

# Investigate the Performance of QAM Communication System by Transforming Linear Phase Filter Design using Parks-McClellan Algorithm into Minimum Phase Filter

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## ABSTRACT

In this paper, the design of a low-pass minimum phase equiripple finite impulse response (FIR) digital filter from the given low-pass linear phase equiripple. The initial linear phase transfer function is obtained by standard Parks-McClellan algorithm. The shorter group delay should be minimal for efficient performance of digital pulse shaping filter like it improves the performance parameters like increased Capacity, reduced BER, better S/N ratio, and Reduced ISI (noise). The small group delay minimum phase filter coefficients were subjected to QAM (32, 64, and 128) system. The minimum phase filter performed well in terms of bit error rate (BER) parameter.

## General Terms

Parks- McClellan Algorithm

## Keywords

Linear phase FIR filter, Minimum phase filter, QAM, BER.

## 1. INTRODUCTION

Sometimes there is a requirement to design a FIR filter that not only does well but it is optimal. Optimization is the ability to state a maximum error on each band of interest. This error is expressed as the entire difference between the ideal or desired frequency response and the actual or resulting frequency response. The resulting filters are recognized as Equi-ripple FIR Filters. In order to minimize the error we need to define an error function  $E(\theta)$  and a weight function  $G(\theta)$  which defines the relative significance of the error at any given frequency  $\theta$ . Then, the error function can be defined as follows [10]

$$E(\theta) = G(\theta) [H_d(\theta) - H(\theta)] \quad (1)$$

Where  $H_d(\theta)$  is the desired amplitude response, and  $H(\theta)$  is the actual amplitude response.

The weight function  $G(\theta)$ , defined as follows [12]

$$G(\theta) = \begin{cases} 1, & \theta \in (\text{passband}) \\ 0, & \theta \in (\text{stopband}) \end{cases} \quad (2)$$

And the resulting amplitude response,  $H(\theta)$  is defined by:

$$H(\theta) = F(\theta) K(\theta) \quad (3)$$

And

$$K(\theta) = \sum_{k=0}^m b[k] \cos(k\theta) \quad (4)$$

Where

$F(\theta) = 1$ , and  $M = \frac{N}{2}$  for Type I FIR filter,  $N =$  Even order of filter.

Now, the main task here is to obtain the coefficients  $b[k]$  that minimize the maximum absolute weighted error  $|E(\theta)|$ , that is, to obtain

$$\epsilon = \max |E(\theta)| \quad (5)$$

Where  $\theta$  is in the operating frequency range of the filter.

## 2. THE ALTERNATION THEOREM

This theorem states that there exist at least  $2 + K$  frequencies  $\theta_i$ ,  $\{0 \leq i \leq K+1\}$  where the maximum error,  $\epsilon$ , occurs. That is,

$$|E(\theta_i)| = \epsilon, \quad 0 \leq i \leq K+1 \quad (6)$$

And

$$E(\theta_{i+1}) = -E(\theta_i), \quad 0 \leq i \leq K \quad (7)$$

The last equation shows that the sign changes  $K+1$  times, resulting in ripple on the band of interest.

## 3. PARKS-MCCLELLAN METHOD

The most common implementation of the Remez Exchange Algorithm is the described by Parks, McClellan and Rabiner [1] [4]. Its objective is to obtain the coefficients  $b[k]$  that minimize  $\epsilon$  and it uses the properties of the Alternation Theorem mentioned above. Parks-McClellan method (also known as the Equiripple, Optimal, or Minimax method) with the Remez exchange algorithm is used to find an optimal equiripple set of coefficients to design an optimal linear phase filter. It minimizes the filter length for a particular set of design constraints. This method is used to design linear phase, symmetric or anti-symmetric filters of any standard type. Better filters result from minimization of maximum

error in both, the stop band and the pass band of the filter which leads to equiripple filters. In this algorithm to design FIR filters, some of its parameters such as the filter length (M), passband and stop band normalized frequencies (wp, ws), ripple in the pass band and stop band (δp, δs) are fixed and the remaining parameters are to be optimized. Parameters M, δp, and δs are fixed while the remaining parameters are optimized. The Parks–McClellan (PM) algorithm is the most widespread method for finest FIR filters design due to its flexibility and computational efficiency.

The first step is to find the order N of the desired filter. The following is an experiential formula for this [11]

$$N = \frac{-20 \log_{10}(\sqrt{\delta_p \delta_s}) - 13}{2.32 |\theta_p - \theta_s|} \quad (8)$$

Where  $\theta_p$  is the passband-edge digital frequency,  $\theta_s$  is the stopband-edge digital frequency,  $\delta_p$  is the passband deviation,  $\delta_s$  is the stopband deviation.

$$\delta_p = (10^{A_p/20} - 1)(10^{A_p/20} + 1) \quad (9)$$

$$\delta_s = 10^{-A_s/20} \quad (10)$$

Where:

$A_p$  And  $A_s$  are pass-band and stop-band attenuations respectively.

#### 4. MINIMUM PHASE FILTER

FIR filters that have symmetric or anti-symmetric impulse responses have no phase distortion (their phase is linear). However, the difficulty with linear-phase filters is that the delay can be too great. The delay of a linear-phase filter is equal to  $\frac{(N-1)}{2}$ , where N is the length of the filter. To obtain a linear-phase FIR filter with a narrow transition-band and high stop-band attenuation requires making the filter long. Therefore, linear-phase FIR filters satisfying challenging specifications will have a large delay. This large delay could be a major drawback.

1. If the filter is used inside a feedback loop in a control system it could cause instability.
2. If the filter is used in a communication system it could cause delays that are longer than is tolerable.

For applications where it is vital to minimize the delay caused by a filter, a minimum-phase filter can be a good choice. Minimum-phase filters have all their zeros inside or on the unit circle. A minimum-phase filter can be obtained from a linear-phase filter by reflecting all of the zeros that are outside the unit circle to inside the unit circle. In other words, those zeros located at  $z = r e^{j\theta}$  are moved to  $z = \frac{1}{r} e^{j\theta}$ . This modification results in a minimum-phase filter that has the same frequency response magnitude  $|H(e^{j\omega})|$  as the linear-phase filter. However, the minimum-phase filter obtained in this way will not be optimal in general. There will generally be a minimum-phase filter that is superior to the one gotten by simply reflecting the zeros in this way.

#### 5. GROUP DELAY OF MINIMUM PHASE FILTER

The minimum phase system has the minimum group delay. Suppose, consider one zero  $\alpha$  of the transfer function  $H(z)$ . Lets place this zero inside unit circle ( $|\alpha| < 1$ ) and see how the group delay is affected.

$$\alpha = |\alpha| e^{i\theta\alpha} \text{ where } \theta\alpha = \text{Arg}(\alpha) \quad (11)$$

Since the zero  $\alpha$  contributes the factor  $1 - \alpha z^{-1}$  to the transfer function, the phase contributed by this term is the following.

$$\Phi_{\alpha}(\omega) = \text{Arg}(1 - \alpha e^{-i\omega}) \quad (12)$$

$$\text{Arg}(1 - |\alpha| e^{i\theta\alpha} e^{-i\omega})$$

$$= \text{Arg}(1 - |\alpha| e^{-i(\omega - \theta\alpha)})$$

$$= \text{Arg}(1 - |\alpha| \cos(\omega - \theta\alpha) + i(|\alpha| \sin(\omega - \theta\alpha)))$$

$$= \text{Arg}(1 - |\alpha|^{-1} - \cos(\omega - \theta\alpha) + i(\sin(\omega - \theta\alpha))) \quad (13)$$

$\Phi_{\alpha}(\omega)$  contributes the following to the group delay [14]

$$\frac{d\Phi_{\alpha}(\omega)}{d\omega} = \frac{\sin^2(\omega - \theta\alpha) + \cos^2(\omega - \theta\alpha) - |\alpha|^{-1} \cos(\omega - \theta\alpha)}{\sin^2(\omega - \theta\alpha) + \cos^2(\omega - \theta\alpha) + |\alpha|^{-2} - 2|\alpha|^{-1} \cos(\omega - \theta\alpha)}$$

$$\frac{d\Phi_{\alpha}(\omega)}{d\omega} = \frac{|\alpha|^{-1} - \cos(\omega - \theta\alpha)}{|\alpha| + |\alpha|^{-1} - 2\cos(\omega - \theta\alpha)} \quad (14)$$

The denominator and  $\theta\alpha$  are invariant to reflecting the zeros  $\alpha$  outside the unit circle, bi. e., replacing  $\alpha$  with  $(\alpha^{-1})$ . However, by reflecting  $\alpha$  outside unit circle, the increases the magnitude of  $|\alpha|$  in the numerator, Thus having  $\alpha$  inside the unit circle minimizes the group delay.

#### 6. MEASUREMENT RESULTS

The measurement results carried out by using MATLAB. As the group delay of the filter play an important role in the performance of system. Smaller the group delay minimum the number of errors occurred [13]. The minimum phase filter gives shorter group delay then simple linear phase filter. QAM (32,64,128) communication system subjected to minimum phase filter's coefficients

**Table1. Performance analysis based on BER parameter (32QAM)**

| EbN | BER(theoretic al) | BER(Origin al) | BER(minimu m Phase) |
|-----|-------------------|----------------|---------------------|
| 0   | 0.1895            | 0.1325         | 0.1301              |
| 1   | 0.1673            | 0.1193         | 0.1166              |
| 2   | 0.1461            | 0.1045         | 0.1016              |
| 3   | 0.1260            | 0.0887         | 0.0855              |

|    |        |        |        |
|----|--------|--------|--------|
| 4  | 0.1066 | 0.0722 | 0.0690 |
| 5  | 0.0880 | 0.0559 | 0.0529 |
| 6  | 0.0702 | 0.0406 | 0.0379 |
| 7  | 0.0536 | 0.0274 | 0.0251 |
| 8  | 0.0388 | 0.0168 | 0.0151 |
| 9  | 0.0262 | 0.0092 | 0.0081 |
| 10 | 0.0162 | 0.0043 | 0.0037 |
| 11 | 0.0090 | 0.0017 | 0.0014 |
| 12 | 0.0044 | 0.0006 | 0.0004 |
| 13 | 0.0018 | 0.0001 | 0.0000 |
| 14 | 0.0006 | 0.0000 | 0.0000 |
| 15 | 0.0002 | 0.0000 | 0.0000 |
| 16 | 0.0000 | 0.0000 | 0.0000 |

**Table2. Performance analysis based on BER parameter (64QAM)**

| EbNo | BER(theoretical) | BER(Original) | BER(minimum Phase) |
|------|------------------|---------------|--------------------|
| 0    | 0.1998           | 0.1018        | 0.0996             |
| 1    | 0.1779           | 0.0898        | 0.0873             |
| 2    | 0.1570           | 0.0767        | 0.0741             |
| 3    | 0.1372           | 0.0630        | 0.0604             |
| 4    | 0.1185           | 0.0494        | 0.0468             |
| 5    | 0.1008           | 0.0364        | 0.0341             |
| 6    | 0.0838           | 0.0249        | 0.0230             |
| 7    | 0.0676           | 0.0156        | 0.0141             |
| 8    | 0.0523           | 0.0088        | 0.0077             |
| 9    | 0.0385           | 0.0043        | 0.0037             |

|    |        |        |        |
|----|--------|--------|--------|
| 10 | 0.0265 | 0.0018 | 0.0015 |
| 11 | 0.0169 | 0.0006 | 0.0005 |
| 12 | 0.0097 | 0.0002 | 0.0001 |
| 13 | 0.0049 | 0.0000 | 0.0000 |
| 14 | 0.0022 | 0.0000 | 0.0000 |
| 15 | 0.0008 | 0.0000 | 0.0000 |
| 16 | 0.0002 | 0.0000 | 0.0000 |

**Table3. Performance analysis based on BER parameter 128QAM**

| EbNo | BER(theoretical) | BER(Original) | BER(minimum Phase) |
|------|------------------|---------------|--------------------|
| 0    | 0.2477           | 0.0805        | 0.0784             |
| 1    | 0.2272           | 0.0695        | 0.0673             |
| 2    | 0.2065           | 0.0579        | 0.0557             |
| 3    | 0.1858           | 0.0461        | 0.0439             |
| 4    | 0.1656           | 0.0348        | 0.0327             |
| 5    | 0.1463           | 0.0244        | 0.0227             |
| 6    | 0.1281           | 0.0158        | 0.0144             |
| 7    | 0.1111           | 0.0092        | 0.0082             |
| 8    | 0.0950           | 0.0047        | 0.0041             |
| 9    | 0.0796           | 0.0021        | 0.0017             |
| 10   | 0.0650           | 0.0008        | 0.0006             |
| 11   | 0.0511           | 0.0002        | 0.0002             |
| 12   | 0.0383           | 0.0000        | 0.0000             |
| 13   | 0.0271           | 0.0000        | 0.0000             |
| 14   | 0.0177           | 0.0000        | 0.0000             |
| 15   | 0.0106           | 0.0000        | 0.0000             |
| 16   | 0.0057           | 0.0000        | 0.0000             |

|    |        |        |        |
|----|--------|--------|--------|
| 17 | 0.0012 | 0.0000 | 0.0000 |
| 18 | 0.0002 | 0.0000 | 0.0000 |
| 19 | 0.0001 | 0.0000 | 0.0000 |

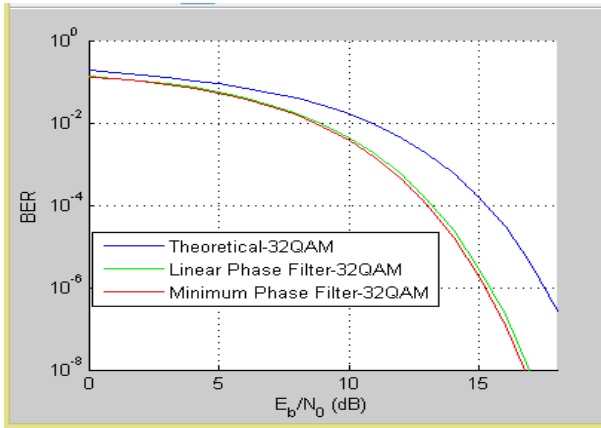


Figure 1. BER versus EbNo Performance Curves for 32QAM

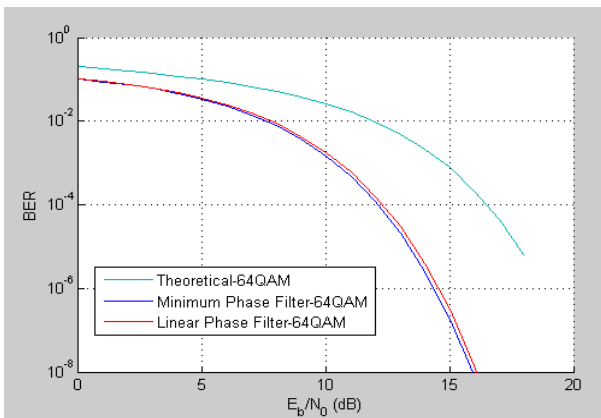


Figure 2. BER versus EbNo Performance Curves for 64QAM

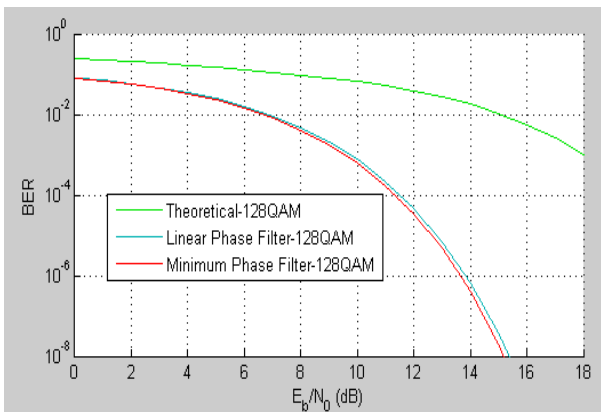


Figure3. BER versus EbNo Performance Curves for 128QAM

## 7. CONCLUSION

In this paper, A Minimum Phase FIR digital filter design approach from a given low-pass FIR transfer function, which

has identical amplitude response, author deal with the equiripple amplitude response and observe that by using a minimum-phase filter obtained from linear phase parks-mcclellan algorithm, got a filter that has a better frequency response magnitude as it reduces the group delay. The group delay plays a crucial role in pulse shaping digital finite impulse response filter. The group delay actually is the time interval between the signal fed into the system (filter) and and comes out of the system. The during processing the signal, some kind of system errors affects the signal (insertion loss) and thus reduced the output SNR and corresponding system performance. Insertion loss is the decrease in power delivered to the load when a filter is inserted between the source and the load. To reduce the insertion loss signal must come out as soon as possible (shorter group delay). The value of group delay should be minimal for efficient performance of digital pulse shaping filter like it improves the performance parameters like increased Capacity, reduced BER, better S/N ratio, and Reduced ISI (noise).The minimum phase filter has all zeros inside the unit circle, thus this arrangement corresponds to the minimum increase in total phase, which corresponds to minimum average total phase delay, which corresponds to maximum compactness in time, for any given(stable) set of poles and zeros with the exact same frequency magnitude response. The coefficients obtained from the minimum phase filter, when subjected to system with QAM, gives better result than original one, in terms BER metric.

## 8. FUTURE SCOPE

In this paper Parks McClellan (PM) Algorithm is used to design filter.A number of algorithms are available in literature like Evolutionary Algorithm, Root Moments, complex cepstrum etc. can be used.

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