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Investigation of Prospective Primary Mathematics Teachers' Perceptions and Images for Quadrilaterals

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Abstract

The object of this study was to show how prospective elementary mathematics teachers define and classify the quadrilaterals and to find out their images. This research was a qualitative study. It was conducted with 36 prospective elementary mathematics teachers studying at 3rd and 4th years in an educational faculty. The data were collected by semi-structured interview. The collected data were analyzed by inductive content analysis. It was determined that prospective teachers made personal definitions for quadrilaterals and based on these definitions it was seen that there were inaccuracies in their perceptions. Some prospective teachers' images in rhombus caused problem on indicating the differences between rhombus and square. It was revealed that the prospective teachers drew defectively, especially rhombus and trapezoid figures. Furthermore, the prospective teachers did not prefer inclusion relations for the classifying quadrilaterals and made partition classifications, in general.

Key Words

Perception of Quadrilateral, Image of Quadrilateral, Classification of Quadrilateral, Prospective Primary Mathematics Teachers.

Teaching geometry is as many researchers stated (Baykul, 1999; Duatepe, 2000; Fujita & Jones, 2007) not only means of comprehending information and relations about point, line, figures, space but also important in the sense of improving spatial thinking and visual skills.

In the education of many subject in mathematics, people form an image in their mind about the concepts. This concept image changes and takes shape in time. Tall and Vinner (1981) defined concept image as the cognitive structure which includes mental image, features and processes about the concept. There are 3 different cases in geometrical concepts. These are the image, definition and properties of the geometric concept (De Villiers, 1998; Fujita, 2012).

According to Tall and Vinner (1981), whether definitions of concepts are directly taught or students construct these definitions; individuals filter these definitions in their minds. These personal definitions may differ from formal definitions. These personal definitions of concepts may cause people to create their own concept images. Even according to Heinze and Ossietzky (2002), the academic definition of the concept is forgotten while solving problems and personally created concept image comes into prominence.

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In addition to this, visual image of every geometric concept may be in the forefront rather than the concept. In this sense typical (prototype) examples are the key factor. These prototype shapes always have an effect in concept image (Fischbein, 1993; Hershkowitz, 1990). As a result of many studies, Fujita (2002) have stated that definition of the geometric figure and characteristics of family relation which include this figure generally cause contradiction. This contradiction results in wrong perception and generalizations with the conceptual perception of prototype figure (Fujita; Fujita & Jones, 2006; Hershkowitz).

There are various studies nearly for every age group about how the geometric figures are perceived, how the figurative display and conceptual relation of the figures are imprinted in one's mind. In this sense, this study aims to present how the elementary prospective teachers define and classify quadrilaterals and what the images related with quadrilaterals are.

When the literature is analyzed, there are studies in which people are expected to name drawn quadrilaterals or mark the quadrilaterals whose names are given (Clements, Swaminathan, Hanibal, & Sarama, 1999; Fujita, 2012; Fujita & Jones, 2007; Monaghan, 2000; Okazaki & Fujita, 2007). However, as the case is so in the study of Okazaki and Fujita, it was stated that there is a problem in defining figures drawn in different ways rather than typical drawings. In a younger age group, similar images about quadrilaterals were observed which resembles to the study of Okazaki and Fujita. For example in a study carried out with kindergarten children, nearly all the students participated in the study asserted that a diamond shaped square is not a square and stated that since it is a slanted square, it cannot be named as square (Dickson, Brown, & Gibson, 1984).

Usage of typical drawings of geometric concepts plays an important role while making conception. However, these formal displays (drawings) cause typical generalizations in making conceptions. It is possible to gather this case in two types of generalizations (Hershkowitz, 1990):

Type 1: Generalization of visual judgment based on typical examples to the other cases (which are not compatible).

Type 2: Use prototype's self-properties in making deduction or decision and impose these self-properties in other types of concept.

In many studies, it was determined that students in different age groups cannot exactly state the relationship between quadrilaterals (De Villiers, 1998; Elia, Gagatsis, Deliyianni, Monoyiou, & Michael, 2009; Fujita & Jones, 2007; Heinze & Ossietzky, 2002; Okazaki & Fujita, 2007).

In the studies carried out on quadrilateral, there are also researchers on how the students define quadrilaterals. According to Okazaki and Fujita (2007), perception about quadrilaterals are directed by typical (prototype) figure image, characteristics about figure and unclear characteristics caused by the prototype image, therefore personal definitions different from academic definitions can be seen.

Classification of polygons is important in making relations between polygons and also in the solution of geometry problems and making proof. In this sense, it is pointed out in the classification of DeVilliers (1994) that people can make two types of classifications. These are hierarchical and partition classifications.

As various researchers have emphasized hierarchical classification supports the formation of their education (De Villiers, 1994, 1998; Fujita, 2012). However, in the studies carried out on quadrilaterals in different age groups, it was determined that most of the participants used partition classification (Berkün, 2011; Erez & Yerushalmy, 2006; Fujita & Jones, 2006; Monaghan, 2000).

Method

Research Design

Phenomenological method, which is one of the qualitative researches, was chosen. The reason for choosing this study design is that as it can be understood from the definition phenomenological design is used for the analysis of the phenomena which are not completely foreign to us but yet cannot be completely understood. Phenomenology also aims to explain how a concept is perceived by participants through the perspective of that phenomena (Creswell, 1998). Phenomenological design was chosen in this study since it was aimed to analyze definition, drawings and differences of prospective teachers in definition and drawing of quadrilaterals and the reasons of these differences.

Participants

The study was carried out with 36 prospective elementary mathematics teachers; 18 of whom were at the 3rd grade and 18 of whom were at the 4th grade in their faculty. The reason for choosing the study group especially among 3rd and 4th grade was that they had mastered in the program to have been taught and they

had nearly completed their knowledge equipment. These prospective teachers were chosen with maximum variation sampling. Since the variation of individuals is important in maximum variation sampling, selection was done according to the success and grade differences of prospective teachers.

Data Collection Tool

Data of the study were collected via semi-structured interview form. The questions prepared for the interview were composed of two sections. In the first section there were questions about how prospective teachers described definitional and graphical characteristics of the quadrilaterals, in the second section there were questions to present relations between quadrilaterals. Pilot study of the interview was conducted. After the pilot interviews, three mathematics educators who carried out the study decided on the final form of interview questions. Before the interview, the researchers informed each prospective teacher that the interviews would be recorded. Apart from the data obtained from interview, drawings of prospective teachers during interview were used in the analysis.

Analysis of Data

Content analysis technique was used in the analysis of data obtained from the study. Analysis of data was made in three stages. In the first stage, analysis of data obtained as voice record was done and data analyses were controlled by two researchers. There was 85% agreement between independent coding of researchers. Since the percentage of agreement should be 70% or above, reliability was enabled in the sense of data analysis (Türnüklü, 2000).

Validity and Reliability

Specific measures were taken in order to increase validity and reliability of the study. While interview form was prepared in order to increase internal validity of the study, literature was also taken into consideration. The data obtained from voice record were transcribed into written form and data were controlled by the researchers. Integrity was provided by controlling topics obtained from data and the relation of sub-topics that form the topic and the relation of each topic with another. In order to increase internal reliability of the study, the findings were presented directly without any comment. The data were coded separately by the researchers and percentage of agreement was calculated after data were gathered.

In order to increase external reliability of the study, research model, study group, data collection tool, data collection process, collection, analysis and interpretation of data were made in details.

Results

In the study, regarding the perception of prospective teachers about quadrilaterals, data were analyzed and studies under 3 main titles as definition of quadrilaterals, drawing of quadrilaterals, and classification of quadrilaterals.

Definition of Quadrilaterals

In the study, prospective teachers were asked to define square, rectangle, parallelogram, rhombus and trapezoid. While correct statement of definition were important in academic sense, personal definitions directed by conceptual perception were also significant. In this sense, academic definition; making correct, imperfect or wrong personal definitions have been important in determining images of prospective teachers about these concepts.

Definitions of required and sufficient properties of quadrilaterals which have no error were accepted as "academic definition". The group of "lacking definition" is the group which cannot totally state the required and sufficient definition of the polygon and does not include error. "Definitions made with extra properties" were evaluated apart from academic definition group. Because making definition by giving every property known about polygons shows that the polygon is not completely conceptualized but should be evaluated separately.

In order to understand categorization of quadrilateral definitions, examples were given for each definition of quadrilateral in table 1.

In the definitions, prospective teachers used statements such as polygons, geometric figure, figure, solid figure apart from quadrilaterals. While polygon, geometric figure and figure is accepted as correct academically, solid figure is not accepted within quadrilateral definitions and was evaluated as wrong definition. Prospective teachers were mostly succeeded in the definition of square (53%) and then rectangle (33%). The most number of wrong statements in academic definition were done for trapezoid (33%).

When definitions of prospective elementary mathematics teachers were evaluated in details, it is seen that some of the significant perceptions are reflected to definitions. In this sense, first of all personal

definitions about square for which academic definitions were mostly made was evaluated and properties of having four sides and equal sides stand out. It is remarkable that some of the prospective teachers stated only one of the properties of sides and angles of the square and regard it adequate for the definition. This situation points out to the fact that they do not include necessary properties about square to their perception. Moreover definition of students such as "it is a figure with four equal sides" shows that they make definition with typically drawn square image and the property of angle is not included within this image.

When rectangle definitions of prospective elementary mathematics teachers are evaluated, it is seen that they make definition stating properties of sides and angles like in definition of square. In this definition, different from square, equality of opposite sides reflects to definitions naturally. Statement of one of the participant, "it has along and a short side", was notable. Because considering in this perspective, square would not be included within quadrilateral family for this participant. Moreover this shows that participant considers typical drawing of the rectangle and makes wrong generalization. This case corresponds to first type misjudgment of Hershkowitz (1990).

When definitions of rhombus are evaluated it was observed that 40% of prospective elementary mathematics teachers could make academic definition. For the other personal definitions, they have generally defined rhombus with the statements of "a figure with 4 equal sides". Generally properties of angles were not reflected to statements. This case shows that teachers generally cannot separate rhombus image from square image. Moreover there are definitions which directly points out square. This case corresponds to second type misjudgment of Hershkowitz (1990).

There are also definitions such as "figure or quadrilateral which is formed with integration of two

Table 1.
Examples for Quadrilateral Definitions

	Academic Definition	Lacking Definition	Definition with extra properties	Wrong Definition
Square	Square is a quadrilateral whose sides have equal length and angles between sides are 90 degrees. (A.A:3,16, 3th Grade)	Square is a quadrilateral which has four sides and angles of sides are 90 degrees. (A.A:2,40, 3th Grade	4 line segments of square must be intersected vertically, all the lengths should be equal, lengths of the sides should be equal and composed of line segments with 90 degrees. For examples, diagonals divide into 2 equal pieces. They are divided into 2 equal pieces with 45 degrees. (A.A:2,64, 3th Grade)	Square is a solid figure whose all sides are equal and every angles are 90 degrees. (A.A:2,12, 4rd Grade)
Rectangle	It is a quadrilateral whose opposite sides are equal and perpendicular. (A.A:2,40, 3th Grade)	A quadrilateral whose opposite sides are equal. (A.A:3,08, 4rd Grade)	Rectangle is a quadrilateral whose internal angles are 90 degrees and opposite sides are parallel and equal. (A.A:1,88, 3th Grade)	Rectangles are closed solid figures whose opposite sides are equal and perpendicular. (A.A:2,71, 3th Grade)
Rhombus	A geometric figure whose opposite sides are parallel and four sides are equal. (A.A:1,88, 3th Grade)	A geometric figure whose all sides are equal. (A.A:2,25, 4rd Grade)	Sides of rhombus are equal. However all the angles are not 90 degree. Opposite sides are equal. And opposite diagonals form angle bisector. Let's say it divides angles equally into two. Sum of interior angles is 360 degrees. In other words, it is a quadrilateral whose four sides are equal to each other. (A.A:3,06, 4rd Grade)	It is a closed solid figure with four sides formed with the integration of two isosceles triangles. (A.A:2,71, 3th Grade)
Parallelogram	Parallelogram is a quadrilateral whose opposite sides are parallel and equal. (A.A:3,45, 3th Grade)	It is a quadrilateral whose two opposite sides are equal and angles are not 90 degrees, whose internal angles are different. (A.A:3,08, 4rd Grade)	A quadrilateral sum of whose opposite angles are 180 degrees and opposite sides are equal and opposite. (A.A:2,45, 3th Grade)	Parallelogram is a solid figure whose opposite sides are equal and parallel. (A.A:1,88, 3th Grade)
Trapezoid	Opposite sides are parallel but other two sides does not have to be parallel, this geometric figure is called trapezoid. (A.A:2,25, 4.Class)	Quadrilaterals whose internal angles are 360 degrees and length of sides are not equal to each other. (A.A:2,49, 4rd Grade)	There are two sides of trapezoid; isosceles trapezoid and right trapezoid. Only the sides below and above are parallel, other sides can either be equal, or one side can be perpendicular or not equal to each other. There are three types of trapezoid. (A.A:2,12, 4rd Grade)	Trapezoid is a geometric figure which has 5 sides. (A.A:2,07, 4rd Grade)

isosceles triangle or two equilateral triangle" about definitions of rhombus. Although these are not academic definitions, they do not lead to a structure different from rhombus in theory. However, this may cause restrictions in presenting specific properties of rhombus. In a rhombus formed with two equilateral triangle individuals may be mistaken in the sense of angle properties. Because in this structure there is only specific type of angles 60^0 and 120^0. Moreover some of the students have made definition of deltoid by stating that rhombus is formed by the integration of 2 isosceles triangle and these definitions were evaluated as wrong definitions.

When parallelogram definitions of prospective elementary mathematics teachers are evaluated, "quadrilaterals whose opposite sides are parallel" was generally a common definition. In some definitions, statement of "equal opposite sides and/or angles" was also made. In fact equal opposite angles or sides are the properties which guarantee parallelism of parallelogram. In this sense, answers of prospective teachers are successful in the sense of parallelogram perception. It was implied that it was not clear for some of the prospective teachers that square and rectangle are also parallelograms. For example, the statement of parallelogram as "special form of square and rectangle" reflects that relation between family classes was not constructed correctly. Another prospective teacher stated that "but in this case opposite sides should be equal not all 4 sides" which means that although there is no problem in the perception at first glance, it causes him to exclude square from parallelogram class. In another definition giving the statement about angles of parallelogram that "it does not have to be 90^0" shows that borders of family of parallelogram were drawn correctly.

When personal definitions apart from academic ones about trapezoid were evaluated, it is seen that in fact prospective teachers who participated in the study do not have a trapezoid perception with a specific characteristic.

Drawings of Quadrilaterals

Prospective elementary mathematics teachers who participated in the study were asked to draw square, rectangle, parallelogram, rhombus, and trapezoid. They were asked to draw 2 more of each figure which is different from the first ones and asked why they thought they were different.

Quadrilaterals drawn by prospective teachers were categorized as correct or wrong drawing by their appearance. Drawings which are classified as correct were regarded in two groups according to paying attention to notation or not. Prospective teachers who have drawn correctly by considering notation were expected to draw the figure correctly as a shape and also show necessary notations from parity of side lengths, parity of angles, right angles and parallel sides. Prospective teachers who draw correct figure formally without considering notation were accepted.

Only three prospective teachers drew rhombus wrongly while every participant drew all the other quadrilaterals correctly. However, it was determined that most of the prospective teachers drew figure without considering notation, in other words without using suitable mathematical indications such as equivalence of angles and sides, parallelism.

When all the drawings were evaluated, it was seen that rectangle and parallelogram were drawn in the way stated in definitions. It was not expected to draw trapezoid correctly. Because prospective teachers have generally wrong definitions for trapezoid among quadrilaterals. Although most of the prospective teachers emphasized in their definitions that trapezoid is a quadrilateral which is drawn without a specific rule, it is remarkable that they have paid attention to the parallelism of 2 sides. This situation shows that prospective teachers perceive trapezoid as a whole as visual image and envisage as such. Being unable to make a definition or making wrong definitions show that they cannot differentiate properties of this figure. This case corresponds to visual level which is the first level of Hiele's levels of geometric thinking. Another point which draws attention to trapezoid drawings is that although they do not indicate isosceles trapezoid with notation, most of them draw it as isosceles trapezoid. This shows that trapezoid images of prospective teachers are in the shape of isosceles trapezoid. This may result from the fact that teachers prototypically draw trapezoid as isosceles trapezoid at school.

It was determined that 70% of prospective teachers typically drew rhombus in diamond shape. There are also drawings which are in format of rotated square. These drawings, as mentioned before, can be proof for why the definition of rhombus is mixed up the definition of square. A rhombus which is drawn as rotated square would lead to wrong generalization that all rhombuses would have the properties of square as an image. This case, as stated before, corresponds to second type misjudgment of Hershkowitz (1990).

There are false drawings only for rhombus among the quadrilaterals. Three prospective teachers who participated in the study draw rhombus as kite.

It was determined that prospective teacher used prototype drawings for square and parallelogram. It was

observed that they drew rectangle in a way that long side would double the short side. Some prospective teachers even stated this case with notation. This case shows us that prospective teachers generally have a rectangle image in their mind in a way that long side would double the short side. This may result from the fact that teachers draw rectangle prototypically at school and do not use different orientations.

In the study prospective teachers were expected to draw three quadrilaterals as different from each other. Second and third drawings were compared with the first one and it was determined on which property they create the difference. In this sense, different drawings were made by differentiation of measurements such as length of sides and angles; difference in the sense of orientations (for example rotated forms of rectangle) and family relations.

It was observed that while prospective mathematics teachers made different drawings generally according to side lengths and then orientations; they did not reflect family relations to their drawings much.

The prospective teachers who draw quadrilaterals in different orientations show that they can go beyond typical prototype figures of quadrilaterals. The prospective teachers who draw quadrilaterals considering angle and length of sides show that they cannot go beyond prototypical images. Teachers who made drawings considering family relations can be thought to have high-level perception about quadrilaterals.

Classification of Quadrilaterals

In the study prospective teachers were expected to classify quadrilaterals. Prospective teachers made classifications according to a specific property such as angle property in the sense of perpendicularity (22 of participants) and equality (14 of participants), equality of sides (19 of participants), property of parallelism (10 of participants), perpendicular intersection (32 of participants), intersection in the mid-point (9 of participants) and length (15 of participants) or according to family relations (4 of participants). There are prospective teachers who have not made any classification as well (5 of participants).

Prospective teachers who made classification according to properties made listing such as those which have or not have right angle or those whose side lengths are equal or not. In this types of classification, quadrilaterals were considered according to single property, multiple classification such as both having right angle and equal sides was not done. Such kind of classifications can be regarded within the group of partition classification, based on the classification of De Villiers (1994).

Some of the prospective teachers who made such kind of partition classification had misjudgments. One of these misjudgments results from the thought of parallelogram cannot have the properties of square and rectangle. One prospective teacher puts parallelogram in the category of those whose side lengths are not equal and differentiated it from square. The same perception was observed for rectangle as well. This case was observed in definitions and drawings as well. It is generally results from the fact that typical parallelogram drawing does not correspond with square and rectangle perception of this image perceived as a whole in mind.

It was observed that classification made according to family relations was not preferred among participants. Such kind of classification corresponds to hierarchical classification of De Villiers (1994). Classification in Figure 1 can be shown as example of this case.

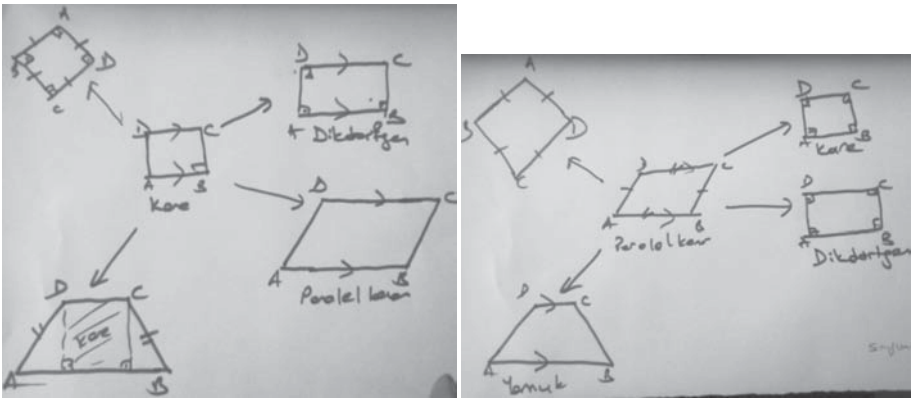

Figure 1.
Classification According to Family Relations (A.A:2.57, 4th Grade)

There are specific misjudgments in these classifications. One of them is the place of trapezoid which was also problematic about definition. In the figure above, trapezoid was related with square and parallelogram in a wrong way. When data of the study was evaluated it is clear that there are specific problems about trapezoid. Although it is correct in the sense of drawing, it was determined that there are misjudgments about properties and definition. This case is also observed in classification. One of the prospective teacher have even used trapezoid in two different categories and there was a rhombus, square in one of them and parallelogram, rectangle in the other.

Discussion

This study was conducted with 36 elementary prospective mathematics teachers who receive education at 3rd and 4th grade at an education faculty. It cannot be concluded that prospective teachers who participated in the study have many mistakes about quadrilaterals or their knowledge is incomplete. However, perceptions which seem correct at first glance cause different formation of other perceptions and this situation may result in misjudgments.

Personal definitions are shaped by elementary prospective mathematics teachers' perceptions. It is seen that personal definitions are more common compared to academic definitions in this study, as it is so in many other studies (De Villiers, 1998; Tall & Vinner, 1981; Vighi, 2003).

This study shows that there are many misjudgments about trapezoid. This problem about trapezoid is mentioned in restricted number of studies (Berkün, 2011). It was determined that trapezoid is drawn randomly and does not include a specific property. It can be studied whether this situation is caused by the semiotic structure of the word "trapezoid".

It is seen that prospective teachers who participated in the study generally use partition classification while classifying quadrilaterals. Side and angle properties are prominent in partition classification. Although this finding is normal based on Hasegawa (1997), in this study it was expected that participants could make hierarchical classification more commonly and form family relations correctly since the study group is at university level and prospective mathematics teachers. However, findings were not as expected and are identical with the findings obtained in other studies (Berkün, 2011; De Villiers, 1994; Monaghan, 2000).

Although this study was carried out with restricted number of elementary prospective mathematics teachers, the existence of perceptions about quadrilaterals points out to the fact that this perception exists in other individuals. However, in this frame, a research with larger and different study groups can be suggested in order to determine the cognitive process.

References/Kaynakça

Baykul, Y. (1999). *İlköğretimde matematik öğretimi*. Ankara: Anı yayıncılık.

Berkün, M. (2011). *İlköğretim 5 ve 7. sınıf öğrencilerinin çokgenler üzerindeki imgeleri ve sınıflandırma stratejileri*. Yayımlanmamış yüksek lisans tezi, Dokuz Eylül Üniversitesi, İzmir.

Clements, D. H., Swaminathan, S., Hanibal, M., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education*, *30* (2), 192- 212.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: Sage Publications.

De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For The Learning of Mathematics*, *14*, 11-18.

De Villiers, M. (1998). To teach definitions in geometry or teach to define? In A.Oliver & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, 2,* 248-255.

Dickson, L., Brown, M., & Gibson, O. (1984). *Children learning mathematics: A teacher's guide to recent research*. London: Cassel Pub.

Duatepe, A. (2000). *An investigation of the relationship between van hiele geometric level of thinking and demographic variable for pre-service elementary school teacher.* Yayımlanmamış yüksek lisans tezi, ODTÜ, Ankara.

Elia, I., Gagatsis, A., Deliyianni, E., Monoyiou, A., & Michael, S. (2009). A structual model of primary school students' operative apprehension of geometrical figures. In M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of The 33rd Conference of the Internatıonal Group for the Psychology of Mathematics Education, 3,* 1-8.

Erez, M. M., & Yerushalmy, M. (1996). "If you can turn a rectangle ınto a square, you can turn a square ınto a rectangle ..." young students experıence. *International Journal of Computers for Mathematical Learning, 11*, 271–299.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*, *24* (2), 139-162.

Fujita, T. (2012). Learners' level of understanding of inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior, 31,* 60-72.

Fujita, T., & Jones, K. (2006). Primary trainee teachers' understanding of basic geometrical figures in scotland. In J. Novotana, H. Moraova, K. Magdelena & N. Stehlikova (Eds.), *Proceedings of The 30th Conference of the International Group for the Psychology of Mathematics Education, 3,* 14-21.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education*, *9* (1&2), 3-20.

Hasegawa, J. (1997). Concept formation of triangles and quadrilaterals in second grade. *Educational Studies in Mathematics, 32* (2), 157-179.

Heinze, A., & Ossietzky, C. (2002). "…Because a square is not a rectangle" students' knowledge of simple geometrical concepts when starting to learn proof. In A. Cockburn & E. Nardi (Eds.), *Proceedings of The 26th Conference of the International Group for the Psychology of Mathematics Education*, *3*, 81-88.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70-95). Cambridge: Cambridge University Press.

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics*, *42* (2),179-196.

Okazaki, M., & Fujita,T. (2007). Prototype phenomena and common cognitive paths in the understanding of the ınclusion relations between quadrilaterals in japan and scotland. In H. Woo, K. Park & D. Seo (Eds.), *Proceedings of The 31st Conference of the Internatıonal Group for the Psychology of Mathematics Education*, *4*, 41-48.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151-16.

Türnüklü, A. (2000). Eğitim bilim araştırmalarında etkin olarak kullanılabilecek nitel bir araştırma tekniği: Görüşme. *Kuram ve Uygulamada Eğitim Yönetimi, 24*, 543-559.

Vighi, P. (2003). The triangle as a mathematical object. *European Research in Mathematics Education III Congress Proceedings*, *Bellaria, Italy, 28 Februrary-3 March,* 1-10.