive market can entail the existence of sleeping patents. The result is clearly wasteful, since invention could be delayed with a consequent cost saving and no effect on the date of innovation. This does not occur because any firm that chose to delay invention would lose the competition for patent rights to the new technology. In equilibrium, it is also not possible to reduce the time during which the patent sleeps by advancing the date of innovation. To do so would lower the market price at the date of innovation since production of the resource would be increased. This would lower profits from the invention, which would force a later invention date. Thus any competitor who attempted to shrink the time during which the patent sleeps could lose the patent race to another competitor who promised a longer period between invention and innovation. The market, or more precisely the patent race, rewards the competitor who invents early, even if expenditures on invention are excessive and innovation purposely lags discovery.

## APPENDIX

I. Proof that

$$\frac{dV(S,T)}{dT} = e^{-rT} N(Q(T)). \tag{A.1}$$

Differentiating $V(S, T)$, taking note that

$$u'(Q) = P$$

and

$$P(t)e^{-rt} = P(Q(T))e^{-rT},$$

where $Q(T)$ is the value of $Q$ immediately before invention, gives

$$\frac{dV(S,T)}{dT} = P(Q(T))e^{-rT} \int_0^T \frac{dQ(t)}{dT}\, dt + u(Q(T))e^{-rT}. \tag{A.2}$$

Since, at the optimum,

$$\int_0^T Q(t)dt = S,$$

and $S$ is a constant, then

$$\int_0^T \frac{dQ(t)}{dT} dt = -Q(T). \tag{A.3}$$

Substituting equation (A.3) into equation (A.2) and noting that

$$N(Q(T)) = u(Q(T)) - P(Q(T))Q(T)$$

gives the desired result.

II. *Proof of Proposition* 4. Marginal monopoly profits from invention are

$$\frac{d\Pi^m}{dT} = e^{-rT}[M(Q(T)) - M(Q^m) - x'(T)e^{rT}] \tag{A.4}$$

where $M(Q) = R(Q) - R'(Q)Q$ and $Q^m$ is defined by

$$R'(Q^m) = \bar{P}.$$

At the socially optimal date, $T^*$, provided $T^* \in (0, \infty)$,

$$-\frac{dx}{dT} e^{rT^*} = N(\bar{Q}) - N(Q(T^*)). \tag{A.5}$$

Substituting equation (A.5) in (A.4) gives

$$e^{rT} \frac{d\Pi^m}{dT}\bigg|_{T^*} = [N(\bar{Q}) - M(Q^m)] - [N(Q(T^*)) - M(Q^m(T^*))],$$

where $Q^m(T^*)$ is the output of the monopolist at the date $T^*$. To evaluate the sign of $d\Pi^m/dT$ at $T^*$, first note that

$$N(\bar{Q}) - M(Q^m) = N(Q^m) + L(Q^m),$$

where $L(Q^m)$ is the deadweight loss from production of the substitute at the monopoly price. Thus

$$e^{rT} \frac{d\Pi^m}{dT}\bigg|_{T^*} = [N(Q^m) - N(Q(T^*))] + M(Q^m(T^*)) + L(Q^m). \tag{A.6}$$

Suppose the monopolist's profit-maximizing date of invention, $T^m$, coincides with or precedes the optimal date, $T^*$. We shall show this leads to a contradiction. The proof involves comparing outputs at the invention date conditional on different invention dates. This calls for the brief use of a notation which includes the invention date as a parameter. Define $Q^m(t \mid T)$ to be the monopoly output at date $t$ conditional on invention at $T$, and define $Q(t \mid T)$ similarly for the socially planned output.

If the monopolist's invention date did precede the socially optimal date, then

$$Q^m(T^* \mid T^*) < Q^m(T^m \mid T^m).$$

This follows since the initial resource stock is fixed and extraction paths with different time horizons do not cross. From Proposition 3(d), the post-invention monopoly output exceeds the pre-invention output;

$$Q^m(T^m \mid T^m) < Q^m.$$

Therefore

$$Q^m(T^*|T^*) < Q^m.$$

Given any time horizon, $T$, for depletion of the resource stock, the initially conservative bias of monopoly implies that at date $T$ the rate of output by a monopolist is no less than the rate of output from the same resource stock in an efficient allocation (see Sweeney (1976)).[5] Thus

$$Q(T^*|T^*) \leqq Q^m(T^*|T^*) < Q^m.$$

Since $Q(T^*|T^*) = Q(T^*)$ in the more concise notation, we have $Q(T^*) < Q^m$ and

$$N(Q(T^*)) < N(Q^m). \tag{A.7}$$

Substituting equation (A.7) (which was obtained under the hypothesis that $T^m \leqq T^*$) into equation (A.6) yields

$$\left.\frac{d\Pi^m}{dT}\right|_{T^*} > 0.$$

That is, monopoly profits are increased by delaying the date of invention beyond the socially-optimal date. This contradicts our original hypothesis, so we may conclude that $T^m \leqq T^*$ is not possible. Proposition 4 follows directly. ||

III. *Proof of Proposition* 7, *Part* (*a*)(*ii*). The proof begins with inequality (29):

$$\frac{P^*(T^*)}{P^m} < \frac{\pi^*(S, T^*)}{\pi^m/r} \frac{1}{\gamma(T^*)/T^*}$$

if $T_1(S) \geqq T^*(S)$. Since, by assumption, $T_1(S_0) = T^*(S_0)$ for $S_0 \leqq S'$,

$$\frac{\gamma(T^*)}{T^*} \geqq \frac{-x'(T(S_0))}{x(T(S_0))} = \frac{\pi^*(S_0, T(S_0))}{\pi^m/r}.$$

Thus

$$\frac{P^*(T^*)}{P^m} < \frac{\pi^*(S, T^*(S))}{\pi^*(S_0, T^*(S_0))}.$$

Since $S_0 \leqq S'$, $P^*(T^*(S_0)) \geqq P^m$. For $S > S'$, $P^*(T^*(S)) < P^m$. Therefore,

$$\frac{\pi^*(S, T^*(S))}{\pi^*(S_0, T^*(S_0))} \leqq \frac{P^*(T^*(S)) - \bar{P}}{P^m - \bar{P}},$$

but

$$\frac{P^*(T^*(S)) - \bar{P}}{P^m - \bar{P}} < \frac{P^*(T^*(S))}{P^m},$$

so that $T_1(S) \geqq T^*(S)$ for $S > S'$ yields a contradiction; we must have $T_1(S) < T^*(S)$ for $S > S'$. For $S \leqq S'$ the remainder of the proof follows directly from part (a)(i) of Proposition 7. ||

*First version received April* 1981; *final version accepted March* 1982 (*Eds.*).

The authors are grateful to Geoffrey Heal and David Newbery for helpful discussions. The work was partially supported by the National Science Foundation (NSF Grant SOC77-08822).

NOTES

1. The research function is non-stochastic. This greatly simplifies the analysis, which can be extended to stochastic research functions.

2. We assume there exists an optimal policy for the monopolist. This is assured if there is some price above which the demand for the resource has elasticity greater than unity (see Salant (1976)).

3. The argument here is that for any equilibrium price trajectory, suppliers of the resource are indifferent with respect to extraction for $t \in [0, T_3]$. But given $P(t)$, the substitute producer is not indifferent and will want to delay production subject to the condition that

$$\int_{T_1}^{T_3} y(t)dt = \int_0^{T_3} Q(P(t))dt - S.$$

This means that in equilibrium, the resource will be exhausted before production of the substitute begins, and the substitute producer will be obligated to maintain $\dot{P}/P = r$ for $t \in [T_2, T_3]$.

4. The significance of this assumption is implicit in the work of Barzel (1967) and is stated explicitly in Stiglitz (1969).

5. Under certain demand conditions it is possible that an unconstrained monopolist will be more profligate initially, but this case can be ruled out if we allow for competitive arbitrage.

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