Perception & Psychophysics
1978, Vol. 24 (5), 457-465

Inverse attribute functions and the proposed modifications of the power law

WILLIAM E. DAWSON and MARK E. MILLER
University of Notre Dame, Notre Dame, Indiana 46556

Three prominent modifications of Stevens' power law, designed to account for the curvature at low stimulus intensities, were examined in the light of previously reported inverse attribute functions and five new experiments scaling loudness and softness by magnitude estimation. One hundred and seven students served as observers in the five experiments. Lack of curvature in the inverse functions at low stimulus intensities, plus the lack of parameter invariance, pointed to the inadequacy of the effective threshold and physiological noise modifications. The additive constant model of zero-point response bias, previously advanced by Irwin and Corballis (1968) and McGill (1960) was found more satisfactory. A discussion of the implications of the research findings for subsequent formulations of the modified power law was included.

From the earliest days of the power law, it was realized that there is often a departure from its simplest form for low stimulus intensities. This departure usually takes the form of a downward curvature in a logarithmic-logarithmic plot. The two most prevalent explanations for this effect, along with several others, were reviewed by Marks and J. C. Stevens in 1968, and they have been assessed by Fagot and his associates in several articles (e.g., Fagot, 1975; Fagot & Stewart, 1968). These two explanations are the effective threshold account and the physiological noise account. Recent reviews of these and other suggested power law modifications appear in Gescheider (1976), Marks (1974) and S. S. Stevens (1975).

The *effective threshold model* is sometimes called a phi-translation model, since correction amounts to a translation of physical stimulus intensity on the abscissa. According to the model, the effect arises from the operation of a threshold-like mechanism. Sensation does not start at very low stimulus intensities, but rather begins only after the so-called "effective threshold" has been reached. Hence the effective stimulus must be measured starting at the effective threshold. Equation 1 in Table 1 is the effective threshold model with R representing sensory intensity, k a multiplicative constant, S stimulus intensity, S_0 effective threshold, and n the exponent characteristic of the modality involved.

The *physiological noise model* is sometimes called a psi-translation model, since the correction amounts to a translation of sensory intensity on the ordinate. According to this model, the curvature effect arises from the existence of some sort of physiological "noise" in the modality involved. The physiological noise presumably gives rise to a background *sensory* noise which obscures or masks low-intensity stimuli and so results in the low-end curvature. A constant added to the sensory or left-hand side of the power law, seen as reflective of sensory noise, permits the straightening out of the data at the lower end. Equation 3 is the *additive constant model* version of the psi-translation. The physiological noise aspect emerges more obviously in Equation 2, a commonly presented version (e.g., see Lochner & Burger, 1961; Marks and J. C. Stevens, 1968), which is derivable from Equation 3 by letting $C = kS_0^n$ and by subtracting it from the right-hand side of that equation.

Although Equations 1 through 3 have been the most widely used as models, and hence are those focused upon in the present paper, occasionally variants of these equations have been suggested by experimenters for their experimental results. For example, Ekman (1959) reported data that were better fit by Equation 1 modified so that S_0 was added rather than subtracted from S. Analogously, Ekman (1961) reported finding a better fit of gustation data using Equation 3 but with $R - C$ rather than $R + C$ on the left-hand side of the equation. For a psychological mechanism, he suggested that C, in the subtractive form, might be due to sensory noise.

Also less investigated, but previously put forth, are models combining both a phi- and psi-translation in the same model. For treatment of these models, one should see Fagot (1966) and Marks and J. C. Stevens (1968).

Which Modification Is Better?

Several attempts have been made to determine

A preliminary report of this paper was read at the Ninth Annual Mathematical Psychology Meetings, New York University, New York City, August 1976. Portions of this paper are based on experimental data taken from the second author's unpublished MA thesis, which was completed at the University of Notre Dame. The authors thank Michael Lilienthal for assistance with the computer-generated figures.

Table 1
Three Models for Modification of the Simple Power Law

Model	Primary Attribute		Inverse Attribute	
Effective Threshold	$R = k(S - S_o)^n$	(1)	$R_{inv} = k^{-1}(S - S_o)^{-n}$	(4)
Physiological Noise	$R = k(S^n - S_o{}^n)$	(2)	$R_{inv} = k^{-1}(S^n - S_o{}^n)^{-1}$	(5)
Additive Constant	$R + C = kS^n$	(3)	$R_{inv} + C = kS^{-n}$	(6)

whether the threshold or the physiological noise model better describes the data. Lochner and Burger (1961), while scaling loudness under masking and nonmasking conditions, found support for the noise model. Fagot and Stewart (1968) also did for the brightness continuum, but their data were fit poorly by both the threshold and noise models in some cases. Marks and J. C. Stevens (1968) pointed out the empirical difficulties of deciding between the two models, indicating the small differences in prediction and the highly variable judgments in direct scaling. Fagot (1975) has recently reexamined the two power law modifications and concluded that the psi-translation, in the form of Equation 2, is the more appropriate.

Irwin and Corballis (1968), in an important paper, approached the issue of model superiority from another direction. They examined loudness *and its inverse attribute*, softness, and found that the additive constant model, i.e., Equation 3, fit both the loudness and softness data. Table 3 of the present paper gives the exponents that they fitted to their data; these exponents are of the same magnitude for loudness and softness but appropriately of opposite sign. Fitting the effective threshold model to the data, on the other hand, led Irwin and Corballis to widely different absolute magnitudes for the two exponents (see Table 3). Hence, they argued in favor of the psi-translation. It should be noted, however, that they found it difficult to give a definitive interpretation of the psychological meaning or significance of the C of Equation 3. They did, however, suggest that a zero-point response bias might be operating. Their experiment examined neither the loudness nor the softness functions over the full range of tolerable sound pressures. For loudness, sound pressure levels varied from 8 to 48 dB; with softness, they ranged only from 68 to 108 dB. Of particular importance for the present paper is the fact that very few studies that have examined the inverse attribute have explored that attribute in the so-called threshold curvature region.

Inverse Attribute Scales

The present paper, following the lead of Irwin and Corballis, examines the inverse scales of loudness and softness. It first looks at what these scales would look like over the full stimulus range if they were truly inverses. Next, it provides a cursory review of a selection of relevant studies from the psychophysical literature that have dealt with inverse scales. Finally, a set of experiments is presented and the various models in Table 1 examined for their ability to describe the results.

If subjective softness is actually a valid inverse of loudness, then Equation 4 or 5 depending upon whether Equation 1 or 2 best describes loudness judgments, should account for softness over the full stimulus range. The threshold and noise models, as Figure 1 shows, predict that curvature will occur at the low stimulus-intensity end of the inverse scale. Irwin and Corballis' model for softness, i.e., Equation 6, is different in that it predicts curvature at the high-intensity end (see Figure 1). Equation 6, unlike Equations 4 and 5, is not a simple reciprocal of the loudness equation adjacent to it in Table 1. The symmetry for the effective threshold and physiological noise models is about a horizontal line, while that for the additive constant model is about a vertical one. The untranslated judgments according to the additive constant model will not be reciprocally related over the full range.

An examination of the direct ratio scaling literature shows that, although in some cases inverse scales are indeed good inverses, more often than not there is curvature in the function, particularly at the upper end when high stimulus intensities are employed. Table 2 summarizes the shape of inverse functions found in a variety of studies. A review of the plotted data in most all of these studies shows that the inverse attribute yields more curvilinear results than does the primary attribute. Few experimenters,

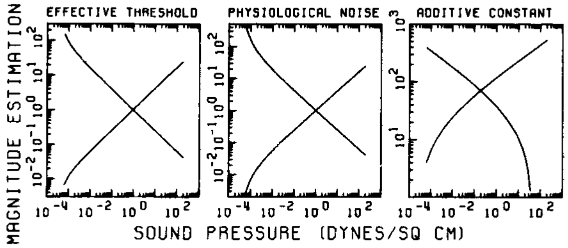

Figure 1. Sample primary and inverse functions for three models of power law modification. Effective threshold and physiological noise models assume k = 1, n = ±0.6, and S_0 = .0004 dyne/cm². The Irwin and Corballis or additive constant model uses the values of k, n, and C reported by them and listed in Table 3.

Table 2
Shapes of Inverse Functions in Psychophysical Experiments

Experimenter	Continua Scaled	Curvilinearity**
Magnitude Estimation		
Torgerson (1960)	Lightness /Darkness	Strong
Stevens & Guirao (1962)	Loudness/Softness	Negligible
Stevens & Harris (1962)	Roughness/Smoothness	Negligible
Schneider & Lane (1963)*	Loudness/Softness	Strong
Robertson (1966)	Brightness/Dimness	Strong
Irwin & Corballis (1968)	Loudness/Softness	Moderate
John (1971)	Loudness/Softness	Strong
Mattiello & Guirao (1974)	Lightness/Darkness	Moderate
Magnitude Production		
Stevens & Guirao (1962)	Loudness/Softness	Negligible
Stevens & Guirao (1963)	Longness/Shortness	Negligible
Stevens & Guirao (1963)	Largeness/Smallness	Negligible
Sensory Modality Matching		
Dawson & Mirando (1976)	Ease/Difficulty of Pronunciation	Negligible
Dawson & Mirando (1976)	Desirability/Undesirability of Occupation	Moderate

**Data from Eisler (1962). **Curvilinearity in inverse function.*

possibly only Robertson (1966), have scaled both a primary attribute and its inverse attribute in the low stimulus-intensity region, where primary attribute curvilinearity occurs. Some of the experiments reported in the present paper examined the loudness and softness scales over a full range of from 10 to 100 dB sound pressure level in an effort to make up for this lack. Of particular interest were the questions: (1) Which of the models, if any, adequately deals with the judgments? (2) What is the shape of the softness function at low intensities? (3) What is the value of C, and that of the S_0 implied by the C, for the softness case with Equation 6?

METHOD

Subjects

One hundred and seven students from introductory psychology classes each served as an observer in one of five experiments. Of these students, 88 were male and 19 female. Each received extra course credit for participation. Ten additional persons were dropped from the experiments when it was found that they were unable to hear the lower sound intensities.

Apparatus

The observers listened to the tones in an Industrial Acoustics Company sound-deadened room. The various sound pressure levels were presented via TDH-39 calibrated headphones connected in series and mounted in neoprene cushions. A Hewlett-Packard HP-400 CD oscillator served as signal source and a Hewlett-Packard HP-350 D attenuator permitted adjustment of stimulus intensity.

Procedure

General method. Since most subjects had not previously encountered the method of magnitude estimation, a preexperiment familiarization task was used in all cases. In this task, the subject was asked to give magnitude estimates of either line length ("longness") or line shortness, depending upon whether the experiment involved loudness or softness, respectively.

Magnitude estimation was employed in all of the experiments. The subjects were told to give numbers in proportion to their impressions of loudness or softness, whichever the experiment required. A variety of stimulus standards and moduli were used in the different experiments in an effort to establish the generality of the curvature effects. All tones used had a frequency of 1,000 Hz. The subject controlled the length and frequency of presentation of each stimulus intensity by operating a key within the sound-deadened room. The stimulus intensities were presented in different irregular orders for each subject, with the standard stimulus occurring first if the experiment employed one. The full series of stimuli was presented to the subject before the series was repeated. In all experiments, the subjects gave three judgments of each stimulus intensity.

The experiments. Experiment 1 required 25 observers to judge the loudness of 10 sound levels. The stimuli ranged from 10 to 100 dB in 10-dB steps. No single stimulus was chosen to occur first as a standard and no modulus was assigned to any stimulus.

Experiment 2 employed another 25 observers for judgments of the loudness of the same stimuli used in Experiment 1. No modulus was assigned, but all persons received the 100-dB tone as the first stimulus or standard.

Experiment 3, the first in which subjects judged softness, asked 21 new observers to report on the softness of the same 10 stimuli. Here the 60-dB tone served as the standard, and all observers were told to call it "10."

Experiment 4 required 18 observers to judge the loudness of tones in each of three different stimulus ranges: 10-100 dB in 10-dB steps (10 stimuli), 10-45 dB in 5-dB steps (8 stimuli), and 65-100 dB in 5-dB steps (8 stimuli). The midrange standards, each called "10," that were used were 55, 27, and 82 dB, respectively. The observers dealt with each range in a separate experimental session, with the order of the three ranges properly counterbalanced across observers. The sessions for each observer were never less than 24 h apart, and all three never required more than 1 week to complete.

Experiment 5 was exactly like Experiment 4, except that a new group of 18 observers now judged the softness of the tones in the three ranges.

RESULTS

Plots of the Data

Geometric means of the judgments of each stimulus level were calculated for each experiment or for each stimulus range within an experiment. They appear in

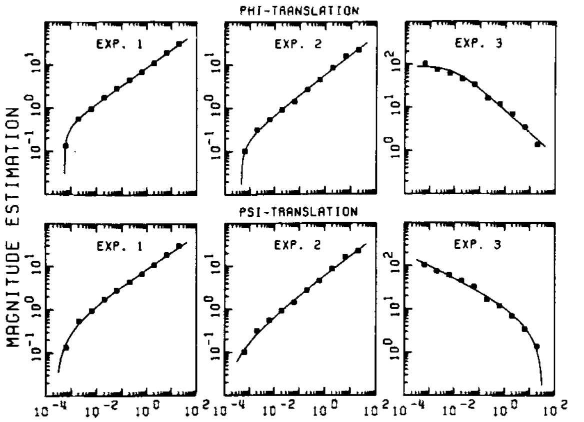

Figure 2. Geometric means for Experiments 1-3 with best-fitting curves for the effective threshold (phi-translation) model and for the physiological noise and additive constant models (psi-translation).

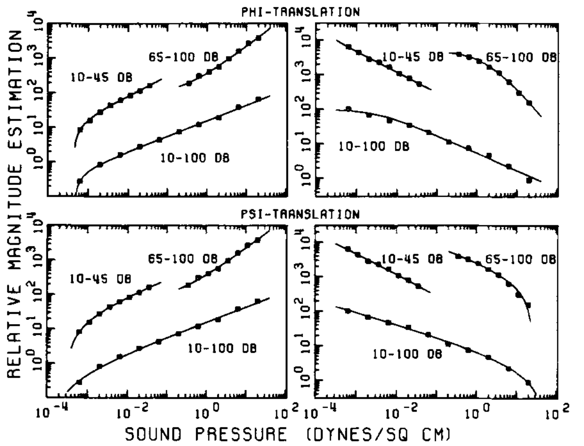

Figure 3. Geometric means for Experiments 4 (loudness) and 5 (softness) with best-fitting curves for the effective threshold (phi-translation) model and for the physiological noise and additive constant models (psi-translation).

untranslated form plotted on log-log axes, in Figures 2 and 3. In general, the loudness functions are curvilinear for low sound pressures, whereas the softness functions are curvilinear for high sound pressures.

Computer programs were used to locate those translations, S_0 in the phi-translation case and C in the psi-translation case, which yielded the best fit to each set of geometric means. Best-fitting S_0 and C were taken to be those values which maximized the square of the Pearson product-moment correlation between stimulus levels and the corresponding judgments, where one or the other of the variables was translated in accord with the threshold correction model or the additive constant model. A similar approach was used by Marks and J. C. Stevens (1968). For the physiological noise model, S_0 was calculated from the best-fitting C using the equation $S_0 = (C/k)^{1/n}$.

The best-fitting curves for the various models, according to the fitted parameters, have been drawn through the geometric means in Figures 2 and 3. The three-parameter models fit the various sets of datum points equally well, and it is difficult here to choose graphically between the effective threshold model and the additive constant or physiological noise models.

Figures 4 and 5 are the analogs to Figures 2 and 3, respectively, differing only in the fact that the appropriate axis translations have been made. For the effective threshold model, S_0 has been subtracted from each S; with the additive constant model, C has been added to each R value. A good quality fit to data by a model is indicated by all points lying

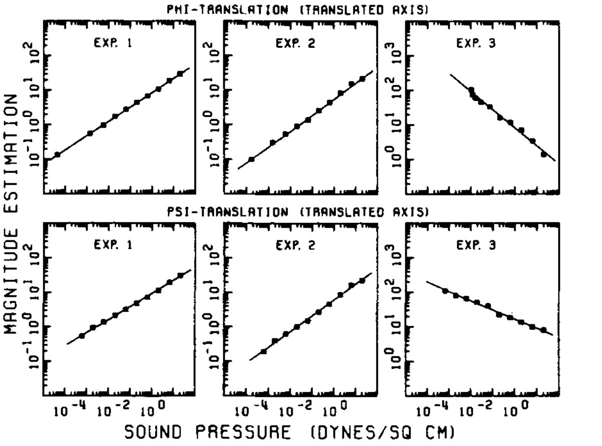

Figure 4. The results of Figure 2 with appropriately translated axes for the power law modification. The phi-translation involves a shift along the abscissa and the psi-translation a shift along the ordinate.

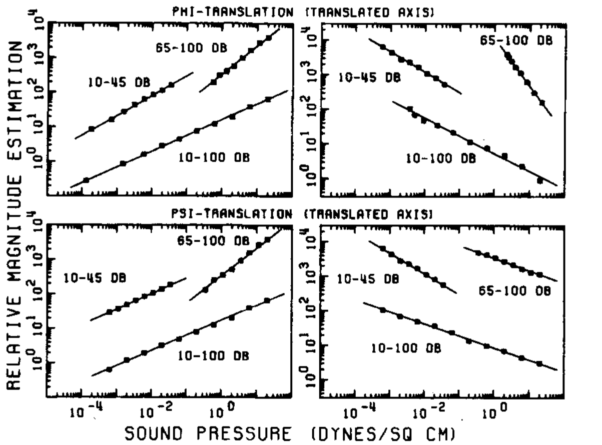

Figure 5. The results of Figure 3 with appropriately translated axes for the power law modifications. The phi-translation involves a shift along the abscissa and the psi-translation a shift along the ordinate.

randomly about or on a straight line. Using this criterion, most of the sets of data are fitted very well by both models. Only the effective threshold model for softness over the full 10- to 100-dB range (see Figures 4 and 5) results in what appears to be a systematic nonlinearity. The softness judgments are better fitted by the additive constant model.

Fitted Parameters

Table 3 lists the best-fitting parameters for the effective threshold model for the experiments along with comparable values for the Irwin and Corballis (1968) study. Values for k for the latter study were obtained graphically from a figure in their article. Table 3 also gives the value for S_0 expressed in decibels (re .0002 dynes/cm^2) and the residual variance, a rough measure of the goodness of fit. Those decibel values followed by two asterisks are based upon the absolute Value of S_0, since no decibel values exist for negative S_0s. We included them for comparison purposes, much as Irwin and Corballis did, but they would only make psychological sense if the value of S_0 were added to S rather than subtracted from it as in Equation 1. The need for the *addition* of S_0 here is reminiscent of Ekman's (1959) findings.

The exponents in Table 3 are appropriately positive for loudness and negative for softness, but they vary widely in magnitude. Lower absolute magnitudes occur for broad stimulus ranges, while higher ones are associated with narrow ranges. Their magnitudes fall on either side of the 0.67 suggested by S. S. Stevens (1972) as the best estimate for the loudness function. Low values of exponent are known to occur when the so-called regression effect (see S. S. Stevens, 1971) is not compensated for. The exponents of Irwin and Corballis resemble those for our experiments, if one looks at those involving similar stimulus ranges. The fact that the 10-100-dB ranges of Experiments 4 and 5 yield exponents more nearly equal (but opposite in sign) than the narrow ranges used by Irwin and Corballis suggests two things. First, when the full range is scaled for loudness and softness, the effective threshold model does find exponents nearly equal in absolute magnitude. Secondly, the stimulus range selected can strongly affect the exponent. Of interest, however, is the fact that for the same 65-100-dB range, the loudness and softness exponents still differ in absolute value.

The effective thresholds for loudness in Table 3 fall mainly in a 6-9-dB range, and thus resemble values found earlier (e.g., Scharf & J. C. Stevens, 1961). Only loudness S_0 for the 65-100-dB range in Experiment 4 seems irregular. Since the loudness function is generally a simple power function in this range, it is likely that this S_0 is due to a systematic or sampling error of some sort. S. S. Stevens (1969, 1975, p. 292) has noted how unwarranted translations can lead to data misinterpretation, and so caution in interpreting it in the present case is probably called for.

The softness effective thresholds pose problems for the reciprocal effective threshold model, i.e., Equation 4. Clearly, these values do not fall in the same range as the loudness ones do. In fact, most of the S_0s are negative and their magnitudes are sensitive to the stimulus range used. Further, the fact that they are negative, both for our data and for that of Irwin and Corballis, suggests that the correction should be an addition and not a subtraction. A sizable, and variable, addition would be required according to these results. Again, the irregular S_0 shows up for that range (10-45 dB) where the function usually follows a simple power function, i.e., is linear in log-log coordinates. Sampling error again may be the cause and the translation inappropriate. The two asterisks following the decibel values in the table are meant to indicate that they

Table 3
Parameters for Phi-Translation and Psi-Translation Models

			Phi-Translation Model (1)					Psi-Translation Models (2 and 3)						
E	A	Range*	k	n	S_0	S_0*	RV	k	n	C	C/k	S_0	S_0*	RV
1	L	10-100	8.49	.414	.000589	9.4	.000173	9.13	.379	.397	.0435	.000253	2.1	.00163
2	L	10-100	5.81	.465	.000465	7.3	.00112	5.98	.453	.0967	.0162	.000111	−5.1	.00204
3	S	10-100	8.17	−.517	−.0101	34.0**	.00397	17.0	−.260	6.72	.397	34.9	104.8	.00324
4	L	10-100	15.7	.451	.000500	8.0	.000569	16.3	.434	.347	.0213	.000140	−3.1	.00147
4	L	10-45	111	.572	.000445	6.9	.000322	80.8	.454	2.04	.0253	.000304	3.6	.000135
4	L	65-100	3.16	.852	−.222	60.9**	.000892	3.17	.846	−.623	−.197	C < 0	C < 0	.000901
5	S	10-100	5.25	−.502	−.00298	23.5**	.00425	8.51	−.344	2.19	.258	51.8	108.3	.000752
5	S	10-45	.747	−.594	.000056	−11.1	.000482	.791	−.591	.371	.469	3.60	85.1	.000477
5	S	65-100	115	−1.40	−1.86	79.4**	.000209	34.3	−.379	10.0	.292	25.6	102.1	.00659
							Irwin & Corballis (1968)							
	L	8-48	220†	.509	.000428	6.6		130†	.268	14.06	.108	.000249	1.9	
	S	68-108	330†	−1.40	−4.58	87.2**		60†	−.261	22.33	.372	44.13	106.9	

Note—*E = experiment, A = attribute scaled (L = loudness, S = softness), and RV = residual variance. C < 0 indicates value not defined, since $S_0 = (C/k)^{1/n}$. *In decibels. **Value based on absolute value of S_0. †Obtained graphically.*

are based on the absolute values of the S_0s.

Table 3 also supplies the parameters for the additive constant and physiological noise models, both for our experiments and for those of Irwin and Corballis. Exponents are properly positive for loudness and negative for softness and hover somewhere around 0.4 in absolute magnitude. The variation in absolute magnitude is somewhat large, but the experiments vary in number of stimuli judged, stimulus range employed, modulus and standard used, and number of ranges judged per subject.

The values for C, the additive constant, also vary considerably, but they are not comparable across each set of data, because C is affected by the modulus/standard pairing employed and this varies across experiments and across stimulus ranges within experiments. A comparison, however, can be made of the values of C/k, because this mathematically remains invariant under changes in the modulus/standard pairing. For the loudness tasks, C/k tends to hover around .03; the value for Irwin and Corballis, based on a graphical k, is larger. The 65-100-dB range in Experiment 4 is again recalcitrant. In the case of softness, C/k is approximately .3 on the average, with the 10-45-dB range of Experiment 5 again being atypical, as it was for the effective threshold model. In general, the value of C/k does not appear to be invariant across loudness and softness tasks, averaging about 10 times larger for softness than for loudness.

The values of S_0 for the physiological noise model were found using $S_0 = (C/k)^{1/n}$ and are also given in Table 3. For each, where possible, the corresponding sound pressure level (SPL in decibels re .0002 dynes/cm²) is given in the adjacent column. For the loudness tasks, S_0s are quite small; the corresponding SPLs range from -5.1 to 3.6, values which tend to be a little lower than previously measured absolute thresholds for loudness. The fitted S_0s for this model are less than those for the effective threshold model. The 65-100-dB range of Experiment 4 does not allow computation of an S_0, since C is negative and hence S_0 is undefined.

For the softness case, S_0s are considerably larger, with the 10-45-dB range remaining atypical. The corresponding SPLs hover near 105 dB, both for our data and for those of Irwin and Corballis. Clearly, this is not the same implicit level of physiological noise as was found for loudness.

A brief comparison of residual variances across the two types of models seems worthwhile. A low residual variance is associated with a better fitting model. The variances listed are based on logarithmically transformed values. That is, deviations are given by d = log magnitude estimation obtained − log magnitude estimation predicted, and residual variance by $(\Sigma d^2)/N$, where N refers to the number of datum points and the summation is over N deviations. A comparison shows that in five cases the effective threshold model gives lower variance, while in four cases the additive constant model is superior. These results support the contention of Marks and J. C. Stevens (1968) that the choice between the two types of translations is difficult because their predictions, within an attribute, are very similar and because the variability in judgment data is high.

DISCUSSION

From the plots of the untranslated data (Figures 2 & 3) it is clear that loudness and softness *judgments* are not simple reciprocals or inverses over the full range of sound pressures. Rather, the loudness data show curvature for low stimulus intensities where the softness data are linear and loudness judgments are linear for high stimulus intensities where the softness data are nonlinear. Thus, although many earlier studies have examined the primary-inverse relationship and have concluded that it is nearly a reciprocal one, this relationship does not appear to hold in the present case across the full stimulus range for loudness and softness, continua having very large stimulus ranges. This result throws into question past usage of inverse attribute data as confirming evidence for the simple, untranslated power law (e.g., S. S. Stevens, 1962).

Since there is no curvature for the low end of the softness judgments, it appears that no correction, compensation, or translation is required there. This implies that *neither* a phi-translation nor a psi-translation is needed, and hence that no "effective threshold" or "physiological noise" mechanism is operating here, in a range of sound pressures where loudness judgments do show curvature. In turn, then, we are led to question whether it is an actual physical threshold or physical noise effect that occurs for loudness judgments at low intensities. Rather, it appears to be some other kind of effect. Whatever the cause, it appears to affect the loudness and softness data at opposite ends of the physical dimension. Phi-mechanisms, like absolute thresholds and masking by physiological noise, seem to be ruled out, since it is not clear why such mechanisms would fail to have an effect when the attribute judged is changed while the physical dimension remains the same. A psi-mechanism, one concerned with psychological aspects, would appear to be more appropriately called for. For example, curvature occurs for both attributes at low sensory magnitudes—low loudness and low softness. Maybe a response bias in the use of numbers in magnitude estimation is involved. Perhaps Krantz's (1969) distinction between energy thresholds and observer thresholds is worth mentioning here. An energy or external threshold, according to Krantz, is one measured in energy terms; an observer threshold is one not measured in energy terms but inferred from judgments by the observer. Clearly, an energy threshold is not occurring, although some sort of observer threshold effect still could be.

Since the judgments are not reciprocally related, the simple reciprocal models of Table 1 are not adequate. That is, the softness results are not in accord with Equations 4 or 5, the reciprocals of Equations 1 and 2, respectively. Nor would the results agree with the reciprocal of Equation 3, i.e., with $(R + C)^{-1} = k^{-1}S^{-n}$.

The softness data are describable by the direct attribute form of the models (Equations 1 through 3), as Figures 2 through 5 show. However, the magnitudes of the fitted parameters vary in ways that should be noted. As already stated, they are not in accord with any of the simple reciprocal expressions, but they show other discrepant variations as well. The effective threshold model finds large differences in exponent n and S_0 values, especially for different ranges in softness. It was this sort of variation that led Irwin and Corballis to dismiss Equation 1. The physiological noise model yields less variation in n and S_0, but the values for the latter hover around 105 dB, a value at odds with those usually associated with either the threshold correction or physiological noise models. Perhaps, it might be argued, one should expect two different thresholds, since one is examining two different attributes. And 105 dB might be a reasonable point below which tones commence to have some softness. But thresholds, at least energy thresholds, albeit psychological phenomena, have been measured in terms of physical dimensions. Therefore, based on the classical concept of threshold, one would expect any threshold effect, specific to a region of the physical dimension, to manifest itself for both the direct and inverse attributes scaled near that region. The fact that they do not suggests that it is not such a threshold, or even a noise masking, effect per se which causes the curvature. As Marks (1974) has stated in regard to S_0, "Although the parameter . . . seems to *relate* to absolute threshold, it may be incorrect to consider [it] to *correspond* to threshold" (p. 23). Although a wide variety of reported research demonstrates clearly that curvature occurs for near-threshold stimulus intensities, and does so for a variety of sensory modalities (see Gesheider, 1976), the obtained inverse attribute data suggest that the nonlinearity is not *directly* produced by phi-mechanisms.

If the effective threshold and physiological noise models are made less acceptable by the results, what about the additive constant model, Equation 3? Clearly, both the loudness and the softness data are well fitted by this model, as the psi-translation portions of Figures 2 through 5 show. The exponent is positive for loudness and negative for softness, properly indicating increasing and decreasing functions, respectively. But what about the values for and the interpretation of C? Merely to attribute this parameter to "response bias" is just to relabel the problem. What sort of bias do the obtained values for C, and its derivatives C/k and S_0, suggest is occurring? Since C varies with the modulus employed, we must look to C/k and S_0 for clues as to what sort of bias may be present. Are they invariant for loudness and softness, across different judged stimulus ranges, across different experiments and experimenters, or across different subjects? As Table 3 shows, these two parameters do vary with attribute, are stable across ranges for an attribute when the ranges include the curvature region, and are stable across experiments and experimenters. We know from the previous work of McGill (1960) that the additive constant varies across individuals while remaining fairly stable across replications for any one individual.

Of greatest importance for an interpretation of a response bias appears to be the variation of C/k, and the associated S_0, across loudness and softness. Values of C/k are greater for softness than for loudness; associated S_0s hover around 3 dB for loudness and 105 dB for softness. Bias appears as judgments of loudness and softness get closer to zero. Perhaps it is a zero-point bias of the type referred to by McGill (1960) and later by Irwin and Corballis (1968). On the basis of his data, McGill suggested that, in magnitude estimation, subjects give interval, not ratio, level judgments. As a result, an additive constant is needed to produce a proper zero point. Ward (1971), more recently, has also found support for the idea that magnitude estimation yields interval scales rather than ratio ones. Rule and Curtis (1973) have also argued for the necessity of the additive constant. If this sort of bias is occurring, then it is not surprising that loudness and softness lead to different values. When subjects judge loudness, they miss the zero point in terms of a small sound pressure level, approximately 3 dB on the average; when they judge softness, zero-point correction involves a large sound pressure level of approximately 105 dB since such a value is closer to zero softness. In this interpretation, Equation 3 can be rewritten as $R + kS_0^n = kS^n$ where kS_0^n becomes the *zero-point response bias*. For loudness, S_0 is small and n positive; for softness, S_0 is large and n negative.

Although the values for S_0 are relatively invariant for the loudness and also for the softness experiments cited, it is not clear that they will remain so for other magnitude estimation tasks employing different conditions, e.g., different standards, moduli, or standard/modulus combinations. What causes the zero-point response bias and whether it remains more or less invariant across task conditions are still not clear. Its cause might emerge from a more thorough study of various standard/modulus combinations. Irrespective of what further studies show, the present one suggests that primary and inverse attribute judgments are not simple reciprocals, that subjects do not just take the reciprocals of their primary attribute impressions and report them for their inverse judgments as Rule, Laye, and Curtis (1974) and Schneider and Lane (1964) suggest they might, that the effective threshold and physiological noise modifications of the power law are inadequate, and that magnitude estimation may lead to interval scales requiring additive zero-point corrections.

The full-range softness data, along with the more commonly reported full-range loudness data, must be taken into account in any subsequent discussions of power law modifications as well as for any new models advanced to describe or explain the direct sensory ratio judgments obtained with magnitude estimation. Two other findings in the psychophysical literature also need to be explained in any adequate account.

First, some studies, e.g., Ekman (1959), have found that S_0 needs to be added, not subtracted, from the right-hand side of the psychophysical equation. Similarly, in some studies, e.g., Ekman (1961), the additive constant on the left-hand side has had to be subtracted rather than added. These results are more easily interpreted as response bias than they are as effects of an effective threshold or of a physiological noise, because they imply negative threshold or negative noise values. On the other hand, they may merely reflect differences in experimental procedure. For example, some of Ekman's (1959) results were based on fractionation data, not magnitude estimation data.

Secondly, any comprehensive account should not neglect the data suggesting that direct ratio judgments have not, to date, shown any upper or terminal threshold effect for any so-called "primary" attribute. S. S. Stevens (1955, 1956) employed sound pressure levels of up to 120 dB without finding any nonlinearity in log-log plots of loudness judgments. S. S. Stevens (1975) reports no nonlinearity for even up to 140 dB. Similarly, Eisler (1965) looked for and failed to find evidence for a terminal threshold effect with foot-pedal subjective force.

REFERENCES

Dawson, W. E., & Mirando, M. Inverse scales of opinion obtained by sensory-modality matching. *Perceptual and Motor Skills*, 1976, **42**, 415-425.

Eisler, H. Empirical test of a model relating magnitude and category scales. *Scandinavian Journal of Psychology*, 1962, **3**, 88-96.

Eisler, H. The ceiling of psychophysical power functions. *American Journal of Psychology*, 1965, **78**, 506-509.

Ekman, G. Weber's law and related functions. *Journal of Psychology*, 1959, **47**, 343-352.

Ekman, G. Methodological note on scales of gustatory intensity. *Scandinavian Journal of Psychology*, 1961, **2**, 185-190.

Fagot, R. F. Alternative power laws for ratio scaling. *Psychometrika*, 1966, **31**, 201-214.

Fagot, R. F. A note on the form of the psychophysical function near threshold. *Bulletin of the Psychonomic Society*, 1975, **6**, 665-667.

Fagot, R. F., & Stewart, M. R. An experimental comparison of stimulus and response translated power functions for brightness. *Perception & Psychophysics*, 1968, **3**, 297-305.

Gescheider, G. A. *Psychophysics: Method and theory*. Hillsdale, N.J: Erlbaum, 1976.

Irwin, R. J., & Corballis, M. C. On the general form of Stevens' law for loudness and softness. *Perception & Psychophysics*, 1968, **3**, 137-143.

John, I. D. The properties of distributions of magnitude estimates of loudness and softness. *Scandinavian Journal of Psychology*, 1971, **12**, 261-270.

Krantz, D. H. Threshold theories of signal detection. *Psychological Review*, 1969, **76**, 308-324.

Lochner, J. P. A., & Burger, J. F. Form of the loudness function in the presence of masking noise. *Journal of the Acoustical Society of America*, 1961, **33**, 1705-1707.

Marks, L. E. *Sensory processes: The new psychophysics*. New York: Academic Press, 1974.

Marks, L. E., & Stevens, J. C. The form of the psychophysical function near threshold. *Perception & Psychophysics*, 1968, **4**, 315-318.

Mattiello, M. L. F., & Guirao, M. Direct estimation of lightness of surface colors. *Journal of the Optical Society of America*, 1974, **64**, 206-209.

McGill, W. A. The slope of the loudness function: A puzzle. In H. Gulliksen & S. Messick (Eds.), *Psychological scaling: Theory and applications*. New York: Wiley, 1960.

Robertson, P. Category judgements and magnitude estimations. Unpublished M.A. thesis, University of Auckland, 1966.

Rule, S. J., & Curtis, D. W. Reevaluation of two models for judgments of perceptual intervals. *Perception & Psychophysics*, 1973, **14**, 433-436.

Rule, S. J., Laye, R. C., & Curtis, D. W. Magnitude judgments and difference judgments of lightness and darkness. *Journal of Experimental Psychology*, 1974, **103**, 1108-1114.

Scharf, B., & Stevens, J. C. The form of the loudness function near threshold. In *Proceedings 3rd International Congress of Acoustics*. Amsterdam: Elsevier, 1961. Pp. 80-82.

Schneider, B., & Lane, H. Ratio scales, category scales, and variability in the production of loudness and softness. *Journal of the Acoustical Society of America*, 1963, **35**, 1953-1961.

Schneider, B. A., & Lane, H. L. Note on the variability hypothesis in category scaling. *Journal of the Acoustical Society of America*, 1964, **36**, 1958.

Stevens, S. S. The measurement of loudness. *Journal of the Acoustical Society of America*, 1955, **27**, 815-820.

Stevens, S. S. The direct estimation of sensory magnitudes—loudness. *American Journal of Psychology*, 1956, **69**, 1-25.

Stevens, S. S. Sensation measurement by inverse judgments. *Science*, 1962, **136**, 329.

Stevens, S. S. On predicting exponents for cross-modality matches. *Perception & Psychophysics*, 1969, **6**, 251-256.

Stevens, S. S. Issues in psychophysical measurement. *Psychological Review*, 1971, **78**, 426-450.

Stevens, S. S. Perceived level of noise by Mark VII and decibels (E). *Journal of the Acoustical Society of America*, 1972, **51**, 575-601.

Stevens, S. S. *Psychophysics: Introduction to its perceptual, neural, and social prospects*. New York: Wiley, 1975.

Stevens, S. S., & Guirao, M. Loudness, reciprocality, and partition scales. *Journal of the Acoustical Society of America*, 1962, **34**, 1466-1471.

Stevens, S. S., & Guirao, M. Subjective scaling of length and area and the matching of length to loudness and brightness. *Journal of Experimental Psychology*, 1963, **66**, 177-186.

Stevens, S. S., & Harris, J. R. The scaling of subjective roughness and smoothness. *Journal of Experimental Psychology*, 1962, **64**, 489-494.

Torgerson, W. S. Quantitative judgment scales. In H. Gulliksen & S. Messick (Eds.), *Psychological scaling: Theory and applications*. New York: Wiley, 1960.

Ward, L. M. *Some psychophysical properties of category judgments and magnitude estimations*. Unpublished PhD dissertation, Duke University, 1971.

(Received for publication July 11, 1977;
revision accepted February 17, 1978.)