# Inversion of field data in fault tectonics to obtain the regional stress-III. A new rapid direct inversion method by analytical means 

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#### Abstract

SUMMARY A new method for determining the reduced stress tensor with four degrees of freedom (the orientations of the three principal stress axes as well as the ratio of principal stress differences) using fault slip data (or focal mechanisms of earthquakes) is presented. From a computational point of view, the inversion of fault slip data is made in a direct way by purely analytical means; as a result, the determination process is extremely fast and adaptable on small microcomputers. From a physical point of view, the method aims at simultaneously (i) minimizing the angles between theoretical shear stress and actual slip vector and (ii) having relative magnitudes of shear stress large enough to induce slip despite rock cohesion and friction. Examples of application to actual fault slip data sets with good or poor variety of fault slip orientations are shown. The double significance of the basic criterion adopted results in a more realistic solution of the inverse problem than the single minimization of the shear-stria angle.


Key words: fault, inversion, stress, tectonics.

## 1 BASIC ASSUMPTION

In methodological aspects of fault tectonic analyses, a striking feature of the last 12 years is the development of numerical palaeostress reconstructions using fault slip data. These methods are based on the stress-shear relationship described by Wallace (1951) and by Bott (1959). The inverse problem, which consists of determining the stress tensor knowing the direction and the sense of slip on numerous faults of various orientations, has first been solved by Carey \& Brunier (1974). Various methodological developments and improvements have then been proposed (e.g., Angelier 1975; Carey 1976; Armijo \& Cisternas 1978; Etchecopar, Vasseur \& Daignières 1981; Angelier 1984; Michael 1984; Reches 1987). Under certain conditions, these computational methods apply to populations of focal mechanisms of earthquakes (Angelier 1984; Gephart \& Forsyth 1984). Other methods, based on geometrical instead of mathematical reasoning, bring valuable results but are not considered herein.

No general presentation or discussion of geological and physical conditions of application and limitations of these methods is given hereafter. Such discussions have been made extensively in previous papers (e.g., Angelier 1984, 1989). In contrast, the aim of the present paper is to propose and discuss a method of direct analytical inversion
of data, based on a simple criterion. The use of that criterion enables one to compute the stress tensor within a single sequence of formulae. No 3-D search or iterative process is involved, so that the computation process is very fast.
However, before describing the mathematical analysis of the problem, it is worthwhile to mention again the assumption that underlies all these methods: fault orientations may be arbitrary (as for inherited faults), but each slip (indicated by striae) has the direction and the sense of the shear stress that corresponds to a single common stress tensor. This principle has been proposed by Wallace (1951) and by Bott (1959). The validity and the limits of this basic assumption have been discussed elsewhere (e.g., Angelier 1984).
The actual stress tensor has six degrees of freedom. The data are directions and senses of slip on fault plane whose orientation is known. Neither adding an isotropic stress nor multiplying the tensor by a positive constant can modify the direction and the sense of slip on any fault. As a consequence, the actual tensor being $\mathbf{T}^{*}$, any tensor $\boldsymbol{T}$ equally solves the problem:
$T=t_{1} T^{*}+t_{2} I$
where $t_{1}$ and $t_{2}$ designate any constants ( $t_{1}$ positive) and $I$ the unit matrix. The values of $t_{1}$ and $t_{2}$ can be determined
provided that one adds rock mechanic parameters to data sets and adopts rupture-friction laws. The problem of computing $t_{1}$ and $t_{2}$ in order to determine the actual stress tensor $\boldsymbol{T}^{*}$ is addressed elsewhere (Angelier 1989). The tensor $\boldsymbol{T}$ has four degrees of freedom so that one may adopt a particular form, which is called the 'reduced stress tensor' (Angelier et al. 1982). All tensors $\boldsymbol{T}$ obtained using equation (1) have the same directions of principal stress and the same 'shape ratio' $\boldsymbol{\Phi}$ :
$\Phi=\frac{\sigma_{2}-\sigma_{3}}{\sigma_{1}-\sigma_{3}}$
where $\sigma_{1}, \sigma_{2}, \sigma_{3}$ respectively designate the moduli of maximum compressional stress $\sigma_{1}$, intermediate stress $\sigma_{2}$ and minimum stress $\sigma_{3}$.

The next sections will describe physical and mathematical aspects of a new analytical method to rapidly determine the reduced stress tensor $\mathbf{T}$, i.e. the orientations of axes $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ as well as the value of ratio $\Phi$. Appendices I, II, III and IV contain additional mathematical information.

## 2 SHEAR STRESS-STRIA ANGLE AND DERIVED CRITERIA

For a fault plane number $k$, called $F_{k}$ in Fig. 1, let $\mathbf{n}_{k}$ and $\mathbf{s}_{k}$ be the unit normal to the fault and the unit stria on the fault, respectively (Fig. 1a). T designs the unknown stress tensor, so that the stress vector $\sigma_{k}$ for $F_{k}$ (Fig. 1b) is given by

$$
\begin{equation*}
\boldsymbol{\sigma}_{k}=\mathbf{T} \mathbf{n}_{k} \tag{3}
\end{equation*}
$$

As Fig. 1(b) shows, the normal stress $\sigma_{\mathrm{N} k}$ is the component of $\boldsymbol{\sigma}_{k}$ on $\mathbf{n}_{k}$. The shear stress $\boldsymbol{\tau}_{k}$ is consequently easily obtained:
$\boldsymbol{\sigma}_{\mathrm{N} k}=\left(\boldsymbol{\sigma}_{k} \cdot \mathbf{n}_{k}\right) \mathbf{n}_{k}$,
$\boldsymbol{\sigma}_{k}=\boldsymbol{\sigma}_{\mathrm{N} k}+\boldsymbol{\tau}_{k}$.

Finally, one may simply consider the angle ( $\boldsymbol{\sigma}_{k}, \boldsymbol{\tau}_{k}$ ) between two vectors in the fault plane $F_{k}$ (Fig. 1c): the unit stria $\mathbf{s}_{k}$ indicating direction and sense of actual fault slip (observed), and the computed shear stress $\boldsymbol{\tau}_{k}$ (related to unknown stress tensor $T$ ). Obviously, this angle should be as small as possible for a tensor $\boldsymbol{T}$ consistent with fault slip datum number $k$; the ideal case being
$\left(s_{k}, \boldsymbol{\tau}_{k}\right)=0$.
One must observe that equation (6) lacks significance for a stress vector $\sigma_{k}$ very close to the direction of $\mathbf{n}_{k}$; in this particular case, small variations of the direction of $\sigma_{k}$ may result in large variations of the direction of $\boldsymbol{\tau}_{k}$. One must also note that by definition equation (6) does not take into account the shear stress amplitude necessary to induce motion on fault plane. We shall come back later to these problems.
Adopting equation (6) as a criterion model, with the basic assumptions of Section 1 and the additional implicit assumptions of the least-squares method, one obtains the best fitting stress tensor for a given fault slip data set by minimizing function $S_{1}$, where $\mathscr{H}$ is the number of faults:
$S_{1}=\sum_{k=1}^{k=\mathscr{H}}\left(\mathbf{s}_{k}, \boldsymbol{\tau}_{k}\right)^{2}$.
Because the sine of the half angle continuously increases from 0 to 1 as the angle increases from 0 to $\pi$, similar results are obtained by minimizing function $S_{2}$ :
$S_{2}=\sum_{k=1}^{k=\mathscr{H}} \sin ^{2} \frac{\left(\mathbf{s}_{k}, \boldsymbol{\tau}_{k}\right)}{2}$.
Equation (8) may be written in a different form:
$S_{2}=\frac{1}{4} \sum_{k=1}^{k=\mathscr{H}} v_{k}^{n 2}$
where $v_{k}^{\prime \prime 2}$ designates the modulus of a vector $\boldsymbol{v}_{k}^{\prime \prime}$ shown in Fig. 2(a) and defined by the following equation including


Figure 1. Fault slip datum and computed siress. Index $K$ designates datum number as in text. (a) Observed fault slip: $\mathbf{F}$, fault plane; $\mathbf{n}$, normal fault (unit vector); $s$, unit slip vector (parallel to striae). (b) Components of computed stress: $\boldsymbol{\sigma}$, applied stress; $\boldsymbol{\sigma}_{\mathrm{N}}$, normal stress; $\boldsymbol{\tau}$, shear stress. (c) Shear-stria angle: $\mathbf{s}$, observed slip; $\tau$, computed shear.


Figure 2. Definition of function upsilon. Index $k$ designates datum number as in text. $\mathbf{F}$, fault plane; $\mathbf{s}$, unit slip vector; and $\boldsymbol{\tau}$, shear stress (as in Fig. 1). (a) Definition of $v^{\prime \prime}$ (equations 9 and 10 in text); $\tau_{U}$, unit vector along shear stress. (b) Definition of $v^{\prime}$ (equation 11 in text). (c) Criterion adopted: definition of $v$ (equations 12 and 13 in text).
$\boldsymbol{\tau}_{\mathrm{Uk} k}$, the unit vector along $\boldsymbol{\tau}_{\boldsymbol{k}}$ :
$\mathbf{s}_{k}=\boldsymbol{\tau}_{\mathrm{Uk}}+\boldsymbol{v}_{\boldsymbol{k}}^{\prime \prime}$.

## 3 A NEW CRITERION AND ITS MECHANICAL SIGNIFICANCE

Using minimization criterion given by equation (8) equivalent with $(9-10)$, one finally obtains the best fitting average stress tensor $\boldsymbol{T}$ that corresponds to a given fault slip data set (see details in Angelier 1984). One just has to write $v_{k}^{\prime 2}$ as a function of the unknown components of the stress tensor, with the coordinates of $n_{k}$ and $s_{k}$ as parameters of the problem. Then, one searches by numerical means the stress tensor $\boldsymbol{T}$ that corresponds to the minimum value of $S_{2}$.

However, it is easy to observe that such formulae cannot lead to equations made with simple linear arrangement of polynomials. For example, the use of equation (10), which contains the unit vector parallel to shear stress $\tau$, demands division by a polynomial (the shear stress modulus). As a result, the search of the smallest sum $S_{2}$ requires the use of iterative processes and imbricated program loops, so that in practice additional numerical problems must be solved and computation time cannot be very short.

Let us examine the effect of substituting the vector $\boldsymbol{v}_{k}^{\prime}$ shown in Fig. 2(b) to the vector $v_{k}^{\prime \prime}$ already defined (10) and shown in Fig. 2(a). The new vector $v_{k}^{\prime}$ is defined by an equation that contains the shear stress itself (instead of a unit vector along the shear stress):
$\mathbf{s}_{k}=\boldsymbol{\tau}_{k}+\boldsymbol{v}_{k}^{\prime}$.
From a computational point of view, the use of $\boldsymbol{v}_{k}^{\prime}$ instead of $v_{k}^{\prime \prime}$ in equation (9) enables one to write expressions which are simple sums of polynomials. As a consequence, obtaining solutions of the problem by direct analytical means becomes possible, as will be shown in the next section.

From a geometrical and mechanical point of view, this substitution has large consequences. Note first that whatever the shear stress modulus, the smallest value of $v_{k}^{\prime}$ corresponds to $\left(s_{k}, \boldsymbol{\tau}_{k}\right)=0$, whereas the largest one corresponds to $\left(s_{k}, \boldsymbol{\tau}_{k}\right)=\pi$. In addition, for a constant shear stress modulus, $v_{k}^{\prime}$ continuously increases with ( $\mathbf{s}_{k}, \boldsymbol{\tau}_{k}$ ), as Fig. 2(b) suggests. However, the value of $v_{k}^{\prime}$ also depends on the amplitude of shear stress $\boldsymbol{\tau}_{\boldsymbol{k}}$ : for constant moderate angles ( $\mathrm{s}_{k}, \boldsymbol{\tau}_{k}$ ), the modulus $\boldsymbol{v}_{k}^{\prime}$ decreases as the modulus $\boldsymbol{\tau}_{k}$ increases until a certain limit where $\boldsymbol{v}_{k}^{\prime}$ and $\boldsymbol{\tau}_{k}$ are perpendicular (Fig. 2b). To that respect, substituting $v_{k}^{\prime}$ to $v_{k}^{\prime \prime}$ in equation (9) introduces a more complex criterion that the single minimization of the shear stress-stria angle discussed before.

In detail, the particular case of a stress vector $\sigma_{k}$ almost parallel to $\boldsymbol{n}_{k}$ illustrates an advantage of the new criterion. The angle ( $\mathrm{s}_{k}, \boldsymbol{\tau}_{k}$ ) widely varies and has no real significance in this case (as pointed out before); in constrast, the modulus $v_{k}^{\prime}$ is stable and close to 1 . Although no accurate rupture/friction law is introduced (see Section 1), the fact that the value considered decreases as the shear stress modulus increases corresponds to a quite reasonable requirement: having shear stress levels large enough to induce fault slip despite rock cohesion and friction.

Before examining these variations in more detail, let us
define the final form of the function adopted. Let $\lambda$ be the largest possible value of shear stress with the particular form of $\boldsymbol{T}$ adopted: this case cotresponds to particular orientations of $\mathbf{n}_{k}$ and $\mathbf{s}_{k}$ relative to stress axes. Because the modulus $\tau_{k}$ of $\boldsymbol{\tau}_{k}$ remains between 0 and $\lambda$ while the angle ( $s_{k}, \tau_{k}$ ) may vary from 0 to $\pi$, the modulus $v_{k}$ of the vector $\boldsymbol{v}_{k}$ (upsilon) defined as follows has bounds of 0 and $2 \lambda$ (Fig. 2c):
$\lambda s_{k}=\boldsymbol{\tau}_{k}+\boldsymbol{v}_{k}$.
Finally, the sum $S_{4}$ to minimize is
$S_{4}=\sum_{k=1}^{k=\mathscr{H}} v_{k}^{2}$.

## 4 THE DIRECT INVERSION

In this section, it is shown that the use of the criterion described in equations (12) and (13) leads to development of a fast analytical procedure. Because derived equations are linear, it is possible to find in a direct way the extremum values of function $S_{\mathbf{4}}$ by annuling its four partial derivatives with respect to the unknowns of the reduced stress tensor (see Section 1). The reduced stress tensor $\boldsymbol{T}$ adopted has a particular deviatoric form discussed by Angelier et al. (1982).

$$
\left(\begin{array}{lll}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{array}\right)
$$

$$
=\left(\begin{array}{ccc}
\cos \psi & \alpha & \gamma  \tag{14}\\
\alpha & \cos \left(\psi+\frac{2 \pi}{3}\right) & \beta \\
\gamma & \beta & \cos \left(\psi+\frac{4 \pi}{3}\right)
\end{array}\right)
$$

Coming back to equation (12) that defines the vector upsilon and to Fig. 2(c), it is easy to conclude that the modulus $v_{k}$ of $v_{k}$ is given by
$v_{k}^{2}=\lambda^{2}+\tau_{k}^{2}-2 \lambda \tau_{k} \cos \left(\mathbf{s}_{k}, \tau_{k}\right)$.
The components of $\sigma_{k}$ and $\tau_{k}$ along the direction of unit vector $s_{k}$ are identical, so that
$\tau_{k} \cos \left(s_{k}, \tau_{k}\right)=s_{k} \cdot \sigma_{k}$.
Combining equations (15) and (16), one obtains $v_{k}^{2}$ as a function of $\tau_{k}^{2}, \lambda$ and scalar product $s_{k} \cdot \sigma_{k}$. This is shown as equation (A1) of Appendix I. Some transformations, which are long to write but not difficult to obtain, are then made in order to define $S_{4}$ as a function of $\alpha, \beta, \gamma, \psi$ [the four unknowns of $T$ in equation (14)] and of the $\mathscr{H}$ sets of coordinates of vectors $n_{k}$ and $s_{k}$. The main steps in these transformations are given in Appendix I.

One finally obtains the long but linear expression of $v_{k}^{2}$ given in equations (A9), (A6) and (A8) of Appendix I, and the corresponding derivatives with respect to $\alpha, \beta, \gamma$ and $\psi$. Annuling these derivatives given in equation (A10) of Appendix I results in the following system of four equations,
with $\alpha, \beta, \gamma$ and $\psi$ as unknowns:
$A \alpha+D \gamma+E \beta=G \cos \psi+J \sin \psi+U$,
$D \alpha+B \gamma+F \beta=H \cos \psi+K \sin \psi+V$,
$E \alpha+F \gamma+C \beta=I \cos \psi+L \sin \psi+W$,
$M \cos 2 \psi+\frac{1}{2} N \sin 2 \psi=(G \alpha+H \gamma+I \beta+Q) \sin \psi$

$$
-(J \alpha+K \gamma+L \beta+P) \cos \psi
$$

where $A, B, \ldots, V, W$, respectively designate the sums of polynomials referred to as $a, b, \ldots, v, w$, in equations (A6) and (A8) of Appendix I; these sums are defined from fault number 1 to fault number $\mathscr{H}$. The system of equations (17) is solved by analytical means, as shown in Appendix II, and provides two sets of values corresponding to extremum values of the sum $S_{4}$ of functions $v_{k}^{2}$ (maximum and minimum, respectively). One thus obtains the values of $\alpha$, $\beta, \gamma$ and $\psi$ that correspond to the smallest sum $S_{4}$ defined in equations (12-13), hence the reduced stress tensor $T$ solution of the problem, under the form of equation (14).

It is necessary to assign a value to the parameter $\lambda$ defined in equation (12) and used in the basic equation (15). The largest possible value for shear stress is the half difference between the maximum and minimum principal stresses, that is, $\sqrt{3} / 2$ with the tensor described in (A16) of Appendix III. With the tensor previously described in (14), this largest value depends on $\alpha, \beta, \gamma$ through a scale factor. Because the method aims at simultaneously minimizing shear-slip angle and having shear stress sufficient to induce slip, a reasonable value of $\lambda$ (Fig. 2c) should be chosen as close as possible to this largest possible value of shear stress, that is $\sqrt{3} / 2$ multiplied by the scale factor. This adjustment is made through few successive tensor determinations ( $\lambda$ in each step equals the largest shear of the previous step). Appendix IV shows why the inverse problem, as formulated, has only four unknowns in agreement with the requirement of equation (1), and why the adjustment of the parameter $\lambda$ remains indispensable due to the particular form of the stress tensor adopted (rotation of axes and magnitude of stress are not analytically independent; otherwise, $\lambda$ would be a simple constant and no adjustment would be required).

Eigenvalues and eigenvectors of $\boldsymbol{T}$ are easily computed (e.g., Angelier et al. 1982, pp. 620-621), thus giving the orientations of the three principal stress axes $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ as well as the shape ratio $\Phi$ of the stress ellipsoid; $\Phi$ has been defined in equation (2).

Because stress amplitudes vary through changes in the orientation of principal axes with the stress tensor described in equation (14), artificial permutations between principal stresses may occur for numerical reasons when fault slip data display a poor variety of orientations. A fast analytical procedure to determine again the value of $\psi$ and thus definitely identify the actual stress axes is added. This additional step is described in Appendix III (see also discussion in Appendix IV).

## 5 EXAMPLES WITH ACTUAL DATA

Four examples are shown in Figs 3 and 4, with increasing complexity of fault slip distributions. Although the method has been applied to large data sets (up to several hundreds


Figure 3. Examples of data sets with approximately symmetrical distributions of fault slips. Upper row (a): site AVB, Agia Varvara, Crete, Greece. Lower row (b): site TYM, Tymbaki, Crete, Greece. Schmidt's projection of lower hemisphere (N, geographic north; M, magnetic north). Fault planes shown as thin lines, with slickenside lineations shown as dots with arrows indicating the sense of motion (outward directed = normal; complexity in design of arrow heads increases with the degree of certainty on fault sense). Computed stress axes shown as stars with five branches $\left(\sigma_{1}\right)$, four branches $\left(\sigma_{2}\right)$ and three branches $\left(\sigma_{3}\right)$. See also Table 1 and Tables A1-A2 of Appendix V.
of fault slips), these examples correspond to small sets (few tens of fault slips), so that data distribution is easily observable in stereodiagrams. To make comparisons easier, all these examples refer to a single tectonic type, that is extensional tectonics with predominating normal dip-slip and oblique-slip faults; obviously, other types (compressional tectonics, strike-slip tectonics or even tilted fault patterns) might have been chosen for illustration as well. In addition, three of these data sets had already been adopted as examples in descriptions of previous methods. The reader may consequently compare, if necessary, the results independently obtained [Angelier (1979) for the data set of Fig. 3(b); Angelier et al. 1982, for the data sets of Figs 3(a) and Fig. 4(a)]. The results obtained with the method under investigation are summarized in Table 1; a more detailed account of numerical data and results is given in Tables A1 and A 2 of Appendix V .

The simplest case is illustrated by a data set from Neogene reefal limestones near Agia Vavara, central Crete, Greece (Fig. 3a). The diagram of measured fault slips illustrates a typical conjugate distribution with a low level of dispersion in both subsets of normal faults and slickenside lineations. The attitudes of the three principal stress axes are consequently expected, taking into account obvious symmetries in this distribution of faults and slips (Huang \& Angelier 1989). The stress axes determined after direct inversion process are consistent with this geometrical interpretation (Table 1 and Fig. 3a). Note, however, that the stress ratio $\Phi$ [defined in equation (2)] is poorly controlled in the absence of oblique faults and slips. The determination of the value 0.47 given in Table 1 principally results from a limited data dispersion which may have little


Figure 4. Examples of data sets with asymmetrical distributions of fault slips. Upper row (a): site MD1, Arroyo Montado, Baja California, Mexico. Lower row (b): site KAM, Kamogawa, Boso Peninsula, Central Japan. Symbols as in Fig. 3. In addition, the third diagram of each row (on right) shows the theoretical distribution of fault shears according to the computed average stress tensor. See also Table 1 and Tables A1-A2 of Appendix V.

Table 1. Results of stress tensor determinations with the new direct inversion method. Angles in degrees (trends and plunges of stress axes $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ ). Ratio $\Phi$ defined in equation (2) of text. Estimator RUP defined in text, Section 5 . $n_{1}$, number of data with RUP $>75$ per cent; $n_{2}$, number of data with 75 per cent $\geq$ RUP $>50$ per cent (see Fig. 7a). See also Tables A1-A2 of Appendix V.

| Site | Axis $\sigma_{1}$ trend | plunge | Axis $\sigma_{2}$ trend | plunge | Axis $\sigma_{3}$ trend | plunge | Ratio <br> $\phi$ | Number of data | Average RUP | $n_{1}$ | $n_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AVB (Fig. 3a) | 76 | 72 | 240 | 17 | 331 | 5 | 0.47 | 33 | 30 per cent | 1 | 4 |
| TYM (Fig. 3b) | 232 | 80 | 55 | 10 | 325 | 0 | 0.23 | 38 | 35 per cent | 1 | 4 |
| MD1 (Fig. 4a) | 17 | 78 | 191 | 12 | 281 | 1 | 0.52 | 20 | 44 per cent | 2 | 5 |
| KAM (Fig. 4b) | 218 | 83 | 119 | 1 | 29 | 7 | 0.31 | 50 | 43 per cent | 5 | 11 |

significance (note that it is effectively impossible to significantly determine the ratio $\Phi$ with a perfect conjugate fault system). At which degree the shape of data dispersion is controlled by the ratio $\Phi$ remains undetermined in the absence of fault slips making large angles with the two conjugate subsets.

An important estimator of data dispersion is the average 'ratio upsilon', or RUP (see Table 1). This estimator RUP is the value of the function upsilon [defined in equation (12) and illustrated in Fig. 2(c)], divided by the largest possible value of shear stress [ $\sqrt{3} / 2$ as in Fig. 2(c), with the reduced stress tensor described in equation (A16) of Appendix III]; this value is given as a percentage for convenience. Possible values of estimator RUP range from 0 per cent (shear stress maximum parallel to slip with the same sense) to 200 per cent (shear stress maximum parallel to slip with opposite sense). Average RUP values below 50 per cent, as in Table 1, generally correspond to good fits between actual fault slip data distribution and computed shear stress distribution, with the criterion of equation (13).

The fault pattern described in Fig. 3(b) is also composed of two major sets of normal faults associated in a kind of conjugate system. However, the dispersion of fault planes as well as of slickenside lineations is much larger than in the first case; in addition, some oblique faults are present. This
second data set has been collected in Neogene marly limestones near Tymbaki, southern Crete, Greece. Because many of these faults belong to a conjugate system, the orientation of axes may still be expected taking into account symmetries in fault slip distribution. The computed stress axes effectively correspond to rough symmetry axes. Because fault slip dispersion occurs and oblique faults are present, the value of $\Phi(0.23$, see Table 1$)$ is certainly more significant than in the previous case. It is important to observe that despite of the larger number of data and the larger apparent dispersion of fault slips, the value of the average estimator RUP remains small ( 35 per cent, see Table 1).

Adopting individual RUP values of 50 and 75 per cent as reasonable arbitrary limits to identify fault slip data that fit the average solution very well (RUP $\leq 50$ per cent) or in acceptable conditions taking into account multiple dispersion factors (RUP $\leq 75$ per cent), one obtains two numbers of data (see Table 1). The number $n_{1}$ corresponds to fault slips that are not in agreement with the average solution (RUP $>75$ per cent). The number $n_{2}$ corresponds to fault slips that may be considered or not consistent with the average solution, depending on the range of allowable dispersion ( 50 per cent $<$ RUP $\leq 75$ per cent). The examples shown in Fig. 3 display low values of $n_{1}$ and $n_{2}$, indicating a
good homogeneity (Table 1): in each case, only five faults slips have individual RUP values greater than 50 per cent, showing that $85-87$ per cent of fault slip data fit very well with a single common stress tensor.

The examples shown in Fig. 3 are simple and display rough symmetries, so that the results can be intuitively checked (with the exception of the ratio $\Phi$ ). On the contrary, Fig. 4 illustrates irregular fault distributions with high levels of apparent data dispersion and a lack of symmetry elements.

The data set of Fig. 4(a) has been collected in Pliocene sediments of the Arroyo Montado, Santa Rosalia Basin, Baja California, Mexico. Although there is a dominating NNW-SSE trend of the fault system, the distribution of slickenside lineations is characterized by obliquity and asymmetry. Despite of this geometrical dispersion, the data set fits rather well with a common stress tensor, with an average estimator RUP of 44 per cent (see Table 1) and seven fault slips with individual estimators RUP larger than 50 per cent. The computed direction of extension is WNW-ESE, oblique to normal fault trends (Fig. 4a). The ratio $\Phi$ of 0.52 means that the magnitude of the intermediate stress is close to the arithmetic mean of extreme stress magnitudes (see equation 2).

The last example, shown in Fig. 4(b), refers to normal dip-slip and oblique slip normal faults observed in the 'Mineoka ophiolite' of Kamogawa, Boso Peninsula, Central Japan. Because there were numerous discontinuities in the rock mass prior to the extensional event under investigation, neoformed faults play a minor role and inherited faults are common. As a result, the distribution of fault planes and slickenside lineations is irregular. The average estimator RUP, however, remains small (43 per cent, see Table 1), despite of the increasing number of data. Individual values of RUP are greater than 50 per cent for 16 faults, but only five fault slips are definitely inconsistent with the average solution, with the assumption made (RUP $>75$ per cent).

## 6 COMPARISON WITH OLDER METHODS

The diagrams of the right side of Fig. 4 show the theoretical orientations of slip vectors according to the stress tensors that have been determined with the new method (Table 1; axes shown in central diagrams of Fig. 4). In these diagrams, the symbols of slickenside lineations indicate the orientations and senses of computed shear stresses on the fault planes. As a result, a simple comparison with the diagrams on the left side of Fig. 4 shows the individual angular deviations from actual slips.

The reduced stress tensors have also been computed for the same data sets (Figs 3 and 4) with a 4-D exploration
method described in previous papers (Angelier 1975, 1984). The basic criterion adopted in this older method requires the minimization of a simple function of the angle between the computed shear stress and the actual slip vector. Determinations made using two different functions referred to as $S_{2}$ and $S_{3}$ (Angelier 1984, p. 5841) yielded very similar results; Table 2 refers to results obtained with function $S_{3}$. A rapid comparison between the reconstructed stress axes (Tables 1 and 2) shows that for each of the four sites, there is no significant difference between the results obtained with the new direct inversion method and with the 4-D exploration method (as a consequence, the stress axes obtained with the oldest method, listed in Table 2, were not plotted in Figs 3 and 4). In addition, the values of the ratio $\Phi$ are similar, provided that the fault slip distribution allows reliable determination of $\Phi$ as in Fig. 4.
The estimator of data dispersion for the 4-D exploration method, called ANG and given in degrees (Table 2), is the shear-slip angle itself (Fig. 1c). The averages values of ANG obtained with the four examples described range from $7^{\circ}$ to $19^{\circ}$, which is fairly acceptable taking into account all sources of data dispersion. Assuming reasonable limits of $22.5^{\circ}$ and $45^{\circ}$ for individual ANG values enables one to rapidly identify fault slips that are not in agreement with the average solution (ANG $>45^{\circ}$ ) and fault slips that may be consistent or not with this average solution depending on the range of uncertainties $\left(22.5^{\circ}<\mathrm{ANG} \leq 45^{\circ}\right)$. The numbers $n_{1}$ and $n_{2}$ displayed in Table 2 refer to these two subsets, respectively. Note that these numbers remain generally small (especially $n_{1}$ ), suggesting that the data sets have a good level of homogeneity (as for the new direct inversion method: compare Tables 1 and 2). The determination of the uncertainties of the results is extremely important, but will not be discussed in this paper (see Angelier et al. 1982). The problem of quantifying the dispersions of results whatever tensor determination technique is used will be addressed in a forthcoming paper (work in progress).

The individual values of the estimator ANG may be directly read in the diagrams of Fig. 4, where the attitudes of actual slip (on left) and of theoretical shear (on right) are shown for each fault. Because the fault planes orientations are considered constant, the difference between the actual distribution and the theoretical one lies on the slip-shear angles solely (Fig. 1c). A more sophisticated method, taking into account uncertainties on fault strikes and fault dips as well as on pitches of slip vectors (called $d, p$ and $i$ respectively) has been described (Angelier et al. 1982). Computation, however, was much longer and required much larger memory so that this heavier method could not be applied on microcomputers.

Table 2. Results of stress tensor determinations with the 4-D exploration method, to be compared with results of Table 1. Legends as in Table 1, except for estimator ANG (angle shear-stria, in degrees; see text, Section 6). $n_{1}$, number of data with ANG $>45^{\circ} ; n_{2}$, number of data with $45^{\circ} \geq$ ANG $>22.5^{\circ}$ (see Fig. 7 b ).

| Site | Axis $\sigma_{1}$ trend | plunge | Axis $\sigma_{2}$ trend | plunge | Axis $\sigma_{3}$ trend | plunge | Ratio <br> $\phi$ | Number of data | Average ANG | $n_{1}$ | $n_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AVB (Fig. 3a) | 73 | 73 | 240 | 17 | 331 | 4 | 0.30 | 33 | $7{ }^{\circ}$ | 0 | 0 |
| TYM (Fig. 3b) | 275 | 82 | 57 | 6 | 147 | 5 | 0.07 | 38 | $11^{\circ}$ | 0 | 5 |
| MD1 (Fig. 4a) | 19 | 72 | 194 | 17 | 285 | 1 | 0.57 | 20 | $19^{\circ}$ | 2 | 6 |
| KAM (Fig. 4b) | 246 | 84 | 118 | 4 | 28 | 5 | 0.32 | 50 | $19^{\circ}$ | 5 | 15 |

A first attempt at solving the inverse problem by pure analytical means has already been described as the first direct inversion method (Angelier 1979). However, because the basis of this earlier method was the minimization of the component of computed shear stress perpendicular to actual slip, the sense of slip had no influence on the minimization process. As a result, this method gives good results provided that the data sets display fault slips in opposite directions and at significant angles from extreme stress axes, but computed axes may be inconsistent with several senses of fault motions in particular cases (especially with major fault orientations absent or inherited fault surfaces close to extreme stress axes). This first direct inversion method was consequently abandoned.

## 7 DISCUSSION

There are two major differences between the new direct inversion method described in this paper (Figs 3 and 4 and Table 1) and the 4-D exploration method (Angelier 1975, 1984 and Table 2). The first difference lies in the mathematical aspects of the technique: whereas the older method consists of comparing numerous tensors in order to find the minimum value of a sum of functions, the mathematical basis of the new method is the analytical resolution of equations that set to zero the partial derivatives of the sum of functions considered ( $S_{4}$, see equation 13). These techniques result in a system of four imbricated computation loops in the first case and in a system of four equations in the second case, because there are four independent unknowns (the orientations of the three principal axes and the ratio $\boldsymbol{\Phi}$ ). The sum of functions to minimize has a complicated shape in four dimensions, so that in the case of the exploration method the search grids must be wide with large numbers of points in order to find the minimum in a reliable way; this requirement results in acceptable but long computation times. On the contrary, the system of four equations of the direct inversion method is solved instantaneously, because the function adopted allows analytical resolution (see Appendix II).

In both cases, the sense of each fault slip is taken into account within the determination process. Also, both methods may be used in heavier processes to determine multiple stress tensors and simultaneously identify corresponding fault slip subsets within heterogeneous fault slip data sets (Angelier 1984, p. 5845). However, the long computation time of the exploration method is multiplied by a large factor, whereas the separation process remains very fast with the new direct inversion method. Note first that it is possible to transfer the criterion described in equations ( $12-13$ ) of this paper in the exploration method (although this would have very little interest). On the contrary, it is impossible to carry out direct inversion by analytical means with a criterion depending on the shear-slip angle solely as in sums $S_{1}, S_{2}$ (equation 7 and 8 , this paper) or $S_{3}$ (Angelier 1984, p. 5841), because the system of equations is complicated and non-linear.

Although the importance of practical aspects such as computation time should not be underestimated, the most important difference between the two methods lies in the physical significance of the criterion adopted. The criterion used in the new direct inversion method (equations 12-13
and Fig. 2c) does not depend on the single shear-slip angle but is also a function of the shear stress amplitude, as equation (15) shows. With the value $\sqrt{3} / 2$ assigned to the parameter $\lambda$ (as discussed in Section 4, taking into account the largest possible shear stress with the type of tensor considered), the variations of $v^{2}$ and $v$ as functions of shear-slip angle and shear stress magnitude have been plotted in Fig. 5. The contour lines illustrate the shape of the function upsilon, or $v$. First, $v$ continuously increases with the shear-slip angle for any given shear stress magnitude (Fig. 5b); however, the variation becomes much tighter as shear stress decreases (besides, $v$ is constant when


Figure 5. Variation of $v^{2}$ (a) and $v$ (b) as functions of the shear-stria angle and of the shear stress magnitude. Contour lines in percentage of maximum value ( 3 and $\sqrt{3}$ respectively). Minimum as star, maximum as cross. Area with values smaller than 5 per cent of the maximum shaded. Abscissae: angle between actual slip and computed shear, in degrees. Ordinates: amplitude of shear stress $\tau$, in percentage of the largest possible shear stress ( $\sqrt{3} / 2$ with the reduced tensor adopted, see equation 14).
shear stress equals zero). Second, $v$ continuously decreases with shear stress for small shear-slip angles and continuously increases with shear stress for large ones; as a result, the minimum value (zero) of the function corresponds to smallest angle and largest shear, whereas the maximum value $(\sqrt{3})$ corresponds to largest angle and largest shear (Fig. 5b).

Summarizing, the use of the criterion adopted in this paper aims at simultaneously satisfying two major requirements for the whole data set: obtaining angles between computed shear stress and actual slip vector as small as possible, and having relative shear stress magnitudes as large as possible in order to overcome cohesion and friction on fault planes. These requirements enable one to determine an average stress tensor which, in turn, provides an explanation for fault activation as well as for directions and senses of fault motion. Whether or not this explanation is acceptable depends on the values of estimators such as the average ratio RUP (Table 1). These estimators enable one not only to determine the homogeneity of the fault slip data set ( $n_{1}$ and $n_{2}$ in Table 1), but also to check the levels of average angular deviation and of average shear stress magnitude.

The significance of the angular deviation (shear-slip angle, ANG; see Fig. 1c) is simple and directly expressed by the contrast between actual and synthetic fault slip diagrams in Fig. 4. The shear stress magnitude has also essential significance, because it plays a major role in fault activation. However, the shear stress considered herein and referred to as $\tau$ in equation (15) and Fig. 5 is not the real shear stress but the 'reduced shear stress' induced by the reduced stress tensor described in equation (14). Because the reduced stress tensor is a linear function of the actual tensor, the related shear stress on a given plane equals the actual one multiplied by the undetermined positive constant $t_{1}$ of equation (1). As a consequence, the use of the basic criterion of the method implies that the relative magnitude of shear stress (proportional to the absolute one) must be large to induce slip despite of friction on faults. Simultaneously, slip must occur parallel to and in the same sense as shear stress. This duality of the basic criterion is illustrated by Fig. 5. However, there is no assumption made about any precise value of actual minimum shear stress required for sliding to occur, or about any definite relationship between shear stress and normal stress (see Appendix IV). In other words, the method adopted is not based on the use of particular laws or constants taken from rock mechanics studies, but simply ensures that the average stress tensor solution of the problem cannot induce anomalously low levels of relative shear stress taking into account the distribution of fault orientations. How the values of unknowns $t_{1}$ and $t_{2}$ of equation (1) can be determined using accurate rupture and friction laws after determination of the reduced stress tensor is discussed elsewhere (Angelier 1989).

To illustrate in a simple way this major difference between the criterion based on the single shear-slip angle and the criterion described above, one may consider the case of a simple synthetic fault slip distribution as in Fig. 6. The single minimization of the shear-slip angle leads one to determine a set of solutions with an unique intermediate stress axis and two extreme stress axes which remain


Figure 6. Synthetic data set showing the difference between the method minimizing the shear-slip angle solely (a) and the new direct inversion method (b). Black arrow or black star: computed $\sigma_{1}$ axis. Open arrow or open star: computed $\sigma_{3}$ axis. Schmidt's projection.
undefined within a certain angle. On the contrary, the use of the new criterion allows determination of an unique solution, because there is only one way to obtain average shear stress as large as possible. This important difference between the two methods is also illustrated by the special case of a stress vector $\boldsymbol{\sigma}$ (Fig. 1b) almost perpendicular to fault plane: because a small variation in its orientation may result in large changes in shear stress orientation (hence in the value of ANG), the new model is more valid (the value of RUP is stable and close to zero).

Finally, the relationship between the estimators ANG (for the method minimizing the shear stress angle solely, see Table 2) and RUP (for the new direct inversion method, see Table 1) is summarized in Fig. 7. Especially, the rough equivalences between the limits adopted for ANG and RUP in order to detect inhomogeneities in data sets with both methods (see $n_{1}$ and $n_{2}$ in Tables 1 and 2) are thus explained. In practice, numerous fault slips generally display approximate proportionality between individual estimators ANG and RUP, so that even with the new method the angles between actual slips and theoretical shear stresses (Fig. 4) provide a consistent picture of data dispersion in first approximation (although they cannot accurately reflect the individual deviations in terms of the new criterion adopted in this paper). New methods for determining


Figure 7. Approximate relationship between shear-slip angle in fault plane [(a): individual estimator ANG as in Table 2] and ratio upsilon [(b): individual estimator RUP as in Table 1]. Actual slip vector: open arrow [length $\sqrt{3} / 2$ as in Fig. 2(c)]. Computed shear stress: black arrow (b) or thick line (a). Note that shear stress magnitude has importance for (b), not for (a). Shaded area: possible locations of shear stress with the constraint shown below. Values larger than $90^{\circ}$ (ANG) or 100 per cent (RUP) not illustrated.
palaeostress axes with neoformed conjugate fault slip subsets (Huang \& Angelier 1989) or sets of mixed neoformed and inherited fault slips (this paper) provide fast and reliable reconstructions of the palaeostress tensor. The new direct inversion method is not only characterized by simplicity and easy computer use but also by the introduction of the reasonable requirement that relative shear stress should be large for slip to occur on fault surfaces. This method is also applicable to the reconstruction of present stress tensors using sets of focal mechanisms of earthquakes, provided that the actual fault plane may be identified among the two nodal planes.

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## APPENDIX I

In this appendix as well as in Appendix III, all indices $k$ that refer in text to fault number are omitted, for clarity. For each fault mechanism, the function defined by equations (15-16) in text is
$v^{2}=\lambda^{2}+\tau^{2}-2 \lambda(s \cdot \sigma)$.
Equations (3) and (14) in text define the components $\sigma_{x}, \sigma_{y}$ and $\sigma_{z}$ of $\sigma$ as
$\sigma_{x}=n_{x} \cos \psi+n_{y} \alpha+n_{z} \gamma, \quad \sigma_{y}=n_{x} \alpha+n_{y} \cos \left(\psi+\frac{2 \pi}{3}\right)+n_{z} \beta, \quad \sigma_{z}=n_{x} \gamma+n_{y} \beta+n_{z} \cos \left(\psi+\frac{4 \pi}{3}\right)$,
where $n_{x}, n_{y}$ and $n_{z}$ designate the direction cosines of the unit vector $n ; s_{x}, s_{y}$ and $s_{z}$ designate the direction cosines of the unit vector $\mathbf{s}$ (perpendicular to $\mathbf{n}$ ). The terms $\boldsymbol{r}^{2}$ and $\mathbf{s} \cdot \boldsymbol{\sigma}$ in the right side of (A1) are given by
$\tau^{2}=\sigma_{x}^{2}+\sigma_{y}^{2}+\sigma_{z}^{2}-\left(n_{x} \sigma_{x}+n_{y} \sigma_{y}+n_{z} \sigma_{z}\right)^{2}$,
$\mathbf{s} \cdot \boldsymbol{\sigma}=s_{x} \sigma_{x}+s_{y} \sigma_{y}+s_{z} \sigma_{z}$.
Combining equations (A2) and (A3) for $\tau^{2}$ and (A2) and (A4) for (s $\cdot \sigma$ ) gives the terms in the right side of (A1) as functions of $\lambda, n_{x}, n_{y}, n_{z}, \alpha, \beta, \gamma$ and $\psi$. First, after transferring expressions (A2) of $\sigma_{x}, \sigma_{y}, \sigma_{z}$ into equation (A3), one obtains

$$
\begin{equation*}
\tau^{2}=n \cos ^{2} \psi-2 m \sin \psi \cos \psi-2(g \alpha+h \gamma+i \beta) \cos \psi-2(j \alpha+k \gamma+l \beta) \sin \psi+a \alpha^{2}+b \gamma^{2}+c \beta^{2}+2(d \gamma \alpha+e \alpha \beta+f \beta \gamma), \tag{A5}
\end{equation*}
$$

$+\frac{3}{4} n_{x}^{2}\left(n_{y}^{2}+n_{z}^{2}\right)+3 n_{y}^{2} n_{z}^{2}$,
where $a, b, c, d, e, f, g, h, i, j, k, l, m$ and $n$ are simple polynomials that contain $n_{x}, n_{y}$ and $n_{z}$ as follows:

$$
\begin{align*}
& a=n_{x}^{2}+n_{y}^{2}-4 n_{x}^{2} n_{y}^{2}, \quad b=n_{x}^{2}+n_{z}^{2}-4 n_{x}^{2} n_{z}^{2}, \quad c=n_{y}^{2}+n_{z}^{2}-4 n_{y}^{2} n_{z}^{2}, \quad d=\left(1-4 n_{x}^{2}\right) n_{y} n_{z}, \quad e=\left(1-4 n_{y}^{2}\right) n_{z} n_{x}, \\
& f=\left(1-4 n_{z}^{2}\right) n_{x} n_{y}, \quad g=-\frac{3}{2} n_{x} n_{y}\left(1-2 n_{x}^{2}\right), \quad h=-\frac{3}{2} n_{x} n_{z}\left(1-2 n_{x}^{2}\right), \quad i=3 n_{x}^{2} n_{y} n_{z}, \quad j=\frac{\sqrt{3}}{2}\left(1-2 n_{y}^{2}+2 n_{z}^{2}\right) n_{x} n_{y},  \tag{A6}\\
& k=\frac{\sqrt{3}}{2}\left(-1-2 n_{y}^{2}+2 n_{z}^{2}\right) n_{x} n_{z}, \quad l=\sqrt{3}\left(n_{z}^{2}-n_{y}^{2}\right) n_{y} n_{z}, \quad m=\frac{3 \sqrt{3}}{4} n_{x}^{2}\left(n_{z}^{2}-n_{y}^{2}\right), \quad n=\frac{3}{2} n_{x}^{2}\left(n_{y}^{2}+n_{z}^{2}\right)-3 n_{y}^{2} n_{z}^{2} .
\end{align*}
$$

## J. Angelier

Similarly, the term $\lambda(\mathbf{s} \cdot \boldsymbol{\sigma})$ of equation (A1) is obtained by transferring expressions (A2) of $\sigma_{x}, \sigma_{y}, \sigma_{z}$ into equation (A4):
$\lambda(\mathbf{s} \cdot \boldsymbol{\sigma})=q \cos \psi+p \sin \psi+u \alpha+v \gamma+w \beta$
where $p, q, u, v$, and $w$ are also simple polynomials that contain $n_{x}, n_{y}, n_{z}, s_{x}, s_{y}$ and $s_{z}$, as well as factor $\lambda$ :
$p=\lambda \frac{\sqrt{3}}{2}\left(n_{z} s_{z}-n_{y} s_{y}\right), \quad q=\lambda \frac{3}{2} n_{x} s_{x}, \quad u=\lambda\left(n_{x} s_{y}+n_{y} s_{x}\right), \quad v=\lambda\left(n_{x} s_{z}+n_{z} s_{x}\right), \quad w=\lambda\left(n_{y} s_{z}+n_{z} s_{y}\right)$.
Summarizing, equation (A1) is written as follows, using equations (A5) to (A8):

$$
\begin{align*}
v^{2}= & n \cos ^{2} \psi-2 m \sin \psi \cos \psi-2(g \alpha+h \gamma+i \beta+q) \cos \psi-2(j \alpha+k \gamma+l \beta+p) \sin \psi \\
& +a \alpha^{2}+b \gamma^{2}+c \beta^{2}+2(d \gamma \alpha+e \alpha \beta+f \beta \gamma)-2(u \alpha+v \gamma+w \beta)+\lambda^{2}+\frac{3}{4} n_{x}^{2}\left(n_{y}^{2}+n_{z}^{2}\right)+3 n_{y}^{2} n_{z}^{2} . \tag{A9}
\end{align*}
$$

The partial derivatives of $v^{2}$ with respect to $\alpha, \beta, \gamma$ and $\psi$ are computed from (A9):
$\frac{1}{2} \frac{\partial\left(v^{2}\right)}{\partial \alpha}=a \alpha+d \gamma+e \beta-g \cos \psi-j \sin \psi-u, \quad \frac{1}{2} \frac{\partial\left(v^{2}\right)}{\partial \gamma}=d \alpha+b \gamma+f \beta-h \cos \psi-k \sin \psi-v$,
$\frac{1}{2} \frac{\partial\left(v^{2}\right)}{\partial \beta}=e \alpha+f \gamma+c \beta-i \cos \psi-l \sin \psi-w$,
$\frac{1}{2} \frac{\partial\left(v^{2}\right)}{\partial \psi}=-m \cos 2 \psi-\frac{n}{2} \sin 2 \psi+(g \alpha+h \gamma+i \beta+q) \sin \psi-(j \alpha+k \gamma+l \beta+p) \cos \psi$.
Let us replace in (A10) the polynomials $a, b, \ldots, v, w$ defined in (A6) and (A8) by the corresponding sums from fault number 1 to fault number $\mathscr{H}$, respectively called $A, B, \ldots, V, W$. Thus, we obtain the half partial derivatives of the sum of functions $v^{2}$; this sum is referred to as $S_{4}$ in text (equation 13). Cancelling these four derivatives together requires the system of four equations ( 17 in text) to be solved (see Appendix II).

## APPENDIX II

In order to solve the system of four equations (17) in text, let us define the following determinants:
$\Delta=\left|\begin{array}{lll}4 & D & E \\ D & B & F \\ E & F & C\end{array}\right|$,
$\Delta_{1}=\left|\begin{array}{ccc}G & D & E \\ H & B & F \\ I & F & C\end{array}\right|, \quad \Delta_{2}=\left|\begin{array}{ccc}A & G & E \\ D & H & F \\ E & I & C\end{array}\right|, \quad \Delta_{3}=\left|\begin{array}{ccc}A & D & G \\ D & B & H \\ E & F & I\end{array}\right|$,
$\Delta_{1}^{\prime}=\left|\begin{array}{ccc}J & D & E \\ K & B & F \\ L & F & C\end{array}\right|, \quad \Delta_{2}^{\prime}=\left|\begin{array}{ccc}A & J & E \\ D & K & F \\ E & L & C\end{array}\right|, \quad \Delta_{3}^{\prime}=\left|\begin{array}{ccc}A & D & J \\ D & B & K \\ E & F & L\end{array}\right|$,
$\Delta_{1}^{\prime \prime}=\left|\begin{array}{ccc}U & D & E \\ V & B & F \\ W & F & C\end{array}\right|, \quad \Delta_{2}^{\prime \prime}=\left|\begin{array}{ccc}A & U & E \\ D & V & F \\ E & W & C\end{array}\right|, \quad \Delta_{3}^{\prime \prime}=\left|\begin{array}{ccc}A & D & U \\ D & B & V \\ E & F & W\end{array}\right|$.
The three unknowns $\alpha, \gamma, \beta$ may be expressed as functions of $\psi$ :
$\alpha=\frac{\Delta_{1} \cos \psi+\Delta_{1}^{\prime} \sin \psi+\Delta_{1}^{\prime \prime}}{\Delta}$,
$\gamma=\frac{\Delta_{2} \cos \psi+\Delta_{2}^{\prime} \sin \psi+\Delta_{2}^{\prime \prime}}{\Delta}$,
$\beta=\frac{\Delta_{3} \cos \psi+\Delta_{3}^{\prime} \sin \psi+\Delta_{3}^{\prime \prime}}{\Delta}$.
By combining equations (A12) and the last equation of system (17) in text, one obtains an equation with the single unknown $\psi$ :
$(M \Delta+\xi) \cos 2 \psi+1 / 2(N \Delta+\omega) \sin 2 \psi+\xi^{\prime} \cos \psi-\xi \sin \psi=0$,
where $M$ and $N$ are the parameters already mentioned in text and in Appendix I while $\xi, \omega, \xi^{\prime}$ and $\xi$ designate
$\zeta=J \Delta_{1}+K \Delta_{2}+L \Delta_{3}=G \Delta_{1}^{\prime}+H \Delta_{2}^{\prime}+I \Delta_{3}^{\prime}$,
$\omega=J \Delta_{1}^{\prime}+K \Delta_{2}^{\prime}+L \Delta_{3}^{\prime}-G \Delta_{1}-H \Delta_{2}-I \Delta_{3}$,
$\xi^{\prime}=J \Delta_{1}^{\prime \prime}+K \Delta_{2}^{\prime \prime}+L \Delta_{3}^{\prime \prime}+P \Delta$,
$\xi=G \Delta_{1}^{\prime \prime}+H \Delta_{2}^{\prime \prime}+I \Delta_{3}^{\prime \prime}+Q \Delta$.
Writing $\cos 2 \psi, \sin 2 \psi, \cos \psi$ and $\sin \psi$ as functions of $\operatorname{tg}(\psi / 2)$ (that will be called $t$ ) provides one way to solve equation (A13) by solving analytically the equation of degree 4 :
$\left(M \Delta+\xi-\xi^{\prime}\right) t^{4}-2(N \Delta+\omega+\xi) t^{3}-6(M \Delta+\xi) t^{2}+2(N \Delta+\omega-\xi) t+\left(M \Delta+\xi+\xi^{\prime}\right)=0$.
For each solution $\psi$ of (A15), the other unknowns $\alpha, \beta, \gamma$ of the stress tensor are determined using equations (A12). Among the few sets of values thus obtained for $\alpha, \beta, \gamma$ and $\psi$, the solution of the inverse problem is simply determined by calculating the corresponding sums of functions $\boldsymbol{v}^{2}$ and by retaining the smallest one. In most cases, two values of $\psi$ are obtained as solutions of equation (A15); the related sets of values $\alpha, \beta, \gamma$ (and $\psi$ ) respectively correspond to the maximum and minimum values of the sum $S_{4}$ defined in equations (12-13) of the text.

## APPENDIX III

This section describes a technique to determine the value of $\psi$ for a given orientation of unclassified principal axes, hence to identify the three principal stresses $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ (with decreasing magnitudes). A new particular deviatoric form of the reduced stress tensor is adopted, so that principal stress magnitudes are not affected by any rotation of the stress tensor. This was not the case with the tensor defined in equation (14) of text. The new form is obtained as a function of $\psi$ and of the direction cosines $x_{i}, y_{i}, z_{i}$ of unclassified principal axes $(i=1,2,3)$.

$$
\left(\begin{array}{ccc}
T_{11} & T_{12} & T_{13}  \tag{A16}\\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{array}\right)=\left(\begin{array}{ccc}
x_{1} & x_{2} & x_{3} \\
y_{1} & y_{2} & y_{3} \\
z_{1} & z_{2} & z_{3}
\end{array}\right)\left(\begin{array}{ccc}
\cos \psi & 0 & 0 \\
0 & \cos \left(\psi+\frac{2 \pi}{3}\right) & 0 \\
0 & 0 & \cos \left(\psi+\frac{4 \pi}{3}\right)
\end{array}\right)\left(\begin{array}{ccc}
x_{1} & y_{1} & z_{1} \\
x_{2} & y_{2} & z_{2} \\
x_{3} & y_{3} & z_{3}
\end{array}\right) .
$$

The minimization criterion adopted is the same as in text: the corresponding function $v^{2}$ has been defined in equation (A1) of Appendix I, with the terms $\tau^{2}$ and (s $\cdot \sigma$ ) defined in (A3) and (A4).
Now, the components of the stress vector of equation (3) in text are
$\sigma_{x}=u \cos \psi+u^{\prime} \sin \psi, \quad \sigma_{y}=v \cos \psi+v^{\prime} \sin \psi, \quad \sigma_{z}=w \cos \psi+w^{\prime} \sin \psi$,
where $u, v, w$ and $u^{\prime}, v^{\prime}, w^{\prime}$ are polynomial functions of the direction cosines $n_{x}, n_{y}, n_{z}$ of $\mathbf{n}$ as well as of the direction cosines $x_{i}, y_{i}, z_{i}$ of principal axes mentioned in (A16). These functions are obtained by combining equation (3) of text and equation (A16).
Substituting expressions (A17) into (A3), (A4) and (A1) of Appendix I gives the function $v^{2}$ as follows:
$v^{2}=a \cos ^{2} \psi+b \sin ^{2} \psi+2 c \sin \psi \cos \psi-2 d \cos \psi-2 e \sin \psi+\lambda^{2}$
where $a, b, c, d$ and $e$ are simple polynomials $\left(s_{x}, s_{y}, s_{z}\right.$ are the direction cosines of $s$ ):

$$
\begin{align*}
& a=u^{2}+v^{2}+w^{2}-\left(n_{x} u+n_{y} v+n_{z} w\right)^{2}, \quad b=u^{\prime 2}+v^{\prime 2}+w^{\prime 2}-\left(n_{x} u^{\prime}+n_{y} v^{\prime}+n_{z} w^{\prime}\right)^{2}, \\
& c=u u^{\prime}+v v^{\prime}+w w^{\prime}-\left(n_{x} u+n_{y} v+n_{z} w\right)\left(n_{x} u^{\prime}+n_{y} v^{\prime}+n_{z} w^{\prime}\right), \quad d=\lambda\left(s_{x} u+s_{y} v+s_{z} w\right), \quad e=\lambda\left(s_{x} u^{\prime}+s_{y} v^{\prime}+s_{z} w^{\prime}\right) . \tag{A19}
\end{align*}
$$

The partial derivative of $v^{2}$ with respect to $\psi$ is computed from (A18), as a function of $\operatorname{tg}(\psi / 2)$ (called $t$ ):
$\frac{\delta\left(v^{2}\right)}{\delta \psi} \frac{\left(1+t^{2}\right)^{2}}{2}=(c+e) t^{4}+2(a-b+d) t^{3}-6 c t^{2}+2(d+b-a) t+c-e$.
As in Appendix I, the extremum values of the sum of functions $\boldsymbol{v}^{2}$ for all faults are obtained by cancelling its partial derivative, thus solving the equation

$$
\begin{equation*}
(C+E) t^{4}+2(A-B+D) t^{3}-6 C t^{2}+2(D+B-A) t+C-E=0 \tag{A21}
\end{equation*}
$$

where $A, B, C, D$ and $E$ are the sums of polynomials $a, b, c, d$ and $e$ respectively. A comparison between the values of $v^{2}$ obtained of (A21) allows identification of the actual solution (as in Appendix II). The last step consists of classifying the principal stress axes according to the respective magnitudes of $\cos \psi, \cos (\psi+2 \pi / 3)$ and $\cos (\psi+4 \pi / 3)$.

## APPENDIX IV

In this appendix, it is shown that the use of the particular stress tensor defined by equation (14) in text and of the minimization criterion summarized by equations (15-16) in text and (A1) in Appendix I is in agreement with a major requirement mentioned in Section 1 with equation (1): an arbitrary choice of the positive scale factor $t_{1}$ and of the isotropic stress $t_{2} l$ must not affect the minimization scheme and therefore the results.
First, adding a normal stress $t_{2} I$ to all diagonal terms of the stress tensor would modify expressions of $\sigma_{x}, \sigma_{y}$ and $\sigma_{z}$ in equation (A2) of Appendix I. However, this change does not affect the scalar product $s \cdot \sigma$ of equation (A4), because $s \cdot \sigma$ and $\mathbf{s} \cdot \boldsymbol{\tau}$ are equivalent as equation (16) in text shows, and variations of isotropic stress cannot affect the shear stress $\boldsymbol{\tau}$ (see Fig. 2 in Angelier 1989). As a consequence, the expression (A1) of $v^{2}$ shown in Appendix I is a function of the shear stress magnitude $\tau$ solely, although $\sigma$ has been introduced for mathematical convenience.
$v^{2}=\lambda^{2}+\tau^{2}-2 \lambda(s \cdot \tau)$.
This implies that a deviatoric form of the stress tensor, such as in equations (14) of text or (A16) of Appendix III, can be adopted with the criterion described in this paper, because it is always possible to choose $t_{2}$ so that
$T_{11}+T_{22}+T_{33}=0$.
Second, multiplying the stress tensor by a positive scale factor, referred to as $t_{1}$ in equation (1) of text, implies that all stress components, including normal and shear stress, are multiplied by $t_{1}$ in equation (A1) of Appendix I and in subsequent algebra. For reasons discussed elsewhere (Section 4 of text; see also below in this appendix), the parameter $\lambda$ defined by equation (12) in text receives the value of the largest possible shear stress, which would be multiplied by $t_{1}$ as well as other stress components. As a consequence, the expression of $v^{2}$, (A1) and (A22) in Appendices I and IV respectively, would remain simply proportional to $t_{1}^{2}$ through the multiplication under consideration. In other words, multiplying the stress tensor by a scale factor implies that all individual functions $v$ are simply multiplied by the same scale factor. This cannot affect the search of the minimum and the result of the direct inversion. Therefore, the particular forms of stress tensor shown in equations (14) of text or (A16) in Appendix III could be adopted, because it is always possible to choose $t_{1}$ so that
$T_{11}^{2}+T_{22}^{2}+T_{33}^{2}=3 / 2$.
Summarizing, the basic criterion and the minimization procedure proposed in this paper are not dependent on the scale factor $t_{1}$ and isotropic stress component $t_{2}$ of equation (1) in text. Because of this independence, particular forms of tensors that fulfil requirements (A23) and (A24) were adopted through choices of $t_{1}$ and $t_{2}$ ('reduced stress tensors', see Section 1). The inverse problem, as formulated, was thus a problem of only four unknowns. Complete determination of stress, including not only the orientations of the three principal axes and the ratio $\Phi$ defined by equation (2) in text, but also the magnitudes of principal stresses, obviously implies determination of $t_{1}$ and $t_{2}$. Such determination cannot be made with fault slip data solely: it requires additional geological information which can be obtained through studies of the depth of overburden and determinations of rupture and friction laws in the rock mass under investigation. This further step in stress reconstruction is discussed elsewhere (Angelier 1989).
The simplicity of the tensor form defined by equation (14) in text made subsequent algebra easier (see Section 4 of text and Appendix I). However, as pointed out in text, a rotation of stress axes affects the magnitude of stress with this particular form. This occurs because the diagonal terms contain the single variable $\psi$ whereas the other terms do not depend on $\psi$. To prevent this undesirable effect, a progressive adjustment of the parameter $\lambda$ defined by equation (12) in text was obtained through a short iterative procedure (see text, Section 4). As a consequence, the function $v$ became proportional to shear stress amplitudes as far as $\lambda$ has the value of the largest possible shear stress, as discussed above (this appendix).

Adopting the tensor form defined by equation (A16) of Appendix III would result in a simpler direct inversion, because in this case a rotation of stress axes would not affect the magnitude of stress. In the right side of (A16), the terms related to the orientations of axes and the terms related to the ratio between principal stress differences are clearly separated and independent, which was not the case in the right side of (14). As a consequence, the adjustment of the value of $\lambda$ (depending on stress orientation which controls stress magnitude) would become unnecessary with this new tensor form, thus leading to a still simpler procedure. In fact, $\lambda$ would simply become constant and equal $\sqrt{3} / 2$ as discussed in Section 4 of the text and suggested in Fig. 2(c). Unfortunately, adopting this new formulation resulted in much more complex algebra, so that author's attempts at solving the inverse problem by analytical means with the same logic as in Section 4 of text and Appendices I-II remained unsuccessful (although there is no reason to believe that it is impossible). For this reason, the iterative adjustment of parameter $\lambda$ had to be maintained, and the new tensor form could not be used except in a final step described in Appendix III and discussed in Section 4 of text. Note that this final step, added for safety, would also become unnecessary if the new tensor form could be adopted in the whole inversion process.

## APPENDIX $V$

Tables A1 and A2 of this appendix provide the complete list of data and results corresponding to the four examples discussed in text (Section 5) and illustrated in Figs 3 and 4. Tables 1 and 2 in text summarized the main results obtained through computer determinations using the new direct inversion method and the 4-D exploration method, respectively. The same names of sites (AVB, TYM, MD1 and KAM) refer to the same data sets in all tables. Figs 3 and 4 show the palaeostress axes obtained after direct inversion (this paper) as well as the distribution of fault slip data orientations for each set. Tables A1 and A2 enable one to check the results.
In Table A1, for each data set, the first four columns completely describe the orientation of each fault slip datum. The next two columns display the individual estimators RUP and ANG (see text, Sections 5 and 6) obtained after the direct inversion of Table 1. The last two columns (rup and ang) display the same estimators obtained after the 4-D exploration of Table 2.
For each of the four data sets, the values of the fifth column (RUP) are significantly smaller on average than the values of the same estimators in the seventh column (rup). This is not surprising, because this estimator corresponds to the minimization criterion adopted in the direct inversion. Likewise, the use of the 4-D exploration yields values of the corresponding estimator (ang) which are significantly smaller on average than for the direct inversion (ANG). These discrepancies bring confirmation

Table A1. List of data (angles in degrees) and individual estimators. Column 1: sense of fault slip, all faults being normal ( N ). Column 2: strike of fault plane (azimuth). Column 3: dip of fault. Column 4: pitch (rake) of slickenside lineations. Columns 5 and 7: estimator RUP (in per cent). Columns 6 and 8: estimator ANG (in degrees). Other explanations in this appendix. (See also text, Sections 4-6, and Tables 1-2.)

| Avg |  |  |  |
| :---: | :---: | :---: | :---: |
| $N$ | 59 | 21N | 74E |
| N | 45 | 65* | 69N |
| $N$ | 42 | 70w | 68N |
| $N$ | 47 | 63N | $84 E$ |
| N | 44 | S6E | 835. |
| N | 72 | 655 | B1E |
| $N$ | 63 | 705 | 81 E |
| N | 46 | 61 N | $83 E$ |
| N | 70 | 23N | 8SE |
| $N$ | 72 | 72N | 89W |
| N | 53 | B6N | 72E |
| N | 60 | 74 N | '31E |
| $N$ | 47 | 77N | 6tE |
| N | 55 | 65N | 60E |
| N | 75 | 525 | 62E |
| N | 39 | 85w | 71 N |
| $N$ | 42 | 71w | 69N |
| N | 59 | 63N | 71 E |
| $N$ | 49 | S8N | 69E |
| N | 47 | 57N | 78E |
| $N$ | 84 | 565 | 67E |
| N | 49 | 40N | 68E |
| N | 69 | 515 | 64E |
| N | 77 | 325 | 67E |
| N | 52 | 54N | 55E |
| N | 59 | 675 | 73E |
| N | 47 | 65N | 61 E |
| N | 61 | 865 | 67E |
| N | 47 | 74N | 49E |
| N | 31 | 69w | 45N |
| N | 59 | 655 | B5W |
| N | 56 | 565 | 79E |
| N | 70 | 80 N | 89w |

${ }^{\text {TYM }}{ }^{\mathrm{N}} 47615$ BOE
92 BON 58w
47 63N クaw
N 112 BBN 59w
N 76 78N 65W
N 48 605 80w
N $7969 \mathrm{~N} \mathrm{B6E}$ N 69565 BBE 51 70S 69w $3850 w 86 \mathrm{~N}$ 6051579 w 81 625 77w 5862 N 84 W 36 60E 765 39 63E B55 7072563 E 51535 日9E 1147 W 7 gN N 59 45N 67w N 11474566 w 27 42W 89N N 36 56 87 N 39 72E 50 S $5961562 W$ 70 58N 74E 63625
3058 E 5 S 3269 ESN $3269 E$ B5N
$4363 E 845$ 2568 E 87 N N 71 4BN B5E
 96 70N 724





Table A2. Numerical values in stress tensor determinations. Same data sets as in Table A1. Explanation in this appendix. (See also text, Sections 4-5, and Table 1.)

that the values of estimators, if considered regardless of the minimization criterion adopted for data inversion, have little significance.

In Table A2, two sets of numerical values resulting from the direct inversion process are displayed for each data set. The left half of the table corresponds to the main inversion described in most of Section 4 and Appendices I-II, while the right half shows the result of the additional step described in the last paragraph of Section 4 and Appendix III. For each trial, in the first two lines, two values of $\psi$, PSI (1) and PSI (2), are computed as solutions of equation (A15) of Appendix II (left half of table) or (A18) of Appendix III (right half), and $\psi$ values corresponding to the smallest sum of functions $v^{2}$ are adopted. UPSILO is the corresponding square root of the mean value of $v^{2}$; it is close to, but not identical with, the mean value of $v$ computed $a$ posteriori (RUP, Tables 1 and A1).

The numerical components of the stress tensor are given in the next three lines according to definition (14) of text (on left) or (A16) of Appendix III (on right). The following line displays the parameter LAMBDA $\lambda$ defined in equation (12) of text and discussed in Section 4 of text and Appendix IV, as well as the value of the greatest possible shear stress called TAUMAX. The last four lines show the results of the determination of eigenvalues ( $\mathrm{S} 1, \mathrm{~S} 2$ and S 3 ) and eigenvectors (SIGMA 1, 2 and 3, with trend D and plunge P in degrees); the value of the ratio $\Phi$ defined by equation (2) of text is added (PHI, last line). Not surprisingly, stress axes orientations are identical in the left and right halves of Table A2.

