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Inventory Control and Trade Credit Revisited

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This paper presents the discounted cash-flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. The DCF approach permits a proper recognition of the financial implication of the opportunity cost and out-of-pocket costs in inventory analysis. This approach also permits an explicit recognition of the exact timing of cash flows associated with an inventory system. As a result, the effect of the delayed payment is appropriately reflected in determining the optimal order size.

Key words: discounted cash-flows, inventory, trade credit

INTRODUCTION

There have been extensive discussions in the literature for possible extensions of the basic economic order-quantity (EOQ) model to the variety of situations where its assumptions are not met. However, an important and yet very unrealistic assumption of the model has been overlooked until very recently: the simultaneity of the arrival of an order and the capital investment in inventories.

Recently Haley and Higgins,[1] Kingsman,[2] Chapman *et al.*,[3] Daellenbach,[4] Ward and Chapman,[5] Daellenbach[6] and Chapman and Ward[7] examined the effect of the trade credit on the optimal inventory policy. Although these studies provide useful insights into the importance of the credit period in inventory-control decisions, there are some limitations on their analyses and conclusions since these studies do not correctly recognize the time value of money in determining the optimal order quantity. In particular, these studies fail to recognize appropriately the effect of the delayed payment in determining the optimal order quantity. As a result, Haley and Higgins,[1] Kingsman,[2] Chapman *et al.*,[3] Ward and Chapman[5] and Chapman and Ward[7] argue that as long as the credit periods are fixed, the cost of holding inventory is reduced in comparison with the basic EOQ model, but the optimal order size is the *same* as that of the basic EOQ model. This conclusion is counter-intuitive since reduced inventory-holding costs should result in a larger optimal order quantity, *ceteris paribus*. As is shown in this paper, this conclusion is indeed fallacious once the time value of money is correctly incorporated into the analysis.

This paper presents an alternative framework for the analysis of the effect of the trade credit on inventory decisions based upon the generally accepted principles of financial analysis. Specifically, this study presents the discounted cash-flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. This approach permits an explicit recognition of the exact timing of cash flows associated with an inventory system.

Previously, Trippi and Lewin[8] employed the DCF approach for the analysis of the basic EOQ model. More recently, Kim *et al.*[9] extended Trippi and Lewin's study by applying the DCF approach to various inventory systems. However, these two studies still suffer from the unrealistic assumption of the simultaneity of the arrival of an order and the capital investment in inventories. Based upon this observation, this paper extends the studies of Trippi and Lewin and Kim *et al.* by explicitly recognizing the effect of the trade-credit period on the optimal order quantity using the DCF approach.

ECONOMIC ORDER QUANTITY WITH TRADE CREDIT

This section presents the DCF approach to the EOQ analysis under four different assumptions on the trade-credit terms. The following notation will be used throughout:

Q = the order quantity;
T = the inventory cycle-time;
C = the purchase cost per unit;
D = the demand rate per unit time;

h = the out-of-pocket inventory-carrying costs as a proportion of the value of inventory per unit time;
r = the opportunity cost (i.e. the discount rate) per unit time.

Case 1: Instantaneous cash flows: the case of the basic EOQ model

Case 1 presents the DCF approach for the basic EOQ model under the assumption of instantaneous inventory-holding costs, which implies the simultaneity of the arrival of an order and the capital investment in inventories. Thus, at the beginning of each inventory cycle, there will be cash outflows of the ordering cost (K) and purchase cost (CDT). Since the out-of-pocket inventory-carrying costs are assumed to be proportional to the value of inventory, the out-of-pocket inventory-carrying costs per unit time at t are $hCD(T-t)$. Then the present value of the out-of-pocket carrying costs is obtained by discounting $hCD(T-t)$ at the cost of capital (r), i.e. $hCD(T-t)(1+r)^{-t}$. If we use continuous discounting for expositional convenience, the discount factor $(1+r)^{-t}$ will become e^{-rt} since $(1+r/m)^m$ approaches e^r as m approaches infinity (i.e. continuous discounting), where m is the number discounted per unit time. Thus the present value of cash flows for the first cycle $[PV(Q)]$ is

$$PV(Q) = -K - CDT - hCD\int_0^T (T-t)e^{-rt}\,dt. \tag{1}$$

Note that

$$\int_0^T (T-t)e^{-rt}\,dt = \frac{1}{r}\left\{T + \frac{1}{r}(e^{-rT}-1)\right\}. \tag{2}$$

Substituting (2) into (1), the present value of cash flows for the first cycle is

$$PV(Q) = -K - CDT - hCD(1/r)\{T + (1/r)(e^{-rT}-1)\}. \tag{3}$$

Then the present value of all future cash flows $[PV(\infty)]$ is

$$PV(\infty) = PV(Q)\sum_{n=0}^{\infty} e^{-nrT}. \tag{4}$$

Using

$$\sum_{n=0}^{\infty} e^{-nrT} = 1/(1-e^{-rT}), \tag{5}$$

and substituting equation (3) into equation (4), we obtain

$$PV(\infty) = -\frac{(r+h)CDT + rK}{r(1-e^{-rT})} + \frac{hCD}{r^2}. \tag{6}$$

Letting $dPV(\infty)/dT = 0$, we obtain the following optimality condition:

$$e^{rT^*} = 1 + r\left\{T^* + \frac{rK}{(r+h)CD}\right\}. \tag{7}$$

Note that the left-hand side of (7) can be approximated as

$$e^{rT^*} \approx 1 + rT^* + \frac{(rT^*)^2}{2}. \tag{8}$$

Then from equations (7), (8) and $Q^* = DT^*$, the optimal order quantity, Q^*, is

$$Q^* = \sqrt{2KD/(r+h)C}. \tag{9}$$

Hence, if there is no credit period, the DCF approach gives an identical solution to that of the traditional inventory analysis.

Case 2: Credit only on units in stock (when $T < M$)

Case 2 assumes the existence of the credit period (M). Specifically, this case deals with the situation where payment is linked to the subsequent use of the materials. During the credit period, the firm makes payment to the supplier immediately after the use of the materials. On the last day of the credit period, the firm pays the remaining balance. If we assume that the credit period is greater than the inventory cycle-length, the present value of cash flows for the first cycle is

$$PV(Q) = -K - CD\int_0^T e^{-rt}\,dt - hCD\int_0^T (T-t)e^{-rt}\,dt. \tag{10}$$

Note that the second term on the right-hand side (RHS) of (10) is the continuously discounted present value of the payment to the supplier, and the third term is the present value of the out-of-pocket inventory-carrying costs. Then from (2), (4), (5), (10) and

$$\int_0^T e^{-rt}\,dt = \frac{1}{r}(1-e^{-rT}), \tag{11}$$

the present value of all future cash flows, $PV(\infty)$, is

$$PV(\infty) = -\frac{hCDT + rK}{r(1-e^{-rT})} + \frac{(h-r)CD}{r^2}. \tag{12}$$

Letting $dPV(\infty)/dT = 0$, we obtain the following optimality condition:

$$e^{rT^*} = 1 + r\left\{T^* + \frac{rK}{hCD}\right\}. \tag{13}$$

Finally, from (8), (13) and $Q^* = DT^*$, the optimal order quantity is

$$Q^* = \sqrt{2KD/hC}. \tag{14}$$

Thus, if payment to the supplier immediately follows the use of the materials, and if the credit period is longer than the inventory cycle-length, only out-of-pocket costs are relevant in determining the optimal order quantity. The opportunity cost does not enter into the EOQ formula since, in this case, the firm finances its inventory investment with the trade credit offered by its supplier. This result is identical to the result of Chapman *et al.*[3] [see equation (14), p. 1058].

In the previous two cases, the traditional cost-minimizing inventory analysis is shown to give identical solutions to those of the DCF inventory analysis. In the rest of the paper, however, it will be shown that the solutions from the traditional inventory analysis are not identical to those of the DCF analysis under the alternative trade-credit terms.

Case 3: Credit only on units in stock (when $T > M$)

Case 3 deals with a similar situation to case 2 above; i.e. the payment schedule is linked to the use of the materials. During the credit period, the firm makes payment to the supplier immediately after the use of the materials. On the last day of the credit period, the firm pays the remaining balance. However, in this case, we assume that the credit period is shorter than the inventory cycle-length. Then the present value of cash flows is

$$PV(Q) = -K - CD\int_0^M e^{-rt}\,dt - CD(T-M)e^{-rM} - hCD\int_0^T (T-t)e^{-rt}\,dt. \tag{15}$$

Then from (2), (4), (5), (11) and (15), $PV(\infty)$ is

$$PV(\infty) = -\frac{hCDT + rK + (T-M)rCDe^{-rM} + CD(1-e^{-rM})}{r(1-e^{-rT})} + \frac{hCD}{r^2}. \tag{16}$$

Letting $dPV(\infty)/dT = 0$, we obtain the following optimality condition:

$$e^{rT^*} = 1 + r\left\{T^* + \frac{rK - (1+rM)CDe^{-rM} + CD}{(re^{-rM}+h)CD}\right\}. \tag{17}$$

Finally, from (8), (17) and $Q^* = DT^*$, the optimal order quantity is

$$Q^* = \sqrt{\frac{2[K - \{(1/r) + M\}CDe^{-rM} + (CD/r)]D}{(re^{-rM}+h)C}}. \tag{18}$$

Thus the DCF approach gives a different solution from that of the traditional cost-minimizing inventory analysis [see Chapman *et al.*,[3] equation (7), p. 1057].

Case 4: Fixed credit

This case deals with the situation where the credit period is fixed. The firm pays the full purchase amount on the last day of the credit period. Then $PV(Q)$ is

$$PV(Q) = -K - CDTe^{-rM} - hCD \int_0^T (T - t)e^{-rt}\, dt. \tag{19}$$

Then from (2), (4), (5), (11) and (19), $PV(\infty)$ is

$$PV(\infty) = -\frac{hCDT + rK + rTCDe^{-rM}}{r(1 - e^{-rT})} + \frac{hCD}{r^2}. \tag{20}$$

Letting $dPV(\infty)/dT = 0$, we obtain the following optimality condition:

$$e^{rT^*} = 1 + r\left\{T^* + \frac{rK}{(re^{-rM} + h)CD}\right\}. \tag{21}$$

Finally, from (8), (21) and $Q^* = DT^*$, the optimal order quantity is

$$Q^* = \sqrt{\frac{2KD}{(re^{-rM} + h)C}}. \tag{22}$$

Equation (22) suggests that, in the presence of the trade credit, the correct opportunity cost is the *discounted* opportunity cost (i.e. the *discounted* discount rate). This result is intuitively appealing, since with the delayed payment, the effective capital cost should be less than that of the instantaneous payment. This result differs from those of Haley and Higgins,[1] Kingsman,[2] Chapman *et al.*[3] Ward and Chapman[5] and Chapman and Ward.[7] Previously, these studies argued that if the credit periods are fixed, the optimal order size is the *same* as that of the basic EOQ model. As shown above, this conclusion is fallacious since these studies do not correctly incorporate the time value of money into the analysis.

SUMMARY

This paper presents the discounted cash-flows (DCF) approach for the analysis of the optimal inventory policy in the presence of trade credit. This approach permits an explicit recognition of the exact timing of cash flows associated with an inventory system. As a result, the effect of the delayed payment is appropriately reflected in determining the optimal order size.

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