

При обґрунтуванні проектних параметрів необхідно здійснювати аналіз напружено-деформованого стану окремих елементів технологічних систем, які є множинами деталей, що знаходяться в умовах контактної взаємодії. Ці задачі є нелінійними, і для них не діє принцип суперпозиції. З цією причиною різко зростає обсяг обчислень. Для подолання відзначеного недоліку розроблені методи та моделі задля оперативного і точного дослідження напружено-деформованого стану складних об'єктів з урахуванням контактної взаємодії. Особливістю постановки задачі є те, що для контактних задач за певних умов розв'язок лінійно залежить від навантаження. Установлені закономірності розподілу контактного тиску. Він зосереджений на областях постійної форми та розмірів. Змінюється тільки масштаб розподілу контактного тиску. Це дає можливість суттєво прискорити проектні дослідження штампового оснащення при збереженні точності чисельного моделювання напружено-деформованого стану.

Розроблений підхід передбачає поєднання переваг чисельних та аналітичних моделей та методів аналізу напружено-деформованого стану елементів розділових штамів із урахуванням контактної взаємодії. Це стосується можливості розв'язання задач для системи контактуючих тіл складної форми, що недоступне із застосуванням аналітичних моделей. З іншого боку, обґрунтована можливість масштабування розв'язків цих задач за зусиллям штампування, що у загальному випадку для нелінійних контактних задач не виконується. Отже, достатньо розв'язати задачу визначення напружено-деформованого стану елементів такого розділового штаму. За іншого значення сили штампування застосовується правило пропорційності. Таким чином, різко зростає оперативність досліджень та забезпечується висока точність одержуваних результатів

**Ключові слова:** контактна взаємодія, розділовий штамп, напружено-деформований стан, контактний тиск

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## INVESTIGATION OF MULTIPLE CONTACT INTERACTION OF ELEMENTS OF SHEARING DIES

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### 1. Introduction

Elements of technological systems (equipment, instruments, tools) have a decisive influence on the quality and manufacturing productivity of parts of machine-building structures. Indeed, in technological systems (such as «machine-fixture-tool-workpiece», «press-die-tool-workpiece», «injection molding machine-press-mold-shaping parts-work material», etc.), there is a process of contact interaction of all their components. And within each individual component,

there is a contact interaction of the individual constituting parts. Thus, the contact interaction of a system of elastic deformable bodies has a decisive influence on the shape, precision and quality of technological operations. Therefore, it is necessary to determine the stress-strain state (SSS) of systems of the bodies that are in multiple contact interaction. It should be borne in mind that such analysis involves the solution of non-linear contact problems. Accordingly, at the stage of design research, it is necessary to solve a large number of problems of this type. As a result, there are two

alternative ways: either to attract a large amount of computing resources, or to use simplified models. In the first case, the efficiency of research is lost. In the second one, the accuracy of the analysis is lost. Accordingly, there is a contradiction between the capabilities of existing methods and models for analyzing the contact interaction between elements of technological systems, on the one hand, and the needs of science and technology, on the other.

Thus, the scientific and applied problem of developing methods and models for the SSS analysis of elements of technological systems based on their contact interaction is formed. These developments should combine both the accuracy and efficiency of SSS modeling. This problem is posed, solved and described on the example of cold-forming shearing dies (SD). The urgency of this problem is due to the need for increasing the efficiency of the strength analysis of SD elements.

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## 2. Literature review and problem statement

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The solution of the problems of the SSS analysis of a system of contacting bodies is a separate scientific direction in mechanics. Basic methods and models in this direction are contained in the monograph [1]. In particular, Hertz's model is widely used [1]. This model allows obtaining an analytical solution for contact problems. However, the scope of this model is limited by basic hypotheses. The point is a representation of gap distribution between the contacting bodies in the form of a quadratic coordinate function in the tangent plane. For example, if to draw attention to the cold-forming SD (Fig. 1) [2], the contact between SD elements occurs mainly on flat surfaces. On the other hand, general formulations have recently been developed, which are deprived of these drawbacks. In particular, these are methods and models of the variational inequality theory presented, in particular, in [3, 4]. In this case, the solution of such problems is reduced, as shown in [5], to the problem of minimization of energy functionals on the sets given by inequalities. At the same time, the models take into account various additional factors, given in [6–18]. Thus, the works [6, 10, 16, 17] consider the contact stiffness caused by surface roughness. In [7], the influence of friction is noted. The works [8, 12–15, 18] take into account adhesion between the surfaces of the contacting bodies. The works [11, 17] attempted to expand the scope of the boundary element method to a wider set of factors. At the same time, in all these works [6–18] it was not succeeded to combine all the important factors in a single model. It was also failed to develop a general numerical solution method suitable for different types of problems. In addition, the micromechanical material structure may be taken into account, as shown in [9–21]. Thus, in [9, 12–15] the main attention is paid to models of adhesion contact. Along with that, fundamentally new methods of research are not proposed in these works. Instead, in [10, 11, 16, 17, 21] new methods of contact interaction analysis are proposed. However, they are formulated for common cases. The papers [18–20] describe both new approaches and developed models for contact interaction analysis, but there is no analysis of the features of distribution of contact forces between individual interacting elements. Also, it is possible to take into account various types of joints (in particular bolted or welded), which are considered in [22–27]. As a rule, the finite element method (FEM) is used for the numerical im-

plementation of functionals, as shown in [28, 29]. In practice, modern software systems such as ANSYS, Femap, Abaqus, etc. are used for this purpose. In addition, alternative variational statements were developed within the variational inequality theory. For example, Kalker's principle and its modifications set out in [5, 17, 18, 30]. However, the paper [5] contains only the general formulation of the Kalker's variational principle for elastic bodies. The paper [17] attempted to extend the formulation of this principle for the case of a nonlinear elastic layer between the contacting bodies, in [18] for the case of adhesion contact, and in [30], for the case of roller contact of bodies. But all these works have no general mechanism for taking into account other arbitrary values of a full set of significant factors in a single variational principle.

For sampling of many of the listed problems, the boundary element method is involved.

However, all of the above methods and models do not effectively solve all the problems arising in the SSS research of SD elements. In particular, the described methods and models for the case of multiple contact require significant computing resources.

Thus, a situation occurs when it is necessary, within traditional approaches, to determine compromise ways to solve complex problems.

The situation is clearly evident, for example, in the SSS study of die tooling elements, as can be seen from the analysis of [31–33]. In the models constructed and described in these works, simplified contact conditions for die elements are often used. On the other hand, the general and strict statement requires significant computing resources.

Therefore, it is advisable to develop alternative formulations that should combine advantages of numerical and analytical means for SSS modeling of die tooling elements, taking into account contact interaction.

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## 3. The aim and objectives of the study

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The aim of the study is to develop a numerical and analytical method for studying the stress-strain state of die elements, taking into account contact interaction, which combines high accuracy and efficiency.

To achieve the aim, the following objectives were accomplished:

- to develop a mathematical model of the SSS of system elements, taking into account contact interaction;
- to determine the dependence of contact pressure between bodies with congruent surfaces on the clamping force;
- to investigate the SSS of SD elements based on contact interaction;
- to analyze the results obtained.

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## 4. Mathematical model of the stress-strain state of the body system taking into account contact interaction

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For the SSS analysis of the body system  $\Omega_i$ ,  $i=1, \dots, N_\Omega$ , it is necessary to satisfy the systems of elasticity theory equations [34]:

$$\left. \begin{aligned} \varepsilon_{ij} &= 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i); \sigma_{ij} = C_{ijkl} \cdot \varepsilon_{kl}, \\ \frac{\partial \sigma_{ij}}{\partial x_j} &= 0; \bar{u}/S_u = \bar{u}^*; \bar{\sigma}/S_\sigma = \bar{\sigma}^*; i, j = 1, 2, 3. \end{aligned} \right\} \quad (1)$$

Here  $\vec{u} = \{u_1, u_2, u_3\}^T$  is the displacement vector of points of the contacting bodies;  $\epsilon, \sigma$  are the strain and stress tensors with the components  $\epsilon_{ij}, \sigma_{ij}$ ;  $C_{ijkl}$  is the component of the elastic constant tensor of the material;  $S_u, S_\sigma$  are parts of surfaces on which essential and natural boundary conditions are given;  $\vec{u}^*, \vec{\sigma}^*$  are displacements and loads.

In addition to the classical boundary conditions in the form of equations, the interaction conditions are formed for contact problems in the form of inequalities:

$$u_i^{(\alpha_p)} \cdot v_i^{(p)} + u_k^{(\alpha_q)} \cdot \tau_k^{(q)} \leq \delta_{pq}. \tag{2}$$

Here  $v_s^{(r)}$  is the component of the normal vector to the body surface with the number  $r$ ;  $p, q$  are the numbers of the contacting bodies  $\Omega_p, \Omega_q$ ;  $\delta_{pq}$  is the gap distribution between the bodies with the numbers  $p, q$ .

The relations (2) are written on parts of the surfaces  $S_c$ .

The relations (1), (2) specify local problem formulation.

An alternative is variational formulation based on the variational inequality theory [3, 4]. According to it, the solution of the problem (1), (2) is equivalent to the problem of minimizing the total energy functional  $I$  of the system of bodies on the set  $K$  of distributions  $u$  satisfying (2):

$$u^\Lambda : I(u^\Lambda) \rightarrow \min, \quad u^\Lambda \in K. \tag{3}$$

Here

$$I = \sum_{\alpha} I_{\alpha}, \quad i = 1, \dots, N_{\alpha},$$

where

$$I_{\alpha} = \frac{1}{2} \int_{(\Omega_{\alpha})} \sigma_{ij}(u^{(\alpha)}) \cdot \epsilon_{ij}(u^{(\alpha)}) - \int_{(S_{\sigma}^{\alpha})} \sigma_i^{(\alpha)} \cdot u_i ds. \tag{4}$$

Thus, the search for a minimum quadratic functional on the set  $K$ , which is given by constraints (2), is carried out.

Generally, the problem (1)–(4) is nonlinear for an arbitrary gap distribution  $\delta_{pq}$ . With regard to die tooling elements, this will lead to a change in load  $P_S$  when changing the shape, thickness and dimensions of the stamped part. This stamping force is calculated by the formula [2]:

$$P_S = k \cdot \sigma_{\tau} \cdot L \cdot t, \tag{5}$$

where  $k$  is the bluntness coefficient of matrix edges, complete dies and punches,  $\sigma_{\tau}$  is the shearing stress (characteristic of the workpiece material);  $L, t$  are the perimeter and thickness of the stamped part.

When changing any value in (5), the value of  $P_S$  also changes. Accordingly, for each value of  $P_S$ , it is necessary to separately solve the nonlinear problem of SSS determination, taking into account contact interaction.

Together with the analysis of die designs, in particular, reconfigurable (Fig. 1) [2], it can be seen that the contact of SD parts on coincident plane surfaces is the most common.

This corresponds to zero gap distribution:

$$\delta_{pq} = 0. \tag{6}$$

In the case of FEM sampling:

$$u = \bigcup_{\alpha} u^{\alpha} = \sum_k u_k \varphi_k,$$

where  $u_k$  are nodal values,  $\varphi_k$  are basic functions, the functional (4) has the following form:

$$I = \frac{1}{2} \sum_{i,j} C_{ij} u_i^0 u_j^0 - P_0 \sum_i \rho_i u_i^0 = a(u^0, u^0) - b(u_i^0). \tag{7}$$

Given the contact of die tooling elements on conforming (congruent) surfaces (planes), the conditions of non-penetration of points of one body in another have the following form:

$$u_r - u_s = 0, \quad r, s \in J_c; \tag{8}$$

$$u_r - u_s > 0, \quad r, s \notin J_c, \tag{9}$$

where  $J_c$  is the set of contacting pairs of nodes;  $C_{ij}$  are components of the stiffness matrix of die tooling elements;  $P_0, \rho_0$  is the array of nodal loads on this model;  $u_i^0$  is the array of nodal variables corresponding to the solution of the problem  $I(u_i^0) \rightarrow \min$  under the conditions (8), (9). Giving  $u_i^{\tau}$  as the solution to a similar problem for  $P_S = \tau P_0$ , we note that:

$$I(u_i^{\tau}) = \tau^2 a(u^0, u^0) - \tau b(u^0). \tag{10}$$

Then for the functional in the directions  $u_k$  that do not contain restrictions (9), we obtain the conditions for its minimum:

$$\partial I / \partial u_k = 0, \tag{11}$$

which show the fairness of the condition  $u_k^{\tau} = \tau u_k^0$ .

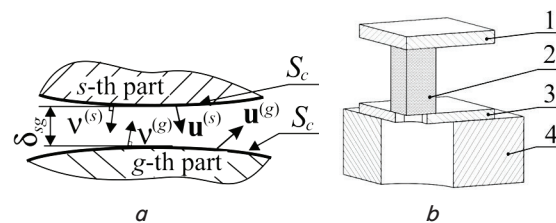


Fig. 1. Contact interaction of elements of shearing dies [2, 28]: *a* – displacement balance in the contact of parts *g* and *s*; *b* – SD design: 1 – upper shoe; 2 – set; 3 – lower shoe; 4 – bolster

In the sections where the restrictions (9) are valid, the solution  $u_k^{\tau} = \tau u_k^0$  due to the automatic upholding of (9) when multiplying all the terms by  $\tau$ , and also under the condition of projection of the minimum  $I$  in the direction  $u_r = u_s$  on the restrictions that coincide with (9). Graphically, this can be presented as shown in Fig. 2.

Fig. 2 introduces the following notation:  $O_0$  is the point of unconditional minimum for  $P_S = P_0$ ;  $T_0$  is the point of conditional minimum for  $P_S = P_0$ ;  $O_{\tau}$  is the point of unconditional minimum for  $P_S = \tau P_0$ ;  $T_{\tau}$  is the point of conditional minimum for  $P_S = \tau P_0$ .

When both the unconditional and conditional minimum is sought, the resulting solutions are bound by proportionality conditions:

$$|O_{\tau} 0| / |O_0 0| = \tau; \quad |T_{\tau} 0| / |T_0 0| = \tau. \tag{12}$$

In addition, the pairs of points  $(u_r^{\tau}, u_s^{\tau})$  either belong to the line  $u_r - u_s = 0$ , or not, and their status does not change when changing  $P_S$ .

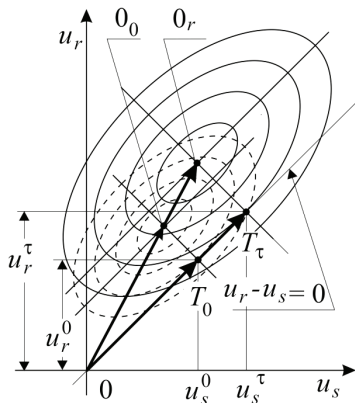


Fig. 2. Level lines of function  $I$  in the section  $u_r-u_s$

**5. Nature of the dependence of the contact pressure between bodies with congruent surfaces on the clamping force**

From the obtained relations we have two consequences:

1) if there is, for the case of contact with conforming surfaces, some solution to the problem of SSS determination, then with the other stamping force, all the components of the solution increase in proportion to  $P_S$ ;

2) there is no change in the set  $J_c$  for  $P_S=var$ , that is, the zone of contact interaction remains, and contact pressure linearly increases.

Taking into account (6), it is possible to formulate the following statement: to solve the set of problems with an arbitrary stamping force it is enough to solve it once with a nominal value. And further, for its arbitrary value, to calculate the contact pressure and SSS components with the linear dependence. This forms the basis of the numerical and analytical method for solving similar problems. It should be noted that it is suitable for the SSS analysis of arbitrary systems of contacting bodies under the conditions (8), (9) in contact areas.

**6. Study of the stress-strain state of die tooling elements**

Consider the shearing die (Fig. 1, 3). The press exerts the stamping force  $P_S$  on it. There are conditions of contact interaction in the interfaces «press bolster-die lower base shoe-replacement set-die upper base shoe» between die elements and guide pins.

Let us consider the problem in two statements. Statement 1 – for a simpler system «replacement set-die lower base shoe-press bolster». Statement 2 – for a more complex system «die upper base shoe-guide pins-die lower base shoe-press bolster».

1. *SSS analysis of elements of the system «press bolster-lower base shoe-set».* The replacement set acts on the lower base shoe, which rests on the press bolster. There are variations in the diameter of the body-size hole in the press bolster – parameter  $p_2$  (within 160–360 mm); the thickness of the die lower base shoe – parameter  $p_1$  (within 15–90 mm). Stamping force  $P_S$  (Fig. 3) – 50 kN, die dimensions – 240×240 mm, set dimensions 100×100 mm. Material of the main parts – 40H steel (elastic modulus  $E=2.1 \cdot 10^5$  MPa,

Poisson’s ratio  $\nu=0.3$ ). Owing to symmetry, 1/4 of the design is considered hereinafter. Fig. 4–7 show characteristic distribution patterns of SSS components of die elements. Analysis of contact pressure and stress distribution indicates that these values are concentrated in angular regions and geometry variation areas. In this case, the shape of the contact zones does not repeat the shape of the initial geometric contact, but is located in the form of a narrow strip on the contact periphery.

2. *SSS analysis of elements of the system «press bolster-lower base shoe-set-guide pins-upper base shoe».* The above results of the studies of Problem 1 are valuable in terms of establishing the variation patterns of SSS characteristics of die tooling elements when varying the individual design and technological parameters or plurality thereof. At the same time, the developed calculation model combines not all the basic die elements. In this regard, it is necessary to conduct the SSS study using a higher (deeper) level model.

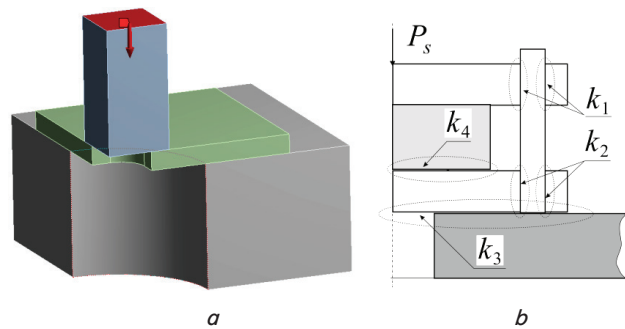


Fig. 3. Elements of the studied technological system: *a* – model; *b* – corresponding characteristic zones of contact interaction  $k_1-k_4$

In particular, the model of the shearing die in the form of the system «press bolster-lower base shoe-set-guide pins-upper base shoe» was constructed for this purpose. This model is shown in Fig. 8, owing to symmetry, 1/4 of the design is considered, the stamping force is 10 kN–1 MN.

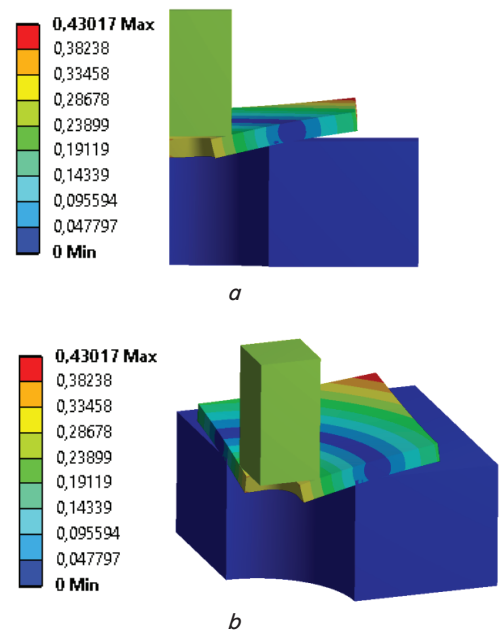


Fig. 4. Distribution of complete displacements in die elements, mm: *a* – side view; *b* – isometry

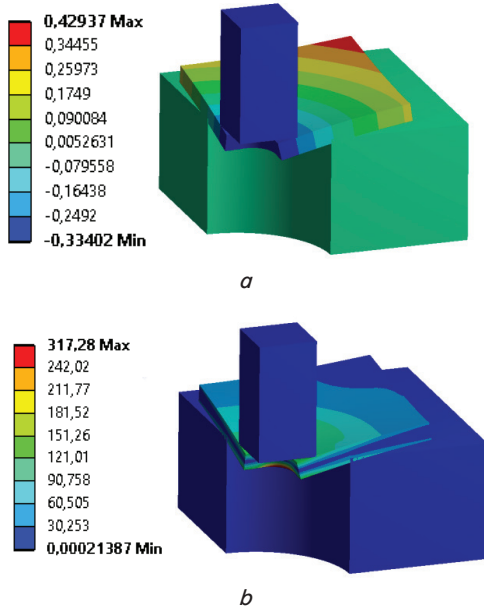


Fig. 5. Results of the SSS study of die elements: *a* – vertical displacement, mm; *b* – von Mises stress, MPa

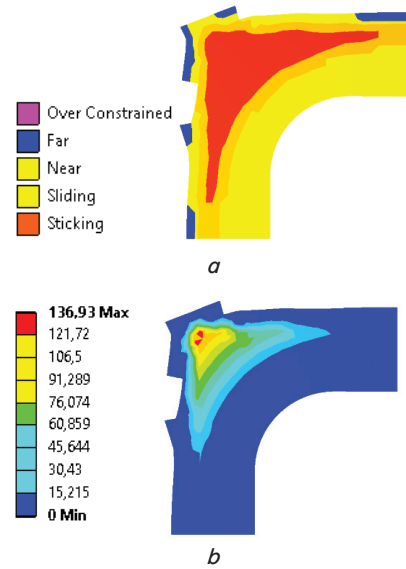


Fig. 7. Results of the SSS study of die elements in the replacement set-base shoe interface: *a* – contact zones; *b* – distribution of contact pressure, MPa

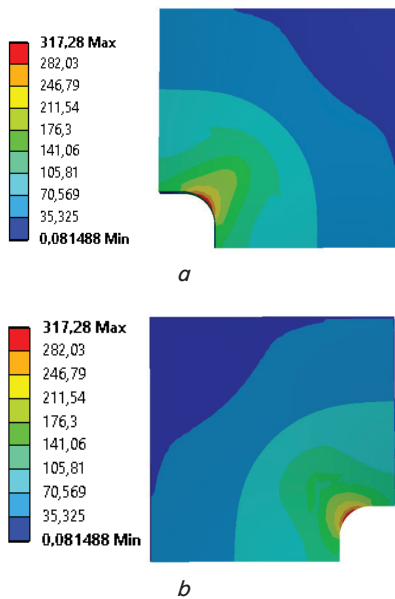


Fig. 6. Distribution of von Mises stress (MPa) in the lower base shoe in the interface plane: *a* – with replacement set, *b* – with press bolster

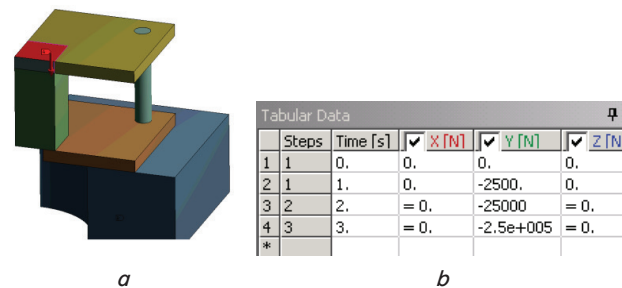


Fig. 8. Shearing die: *a* – model; *b* – load history: «time» = 1 s –  $P_S = 10$  kN; «time» = 2 s –  $P_S = 100$  kN; «time» = 3 s –  $P_S = 1$  MN

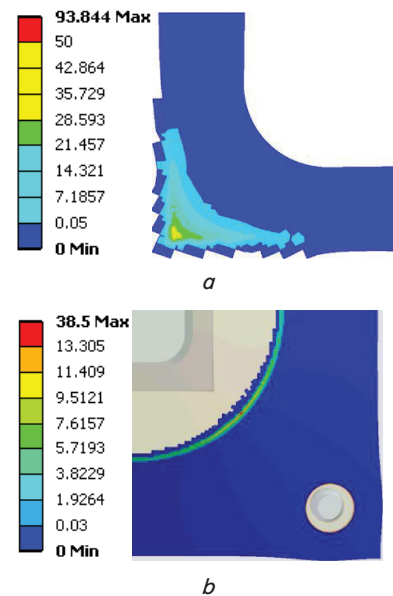


Fig. 9. Distribution patterns of contact pressure (MPa) in the zones: *a* –  $k_4$ ; *b* –  $k_3$  ( $P_S = 10$  kN)

Fig. 9 shows the distribution diagrams of contact pressure in the zones  $k_3, k_4$ . From the given distributions it can be seen that the established patterns, namely regarding the substantially uneven distribution of contact pressure in the interfaces of die tooling elements, are fully confirmed. The same can be said about the distribution of complete displacements, von Mises stresses (Fig. 10). From the whole system, it is possible to separate SSS patterns of separate elements (Fig. 11–15).

These patterns also give grounds for confirming the predicted features of the SSS, based on partial models: about stress concentration in the zones of contact interaction of individual die elements and significant levels of elastic displacement.

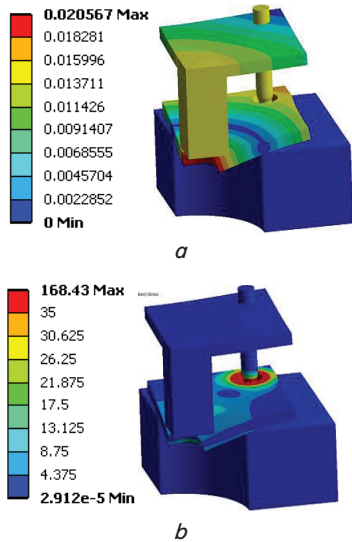


Fig. 10. Distribution patterns of SSS components ( $P_S=10$  kN):  $a$  – complete displacement, mm;  $b$  – von Mises stress, MPa

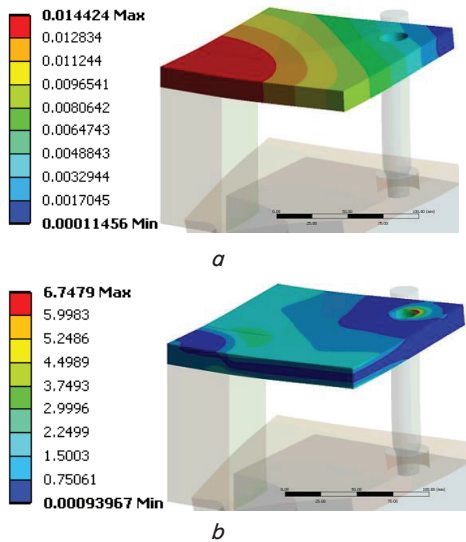


Fig. 11. Distribution patterns of SSS components in the upper base shoe ( $P_S=10$  kN):  $a$  – complete displacement, mm;  $b$  – von Mises stress, MPa

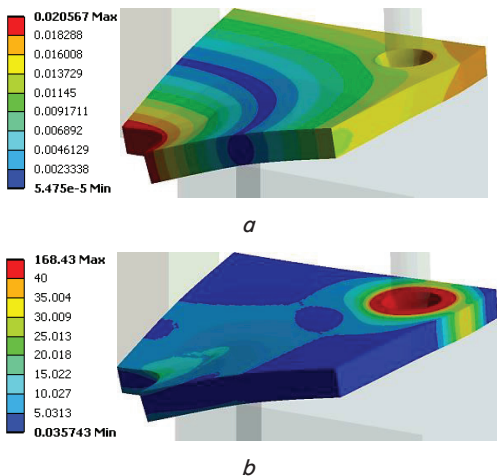


Fig. 12. Distribution patterns of SSS components in the lower base shoe ( $P_S=10$  kN):  $a$  – complete displacement, mm;  $b$  – von Mises stress, MPa

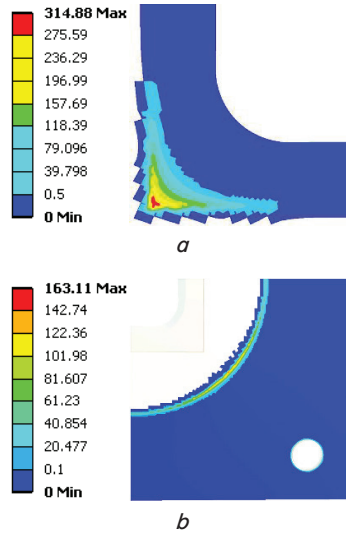


Fig. 13. Distribution patterns of contact pressure in the zones:  $a$  –  $k_4$ ;  $b$  –  $k_3$  ( $P_S=100$  kN)

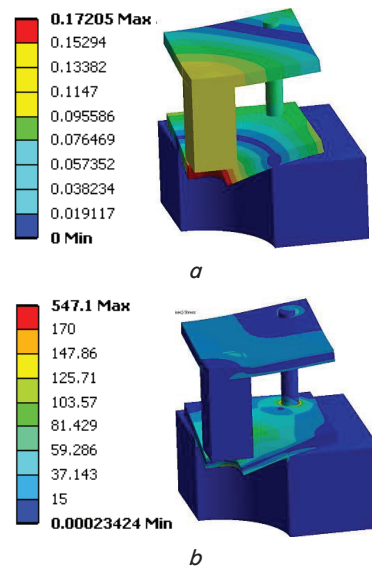


Fig. 14. Distribution patterns of SSS components ( $P_S=100$  kN):  $a$  – complete displacement, mm;  $b$  – von Mises stress, MPa

Thus, we can conclude that a qualitative distribution pattern of the SSS components can be analyzed by the results of studying a more complete die model. At the same time, the quantitative dependencies of the SSS characteristics on the variable parameters can be determined by the calculation results with the use of partial models, which are less cumbersome. Therefore, these dependences require less computing resources for simulation. As a result, a balance between the accuracy of the results obtained and complexity of the developed models is ensured.

The obtained results provide the basis for analyzing the distribution patterns of contact pressure of die tooling elements, specified in [28, 35]. This applies to mechanical systems in which the primary contact on so-called «conforming» (congruent) surfaces is realized. The weak dependence of the area of the contact zones on the magnitude of the acting forces is characteristic. In addition, contact pressure is proportional to the acting forces.

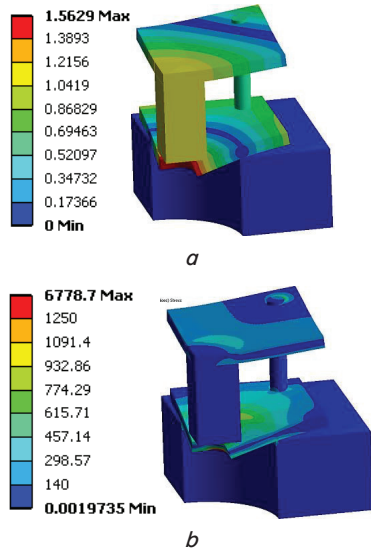


Fig. 15. Distribution patterns of SSS components ( $P_S=1$  MN): *a* – complete displacement, mm; *b* – von Mises stress, MPa

**7. Discussion of the results of the study of contact pressure distribution between elements of shearing dies**

The results of the complex research of elements of shearing dies are the basis for determining the characteristics of contact pressure distribution between elements of the investigated dies, as well as components of their SSS. This is, in particular, the independence of the contact zones and laws of contact pressure distribution, as well as SSS components, of stamping forces. In addition, the contact pressure and SSS components are directly proportional to the stamping force  $P_S$ .

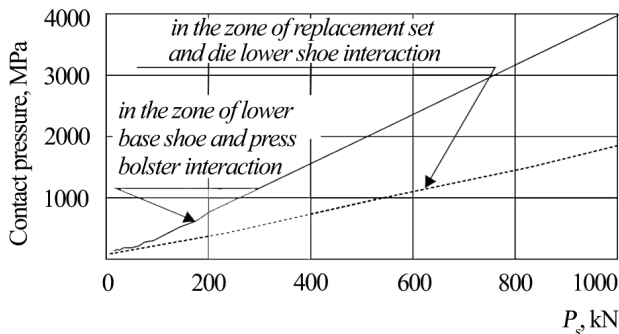


Fig. 16. Dependence of the maximum contact pressure  $p_{max}$  (MPa) on the stamping force  $P_S$  (kN) in the interfaces  $k_3, k_4$  (Fig. 3)

At the same time, these and other qualitative features in solving specific applied problems require quantitative illustration. Fig. 16–20 give the following integral dependences obtained by processing the above results. Fig. 16 shows the dependence of the contact pressure level on the forces  $P_S$ . It is evident that the specified property of contact pressure is observed with a satisfactory accuracy (the error is up to 5%). This is a very important fact, since the SSS at any stamping force can be obtained from the results of a single calculation for a given  $P_S$  by linear dependencies. Fig. 17–19 show the dependences of deflections, pressure and stress on the lower shoe thickness and body-size hole diameter. It can be seen that the

design and technological factors have a significant impact on rigidity and strength characteristics of die elements.

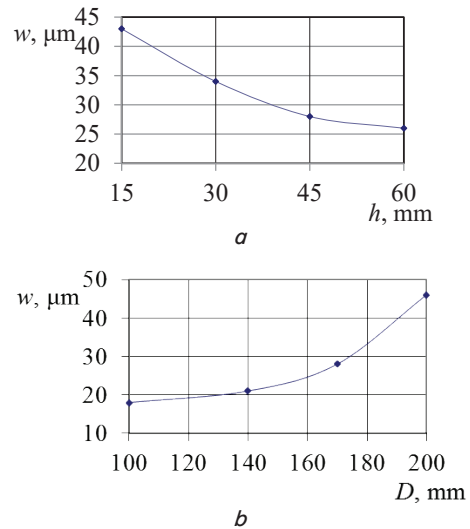


Fig. 17. Dependence of the maximum deflections in the base shoe  $w, \mu\text{m}$ : *a* – with different thicknesses of the lower base shoe  $h$  ( $D=170$  mm); *b* – when varying the body-size hole diameter ( $h=45$  mm)

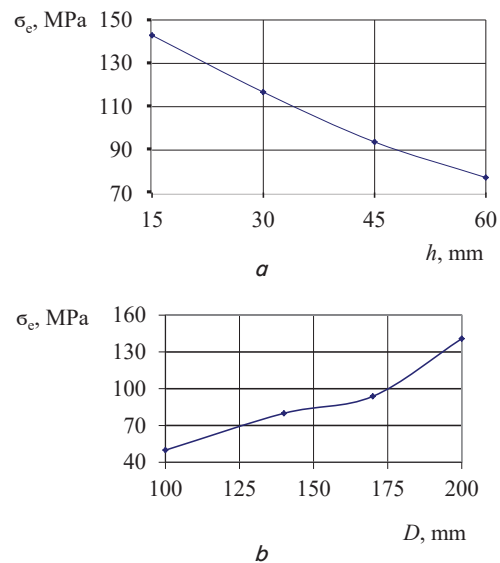


Fig. 18. Dependence of the maximum equivalent (von Mises) stress in the base shoe  $\sigma_e, \text{MPa}$ : *a* – with different thicknesses of the lower base shoe  $h$  ( $D=170$  mm); *b* – when varying the body-size hole diameter ( $h=45$  mm)

If we take into account absolute values of restrictions on deflections of the base shoe and stress level, then its parameters should be chosen from the conditions of a specific problem. Thus, the level of deflections of  $30 \mu\text{m}$  is ensured with the shoe thickness  $h=45$  mm and body-size hole diameter  $D=160$  mm. The stress level up to  $100 \text{MPa}$  is provided by  $h=45$  mm and  $D=175$  mm. Proceeding from the sensitivity of rigidity and strength characteristics of die elements to the change in variable parameters, the recommended ranges are: for deflections –  $h \geq 45$  mm and  $D \leq 160$  mm, and for stresses –  $h \geq 45$  mm and  $D \leq 175$  mm. There is a significant but

rather smooth and linearized dependence of pressure on these parameters. Pressure increases with the body-size hole diameter. With an increase in the thickness of the lower base shoe, pressure decreases.

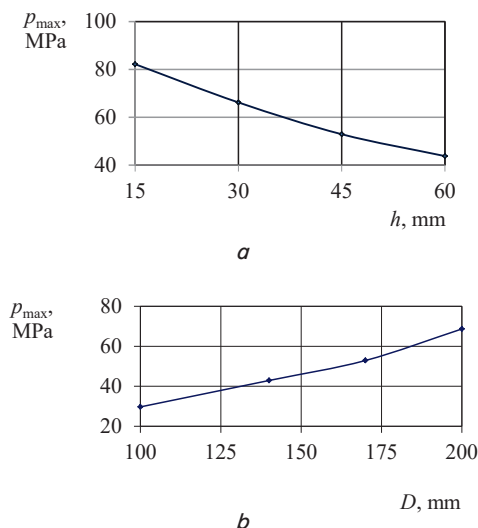


Fig. 19. Dependence of the maximum contact pressure in the zones of contact interaction of elements of the studied system «base shoe-bolster»  $p_{max}$ , MPa: *a* – with different thicknesses of the lower base shoe  $h$  ( $D=170$  mm); *b* – when varying the body-size hole diameter ( $h=45$  mm)

The dependencies of controlled strength and rigidity characteristics of die elements on individual parameters can be generalized to the case of dependencies on a set of parameters (Fig. 20).

These dependencies also allow justifying a set of parameters that provide certain strength and rigidity characteristics of the dies. At the same time, this gives grounds for concluding that it is possible to fill in specialized databases that for certain separation dies allow to promptly and precisely solve two types of problems at the design stage. First, it is an opportunity to determine strength and rigidity characteristics of die elements with a given set of design and technological parameters.

Secondly, it is an opportunity to justify such a set of design and technological parameters, which provides the necessary level of strength, durability, rigidity and accuracy of cold-stamping shearing dies.

As a result, the general approach, methods, models and means of design studies of die tooling elements that combine, in contrast to traditional, efficiency and accuracy, are developed. This is due to the feature of the investigated objects, for which the linear dependence of contact pressure on die loads is valid. However, it should be noted that the determined patterns are valid only for systems of contacting bodies with congruent surfaces. It is also important that the load has only one component (in the studied case, force  $P_S$ ). Therefore, it is expedient to consider multicomponent load in further research.

### 8. Conclusions

1. The complex mathematical model of the SSS of die tooling elements is developed, taking into account multiple contact interaction. The model is based on the variational inequality theory, and the finite element method is used for sampling. This allows, in contrast to traditional approaches, considering not two but more parts in a single set of contacting die elements, that is, improving the accuracy and adequacy of mathematical modeling of SSS.

2. For the system of elements of shearing dies contacting on conforming surfaces, the validity of linear dependence of the level of contact pressure on the stamping force and the invariance of the contact area during their variation is determined. The determined patterns allow, in contrast to traditional approaches, carrying out not a single solution of the problems of SSS analysis, but a group solution for a whole series of materials, thickness and parameters of stamped parts.

3. On the example of SSS research of elements of shearing dies, the influence of accounting of multiple contact on the SSS of dies is demonstrated. In particular, it was revealed that in the case of multiple contact of die tooling elements on conforming (congruent) surfaces, contact areas do not depend on stamping force, and pressure varies directly with the stamping force. SSS components of die tooling elements vary in the same way.

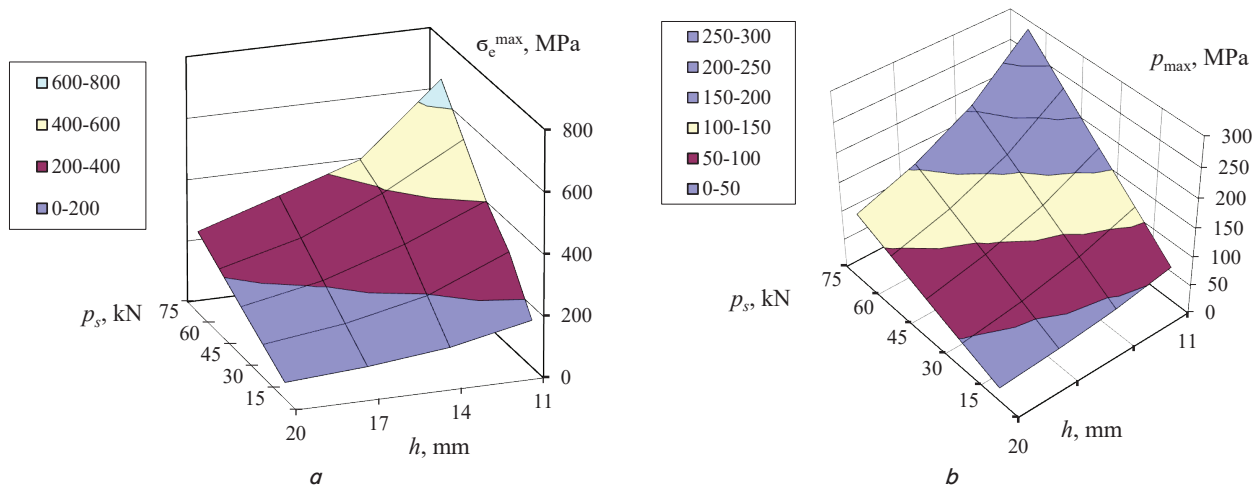


Fig. 20. Results of the study: *a* – maximum von Mises stress in the base shoe  $\sigma_e$ , MPa; *b* – maximum contact pressure in the zones of contact interaction of elements of the studied system «base shoe-bolster»  $p_{max}$ , MPa



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