

# Inverter VAR Control for Distribution Systems with Renewables

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**Abstract**—Motivated by the need to cope with rapid and random fluctuations of renewable generation, we present a model that augments the traditional Volt/VAR control through switched controllers on a slow timescale with inverter control on a fast timescale. The optimization problem is generally nonconvex and therefore hard to solve. We propose a simple convex relaxation and prove that it is exact provided oversatisfaction of load is allowed. Hence Volt/VAR control over radial networks is efficiently solvable. Simulations of a real-world distribution circuit illustrates that the proposed inverter control achieves significant improvement over the IEEE 1547 standard in terms of power quality and power savings.

## I. INTRODUCTION

Traditional VAR Control is achieved by reconfiguring shunt capacitors, at a slow timescale, mainly to adapt to the gradual changes of the aggregate load on a distribution system. Intermittent sources such as solar and wind introduces rapid, large, and random fluctuations in supply. As renewable penetration increases, faster controllers such as inverters will be needed to provide voltage regulation by controlling reactive power flow in the circuit [1]. The advantage of an inverter relative to a conventional circuit controllers, e.g. shunt capacitors, is that they are much faster, can vary the supplied reactive power continuously, and have very low operation costs. Large photovoltaic systems are being deployed by many owners of commercial buildings who have extensive rooftop sites on large warehouses. This research was motivated by Southern California Edison’s (SCE) plan to install 500MW of large commercial rooftop PV plants (each a couple Megawatts) by 2015. In this scenario, centralized control scheme is feasible due to relatively light requirement on telecommunication.

Volt/VAR control (VVC) in a distribution system has been extensively studied in the literature. Most effort focus on finding optimal switching schedules for shunt capacitors and Under Load Tap Changer (ULTCs) to minimize system losses [2], [3], [4]. Inverter control have been considered in some recent works, e.g., [5], [6]. In [5], the authors consider centralized reactive power flow control of inverters and use DC power flow approximations. Although recent studies show that deployment of Conservation Voltage Reduction (CVR) plans on distribution feeders of United States can provide a 3.04% reduction in the annual national energy consumption [7], almost all existing VVC solutions in the literature ignore this and only aim to minimize systems losses.

In this paper, we formulate a VCC problem that minimizes both the power loss and the power consumption by reducing a weighted average of voltages across the feeder (Section II). The overall two-timescale optimization over both switching of the shunt capacitors and ULTCs and the VAR control at the inverters can be formulated as a dynamic program, but here we focus only on the fast timescale control of the inverters. The optimization problem over the inverters is generally a static nonconvex problem and therefore hard to solve. In Section III, we propose a simple convex relaxation that can be solved very efficiently. We prove that the relaxation is exact provided oversatisfaction of load is allowed, i.e., any optimal solution of the relaxed problem is optimal for the original (possibly nonconvex) VCC problem. This implies that the optimal power flow problem over a tree network can be efficiently solved. A similar observation through duality with a different formulation is independently made in [8], [9]. In Section IV, we present a simulation study of a real-world distribution circuit in the Southern California Edison’s service area and compare the performance of the proposed method and the IEEE 1547 standard. Inverters under current IEEE 1547 are operated at unity power factor and do not participate in VAR control. With our proposed scheme, not only is power consumption reduced significantly, more importantly, the circuit stays much less time in “infeasible regions” where primary voltage and VAR flow constraints are violated. Hence optimal inverter VAR control can significantly improve power quality as well as saving energy consumption.

## II. PROBLEM FORMULATION

In this section, we describe our model and formulate the optimal Volt/VAR control (VCC) problem.

### A. Two timescales

As mentioned earlier, there are two types of control devices with two different control timescales: shunt capacitors and the voltage controllers that are controlled on a slow timescale, say, hourly, and inverters that can be controlled on a fast timescale, say, minutes. In practice, the switched controllers are typically re-configured only a few times each day due to their limited life cycle. As the aggregate load changes slowly, the slow timescale control has been sufficient to provide voltage support. As renewable penetration such as solar PV and wind generation increases, fast timescale control of the

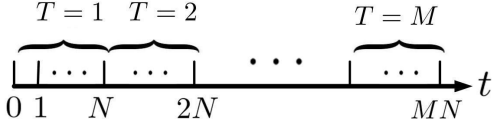


Fig. 1. Two-timescale discretization of a day for switched controllers and inverters.

inverters will become indispensable in order to adapt to the large, rapid, and random fluctuations of their output.

Hence, in our model, we divide each day into  $M$  slots and index these slots by  $T$ . Each of these  $M$  slots is further divided into  $N$  slots indexed by  $t$ , as shown in Figure 1. For instance we can choose each  $T$ -slot to be an hour and each  $t$ -slot a minute, i.e.  $M = 24, N = 60$ . We will assume that the state of the network (the voltages, real and reactive power at each bus), and the input to the network (real and reactive power generated or consumed at each bus) remain unchanged within each  $t$ -slot, and may change only from  $t$  to  $t + 1$ . As we will explain below, the inverter control will be applied for each  $t$  to match renewable output fluctuations, and the configuration of the shunt capacitors and the substation voltage will be adjusted for each  $T$ -slot to match load fluctuations.

### B. Power flow equations and constraints

We will use the network model (Dist-Flow branch equations) first introduced in [2]. Let  $G(\{0\} \cup V, E)$  be a graph representing a radial distribution circuit. Each node in  $\{0\} \cup V$  is a bus and each link in  $E$  is a line. We index the nodes by  $i = 0, 1, \dots, |V|$ .

Node 0 denotes the substation bus and other nodes in  $V$  denote branch buses. Let  $v_i(t)$ ,  $i \geq 0$ , denote the *square* of the voltage magnitude at node  $i$  at time  $t$ . Node 0 is special in that its voltage  $v_0$  is adjusted on the slow timescale  $T$  while other voltages  $v_i(t)$ ,  $i \geq 1$ , are adjusted every  $t$ .

There are two types of nodes in  $V$  that generate reactive power. Let  $V_c$  be the set of nodes with switched shunt capacitors that are controlled at the slow timescale  $T$ , and the remaining nodes in  $V \setminus V_c$  have inverters that are controlled at the fast timescale  $t$ . For notational simplicity only, we assume without loss of generality that each node can have either a shunt capacitor or an inverter, but not both nor neither. Let  $p_i^g(t)$  and  $q_i^g(t)$  be the real and reactive power generation, respectively, at node  $i$  at time  $t$ . For nodes  $i \in V \setminus V_c$ ,  $p_i^g(t)$  represent real power generated by renewable sources such as solar PV that are connected to the grid via inverters. For nodes  $i \in V_c$  that has (only) switched shunt capacitors,  $p_i^g(t) \equiv 0$  for all  $t$ . For each  $i \in V$ , let  $p_i^c(t)$  and  $q_i^c(t)$  be the real and reactive power demand, respectively, at time  $t$ . If there is no load at node  $i$ , we assume  $p_i^c(t) \equiv q_i^c(t) \equiv 0$  for all  $t$ . Here,  $p_i^c(t)$ ,  $q_i^c(t)$ , and  $p_i^g(t)$  are assumed to be given quantities, whereas the reactive power generated  $q_i^g(t)$  are the control variables.

The shunt capacitor and inverter settings  $q_i^g(t)$ ,  $i \geq 1$ , and the substation voltage  $v_0(t)$ , together with the inputs

$(p_i^c(t), p_i^g(t), q_i^c(t))$  determine the voltages and real and reactive power flows on the network. Let  $P_{ij}(t)$  and  $Q_{ij}(t)$  be the real and reactive power flows from nodes  $i$  to  $j$  over link  $(i, j)$ . Then, from [2], these variables satisfy the following recursion (Dist-Flow equations): for each link  $(i, j)$  in the distribution circuit,

$$P_{ij}(t) = \sum_{k:(j,k) \in E} P_{jk}(t) + r_{ij} \frac{P_{ij}^2(t) + Q_{ij}^2(t)}{v_i(t)} + p_j^c(t) - p_j^g(t) \quad (1)$$

$$Q_{ij}(t) = \sum_{k:(j,k) \in E} Q_{jk}(t) + x_{ij} \frac{P_{ij}^2(t) + Q_{ij}^2(t)}{v_i(t)} + q_j^c(t) - q_j^g(t) \quad (2)$$

$$v_j(t) = v_i(t) - 2(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)) + (r_{ij}^2 + x_{ij}^2) \frac{P_{ij}^2(t) + Q_{ij}^2(t)}{v_j(t)} \quad (3)$$

The end points of the feeder can be modeled through the boundary condition that there is zero power flow downstream from them. For this purpose, all the leaf nodes in our model  $G = (\{0\} \cup V, E)$  and the edges incident on these leaf nodes are actually artificial nodes and edges added to each leaf node (bus) in the real distribution circuit. Denote by  $V' \subset V$  the set of leaf nodes. The the boundary constraints will force the real and reactive power flows to be zero on each edge incident to  $V'$ : for all  $t$

$$\forall (i, j) \in E \text{ with } j \in V' : \quad P_{ij}(t) = Q_{ij}(t) = 0 \quad (4)$$

The primary purpose of VVC on distribution circuits is to maintain voltages in an acceptable range at customer level without creating excessive VAR demand on transmission and subtransmission systems under all operating conditions. This is formulated as constraints on the voltage variables  $v_i$ : for all  $i \notin V'$  for all  $t$ ,

$$\underline{v} \leq v_i(t) \leq \bar{v} \quad (5)$$

The total reactive power demand on the feeder is also constrained to within a range, modeled by

$$\underline{Q}_0(t) \leq \sum_{j:(0,j) \in E} Q_{0j}(t) \leq \bar{Q}_0(t) \quad (6)$$

The lower limit  $\underline{Q}_0(t)$  and the upper limit  $\bar{Q}_0(t)$  on the reactive power flow  $\sum_j Q_{0j}(t)$  through substation need to be carefully set for different seasons and different times of day considering transmission VAR emergency cases, but this is outside the scope of this paper. For our purpose, we assume these limits are given.

### C. Switched controller model

We consider switched shunt capacitors and substation Under Load Tap Changer (ULTC) in our VVC scheme. Shunt capacitors generate reactive power when they are on. Since shunt

capacitor settings are changed on slow timescale for each  $T$ , we represent this by:

$$q_i^g(t) = c_i(T)q_i \quad \forall i \in V_c, t \in T$$

where  $c_i(T) \in \{0, 1\}$  is the switching control for period  $T$ . That is, the capacitor at node  $i$  generates reactive power of  $q_i$  if it is on and no reactive power if it is off. The substation ULTC regulates the voltage  $v_0(T)$  at the substation bus in discrete steps corresponding to different tap levels. It is also controlled at the slow timescale.

Let  $S$  denote the set of possible states for the discrete controllers  $(v_0(T), c_i(T), \forall i \in V_c)$ . With  $k$  number of taps for the substation's ULTC and  $m$  number of switched capacitors, we will have  $|S| = k \times 2^m$  different states. Let  $s(T) := (v_0(T), c(T)) := (v_0(T), c_i(t), i \in V_c)$  be the control at time  $T$ . Then  $s(T) \in S$ . The traditional Volt/VAR control is to choose  $s(T)$  so as to minimize a certain cost function of the form  $J(s) + C(s', s)$  where  $C(s', s)$  represents the cost of switching from the configuration  $s'$  in the previous time period to the new configuration  $s$  in the current period, and  $J(s)$  represents the cost in the new state  $s$ , e.g., the loss in the distribution circuit. Hence, the slow timescale control can be formulated as the following optimization problem:

$$\min_{s(T) \in S} \sum_{T=1, \dots, M} \{J(s(T)) + C(s(T-1), s(T))\}$$

Given the cost functions  $J$  and  $C$ , this can be solved using dynamic programming.

Typically each distribution feeder has a few number of capacitors and hence searching through the state space for the optimal setting of the discrete controllers should be computationally tractable at the slow timescale with the current processing power<sup>1</sup>.

#### D. Inverter model

Besides the slow timescale control, nodes  $i \in V \setminus V_c$  have inverters that are controlled at the fast timescale  $t$ . We use the inverter model of [5], [11]. For our purposes, the main implication is that the magnitude of the reactive power  $q_i^g(t)$  generated at an inverter is upper bounded by a quantity that depends on the real power generated at node  $i$ : for all  $i \in V \setminus V_c$ ,

$$|q_i^g(t)| \leq \bar{q}_i(t) \quad (7)$$

where the upper bound  $\bar{q}_i(t) := \sqrt{s_i^2 - (p_i^g(t))^2}$  is assumed given for each  $t$ .<sup>2</sup> Here  $s_i$  represents the rated apparent power capacity of PV panel at bus  $i$  and  $p_i^g(t)$  is the real power generated at time  $t$ .

<sup>1</sup>For a large number of discrete controllers, we could consider reactive power injection of capacitors and the substation voltage as continuous variables, find the optimal solution of the resulting optimization problem and then determine On/Off and tap setting for the switched capacitors and substation ULTC by projecting the solution to the discrete state space using thresholding.

<sup>2</sup>There is no loss of generality by assuming all nodes not in  $V_c$  have inverters, because if node  $i$  has neither shunt capacitors nor inverter, we can assume  $i \in V \setminus V_c$  with  $\bar{q}_i(t) = 0$  for all  $t$ .

#### E. Real power consumption model

We will now approximate total real power consumption of loads as a function of voltage on the feeder. Recall that  $v_i$  represents the *square* of the voltage magnitude at node  $i$ . A model of a voltage-dependent load  $p_i^c$  is the following: for  $0 \leq n \leq 2$

$$p_i^c(t) = p_i^c \cdot (v_i(t))^{n_i/2} \quad (8)$$

$$q_i^c(t) = q_i^c \cdot (v_i(t))^{n_i/2} \quad (9)$$

where  $\sqrt{v_i(t)}$  is the per unit value of the load's voltage magnitude at time  $t$ , and the constants  $p_i^c$  and  $q_i^c$  are the real and reactive power consumed by the load at the reference voltage, assumed given.<sup>3</sup> Three special cases are of particular interest:  $n = 0$  for constant power loads,  $n = 1$  for constant current loads, and  $n = 2$  for constant impedance loads.

The real power consumption at a load at per unit voltage  $v_i(t)$  can be approximated as:  $p_i^c \cdot (v_i(t))^{n_i/2} \approx p_i^c (1 + \frac{n_i}{2}(v_i(t) - 1)) = \sum_i \alpha_i v_i + \text{constant}$ , where  $\alpha_i = (n_i/2)p_i^c$  and the constant can be neglected in the power consumption minimization (see below). Here we have used the fact that for all loads,  $v_i(t) \approx 1$ . It is also common to model a load as a combination of above three models:

$$p_i^c(t) = p_i^c \left( a_i^0 + a_i^1 \sqrt{v_i(t)} + a_i^2 v_i(t) \right)$$

$$q_i^c(t) = q_i^c \left( a_i^0 + a_i^1 \sqrt{v_i(t)} + a_i^2 v_i(t) \right)$$

where  $a_i^0 + a_i^1 + a_i^2 = 1$ . The real power consumption of such a load can again be approximated as  $\sum_i \alpha_i v_i + \text{constant}$ , where  $\alpha_i = p_i^c (a_i^1/2 + a_i^2)$ . Hence for our purposes, minimizing real power consumption (CVR) is equivalent to minimizing a weighted sum of the  $v_i$  values over all voltage dependent loads.

#### F. VVC optimization problem

We are now ready to state the the VVC optimization problem.

As explained in Section II-C, the switched controllers are controlled at the timescale  $T$ . Within each period  $T$ , the control  $s(T)$  is fixed and the reactive power  $q_i^g(t), i \in V \setminus V_c$  generated by inverters are adjusted at the fast timescale  $t \in T$ . The daily operation is then represented by a hierarchical optimization problem, where the slow timescale control solves

$$\min_{s(T) \in S} \sum_{T=1, \dots, M} \{J(s(T)) + C(s(T-1), s(T))\} \quad (10)$$

and the cost  $J(s(T))$  in period  $T$  is the sum of cost  $J(s(T), t)$  in each period  $t \in T$  under the fast timescale control:

$$J(s(T)) = \sum_{t \in T} J(s(T), t)$$

Here, for each  $t$ , the optimal cost  $J(s(T), t)$  is the sum of the loss in the distribution circuit and the weighted voltages as

<sup>3</sup>If there is no load at node  $i$ , then  $p_i^c = q_i^c = 0$ .

explained in Section II-E: given the switched control  $s(T) = (v_0(T), c(T))$ ,

$$J(S_T, t) = \min \sum_{(i,j) \in E} r_{ij} \frac{P_{ij}^2(t) + Q_{ij}^2(t)}{v_i(t)} + \sum_i \alpha_i v_i(t) \quad (11)$$

subject to (18)–(9) (12)

$$v_0(t) = v_0(T) \quad (13)$$

$$q_i^g(t) = c_i(T) q_i, \quad i \in V_c \quad (14)$$

over  $P(t), Q(t), v(t), q^g(t)$  (15)

In this paper, we focus on the fast timescale inverter control, i.e., we assume a given  $s(T)$  and consider solving (11)–(15) for a fixed  $t \in T$ .

### III. FAST-TIMESCALE CONTROL: INVERTER OPTIMIZATION

In this section we propose a convex relaxation for the problem (11)–(15) that is solved at each time  $t$  within a larger time period  $T$ , given a switched control  $s(T)$ . Since the problem is solved for each time  $t$  separately, given  $s(T)$ , we will drop  $t$  from the notation in this section.

Define the variables:

$$\forall (i, j) \in E : \ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \quad (16)$$

Note that this variable corresponds to the square of the current magnitude at each link, i.e.  $|I_{ij}|^2$ , that appears in the power loss and voltage drop equations. If we substitute (16) into (11)–(15), then, with the additional optimization variables  $\ell_{ij}$ , the objective function becomes linear and all constraints become linear except the equality constraint (16). This nonlinear equality constraint is in general the source of non-convexity, which we propose to relax (21). This will be equivalent to relaxing the magnitude of currents on all links. We will later prove these inequalities will be tight in the optimal solution.

Given the switched control  $s(T) = (v_0(T), c(T))$ , consider the following relaxation of the original problem (11)–(15), dropping  $t$  from the notation:

$$\text{minimize} \quad \sum_{(i,j) \in E} r_{ij} \ell_{ij} + \sum_i \alpha_i v_i \quad (17)$$

$$\text{subject to} \quad P_{ij} = \sum_{k:(j,k) \in E} P_{jk} + r_{ij} \ell_{ij} + p_j^c - p_j^g \quad (18)$$

$$Q_{ij} = \sum_{k:(j,k) \in E} Q_{jk} + x_{ij} \ell_{ij} + q_j^c - q_j^g \quad (19)$$

$$v_j = v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) \ell_{ij} \quad (20)$$

$$\forall (i, j) \in E : \ell_{ij} \geq \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \quad (21)$$

$$p_i^c \leq \bar{p}_i^c, \quad q_i^c \leq \bar{q}_i^c \quad (22)$$

$$p_i^g \leq \bar{p}_i^g \leq \bar{p}_i^g, \quad q_i^g \leq \bar{q}_i^g \leq \bar{q}_i^g \quad (23)$$

$$v_i \leq \bar{v}_i \leq \bar{v}_i \quad (24)$$

$$\text{over} \quad X := (P, Q, p^g, p^c, q^g, q^c, v, \ell) \quad (25)$$

Here,  $v_0 = v_0(T)$ ,  $q_i^g = c_i(T) q_i, \forall i \in V_c$  are determined by the slow timescale control, and  $\forall (i, j)$  with  $j \in V'$  :  $P_{ij} = Q_{ij} = 0$ . We have made two relaxations. First the equalities in the original problem are relaxed to inequalities in (21). Second we allow over-satisfaction of all active and reactive loads (22)–(23). This assumption has been made in several other works (see [9] and references therein for a justification), as it is expected to reach to the same solution point with and without the over-satisfaction assumption for loads. Our key result is

*Theorem 1:* The relaxed problem (17)–(25) is convex. Moreover, it is exact, i.e., an optimal solution of (17)–(25) is also optimal for the original problem (11)–(15).

*Proof:* (Sketch) We first note that the relaxed problem is a Second Order Cone Program (SOCP) and hence convex, since the non-linear inequalities (21) can be written as the following second order cone constraint:

$$\left\| \begin{array}{c} 2P_{ij} \\ 2Q_{ij} \\ \ell_{ij} - v_i \end{array} \right\|_2 \leq \ell_{ij} + v_i$$

To prove that the relaxation is exact, it suffices to show that any optimal solution of (17)–(25) has equality in (21). Assume for the sake of contradiction that  $X^* := (P^*, Q^*, p^{g*}, p^{c*}, q^{g*}, q^{c*}, v^*, \ell^*)$  is optimal but  $(i, j) \in E$  has strict inequality, i.e.,

$$\ell_{ij}^* > \frac{P_{ij}^{*2} + Q_{ij}^{*2}}{v_i^*}$$

Now for some  $\varepsilon > 0$ , consider another point  $\tilde{X} = (\tilde{P}, \tilde{Q}, \tilde{p}^g, \tilde{p}^c, \tilde{q}^g, \tilde{q}^c, \tilde{v}, \tilde{\ell})$  defined by:

$$\tilde{v} = v^*, \quad \tilde{p}^g = p^{g*}, \quad \tilde{q}^g = q^{g*}$$

$$\tilde{\ell}_{ij} = \ell_{ij}^* - \varepsilon, \quad \tilde{\ell}_{-ij} = \ell_{-ij}^*$$

$$\tilde{P}_{ij} = P_{ij}^* - r_{ij} \varepsilon / 2, \quad \tilde{P}_{-ij} = P_{-ij}^*$$

$$\tilde{Q}_{ij} = Q_{ij}^* - x_{ij} \varepsilon / 2, \quad \tilde{Q}_{-ij} = Q_{-ij}^*$$

$$\tilde{p}_i^c = p_i^{c*} + r_{ij} \varepsilon / 2, \quad \tilde{p}_j^c = p_j^{c*} + r_{ij} \varepsilon / 2, \quad \tilde{p}_{-(i,j)}^c = p_{-(i,j)}^{c*}$$

$$\tilde{q}_i^c = q_i^{c*} + x_{ij} \varepsilon / 2, \quad \tilde{q}_j^c = q_j^{c*} + x_{ij} \varepsilon / 2, \quad \tilde{q}_{-(i,j)}^c = q_{-(i,j)}^{c*}$$

where the negative indices mean excluding elements from a vector. It can be verified that  $\tilde{X}$  satisfies all constraints (18)–(25) and hence is a feasible point. Moreover, since  $\tilde{\ell}_{ij} = \ell_{ij}^* - \varepsilon$ , the  $\tilde{X}$  has a strictly smaller objective value than  $X^*$ . This contradicts the optimality of  $X^*$ . ■

Theorem 1 says that the optimal power flow problem over a radial network can be efficiently solved, provided over-satisfaction of load is allowed. A similar observation through duality with a different formulation is independently made in [8], [9]. Taylor [13] has proposed a SOCP relaxation for an approximation of DistFlow equations in radial networks, by omitting the last non-linear term in voltage drop equations (3).

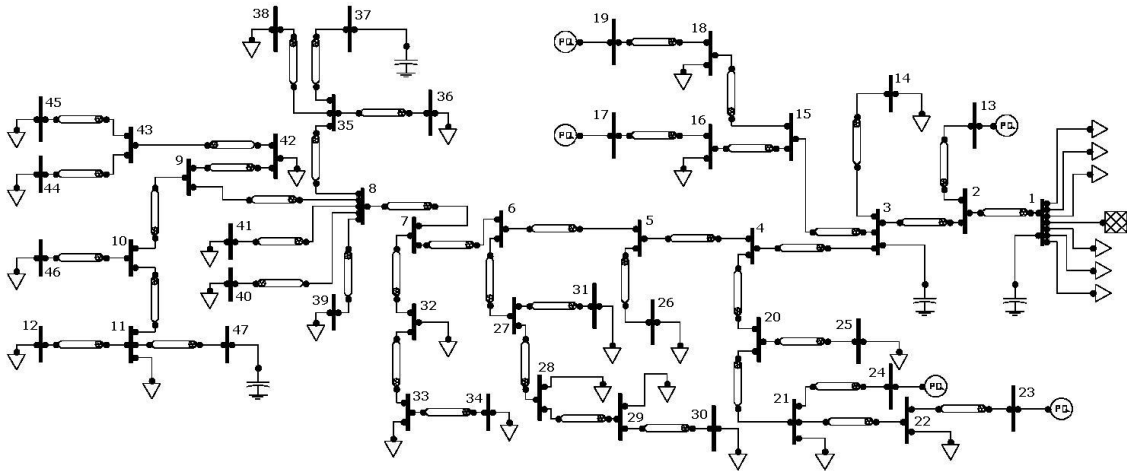


Fig. 2. Schematic diagram of a distribution feeder with high penetration of Photovoltaics. Bus No. 1 is the substation bus and the 6 loads attached to it model other feeders on this substation.

TABLE I  
NETWORK OF FIG. 2: LINE IMPEDANCES, PEAK SPOT LOAD KVA, CAPACITORS AND PV GENERATION'S NAMEPLATE RATINGS.

Network Data																	
Line Data				Line Data				Line Data				Load Data		Load Data		PV Generators	
From Bus.	To Bus.	R ( $\Omega$ )	X ( $\Omega$ )	From Bus.	To Bus.	R ( $\Omega$ )	X ( $\Omega$ )	From Bus.	To Bus.	R ( $\Omega$ )	X ( $\Omega$ )	Bus No.	Peak MVA	Bus No.	Peak MVAR	Bus No.	Nameplate Capacity
1	2	0.259	0.808	8	41	0.107	0.031	21	22	0.198	0.046	1	30	34	0.2	13	1.5MW
2	13	0	0	8	35	0.076	0.015	22	23	0	0	11	0.67	36	0.27	17	0.4MW
2	3	0.031	0.092	8	9	0.031	0.031	27	31	0.046	0.015	12	0.45	38	0.45	19	1.5 MW
3	4	0.046	0.092	9	10	0.015	0.015	27	28	0.107	0.031	14	0.89	39	1.34	23	1 MW
3	14	0.092	0.031	9	42	0.153	0.046	28	29	0.107	0.031	16	0.07	40	0.13	24	2 MW
3	15	0.214	0.046	10	11	0.107	0.076	29	30	0.061	0.015	18	0.67	41	0.67		
4	20	0.336	0.061	10	46	0.229	0.122	32	33	0.046	0.015	21	0.45	42	0.13		
4	5	0.107	0.183	11	47	0.031	0.015	33	34	0.031	0	22	2.23	44	0.45		
5	26	0.061	0.015	11	12	0.076	0.046	35	36	0.076	0.015	25	0.45	45	0.2		
5	6	0.015	0.031	15	18	0.046	0.015	35	37	0.076	0.046	26	0.2	46	0.45		
6	27	0.168	0.061	15	16	0.107	0.015	35	38	0.107	0.015	28	0.13				
6	7	0.031	0.046	16	17	0	0	42	43	0.061	0.015	29	0.13				
7	32	0.076	0.015	18	19	0	0	43	44	0.061	0.015	30	0.2				
7	8	0.015	0.015	20	21	0.122	0.092	43	45	0.061	0.015	31	0.07				
8	40	0.046	0.015	20	25	0.214	0.046	32	33			32	0.13				
8	39	0.244	0.046	21	24	0	0	33	34			33	0.27				

#### IV. SIMULATION RESULTS

In this section we present an example to illustrate the effectiveness of our fast-timescale VVC control, compared with the current PV integration standards IEEE 1547 which requires all the inverters to operate at unity power factor and not participate in VAR control of the distribution circuit [11].

We use the 47-bus distribution feeder shown in Fig. 2 in our simulations. This circuit is a simplified model of an industrial distribution feeder of SCE with high penetration of renewables, integrated with 5 large PV plants. The network data, including line impedances, peak MVA demand of loads, and the nameplate capacity of the shunt capacitors and the PV generations are listed in table I. For all the loads in this industrial area, we assume a constant power factor of 0.8.

To focus on the fast-timescale control, we fix the trajectory of slow-timescale control ( $v_0(T), c(T), T = 1, \dots, M$ ) to be the settings used in practice, and only solve problem (11)–(12) for each time  $t$ . We use real data for load and solar generation over the course of a year. Using minute based data for one year, the empirical joint probability distribution of the load and solar

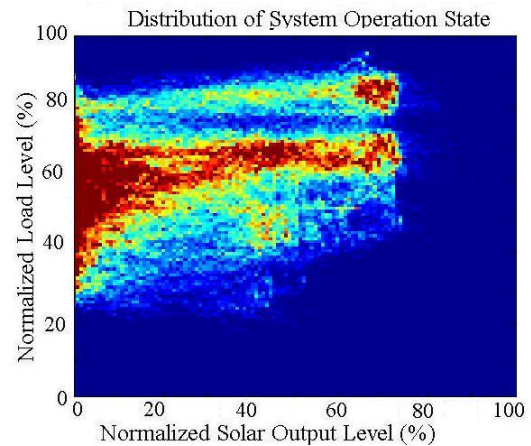


Fig. 3. Joint distribution of the normalized solar output level and the normalized load level

output levels is shown in Figure 3. We have used a hot color scale, with hotter colors representing a higher probability for

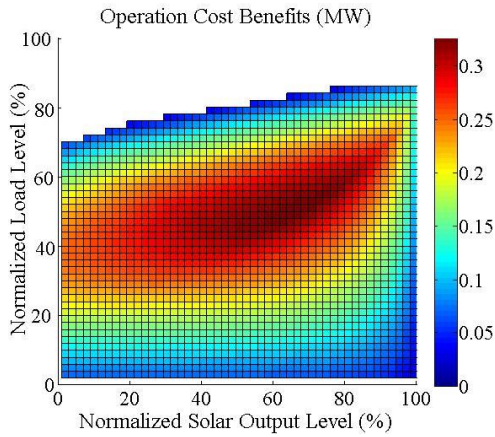


Fig. 4. Overall power savings in MW, for different load and solar output levels assuming a 3% voltage drop tolerance.

the system to spend time in. The axes are scaled to the peak power demand and total capacity of installed PV generation, respectively. We assume  $n_i = 1$  for all loads, corresponding to a constant current load model for this feeder. We further constraint the reactive power flow through substation bus to be less than 2MVAR at all times. To compare the cost of the operation under the IEEE 1547 standard and our proposed algorithm, we choose a fixed voltage drop tolerance.

We implemented the proposed convex relaxation of the original VVC optimization problem and solved it using CVX optimization toolbox [10] in Matlab. In all our simulations, we checked the inequality constraint in condition (21) for optimal solutions of the relaxed problem and confirmed that the inequality constraints were all active. Figure 4 shows the difference in the objective function between current standard IEEE 1547 standard and our proposed solution for different load and solar output levels, given a 3% voltage drop tolerance. This can be interpreted as the overall power savings achieved by optimal control of inverter's VAR dispatch. Interestingly, in typical loading conditions, optimal inverter control can save more than 200KW in overall power consumption, even at nights. The missing region in figure 4 is the set of the infeasible settings of load and solar for the IEEE 1547 operation mode in the relaxed problem formulation, therefore also infeasible for the original problem. The feasible operation region for the proposed method is considerably a superset of that of IEEE 1547 operation mode. This demonstrates significant power quality benefits achieved by optimal inverter control. Table II summarizes the overall time that the feeder can save not spending outside feasible region, and the overall operation cost benefits for the whole year, for different voltage drop tolerances. The results show more than 3% savings achieved by optimal reactive power dispatch for the inverters.

## V. CONCLUSION

Expanding renewable capacity of today's distribution circuits requires design of more advanced and efficient operation and control schemes. We proposed a general hierarchial

TABLE II  
SIMULATION RESULTS FOR SOME VOLTAGE TOLERANCE THRESHOLDS

Voltage Drop Tolerance	Annual Hours Saved Spending Outside Feasibility Region	Average Power Saving
3%	842.9h	3.93%
4%	160.7h	3.67%
5%	14.5h	3.62%

framework for optimal reactive power dispatch of inverters and switched controllers in radial networks by considering different time scale of variations in load and the output of intermittent renewable sources. We introduced a centralized solution for the fast time-scale subproblem and illustrated its effectiveness using simulations on a typical distribution feeders with high penetration of Photovoltaics and demonstrating significant power quality and efficiency benefits.

## ACKNOWLEDGMENT

We would like to thank Jeff Gooding, Russel Neal, Percy Haralson, Michael Montoya, Bryan Pham, Robert Yinger and Marshall Parsons in the Advanced Technology Division of the Southern California Edison's (SCE) Transmission and Distribution Business Unit (TDBU) for multiple fruitful discussions.

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