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Inverse analysis of impact force

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ABSTRACT

An inverse problem is studied for estimating the magnitude and direction of impact force acting on a body of arbitrary shape from strain responses measured at several points of the body. The noncausal Wiener filtering and the singular value decomposition are employed to improve the ill-conditioned nature of the inverse problem. The impact force acting on a simply supported beam subjected to transverse impact is estimated to verify the effectiveness of the inverse analysis.

INTRODUCTION

The impact of two bodies is a dynamic contact problem in which impact force is induced between the bodies. The accurate evaluation of the impact force is a fundamental issue for investigating impact phenomena. However, the experimental evaluation of the impact force, as well as the theoretical prediction, has been difficult except for some limited cases such as the longitudinal impact of rods.

To overcome this difficulty, methods based on the concept of the inverse analysis are recently studied by many authors, e.g. Doyle[1,2,4], Michaels and Pao[3], Chang and Sun[5], Bateman et al.[6], Buttle and Scruby[7]. The authors[8–13] also have studied a method to evaluate the impact force acting on a body of arbitrary shape, in which the ill-conditioned nature of the inverse problem is taken into account in order to obtain accurate estimates of the impact force.

In this paper, we apply the inverse analysis to the evaluation of the magnitude and direction of the impact force acting on a simply supported beam subjected to transverse impact of a ball in arbitrary direction.

INVERSE ANALYSIS OF IMPACT FORCE

Basic Scheme

Let $f_i(t), (i = 1, 2, 3)$ denote directional components of the impact force acting on a body of arbitrary shape, and let $e_j(t), (j = 1, \cdots, n; n \geq 3)$ denote the strain responses at n points away from the impact site. If we assume that the strain responses are linearly dependent on the impact force, we may write

$$e_j(t) = \sum_{i=1}^{3} \int_0^t h_{ji}(t-\tau) f_i(\tau) d\tau, \quad (j = 1, \cdots, n), \tag{1}$$

where $h_{ji}(t)$, $(i = 1, 2, 3;\ j = 1, \cdots, n)$ denote the impulse response functions. Taking the Fourier transforms of both sides of Equation (1), we obtain

$$\left\{ \begin{array}{c} E_1(\omega) \\ E_2(\omega) \\ \vdots \\ E_n(\omega) \end{array} \right\} = \left[\begin{array}{ccc} H_{11}(\omega) & H_{12}(\omega) & H_{13}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & H_{23}(\omega) \\ \vdots & \vdots & \vdots \\ H_{n1}(\omega) & H_{n2}(\omega) & H_{n3}(\omega) \end{array} \right] \left\{ \begin{array}{c} F_1(\omega) \\ F_2(\omega) \\ F_3(\omega) \end{array} \right\}, \tag{2}$$

or in short

$$\boldsymbol{E} = \boldsymbol{H}\boldsymbol{F}, \tag{3}$$

where $F_i(\omega)$, $E_j(\omega)$ and $H_{ji}(\omega)$ denote the Fourier transforms of $f_i(t)$, $e_j(t)$ and $h_{ji}(t)$, respectively. We call $H_{ji}(\omega)$ the transfer function.

Although the impact force is usually difficult to measure directly, the strain responses can be measured rather easily, e.g. by using strain gages. Therefore, if the transfer functions have been identified in advance, we can estimate the impact force from the measured strain responses by solving Equation (3) for $\boldsymbol{F}$. The Fourier transformations and inversions can be computed by utilizing the fast Fourier transform (FFT) algorithm. The authors[9,12,14] have shown that the use of exponential function as the window is appropriate for the numerical deconvolution by means of FFT.

Identification of the Transfer Function

In order to apply the above method, the transfer functions $H_{ji}(\omega)$ should be identified in advance. For the bodies of arbitrary shape, it is difficult to identify the transfer functions analytically. It is rather appropriate to identify the transfer functions experimentally by conducting a calibration. The calibration can be done in the following manner:

1. Apply an impact force to the body in the direction $i = 1$, where the impact force can be measured by a certain available technique.
2. Measure the impact force $f_1(t)$ and the strain responses $e_j(t)$, $(j = 1, \cdots, n)$.

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3. Identify the transfer functions $H_{j1}(\omega), (j = 1, \cdots, n)$ from the Fourier transforms of the data $f_1(t)$ and $e_j(t), (j = 1, \cdots, n)$.
4. Repeat the above procedures for the directions $i = 2$ and 3.

The deconvolution of experimental data is known to be an ill-conditioned problem (see e.g. Baumeister[15]). In order to obtain a stable solution of the problem, the authors[11, 12] have shown a method based on the noncausal Wiener filtering theory. The method can be realized by identifying the transfer function according to the following equation:

$$H_{ji}(\omega) = \frac{\sum_{k=1}^{K} E^*_{jk}(\omega) E_{jk}(\omega)}{\sum_{k=1}^{K} E^*_{jk}(\omega) F_{ik}(\omega)}, \tag{4}$$

where $F_{ik}(\omega)$ and $E_{jk}(\omega)$, $(k = 1, \cdots, K)$ denote the FFTs of the impact force $f_{ik}(t)$ and the strain response $e_{jk}(t)$, respectively, which are obtained by conducting the calibration K-times over, and $*$ denotes the complex conjugates. For details of the method, refer to Inoue et al.[11, 12].

The Least Squares Method

In general, there might be a case when Equation (3) cannot be solved for $\boldsymbol{F}$ in the strict sense. The authors[13] have shown that a method based on the singular value decomposition (SVD, see e.g. Horn and Johnson[16]) is effective for obtaining an approximate solution.

The SVD of the matrix $\boldsymbol{H}$ is defined by

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H, \tag{5}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are n-by-n and 3-by-3 unitary matrices, respectively, and $\boldsymbol{\Sigma}$ is an n-by-3 real matrix composed of the singular values of $\boldsymbol{H}$. In addition, the superscript H denotes the Hermitian adjoint matrix. According to the theory of the SVD, the least squares solution of Equation (3) is given by

$$\tilde{\boldsymbol{F}} = \boldsymbol{H}^+\boldsymbol{E}, \tag{6}$$

where $\boldsymbol{H}^+$ is the Moore-Penrose generalized inverse of $\boldsymbol{H}$ defined by

$$\boldsymbol{H}^+ = \boldsymbol{V}\boldsymbol{\Sigma}^+\boldsymbol{U}^H, \tag{7}$$

where $\boldsymbol{\Sigma}^+$ is the transpose of $\boldsymbol{\Sigma}$ in which the non-zero singular values of $\boldsymbol{H}$ are replaced by their reciprocals.

Equation (6) provides a unique solution such that both $\|\boldsymbol{E} - \boldsymbol{H}\tilde{\boldsymbol{F}}\|_2$ and $\|\tilde{\boldsymbol{F}}\|_2$ are minimal. Therefore, a unique estimate of the impact force can always be obtained in the sense of least squares.

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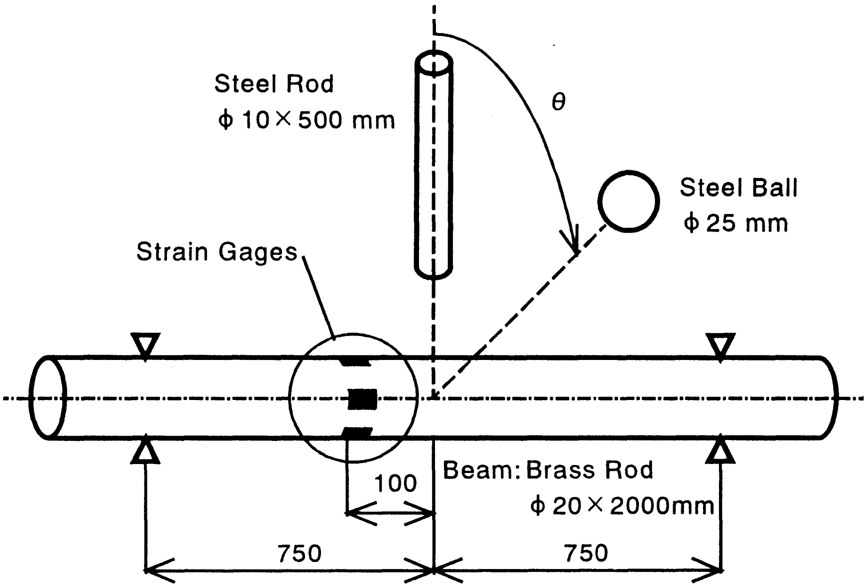

Figure 1. Simply supported beam subjected to transverse impact.

MEASUREMENT OF IMPACT FORCE ACTING ON A SIMPLY SUPPORTED BEAM

Experimental Setup

We applied the inverse analysis described above to the experimental evaluation of the impact force acting on a simply supported beam. The experimental setup is shown in Figure 1. A brass rod was used as the simply supported beam. The impactors used were a steel rod and a steel ball. The impact force was applied to the center of the beam by freely dropping the impactor in the direction perpendicular to the neutral axis of the beam. The direction of the impact was varied by rotating the beam about its neutral axis to a specified angle θ. The strain responses of the beam were measured by using strain gages (Kyowa Electronic Instruments KSP-2-E4) adhered at 100 mm away from the impact site. In this experiment, the number of directional components of the impact force is two. We defined the direction of the components of the impact force $f_i(t), (i = 1, 2)$ and the strain responses $e_j(t), (j = 1, 2, 3, 4)$ as illustrated in Figure 2(a).

The procedures for estimating the impact force are summarized as follows:

1. Impact the beam with the steel rod in the direction $i = 1$ and measure the impact force $f_1(t)$ and the strain responses $e_j(t), (j = 1, 2, 3, 4)$

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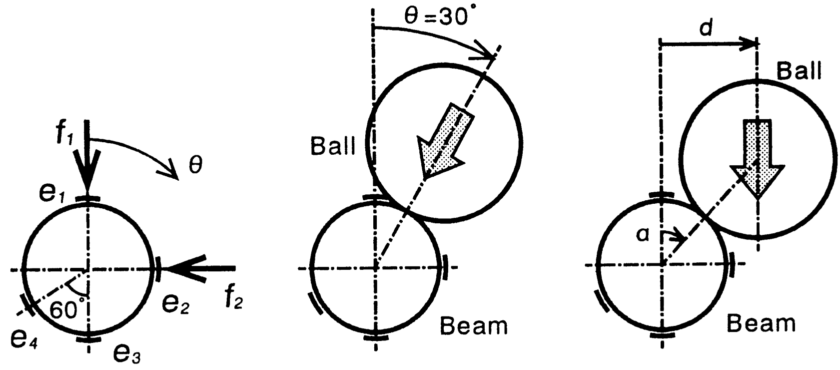


(a) Components of the impact force and strain responses.

(b) Case-A

(c) Case-B

Figure 2. Impact of the beam with the steel ball.

using strain gages.

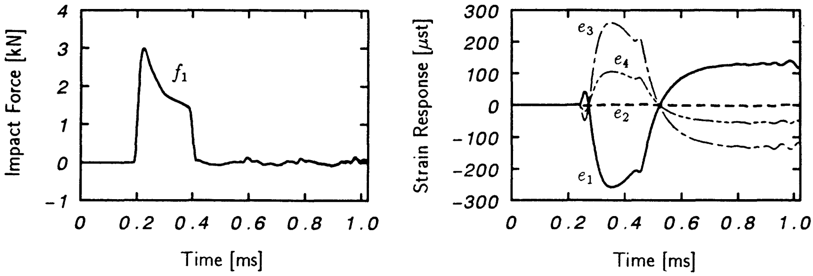
2. Identify the transfer functions $H_{j1}(\omega), (j = 1,2,3,4)$ from the data obtained in the first step.
3. Impact the beam with the steel rod in the direction $i = 2$ and identify the transfer functions $H_{j2}(\omega), (j = 1,2,3,4)$.
4. Impact the beam with the steel ball in arbitrary direction and measure the strain responses $e_j(t), (j = 1,2,3,4)$.
5. Estimate the impact force $f_i(t)$, $(i = 1,2)$ from the strain responses $e_j(t)$, $(j = 1,2,3,4)$ by using the transfer functions $H_{ji}(\omega)$, $(i = 1,2; j = 1,2,3,4)$ obtained in the second and the third steps.

In the calibration (the first and the third steps), the impact force was measured by using strain gages (KSP-2-E4) adhered to the steel rod according to the theory of one-dimensional longitudinal impact of rods.

The acquisition and processing of the data were accomplished by using DC strain amplifier (DC–200 kHz, ±3 dB), digital memory (12-bits resolution) and personal computer (NEC PC-9801). The sampling rate was $\Delta t = 1$ μs and the data length was $N = 1024$. The exponential window function $\exp(-\gamma t), \gamma = 5/(N\Delta t)$ was employed for the numerical Fourier transformation and inversion using the FFT (see Inoue et al.[14]).

<u>Results of the Calibration</u>

The beam was impacted with the steel rod in the direction $i = 1$. The rod was dropped from 200-mm high (the impact velocity was 1.98 m/s). The impact force $f_1(t)$ and the strain responses $e_j(t), (j = 1,2,3,4)$ measured



(a) Impact force (b) Strain response

Figure 3. A typical result of the calibration for the direction $i = 1$.

are shown in Figures 3(a) and (b), respectively. Ten sets of data similar to those shown in the figures were acquired and used for identifying the transfer functions $H_{j1}(\omega), (j = 1, 2, 3, 4)$ according to Equation (4) with $K = 10$. The transfer functions $H_{j2}(\omega), (j = 1, 2, 3, 4)$ were also identified by conducting the calibration for the direction $i = 2$ in a similar manner.

Estimation of the Impact Force by Ball

First, the beam was impacted with the steel ball in the direction $\theta = 30°$ as shown in Figure 2(b). Figure 4 shows the impact force estimated from the strain responses in the case when the impact velocity is 3.13 m/s. In this figure, the impact force predicted by the method of Schwieger[17] is also shown by the dotted line. The estimated magnitude agrees well with the predicted one. In addition, the direction has been estimated to be $\theta = 30°$ during the impact. For other impact velocities, similar results were obtained. It can be concluded that the magnitude and direction of the impact force can be estimated satisfactorily by the inverse analysis.

Next, the beam was impacted with the same ball in a manner shown in Figure 2(c). The impact force in the case when $d = -5, -10, -15, -20$ mm and the impact velocity is 3.13 m/s were estimated as shown in Figure 5. The maximum impact force decreases with increase of d, while the duration of impact is unchanged. The direction is similar to the angle α shown in Figure 2(c), which implies little effect of friction on the impact force.

CONCLUSIONS

An inverse analysis has been studied for experimentally evaluating the magnitude and direction of the impact force acting on a body of arbitrary shape. It has been shown that the present method provides accurate estimates of the impact force. The inverse analysis will be an effective tool for investigating dynamic contact phenomenon.

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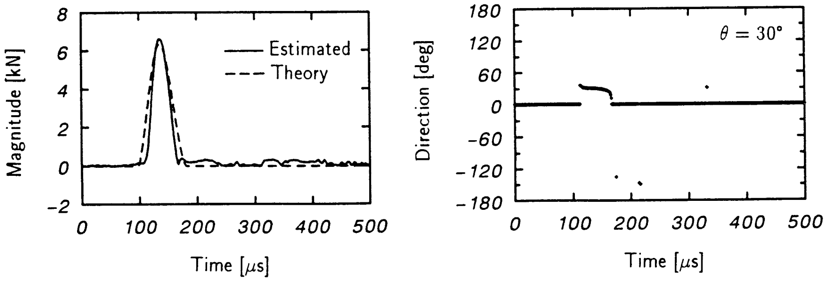

Figure 4. Estimated impact force for the Case-A.

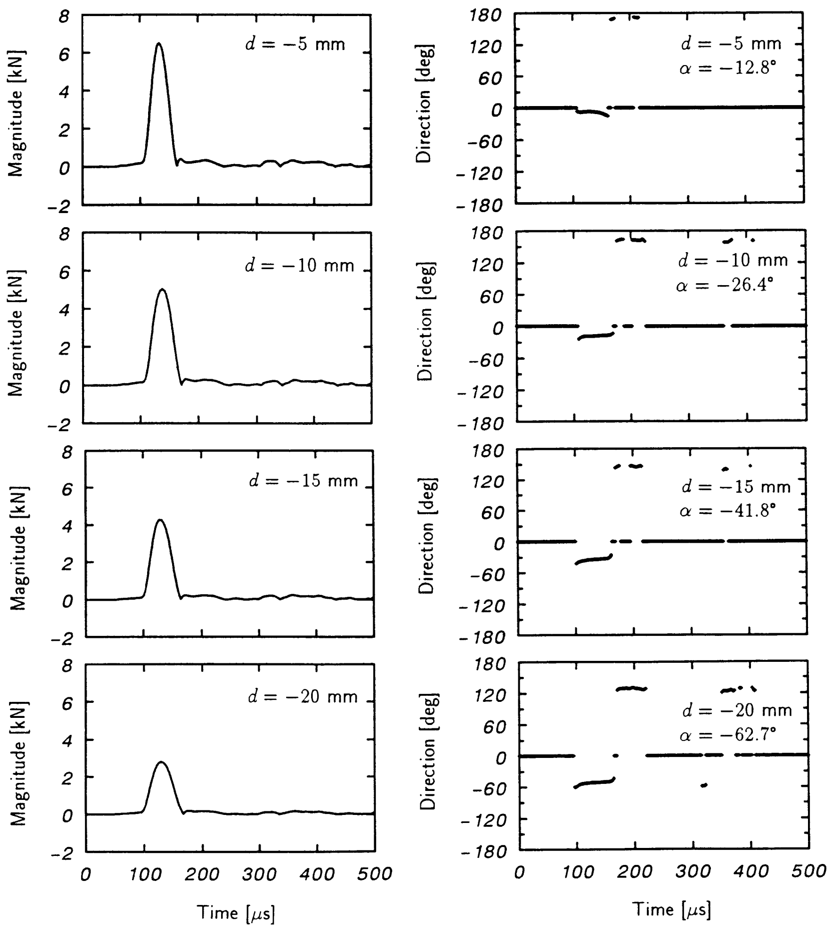

Figure 5. Estimated impact force for the Case-B.

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