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INVENTORY MODEL FOR TIME DEPENDENT HOLDING COST AND DETERIORATION WITH SALVAGE VALUE AND SHORTAGES

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Abstract

In this paper, a deterministic inventory model is developed for deteriorating items in which shortages are allowed and salvage value is incorporated to the deteriorated items. In this model the demand rate is constant, deterioration rate is time dependent with weibull's distribution and holding cost is a linear function of time. The model is solved analytically by minimizing the total inventory cost. Numerical analysis is provided to illustrate the solution and application of the model. The model can be applied to optimizing the total inventory cost for the business enterprises where holding cost and deterioration rate both are time dependent and salvage value is incorporated to the deteriorated items.

Keywords: Inventory, deteriorating items, shortages, time dependent deterioration, salvage value, weibull's distribution, time varying holding cost.

Mathematics Subject Classification: 90B05

1. Introduction

Deterioration is defined as decay or damage such that the item cannot be used for its original purposes. The effect of deterioration is very important in many inventory systems. Food item, Pharmaceuticals and radioactive substances are example of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system, as deterioration of an item and holding cost of inventory depends upon the time. It would be more reasonable and realistic if we assume the deterioration and holding cost both as a time dependent function.

Ghare and Schrader [1963] initially worked in this field and they extended Harris [1915] EOQ model with deterioration and shortages. Goyal and Giri [2001] gave a survey on recent trends in the inventory modeling of deteriorating items. Lee and Wu [2004] developed a note on EOQ model for items with mixtures of exponential distribution deterioration, shortages and time varying demand. Ajanta Roy [2008] developed an inventory model for deteriorating items with time varying holding cost and price dependent demand. Huang and Hsu [2008] presented a simple algebraic approach to find the exact optimal lead time and the optimal cycle time in the constant markets demand situations. Liao, J.J. [2008] gave an EOQ model with non instantaneous receipt and exponential deteriorating item under two level trade credits. **Skouri,** Papachristos and Goyal [2008] proposed new credit period scheme under which the supplier offers to the buyer a fixed base level credit period in settling the account plus additional delay time, depending on the quantity order.

Sarala Pareek and Vinod Kumar [2009] developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri, Konstantaras, Papachristos, and Ganas [2009] developed an inventory models with ramp type demand rate, partial backlogging and Weibell`s deterioration rate. Mishra and Singh [2010] developed a deteriorating inventory model for waiting time partial backlogging when demand is time dependent and deterioration rate is constant. They made Abad [1996, 2001] more realistic and applicable in practice. Kuo-Chen Hung [2011] gave an inventory model with generalized type demand, deterioration and backorder rates Mishra & Singh [2011] developed a deteriorating inventory model for time dependent demand and holding cost with partial backlogging.

In this paper, we made the paper of Sarala Pareek & Vinod Kumar [2009] and Mishra & Singh [2011] more realistic by considering that the salvage value is incorporated to the deteriorated items and holding cost is linear function of time and developed an inventory model for deteriorating items with time dependent deterioration rate in which demand rate is constant. Shortages are allowed and fully backlogged. Deteriorating items have salvage value. The aim of this model is to find an optimal order quantity which minimizes the total inventory cost.

2. Assumption and Notations

The mathematical model is based on the following assumptions and notations.

2.1 Assumptions

- Demand rate is constant and known.
- The lead time is zero or negligible.
- The replenishment rate is infinite.
- Shortages are allowed and completely backlogged.
- Deterioration rate is time dependent and follow weibull's distribution i.e., $\theta(t) = t^{\beta-1}\alpha\beta$ where α $(0 \le \alpha < 1)$ denote scale parameter and $\beta > 1$denote shape parameter.
- The salvage value γ $(0 \le \gamma < 1)$ is associated to deteriorated units during the cycle time.
- The holding cost is a linear function of time i.e., H(t) = a + b t (a>0,b>0)
- The deteriorated units cannot be repaired or replaced during the period under review.

2.2 Notations

- *C*: purchase cost per unit.
- $C_{3:}$ ordering cost per order.
- *H (t)*: holding cost per unit per unit time, H (t) = a +b t.

- θ(t): the deterioration rate at time t, where $\theta(t) = t^{\beta-1}\alpha\beta$
- γ : salvage value associated with deteriorated item.
- t_1 : the time at which the inventory level reaches zero, $t_1 \geq 0$.
- T: the length of cycle time.
- IM: the maximum inventory level during [0, T].
- IB: the maximum inventory level during shortage period.
- Q : (= IM + IB) the order quantity (Inventory level) during a cycle of length T.
- Q_1 (t) : the level of positive inventory at time t, $0 \leq t \leq t_1$.
- Q_2 (t) : the level of negative inventory at time t, $t_1 \leq t \leq T$.
- *R:* demand rate.
- *IHC*: holding cost per order.
- SC: shortage cost per unit per unit time.
- CD: deterioration cost per order.
- B: backordered inventory
- SV: salvage value per unit time.
- π : shortage cost per unit short.
- $TC(t_1, T)$: total cost per time unit.

3. Mathematical Formulation

The rate of change of inventory during positive stock period [0,t_1] and shortage period [t_1,T] is governed by the following differential equations

$$\frac{dQ_1(t)}{dt} + \theta(t)Q_1(t) = -R \quad (0 < t < t_1). \qquad \text{... (1)}$$

$$\frac{dQ_2(t)}{dt} = -R \qquad (\ t_1 < t < T\ ) \qquad \text{... (2)}$$

With boundary condition

Q_1 (t) =Q_2 (t) =0 at t=t_1 , Q_1 (t) =IM at t=0 and Q_2 (t) =IB at t=T

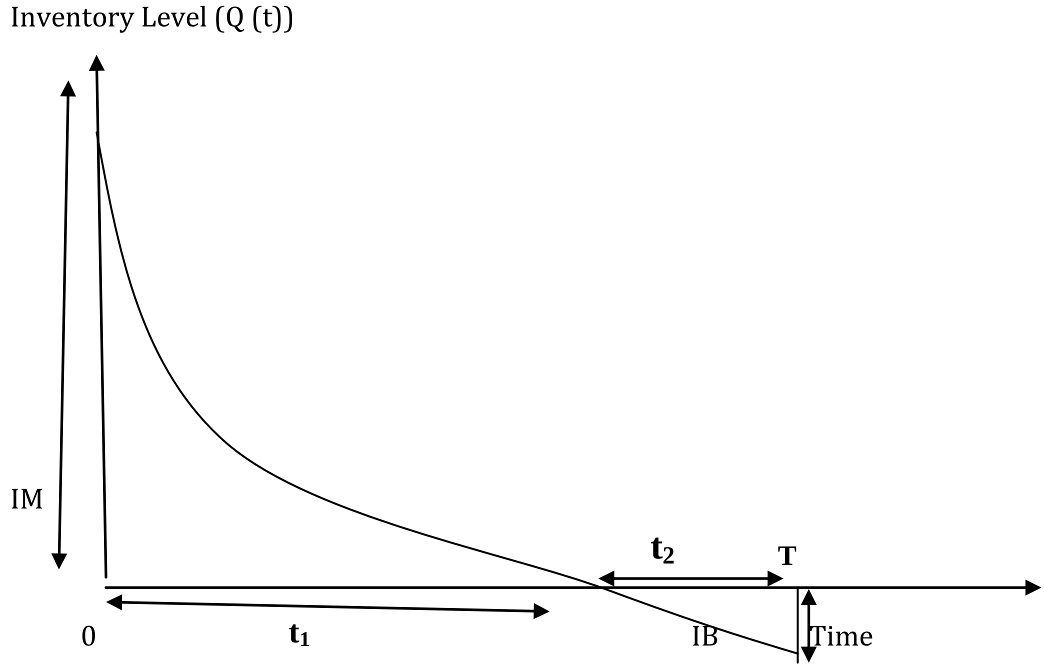


Fig -1: Graphical Representation of the state of Inventory

4. Analytical Solution of the Model

Case I: Inventory level without shortage

During the period [0, t_1], the inventory depletes due to the deterioration and demand. Hence, the inventory level at any time during [0, t_1] is described by differential equation

$$\frac{dQ_1(t)}{dt}+\theta(t)Q_1(t)=-R\;(0<t<t_1). \qquad \text{... (3)}$$

With the boundary conditions are $Q_1(0)=IM$ and $Q_1(t_1)=0$.

The solution of the linear differential equation (3) is:

$$Q_1(t)=R\left[t_1-t+\frac{t^{\beta+1}\alpha\beta}{\beta+1}+\frac{\alpha t_1^{\beta+1}}{\beta+1}-\alpha t_1 t^{\beta}\right]$$ (Neglecting the α^2 and higher powers terms) ... (4)

Case II: Inventory level with shortage

During the interval [t_1 ,T] the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during [t_1 ,T] can be represented by the differential equation

$$\frac{dQ_2(t)}{dt} = -R \quad \text{Where } (\; t_1 < t < T \;) \qquad \dots (5)$$

With boundary condition $Q_2\,(t_1) = 0$ and $Q_2\,(T) = IB$

The solution of differential equation (5) is

$$Q_2(t) = R(t_1 - t) \qquad \dots (6)$$

Therefore the total cost per replenishment cycle consists of the following components:

1. Inventory holding cost

$$IHC = \int_0^{t_1} H(t) Q_1(t)\, dt$$

$$IHC = \int_0^{t_1} R\left[t_1 - t + \frac{t^{\beta+1}\alpha\beta}{\beta+1} + \frac{t_1^{\beta+1}\alpha}{\beta+1} - \alpha t_1 t^{\beta} \right](a+bt)dt$$

$$IHC = aR\left[\frac{t_1^2}{2} + \frac{t_1^{\beta+2}\alpha\beta}{(\beta+1)(\beta+2)} \right] + bR\left[\frac{t_1^3}{6} + \frac{t_1^{\beta+3}\alpha\beta}{(\beta+3)(\beta+2)} \right] \qquad \dots (7)$$

2. Shortage cost during [t_1, T]

$$SC = \pi \int_{t_1}^{T} Q_2(t) dt \qquad (\pi \text{ is the shortage cost per unit short})$$

$$SC = \pi \int_{t_1}^{T} R(t - t_1) dt$$

$$SC = \frac{1}{2} \pi R (T - t_1)^2 \qquad \dots (8)$$

3. Stock loss due to deterioration = total positive inventory – demand

$$D = R\left[t_1 + \frac{t_1^{\beta+1}\alpha}{\beta+1} - t_1 \right] = \frac{R t_1^{\beta+1}\alpha}{\beta+1} \qquad \dots (9)$$

4. Deterioration cost

$$CD = C\ R\left[\frac{t_1^{\beta+1}\alpha}{\beta+1}\right] \quad \text{... (10)}$$

5. Ordering cost per order

$$OC = A \quad \text{... (11)}$$

6. Salvage value of deteriorated units per time unit

$$SV = \gamma CR\left[t_1 + \frac{t_1^{\beta+1}\alpha}{\beta+1} - t_1\right] \quad \text{... (12)}$$

Thus objective function of this inventory system, total cost function per time unit

$$TC(t_1,T) = \frac{1}{T}\left[IHC + SC + OC + CD - SV\right]$$

Putting the values of above cost components in this total cost equation then

$$TC(t_1,T) = \frac{1}{T}\Big[\quad aR\left[\frac{t_1^2}{2} + \frac{t_1^{\beta+2}\alpha\beta}{(\beta+1)(\beta+2)}\right] + bR\left[\frac{t_1^3}{6} + \frac{t_1^{\beta+3}\alpha\beta}{(\beta+3)(\beta+2)}\right] \quad + \quad \frac{1}{2}\pi R(T-t_1)^2 + \quad A \quad +(1-\gamma)C\,R\left[\frac{t_1^{\beta+1}\alpha}{\beta+1}\right] \quad \text{... (13)}$$

The necessary condition for the total cost per time unit, to be minimize is

$$\frac{\partial TC}{\partial t_1} = 0 \text{ and } \frac{\partial TC}{\partial T} = 0 \quad \text{... (14)}$$

Provided $\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0$ and $\left(\frac{\partial^2 TC}{\partial t_1^2}\right)^2 > 0$ ... (15)

Since the nature of the cost function is highly non linear thus the convexity of the function shown graphically in the next section.

5. Numerical Illustration and Sensitivity Analysis

Consider an inventory system with the following parametric values in proper units: R=2000, a=2.0, b=1.5, C=10, π=50, A=50, α=0.15, β=1.5, γ=0.10. The output of the program by using maple mathematical software is T=1.422144, t1=0.0160969 and TC= 70.30235743. i.e.

the value of t1 at which the inventory level become zero is 0.016. The effect of changes in the parameter of the inventory model is as follows

Table-1 Effect of changes in the parameter of the inventory model

Parameter	% Change	T	t_1	TC
R	+40%	1.419974156	0.01167069490	70.41517305
	+20%	1.421459571	0.01469953214	70.33800196
	-20%	1.424000822	0.01989201849	70.20544017
	-40%	1.427021837	0.02607862859	70.04716039
α	+40%	1.421875009	0.01563498949	70.31200099
	+20%	1.422006363	0.01586102779	70.30726674
	-20%	1.422288646	0.01634342295	70.29726113
	-40%	1.422440702	0.01660140470	70.29196484
β	+40%	1.422858828	0.01732135614	70.27687361
	+20%	1.422670438	0.01701936540	70.28255366
	-20%	1.420894310	0.01362119166	70.36365589
	-40%	1.418544038	0.008423916848	70.50600603
γ	+40%	1.422175497	0.01615063241	70.30124325
	+20%	1.422159738	0.01612370872	70.30180147
	-20%	1.422128475	0.01607025426	70.30291108
	-40%	1.422112970	0.01604372129	70.30346243
a	+40%	1.420114479	0.01190742938	70.44910654
	+20%	1.420985301	0.01369695205	70.37548717
	-20%	1.423751973	0.01946766013	70.22971249
	-40%	1.426105064	0.02450166145	70.15754767
b	+40%	1.422121960	0.01606311676	70.30295457
	+20%	1.422132987	0.01607998162	70.30265333
	-20%	1.422155194	0.01611392154	70.30206671
	-40%	1.422166377	0.01613099769	70.30178115

The above numerical illustration shows that the model is quite stable by changing in the parameter of the model.

If we plot the total cost function (13) with some values of t_1 and T s.t., t_1= 0.01 to 0.09 with equal interval T = 0.5 to 4, fixed T at 1.422 and t1 varies from 0.01 to 0.05, fixed t1 at 0.01 and T varies from 1.00 to 1.90 then we get strictly convex graph of total cost function (TC) given by the figure 2, 3 and 4 respectively.

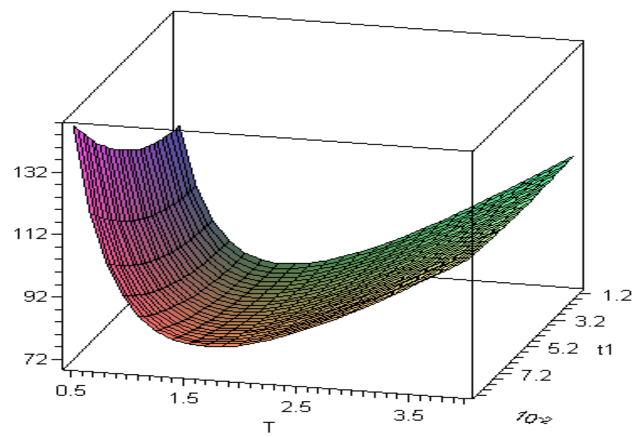
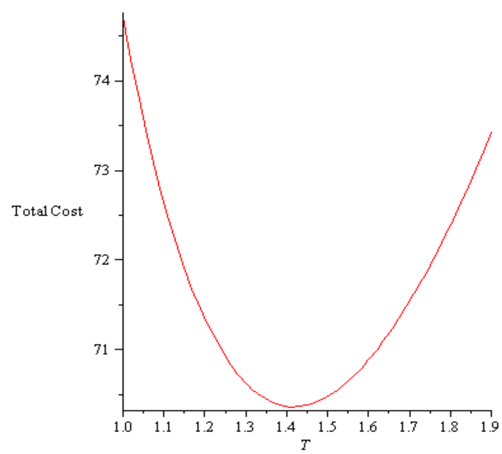


Figure-2 (Total Cost vs. t1 and T)



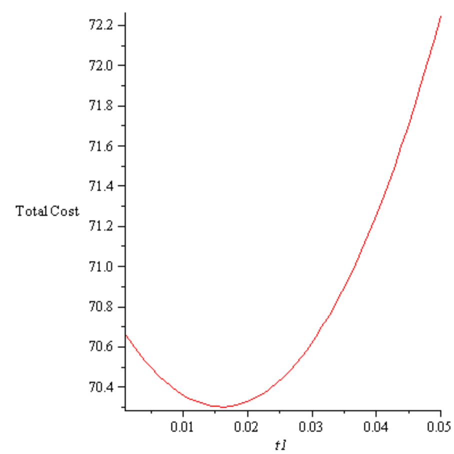


Figure-3 (Total Cost vs. T at t1=0.01) **Figure-4 (Total Cost vs. t1 at T=1.42)**

6. Conclusion

As almost all items undergo either direct spoilage or physical decay in the course time i.e. deterioration is natural feature in the inventory system that's why the deterioration factor taken into consideration in the present model with some realistic assumptions like time dependent deterioration and holding cost with salvage value and give analytical solution of the model that minimize the total inventory cost. The model is very practical for the industries in which the deterioration and holding cost both are depends upon the time. The sensitivity of the model has checked with respect to the various parameter of the system. It is observed that the solution of the model is quite stable. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, Probabilistic demand rate, etc.

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