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INVESTIGATION OF WAVES PROPAGATING IN ISOTHERMAL PLASMA AROUND DE SITTER BLACK HOLE

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ABSTRACT

We investigate the wave properties for isothermal plasma state around to the de Sitter black hole's horizon using 3+1 split of spacetime. The corresponding Fourier analyzed perturbed perfect GRMHD equations are used to obtain the complex dispersion relations. We obtain the real values of the wave number k, from these relations, which are used to evaluate the quantities like phase and group velocities etc. These have been analyzed graphically in the neighborhood of the horizon.

Key Words: 3+1 Formalism; Rindler coordinates; Near-horizon magnetohydrodynamics

I. INTRODUCTION

It is known that black holes greatly affect the surrounding highly magnetized plasma medium with their massive gravitational fields. Therefore plasma physics around a black hole has become a subject of great importance in astrophysics. In the immediate neighborhood of a black hole, general relativity applies. To formulate plasma physics problems in the context of general relativity is very significant at the moment. An isolated black hole can have an electromagnetic field, if it is endowed with a net electric charge (Israel, 1967, 1968; Hawking, 1972; Robinson, 1974). Since a collapsed object can have a very strong effect on an electromagnetic field, it is of concern to determine this effect using general relativistic magnetohydrodynamics (GRMHD) equations when a black hole is placed in an external electromagnetic field. A covariant formulation of the theory based on the fluid equations in curved spacetime has so far proved unproductive because of the curvature of four-dimensional spacetime in the region surrounding a black hole. The $3 + 1$ formulation of general relativity, developed by Thorne & Macdonald (1982), Macdonald & Thorne (1982), and Price & Thorne (1986), provides a method in which the electromagnetic equations and the plasma physics at least look somewhat similar to the usual formulations in flat spacetime while taking accurate account of general relativistic effects such as curvature.

Works connected with black holes have been facilitated by the replacement of the hole's event horizon with a membrane endowed with electric charge, electrical conductivity and finite temperature, and entropy in Thorne, Price & Macdonald (1986). Mathematically the membrane paradigm is analogous to the standard, full general relativistic theory of black holes so far as physics outside the event horizon is concerned, and moreover, the formulation of all physics in this region turns out to be very much simpler than it would be using the standard covariant approach of general relativity.

Arnowitt, Deser & Misner (1962) have developed the 3+1 split of spacetime at first to study the quantization of the gravitational field. Since then, their formulation has been applied in studying numerical relativity (Evans, Smarr & Wilson, 1986). TPM extended the ADM formalism to include electromagnetism and applied it to study

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electromagnetic effects near the Kerr black hole. As a result, their work has inaugurated many opportunities for studying electromagnetic effects on plasmas in the black hole environment.

Numerous endeavors have been taken to exploit the 3+1 formalism over the last few decades. Zhang (1989a,b) considered the case of perfect GRMHD waves near to Kerr black hole and discussed the linearized waves for the cold (negligible particle pressure) plasma propagating in two-dimensions. Holcomb & Tajima (1989), Holcomb (1990) and Dettman, Frankel & Kowalenko (1993) investigated some properties of wave propagation in the Friedmann universe. Khana (1998) derived the GRMHD equations for two-fluid plasma in Kerr black hole. Anón et al. (2006) investigated various test simulations and discussed magneto-rotational instability of accretion disks. Anile (1989) worked on relativistic shock waves in magneto-fluids in cold relativistic plasma. Komissarov (2002) discussed the Blandford-Znajek monopole solution in black hole electrodynamics. Buzzi, Hines & Treumann (1995a,b) described a general relativistic version of two-fluid plasma physics in TPM formulation and developed a linearized treatment of plasma waves in analogy with the special relativistic formulation of Sakai & Kawata (1980).

In recent times, Sharif & Sheikh (2007) (SS) investigated the behavior of isothermal plasma waves in the vicinity of the Schwarzschild black hole horizon. In this paper we apply TPM formalism of the GRMHD equations to study the dynamical magnetosphere of the de Sitter (dS) space and investigate the nature of the waves.

Physicists have a growing interest in dS space over the last few decades. In the 1970s, the attention was due to the large symmetry group of dS space, which made the field theory in dS space less ambiguous than, for example, in the Schwarzschild spacetime. Researches in the 1980s focused the role it played during inflation-accelerated expansion in the very early universe. The universe is currently asymptotic dS and approach a pure dS space. Recent cosmological observations (Bahcall et al., 1999; Reiss et al., 1998; Permutter et al., 1999) suggest the possibility of existing a positive cosmological constant ($\Lambda > 0$) in our universe and this possibility gives the picture, among many others, of some features closely related to black holes: the existence of cosmological event horizons. These causal horizons exist even in the absence of matter, namely in empty dS space, and hide all the events which are not accessible for geodesic observers. In addition, the success of the ADS/CFT correspondence (Maldacena, 1998; Witten, 1998; Gubser, Klebanov & Polyakov, 1998; Aharony et al., 1999) has led to the intense study of dS space in the context of the quantum gravity (Witten, 2001). The attention has been to obtain an analogue of the ADS/CFT correspondence in dS space, i.e. dS/CFT correspondence (Strominger, 2001a,b; Klemm, 2002; Hull, 1998; Park, 1998, 1999) in the light of which there has been an extensive study of the semiclassical aspects of dS and asymptotic dS spacetimes (Meldved, 2002; Parikh, 2002; Bousso, Maloney & Strominger, 2002). In view of these reasons, it may be of special interest to investigate wave properties of isothermal plasma in the dS space.

This paper is arranged as follows. In section 2, we describe the de Sitter space and its planar analogue. Section 3 is furnished with the presentation of GRMHD equations for isothermal plasma. We obtain the dispersion relations for the case of rotating magnetized background in section 4. In section 5, we present our study of non-magnetized plasma in rotating background. Section 6 is assigned to the cases of non-rotating magnetized and non-magnetized background. Lastly, we summarize and discuss the results. We use units $G = c = 1$.

II. DE SITTER SPACETIME AND ITS PLANAR ANALOGUE

In de sitter space, the simplest solution for the Einstein field equations with $T_{\mu\nu} = 0$ is written as

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -\left(1-\frac{r^2}{\ell^2}\right)dt^2 + \left(1-\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2 d\Omega_2^2. \end{aligned} \tag{1}$$

Here, ℓ is the curvature radius of the dS space [$\Lambda = \frac{3}{\ell^2}$ is the positive cosmological constant], $d\Omega_2^2$ represents a unit 2-sphere, and the nonangular coordinates range according to $0 \leq r \leq \ell$ and $-\infty < t < \infty$. The boundary at $r = \ell$ describes a cosmological horizon for an observer located at $r = 0$.

An absolute three-dimensional space defined by the hypersurfaces of constant universal time t is described by the metric

$$ds^2 = g_{ij}dx^i dx^j = \left(1-\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2 d\Omega_2^2. \tag{2}$$

The indices i, j range over 1, 2, 3 and refer to coordinates in absolute space. The Fiducial Observers (FIDO's), the observers remaining at rest with respect to this absolute space, measure their proper time τ using clocks that they carry with them and make local measurements of physical quantities. Then all their measured quantities are defined as FIDO locally measured quantities and all rates measured by them are measured using FIDO proper time. The FIDO's use a local Cartesian coordinate system with unit basis vectors tangent to the coordinate line

$$\mathbf{e}_{\hat{r}} = \left(1 - \frac{r^2}{\ell^2}\right)^{1/2} \frac{\partial}{\partial r}, \qquad \mathbf{e}_{\hat{\theta}} = \frac{1}{r}\frac{\partial}{\partial \theta}, \qquad \mathbf{e}_{\hat{\varphi}} = \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}. \tag{3}$$

For a spacetime viewpoint rather than a 3 + 1 split of spacetime, the set of orthonormal vectors also includes the basis vector for the time coordinate given by

$$\mathbf{e}_{\hat{0}} = \frac{d}{d\tau} = \frac{1}{\alpha}\frac{\partial}{\partial t}, \tag{4}$$

where α is the lapse function (or redshift factor) defined by

$$\alpha(r) \equiv \frac{d\tau}{dt} = \left(1 - \frac{r^2}{\ell^2}\right)^{1/2}. \tag{5}$$

The gravitational acceleration felt by a FIDO is given by

$$\mathbf{a} = \nabla \ln\alpha = -\frac{1}{\alpha}\frac{r}{\ell^2}\,\mathbf{e}_{\hat{r}}, \tag{6}$$

while the rate of change of any scalar physical quantity or any three-dimensional vector or tensor, as measured by a FIDO, is defined by the convective derivative

$$\frac{D}{D\tau} \equiv \left(\frac{1}{\alpha}\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right), \tag{7}$$

$\mathbf{V}$ being the velocity of a fluid as measured locally by a FIDO.

For a good approximation near the horizon, we write the dS metric in the Rindler coordinate system as follows:

$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right) dt^2 + dx^2 + dy^2 + dz^2, \tag{8}$$

where

$$x = \ell\left(\theta - \frac{\pi}{2}\right), \qquad y = \ell\varphi, \qquad z = 2\ell\left(1 - \frac{r^2}{\ell^2}\right)^{1/2}. \tag{9}$$

The standard lapse function in Rindler coordinates becomes $\alpha = (z/2r_h)$, where $r_h = \ell$ is the location of the cosmological event horizon.

III. 3+1 PERFECT GRMHD EQUATIONS AROUND DE SITTER SPACETIME

Maxwell's equations in 3+1 formalism take the following form:

$$\nabla\cdot\mathbf{B} = 0, \tag{10}$$

$$\nabla\cdot\mathbf{E} = 4\pi\rho_e, \tag{11}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times(\alpha\mathbf{E}), \tag{12}$$

$$\frac{\partial\mathbf{E}}{\partial t} = \nabla\times(\alpha\mathbf{B}) - 4\pi\alpha\mathbf{j}, \tag{13}$$

where ρ_e and $\mathbf{j}$ are electric charge and current density, respectively. For the perfect MHD (i.e., MHD with perfectly conducting) assumption there exists no electric field in the fluid's rest frame, i.e.,

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0. \tag{14}$$

Under this condition the equation for the evolution of magnetic field (12) becomes

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\alpha \mathbf{V} \times \mathbf{B}) \\ &= (\mathbf{B} \cdot \nabla)(\alpha \mathbf{V}) - \mathbf{B} \nabla \cdot (\alpha \mathbf{V}) - (\alpha \mathbf{V} \cdot \nabla) \mathbf{B}. \end{aligned} \tag{15}$$

The conservation of mass, energy and momentum equations are written, respectively, as follows (Gubser, Klebanov & Polyakov, 1998):

$$\begin{aligned} &\frac{\partial(\rho_o \mu)}{\partial t} + \{(\alpha \mathbf{V}) \cdot \nabla\}(\rho_o \mu) \\ &+\rho_o \mu \gamma^2 \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial t} + \rho_o \mu \gamma^2 \mathbf{V} \cdot (\alpha \mathbf{V} \cdot \nabla) \mathbf{V} + \rho_o \mu \{\nabla \cdot (\alpha \mathbf{V})\} = 0, \end{aligned} \tag{16}$$

$$\begin{aligned} &\left\{ \left(\rho_o \mu \gamma^2 + \frac{\mathbf{B}^2}{4\pi} \right) \delta_{ij} + \rho_o \mu \gamma^4 V_i V_j - \frac{1}{4\pi} B_i B_j \right\} \frac{DV^j}{D\tau} \\ &+\rho_o \mu \gamma^2 V_i \frac{D\mu}{D\tau} - \left(\frac{\mathbf{B}^2}{4\pi} \delta_{ij} - \frac{1}{4\pi} B_i B_j \right) V^j{}_{,k} V^k \\ &= -\rho_o \mu \gamma^2 a_i - p_{,i} + \frac{1}{4\pi} (\mathbf{V} \times \mathbf{B})_i \nabla \cdot (\mathbf{V} \times \mathbf{B}) - \frac{1}{8\pi \alpha^2} (\alpha \mathbf{B})^2_{,i} \\ &+\frac{1}{4\pi\alpha} (\alpha B_i)_{,j} B^j - \frac{1}{4\pi\alpha} [\mathbf{B} \times \{\mathbf{v} \times (\nabla \times (\alpha \mathbf{v} \times \mathbf{B}))\}]_i, \end{aligned} \tag{17}$$

$$\begin{aligned} &\gamma^2 \frac{D(\mu\rho_0)}{D\tau} - \frac{1}{\alpha} \frac{\partial p}{\partial t} + 2\rho_0 \mu \gamma^4 \mathbf{V} \cdot \frac{D\mathbf{V}}{D\tau} + 2\rho_0 \mu \gamma^2 (\mathbf{V} \cdot \mathbf{a}) + \rho_0 \mu \gamma^2 (\nabla \cdot \mathbf{V}) \\ &+\frac{1}{4\pi\alpha} \left[(\mathbf{V} \times \mathbf{B}) \cdot (\nabla \times (\alpha \mathbf{B})) + (\mathbf{V} \times \mathbf{B}) \cdot \frac{\partial}{\partial t} (\mathbf{V} \times \mathbf{B}) \right] = 0. \end{aligned} \tag{18}$$

Here a subscript i on a vector quantity refers to the i component of that vector. Equation (18) is derived by using Thorne & Macdonald (1982)

$$\epsilon = \{\mu\rho_0 - p(1 - \mathbf{V}^2)\}\gamma^2, \qquad \mathbf{S} = \mu\rho_0\gamma^2 \mathbf{V}, \qquad \overleftrightarrow{\mathbf{W}} = \mu\boldsymbol{\rho_0}\boldsymbol{\gamma^2}\mathbf{V} \otimes \mathbf{V} + p \overleftrightarrow{\gamma} \tag{19}$$

in Evans, Smarr & Wilson (1986)

$$\frac{d\epsilon}{d\tau} + \theta\epsilon + \frac{1}{2\alpha} W^{ij} (\pounds_t \gamma_{ij}) = -\frac{1}{\alpha^2} \nabla \cdot (\alpha^2 \mathbf{S}) + \frac{1}{\alpha} (\nabla \boldsymbol{\beta}) : \overleftrightarrow{\mathbf{W}} + \mathbf{E} \cdot \mathbf{j}. \tag{20}$$

Here ϵ, $\mathbf{S}$, $\overleftrightarrow{\mathbf{W}}$, $\overleftrightarrow{\gamma}$, θ, $\boldsymbol{\beta}$, $\otimes$ and $\pounds_t$ represent the mass energy density, energy flux, stress tensor, the three metric in absolute space, the expansion rate of the FIDO's four-velocity, the shift vector, the tensor product and the time derivative along shifting congruence (Lie derivative with respect to global time in a standard style). $\frac{d}{d\tau} \equiv \frac{1}{\alpha} \frac{\partial}{\partial t}$ is the rate of change of a three-dimensional vector which lies in the absolute space according to the FIDO. The $\mu \equiv (\rho + p)/\rho_o$ is the specific enthalpy of the fluid, where ρ is the total density of mass-energy and p is the pressure as seen in the fluid's rest frame. The ρ_o is the fluid's rest-mass density and $\gamma \equiv (1 - \mathbf{V}^2)^{-1/2}$ is the fluid's Lorentz factor as seen by the FIDO's. Equations (15)-(18) are the perfect GRMHD equations for the dS black hole.

We consider for ease the "isothermal plasma"(consider the existence of pressure), for which the equation of state can be expressed as

$$\mu = \frac{\rho + p}{\rho_0} = \text{constant}. \tag{21}$$

Using (21) in (15)-(18) we get the perfect GRMHD for isothermal plasma close to the event horizon of dS black hole. We characterize the perturbed flow in the magnetosphere by its velocity $\mathbf{V}$ and magnetic field $\mathbf{B}$ as measured by the FIDO's, pressure of the fluid p and the fluid's density ρ. The first order perturbations in these quantities are denoted by $\delta\mathbf{V}$, $\delta\mathbf{B}$, δp and $\delta\rho$. Accordingly, the perturbed variables take the following form:

$$\mathbf{B} = \mathbf{B}^o + \delta\mathbf{B} = \mathbf{B}^o + B\mathbf{b},\ \mathbf{V} = \mathbf{V}^o + \delta\mathbf{V} = \mathbf{V}^o + \mathbf{v},\ \rho = \rho^o + \delta\rho = \rho^o + \rho\tilde{\rho},\ p = p^o + \delta p = p^o + p\tilde{p} \tag{22}$$

where $\mathbf{B}^o$, $\mathbf{V}^o$, p^o and ρ^o are unperturbed quantities. The waves can propagate in z-direction due to gravitation with respect to time t and thus perturbed quantities must depend on z and t.

IV. ROTATING MAGNETIZED SURROUNDINGS

We use the linear perturbation and Fourier analyze techniques to reduce GRMHD equations to ordinary differential equations. The magnetosphere has the perturbed flow along x-z plane in this surroundings. The FIDO-measured fluid four-velocity can be described in this plane by

$$\mathbf{V} = V(z)\mathbf{e}_x + u(z)\mathbf{e}_z, \tag{23}$$

while the Lorentz factor γ takes the form

$$\gamma = \frac{1}{\sqrt{1 - u^2 - V^2}}. \tag{24}$$

The rotating magnetic field can be expressed in the x-z plane as

$$\mathbf{B} = B[\lambda(z)\mathbf{e}_x + \mathbf{e}_z]. \tag{25}$$

The variables λ , u and V are related by

$$V = \frac{V_F}{\alpha} + \lambda u, \tag{26}$$

where V_F is an integration constant.
We use the perturbation variables which are defined as

$$\begin{aligned}&\tilde{\rho}(t,z) = c_1 e^{-i(\omega t - kz)}, \tilde{p}(t,z) = c_2 e^{-i(\omega t - kz)}, v_z(t,z) = c_3 e^{-i(\omega t - kz)},\\ &v_x(t,z) = c_4 e^{-i(\omega t - kz)}, b_z(t,z) = c_5 e^{-i(\omega t - kz)}, b_x(t,z) = c_6 e^{-i(\omega t - kz)}\end{aligned} \tag{27}$$

where c_1, c_2, c_3, c_4, c_5 and c_6 are arbitrary constants. Here k and ω are defined as the wave number and angular frequency of the wave respectively.
Using linear perturbation (22), we write down the GRMHD equations (15)-(18) with the help of (21) in the following form:

$$\frac{\partial(\delta\mathbf{B})}{\partial t} = \nabla \times (\alpha\mathbf{v} \times \mathbf{B}) + \nabla \times (\alpha\mathbf{V} \times \delta\mathbf{B}), \tag{28}$$

$$\nabla \cdot (\delta\mathbf{B}) = 0, \tag{29}$$

$$\begin{aligned}&\{(\alpha\mathbf{V}\cdot\nabla) + \nabla\cdot(\alpha\mathbf{V}) + \gamma^2\mathbf{V}\cdot(\alpha\mathbf{V}\cdot\nabla)\mathbf{V}\}(\delta\rho + \delta p) + (\rho + p)\gamma^2\mathbf{V}\cdot\frac{\partial\mathbf{v}}{\partial t}\\ &-\alpha(\rho + p)\mathbf{v}\cdot\nabla\ln u + \alpha(\rho + p)(\nabla\cdot\mathbf{v}) + 2(\rho + p)\gamma^2(\mathbf{V}\cdot\mathbf{v})(\alpha\mathbf{V}\cdot\nabla)\ln\gamma\\ &+(\rho + p)\gamma^2(\alpha\mathbf{V}\cdot\nabla\mathbf{V})\cdot\mathbf{v} + (\rho + p)\gamma^2\mathbf{V}\cdot(\alpha\mathbf{V}\cdot\nabla)\mathbf{v} + \frac{\partial(\delta\rho + \delta p)}{\partial t} = 0,\end{aligned} \tag{30}$$

$$\{((\rho+p)\gamma^2+\frac{\mathbf{B}^2}{4\pi})\delta_{ij}+(\rho+p)\gamma^4V_iV_j-\frac{1}{4\pi}B_iB_j\}\frac{1}{\alpha}\frac{\partial v^j}{\partial t}+\frac{1}{4\pi}[\mathbf{B}\times\{\mathbf{V}\times\frac{1}{\alpha}\frac{\partial(\delta\mathbf{B})}{\partial t}\}]_i$$
$$+(\rho+p)\gamma^2 v_{i,j}V^j+(\rho+p)\gamma^4V_iV^jV^kv_{j,k}-\frac{1}{4\pi\alpha}\{(\alpha\delta B_i)_{,j}-(\alpha\delta B_j)_{,i}\}B^j$$
$$=-\gamma^2\{(\delta p+\delta\rho)+2(\rho+p)\gamma^2(\mathbf{V}\cdot\mathbf{v})\}a_i-(\delta p)_{,i}+\frac{1}{4\pi\alpha}\{(\alpha B_i)_{,j}-(\alpha B_j)_{,i}\}\delta B^j$$
$$-(\rho+p)\gamma^4(v_iV^j+v^jV_i)V_{k,j}V^k-\gamma^2\{(\delta\rho+\delta p)V^j+2(\rho+p)\gamma^2(\mathbf{V}\cdot\mathbf{v})V^j$$
$$+(\rho+p)v^j\}V_{i,j}-\gamma^4V_i\{(\delta\rho+\delta p)V^j+4(\rho+p)\gamma^2(\mathbf{V}\cdot\mathbf{v})V^j+(\rho+p)v^j\}V_{j,k}V^k, \quad (31)$$

$$\frac{2}{\alpha}(\rho+p)\gamma^4\mathbf{V}\cdot\frac{\partial\mathbf{v}}{\partial t}-2(\rho+p)\gamma^4(\mathbf{V}\cdot\mathbf{v})(\mathbf{V}\cdot\nabla)\ln u+6(\rho+p)\gamma^6(\mathbf{V}\cdot\mathbf{v})\{\mathbf{V}\cdot(\mathbf{V}\cdot\nabla)\mathbf{V}\}$$
$$+(\rho+p)\gamma^4\mathbf{V}\cdot(\mathbf{v}\cdot\nabla)\mathbf{V}+2(\rho+p)\gamma^4\mathbf{V}\cdot(\mathbf{V}\cdot\nabla)\mathbf{v}+2(\delta\rho+\delta p)\gamma^2\mathbf{V}\cdot\mathbf{a}$$
$$+2(\rho+p)(\mathbf{V}\cdot\mathbf{v})(\mathbf{V}\cdot\mathbf{a})+(\rho+p)\gamma^2(\nabla\cdot\mathbf{v})+(\rho+p)\gamma^2\mathbf{v}\cdot\mathbf{a}+\gamma^2(\mathbf{V}\cdot\nabla)(\delta\rho+\delta p)$$
$$-\frac{1}{\alpha}\frac{\partial(\delta p)}{\partial t}-(\rho+p)\gamma^2(\mathbf{v}\cdot\nabla)\ln u+\frac{\gamma^2}{\alpha}\frac{\partial(\delta\rho+\delta p)}{\partial t}+2(\delta\rho+\delta p)\gamma^4\mathbf{V}\cdot(\mathbf{V}\cdot\nabla)\mathbf{V}$$
$$+(\delta\rho+\delta p)\gamma^2(\nabla\cdot\mathbf{V})+2(\rho+p)\gamma^4(\mathbf{V}\cdot\mathbf{v})(\nabla\cdot\mathbf{V})+\frac{1}{4\pi\alpha}[(\mathbf{V}\times\mathbf{B})\cdot(\nabla\times(\alpha\delta\mathbf{B}))$$
$$+(\mathbf{v}\times\mathbf{B})\cdot(\nabla\times(\alpha\mathbf{B}))+(\mathbf{V}\times\delta\mathbf{B})\cdot(\nabla\times(\alpha\mathbf{B}))+(\mathbf{V}\times\mathbf{B})\cdot\left(\mathbf{V}\times\frac{\partial(\delta\mathbf{B})}{\partial t}\right)$$
$$+(\mathbf{V}\times\mathbf{B})\cdot(\frac{\partial\mathbf{v}}{\partial t}\times\mathbf{B})]+2(\rho+p)\gamma^4\mathbf{v}\cdot(\mathbf{V}\cdot\nabla)\mathbf{V}=0. \quad (32)$$

The component form of (28)-(32) can be written as follows

$$\frac{\partial b_x}{\partial t}+\alpha u b_{x,z}=\alpha'(v_x-\lambda v_z+Vb_z-ub_x)$$
$$+\alpha(v_{x,z}-\lambda v_{z,z}-\lambda' v_z+V'b_z+Vb_{z,z}-u'b_x), \quad (33)$$

$$\frac{\partial b_z}{\partial t}+\alpha u b_{z,z}=0, \quad (34)$$

$$b_{z,z}=0, \quad (35)$$

$$\rho\frac{\partial\tilde{\rho}}{\partial t}+p\frac{\partial\tilde{p}}{\partial t}+(\rho+p)(V\frac{\partial v_x}{\partial t}+u\frac{\partial v_z}{\partial t})+\alpha u\rho\tilde{\rho}_{,z}+\alpha up\tilde{p}_{,z}+\alpha(\rho+p)\{\gamma^2Vuv_{x,z}$$
$$+(1+\gamma^2u^2)v_{z,z}\}-\frac{1}{\gamma}(\tilde{\rho}-\tilde{p})(\alpha u\gamma p)_{,z}+\alpha(\rho+p)\gamma^2u\{(1+2\gamma^2V^2)V'$$
$$+2\gamma^2uVu'\}v_x-\alpha(\rho+p)\{(1-2\gamma^2u^2)(1+\gamma^2u^2)\frac{u'}{u}-2\gamma^4u^2VV'\}v_z=0, \quad (36)$$

$$\{(\rho+p)\gamma^2(1+\gamma^2V^2)+\frac{B^2}{4\pi}\}\frac{1}{\alpha}\frac{\partial v_x}{\partial t}+\{(\rho+p)\gamma^4uV-\frac{\lambda B^2}{4\pi}\}\frac{1}{\alpha}\frac{\partial v_z}{\partial t}$$
$$+\{(\rho+p)\gamma^2(1+\gamma^2V^2)-\frac{B^2}{4\pi}\}uv_{x,z}+\{(\rho+p)\gamma^4uV+\frac{\lambda B^2}{4\pi}\}uv_{z,z}$$
$$-\frac{B^2}{4\pi\alpha}b_x\{\alpha'(1-u^2)-\alpha uu'\}+(\rho\tilde{\rho}+p\tilde{p})\gamma^2u\{(1+\gamma^2V^2)V'+\gamma^2uVu'\}$$
$$+[(\rho+p)\gamma^4u\{(1+4\gamma^2V^2)uu'+4VV'(1+\gamma^2V^2)\}+\frac{B^2u\alpha'}{4\pi\alpha}]v_x$$
$$+\Big[(\rho+p)\gamma^2[\{(1+2\gamma^2u^2)(1+2\gamma^2V^2)-\gamma^2V^2\}V'$$
$$+2\gamma^2(1+2\gamma^2u^2)uVu']+\frac{B^2u}{4\pi\alpha}(\lambda\alpha)'\Big]v_z-\frac{B^2}{4\pi}(1-u^2)b_{x,z}=0, \quad (37)$$

$$\{(\rho+p)\gamma^2(1+\gamma^2u^2)+\frac{\lambda^2B^2}{4\pi}\}\frac{1}{\alpha}\frac{\partial v_z}{\partial t}+\{(\rho+p)\gamma^4uV-\frac{\lambda B^2}{4\pi}\}\frac{1}{\alpha}\frac{\partial v_x}{\partial t}+\{(\rho+p)$$
$$\gamma^2(1+\gamma^2u^2)-\frac{\lambda^2B^2}{4\pi}\}uv_{z,z}+\{(\rho+p)\gamma^4uV+\frac{\lambda B^2}{4\pi}\}uv_{x,z}+\frac{\lambda B^2}{4\pi}(1-u^2)b_{x,z}$$
$$+\frac{B^2}{4\pi\alpha}\{-(\alpha\lambda)'+\alpha'\lambda-u\lambda(\alpha u)'\}b_x+(\rho\tilde{\rho}+p\tilde{p})\gamma^2[a_z+u\{(1+\gamma^2u^2)u'+\gamma^2uVV'\}]$$
$$+[(\rho+p)\gamma^4\{u^2V'(1+4\gamma^2V^2)+2V(a_z+(1+2\gamma^2u^2))\}+\frac{\lambda B^2\alpha'u}{4\pi\alpha}]v_x+[(\rho+p)\gamma^2$$
$$\times\{u'(1+\gamma^2u^2)(1+4\gamma^2u^2)+2u\gamma^2\{a_z+(1+2\gamma^2u^2)VV'\}\}-\frac{\lambda B^2u}{4\pi\alpha}(\alpha\lambda)']v_z=0, \quad (38)$$

$$\gamma^2\frac{\rho}{\alpha}\frac{\partial\tilde{\rho}}{\partial t}+(\gamma^2-1)\frac{p}{\alpha}\frac{\partial\tilde{p}}{\partial t}+\frac{B^2}{4\pi\alpha}(u\lambda-V)(u\frac{\partial b_x}{\partial t}-V\frac{\partial b_z}{\partial t}+\lambda\frac{\partial v_z}{\partial t}-\frac{\partial v_x}{\partial t})$$
$$+(\rho+p)\gamma^2[\gamma^2\frac{2}{\alpha}(V\frac{\partial v_x}{\partial t}+u\frac{\partial v_z}{\partial t})+2\gamma^2uVv_{x,z}+(1+2\gamma^2u^2)v_{z,z}]$$
$$+v_z[(\rho+p)\gamma^2\{6\gamma^2u^2(uu'+VV')+\gamma^2(uu'+VV')+a_z(1+2\gamma^2u^2)$$
$$-\frac{u'}{u}+2\gamma^2uu'\}+\frac{\lambda B^2}{4\pi\alpha}(\alpha\lambda)']+v_x[(r+p)\gamma^4\{2uVa_z+6\gamma^2uV(uu'+VV')$$
$$+2uV'\}-\frac{B^2}{4\pi\alpha}(\alpha\lambda)']+\tilde{\rho}\gamma^2[2pu\{a_z+\gamma^2(uu'+VV')\}+\rho'u+\rho u']$$
$$+\tilde{p}\gamma^2[2\rho u\{a_z+\gamma^2(uu'+VV')\}+p'u+pu']+\gamma^2u(\rho\tilde{\rho}_{,z}+p\tilde{p}_{,z})$$
$$+\frac{B^2}{4\pi\alpha}\{-(\alpha\lambda)'b_zV+(\alpha\lambda)'b_xu+(u\lambda-V)(\alpha b_{x,z}+\alpha'b_x)\}=0. \quad (39)$$

From the Fourier analyzed of (33)-(39) with (27) we obtain

$$-c_3\{(\alpha\lambda)'+ik\alpha\lambda\}+c_4(\alpha'+ik\alpha)-c_6\{(\alpha u)'-i\omega+ik\alpha u\}=0, \quad (40)$$

$$c_5(-\frac{i\omega}{\alpha}+iku)=0, \quad (41)$$

$$ikc_5=0, \quad (42)$$

$$c_1\{\rho(-i\omega+iku\alpha)-\alpha'up-\alpha u'p-\alpha up'-\alpha up\gamma^2(uu'+VV')\}$$
$$+c_2\{\rho(-i\omega+iku\alpha)+\alpha'up+\alpha u'p+\alpha up'+\alpha up\gamma^2(uu'+VV')\}+c_3(\rho+p)$$
$$\times\left[-i\omega\gamma^2u+ik\alpha(1+\gamma^2u^2)-\alpha\{(1-2\gamma^2u^2)(1+\gamma^2u^2)\frac{u'}{u}-2\gamma^4u^2VV'\}\right]$$
$$+c_4(\rho+p)\gamma^2[(-i\omega+ik\alpha u)V+\alpha u\{(1+2\gamma^2V^2)V'+2\gamma^2uVu'\}]=0, \quad (43)$$

$$c_1\rho\gamma^2u\{(1+\gamma^2V^2)V'+\gamma^2uVu'\}+c_2p\gamma^2u\{(1+\gamma^2V^2)V'+\gamma^2uVu'\}$$
$$+c_3[-\{(\rho+p)\gamma^4uV-\frac{\lambda B^2}{4\pi}\}\frac{i\omega}{\alpha}+iku\{(\rho+p)\gamma^4uV+\frac{\lambda B^2}{4\pi}\}+(\rho+p)\gamma^2$$
$$\{\{(1+2\gamma^2u^2)(1+2\gamma^2V^2)-\gamma^2V^2\}V'+2\gamma^2(1+2\gamma^2u^2)uVu'\}+\frac{B^2u}{4\pi\alpha}(\alpha\lambda)']$$
$$+c_4[-\{(\rho+p)\gamma^2(1+\gamma^2V^2)+\frac{B^2}{4\pi}\}\frac{i\omega}{\alpha}+iku\{(\rho+p)\gamma^2(1+\gamma^2V^2)-\frac{B^2}{4\pi}\}$$
$$+(\rho+p)\gamma^4u\{(1+4\gamma^2V^2)uu'+4(1+\gamma^2V^2)VV'\}-\frac{B^2u\alpha'}{4\pi\alpha}]$$
$$-c_6\frac{B^2}{4\pi}\{ik(1-u^2)+(1-u^2)\frac{\alpha'}{\alpha}-uu'\}=0, \quad (44)$$

$$c_1\gamma^2\rho[a_z + u\{(1+\gamma^2u^2)u' + \gamma^2VuV'\}] + c_2[\gamma^2p\{a_z + u\{(1+\gamma^2u^2)u' + \gamma^2uVV'\}\}$$
$$+ikp + p'] + c_3[-\frac{i\omega}{\alpha}\{(\rho+p)\gamma^2(1+\gamma^2u^2) + \frac{\lambda^2B^2}{4\pi}\} + iku\{(\rho+p)\gamma^2(1+\gamma^2V^2)$$
$$-\frac{\lambda^2B^2}{4\pi}\} + \{(\rho+p)\gamma^2\{u'(1+\gamma^2u^2)(1+4\gamma^2u^2) + 2u\gamma^2\{(1+2\gamma^2u^2)VV' + a_z\}\}$$
$$-(\alpha\lambda)'\frac{\lambda B^2u}{4\pi\alpha}\}] + c_4[-\frac{i\omega}{\alpha}\{(\rho+p)\gamma^4uV - \frac{\lambda B^2}{4\pi}\} + iku\{(\rho+p)\gamma^4uV + \frac{\lambda B^2}{4\pi}\}$$
$$+\{\gamma^4(\rho+p)\{u^2V'(1+4\gamma^2V^2) + 2V\{a_z + uu'(1+2\gamma^2u^2)\}\} + \frac{\lambda B^2\alpha'u}{4\pi\alpha}\}]$$
$$+c_6\frac{B^2}{4\pi}\{ik\lambda(1-u^2) + \lambda(1-u^2)\frac{\alpha'}{\alpha} - \lambda uu' + \frac{(\alpha\lambda)'}{\alpha}\} = 0, \quad (45)$$

$$c_1\gamma^2[\rho(-\frac{i\omega}{\alpha} + iku) + 2\rho u\{a_z + \gamma^2(uu' + VV')\} + \rho u' + \rho' u]$$
$$+c_2[p\{\frac{-i\omega}{\alpha}(\gamma^2-1) + ik\gamma^2u\} + 2\rho\gamma^2u\{a_z + \gamma^2(uu' + VV')\} + p\gamma^2u' + p'\gamma^2u]$$
$$+c_3[(\rho+p)\{-2\gamma^4u\frac{i\omega}{\alpha} + ik\gamma^2(1+2\gamma^2u^2) - \gamma^2\frac{u'}{u} + 6\gamma^6u^2(uu' + VV')$$
$$+\gamma^4(uu' + VV') + 2\gamma^4uu' + \gamma^2a_z(1+2\gamma^2u^2)\} + \frac{B^2}{4\pi\alpha}\{\lambda(\alpha\lambda)' - u(\alpha\lambda)'(u\lambda - V)$$
$$-i\lambda(u\lambda - V)(\omega + ku\alpha)\}] + c_4[2\gamma^4(\rho+p)\{V(-\frac{i\omega}{\alpha} + iku)$$
$$+\{3\gamma^2uV(uu' + VV') + uV' + uVa_z\}\} + \frac{B^2}{4\pi\alpha}\{-(\alpha\lambda)' + \alpha'u(u\lambda - V) + i(\omega + ku\alpha)$$
$$\times(u\lambda - V)\}] + \frac{B^2}{4\pi\alpha}c_6[u(\alpha\lambda)' + \{\alpha' - u(\alpha u)' + ik(1-u^2)\}(u\lambda - V)] = 0. \quad (46)$$

From (41) or (42) we obtain c_5 is zero which gives $b_z = 0$. Equating the determinant of the coefficients of c_1, c_2, c_3, c_4 and c_6 of (40), (43)-(46) to zero, we get a complex dispersion relation of the form

$$A_1(z,\omega)k^4 + B_1(z,\omega)k^3 + C_1(z,\omega)k^2 + D_1(z,\omega)k + E_1(z,\omega) + i\{A_2(z,\omega)k^5$$
$$+B_2(z,\omega)k^4 + C_2(z,\omega)k^3 + D_2(z,\omega)k^2 + E_2(z,\omega)k + F_2(z,\omega)\} = 0 \quad (47)$$

We investigate the different types of modes of waves when $B > 0$ and the wave number is in arbitrary direction to $\mathbf{B}$. We use the lapse function $\alpha = \frac{z}{2r_h}$ where $r_h = \ell(= \sqrt{\frac{3}{\Lambda}})$, Λ is a positive constant (Rahman & Al, 2008). Here we consider a black hole of mass $M \sim 1M_\odot$ (Buzzi, Hines & Treumann, 1995a). From the mass conservation law in three-dimensions we get $u = \frac{1}{\sqrt{2+z^2}}$. For simplicity, we also assume that $u = V$, $\rho = 1$ and $B^2 = 8\pi$. We obtain $\lambda = 1 - \frac{\sqrt{2+z^2}}{z}$ by taking $V_F = 1$ from (26), which shows that the magnetic field diverges close to the horizon.

Using these values in the dispersion relation we get values for k, from which we plot surfaces for the wave number k, the phase velocity $v_p \equiv \frac{\omega}{k}$, group velocity $v_g \equiv (n + \omega\frac{dn}{d\omega})^{-1}$ ($n(= 1/v_p)$ is the refractive index), and $\frac{dn}{d\omega}$, which determines whether the dispersion is normal or not.

We get four real values of k from the real part of (47). Out of these two are real and interesting. The other two values are not interesting in the judgment that these turn out to be imaginary in the whole region. The imaginary part of (47) gives five values of k, out of which one is real but not interesting and others are complex conjugate. The two dispersion relations obtained from the real part are shown in the Fig. 1 and Fig. 2.

The Fig. 1 implies that the wave number is infinite at the event horizon ($z = 0$) due to immense gravitational field and the wave number takes also negative values. The wave number decreases as we go away from the horizon. Hence we observe the damping modes in that direction. The phase and group velocities are also negative (for some values of z and ω) and the phase velocity is greater than the group velocity for some region. Since $\frac{dn}{d\omega} < 0$ and k, v_p, v_g are negative, the region is of anomalous dispersion and the medium has the properties of metamaterials.

We see from Fig. 2 that the waves gain energy with the increase in angular frequency but lose when we move from the horizon and hence damping modes arise in this direction. But in the vicinity of the horizon the wave number is very large because of strong gravitational field, so no wave exists there. The phase and group velocities increase as we depart from the horizon. The group velocity is greater than the phase velocity. The group velocity is negative in some region. Since $\frac{dn}{d\omega} \leq 0$ in most region, the dispersion is anomalous here.

V. ROTATING NON-MAGNETIZED SURROUNDINGS

In non-magnetized surroundings $\mathbf{B} = \mathbf{0}$. Then (40)-(46) reduce to

$$\begin{aligned}
&c_1\{(-i\omega + i\alpha ku)\rho - \alpha' up - \alpha u' p - \alpha up' - \alpha\gamma^2 up(VV' + uu')\} \\
&+c_2\{(-i\omega + i\alpha ku)p + \alpha' up + \alpha u' p + \alpha up' + \alpha\gamma^2 up(VV' + uu')\} \\
&+c_3(\rho + p)\left[-i\omega\gamma^2 u + ik\alpha(1 + \gamma^2 u^2) - \alpha\{(1 - 2\gamma^2 u^2)(1 + \gamma^2 u^2)\frac{u'}{u} - 2\gamma^4 u^2 VV'\}\right] \\
&+c_4(\rho + p)\left[i\gamma^2 V(-\omega + k\alpha u) + \alpha\gamma^2 u\{(1 + 2\gamma^2 V^2)V' + 2\gamma^2 uVu'\}\right] = 0, \qquad (48)
\end{aligned}$$

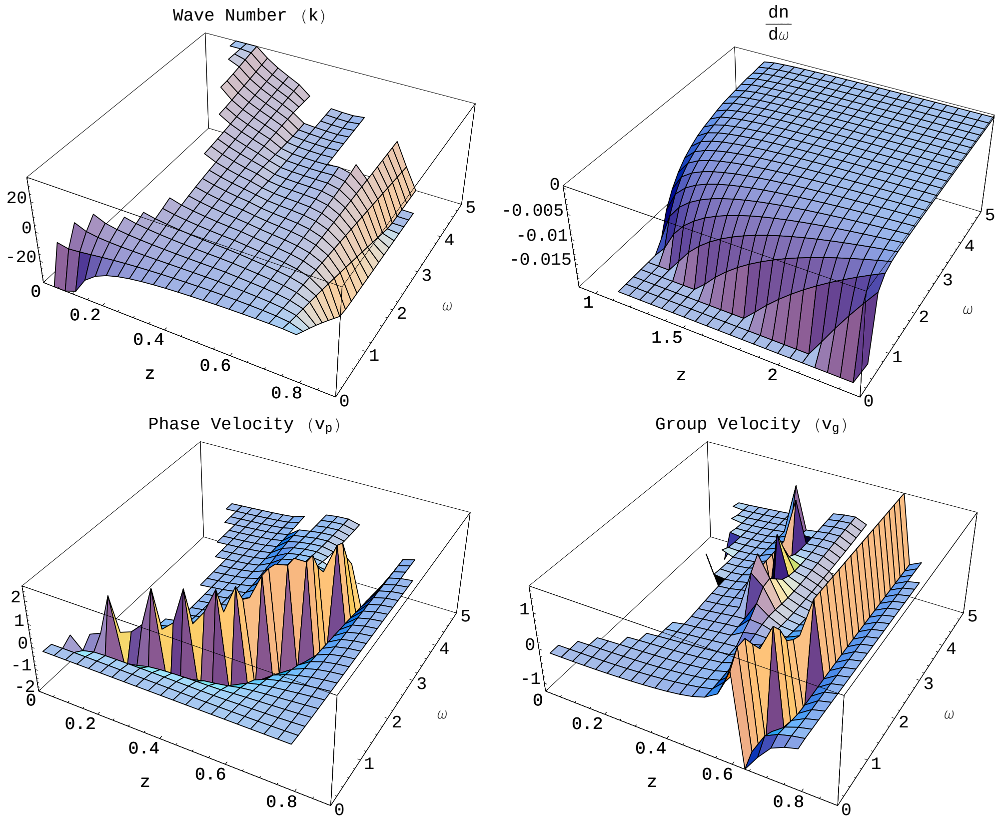

Fig. 1.— The region shows anomalous dispersion. k, v_p, v_g and $\frac{dn}{d\omega} < 0$, the medium has the properties of metamaterials

$$\begin{aligned}
&c_1\rho\gamma^2 u\{(1 + \gamma^2 V^2)V' + \gamma^2 uVu'\} + c_2 p\gamma^2 u\{(1 + \gamma^2 V^2)V' + \gamma^2 uVu'\} \\
&+c_3(\rho + p)\gamma^2\left[\gamma^2 uV(\frac{-i\omega}{\alpha} + iku) + \{(1 + 2\gamma^2 V^2)(1 + 2\gamma^2 u^2)\right.
\end{aligned}$$

$$-\gamma^2V^2\}V' + 2\gamma^2uVu'(1+2\gamma^2u^2)\big] + c_4(\rho+p)\gamma^2$$
$$\left[(1+\gamma^2V^2)(\frac{-i\omega}{\alpha} + iku) + \gamma^2u\{(1+4\gamma^2u^2)uu' + 4(1+\gamma^2V^2)VV'\}\right] = 0, \quad (49)$$

$$c_1\rho\gamma^2\{a_z + uu'(1+\gamma^2u^2) + \gamma^2u^2VV'\} + c_2[\rho\gamma^2\{a_z + uu'(1+\gamma^2u^2) + \gamma^2u^2VV'\}$$
$$+p' + ikp] + c_3(\rho+p)\gamma^2\left[(1+\gamma^2u^2)(-\frac{i\omega}{\alpha} + iku) + u'(1+\gamma^2u^2)(1+4\gamma^2u^2)\right.$$
$$\left.+2u\gamma^2\{(1+2\gamma^2u^2)VV' + a_z\}\right] + c_4(\rho+p)\gamma^4\left[iuV(-\frac{\omega}{\alpha} + ku)\right.$$
$$\left.+u^2V'(1+4\gamma^2u^2) + 2V\{(1+2\gamma^2u^2)uu' + a_z\}\right] = 0, \quad (50)$$

$$c_1\left[i\rho\gamma^2(-\frac{\omega}{\alpha} + ku) + 2\rho\gamma^2u\{a_z + \gamma^2(uu' + VV')\} + \rho\gamma^2u' + \rho'\gamma^2u\right]$$
$$+c_2\left[p\{\frac{-i\omega}{\alpha}(\gamma^2-1) + ik\gamma^2u\} + 2p\gamma^2u\{a_z + \gamma^2(uu' + VV')\} + p\gamma^2u' + p'\gamma^2u\right]$$
$$+c_3(\rho+p)\{-2\gamma^4u\frac{i\omega}{\alpha} + i\gamma^2k(1+2\gamma^2u^2) - \gamma^2\frac{u'}{u} + 6\gamma^6u^2(uu' + VV')$$
$$+\gamma^4(uu' + VV') + 2\gamma^4uu' + \gamma^2a_z(1+2\gamma^2u^2)\} + c_4(\rho+p)$$
$$\times\left[2i\gamma^4V(-\frac{\omega}{\alpha} + ku) + 2\gamma^4\{3\gamma^2uV(uu' + VV') + uV' + uVa_z\}\right] = 0. \quad (51)$$

where the FIDO-measured fluid four-velocity **V**, Lorentz factor γ are given by (23) and (24) respectively. The determinant of the coefficients of c_1, c_2, c_3 and c_4 in (48)-(51) yields a complex dispersion relation of the form

$$A_1(z,\omega)k^4 + B_1(z,\omega)k^3 + C_1(z,\omega)k^2 + D_1(z,\omega)k + E_1(z,\omega)$$
$$+i[A_2(z,\omega)k^3 + B_2(z,\omega)k^2 + C_2(z,\omega)k + D_2(z,\omega)] = 0 \quad (52)$$

To analyze the numerical solution mode we take the same assumption as previous case. From the real part of (52) we get only two real values of k which are shown in the Fig. 3 and Fig. 4. The imaginary part gives only one real value of k shown in Fig. 5.

We see from the Fig. 3 that the wave number is huge large close to the event horizon and the waves lose energy as we go away from the event horizon of dS black hole. This shows that the increase in ω increases k and the waves are in growing mode as z decreases. The group and phase velocities admit the same pattern. Since $\frac{dn}{d\omega} < 0$, the region is not of normal dispersion.

The Fig. 4 shows that the wave number k decreases as z increases i.e. the waves are growing energy with increase in ω and decrease in z but damping occurs when z raises. So waves drop energy when we depart from event horizon. At the event horizon , we see that the wave number turns into infinite which means that the waves disappear due to the effect of immense gravity. The phase velocity is greater than the group velocity and these are negative in some region. Since $k, v_p, v_g < 0$ and $\frac{dn}{d\omega} \leq 0$ for some region, the dispersion is not normal.

We see from Fig. 5 that the wave number is directly proportional to angular frequency but inversely proportional to the distance from the event horizon. The waves gain energy with the increase in angular frequency but lose when we move from the horizon and hence damping modes occur in this direction. But in the vicinity of the horizon, the wave number is very large due to strong gravitational field, so no real wave exists there. The phase and group velocities admit the same behavior excepts for a few point. Since $\frac{dn}{d\omega} \leq 0$, the dispersion is not normal in this region.

VI. NON-ROTATING MAGNETIZED/NON-MAGNETIZED SURROUNDINGS

The magnetosphere has the perturbed flow only along z-axis in non-rotating surroundings. For this case, (40)-(46) reduce to

$$-\frac{i\omega}{\alpha}c_5 = 0 \quad (53)$$

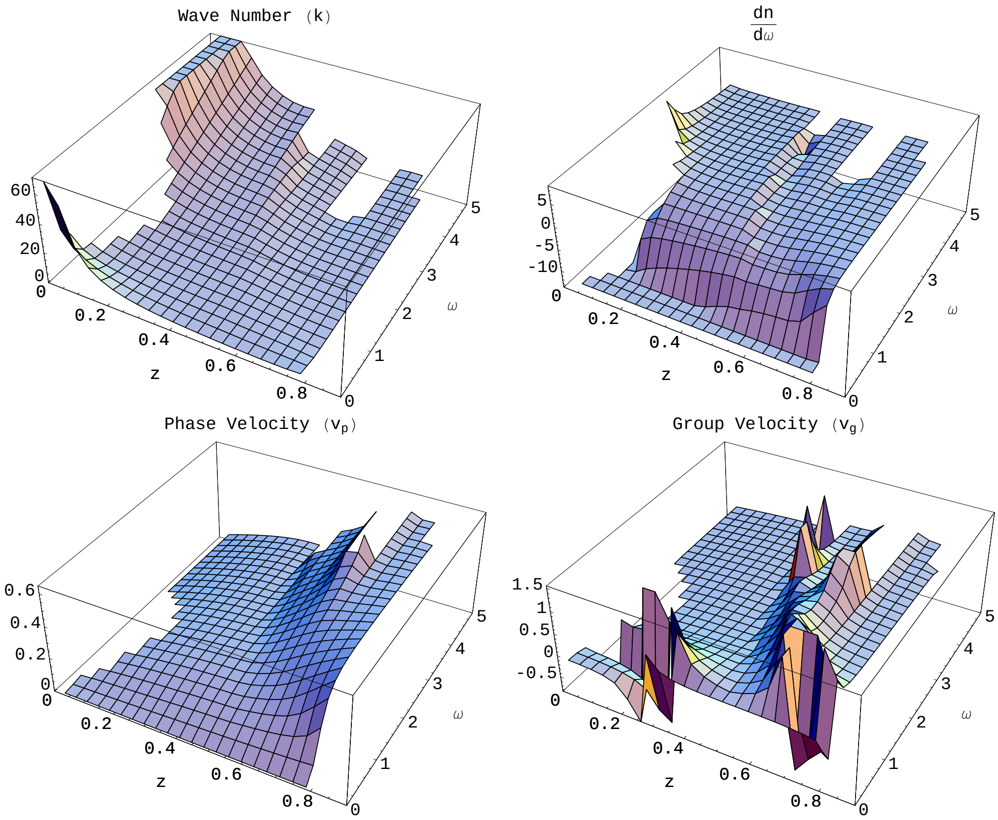

Fig. 2.— The region shows anomalous dispersion. $v_p < v_g$ and $\frac{dn}{d\omega} \leq 0$ in most region

$$-ikc_5 = 0 \tag{54}$$

$$\begin{aligned} &c_1\{-i\rho\omega + ik\rho\alpha u + (u\alpha p)' - \alpha u^2\gamma^2 pu'\} + c_2\{-ip\omega + ikp\alpha u - (u\alpha p)' + \alpha u^2\gamma^2 pu'\} \\ &+c_3(\rho+p)\left\{ik\alpha(1+\gamma^2u^2) - \alpha(1-2\gamma^2u^2)(1+\gamma^2u^2)\frac{u'}{u} - i\omega\gamma^2 u\right\} = 0, \end{aligned} \tag{55}$$

$$\begin{aligned} &c_1\rho\gamma^2\{a_z + uu'(1+\gamma^2u^2)\} + c_2[\rho\gamma^2\{a_z + uu'(1+\gamma^2u^2)\} + ikp + p'] + c_3(\rho+p)\gamma^2 \\ &\times\left[(1+\gamma^2u^2)\left(-\frac{i\omega}{\alpha} + iuk\right) + \{u'(1+\gamma^2u^2)(1+4\gamma^2u^2) + 2\gamma^2 ua_z\}\right] = 0, \end{aligned} \tag{56}$$

$$\begin{aligned} &c_1\left\{i\rho\gamma^2\left(-\frac{\omega}{\alpha} + uk\right) + \gamma^2 u\rho' + 2\rho\gamma^2 ua_z + \rho(1+\gamma^2u^2)\gamma^2 u'\right\} \\ &+c_2\left\{-\frac{i\omega}{\alpha}p(\gamma^2-1) + ik\gamma^2 up + \gamma^2 up' + 2\gamma^2 upa_z + p(1+2\gamma^2u^2)\gamma^2 u'\right\} \\ &+c_3(\rho+p)\gamma^2\left\{-\frac{2i\omega}{\alpha}\gamma^2 u + (ik + 3\gamma^2 uu' + a_z)(1+2\gamma^2u^2) - \frac{u'}{u}\right\} = 0 \end{aligned} \tag{57}$$

where FIDO-measured fluid four-velocity $\mathbf{V} = u(z)\mathbf{e}_z$, Lorentz factor $\gamma = \frac{1}{\sqrt{1-u^2}}$ and FIDO-measured magnetic field $\mathbf{B} = B(z)\mathbf{e}_z$.

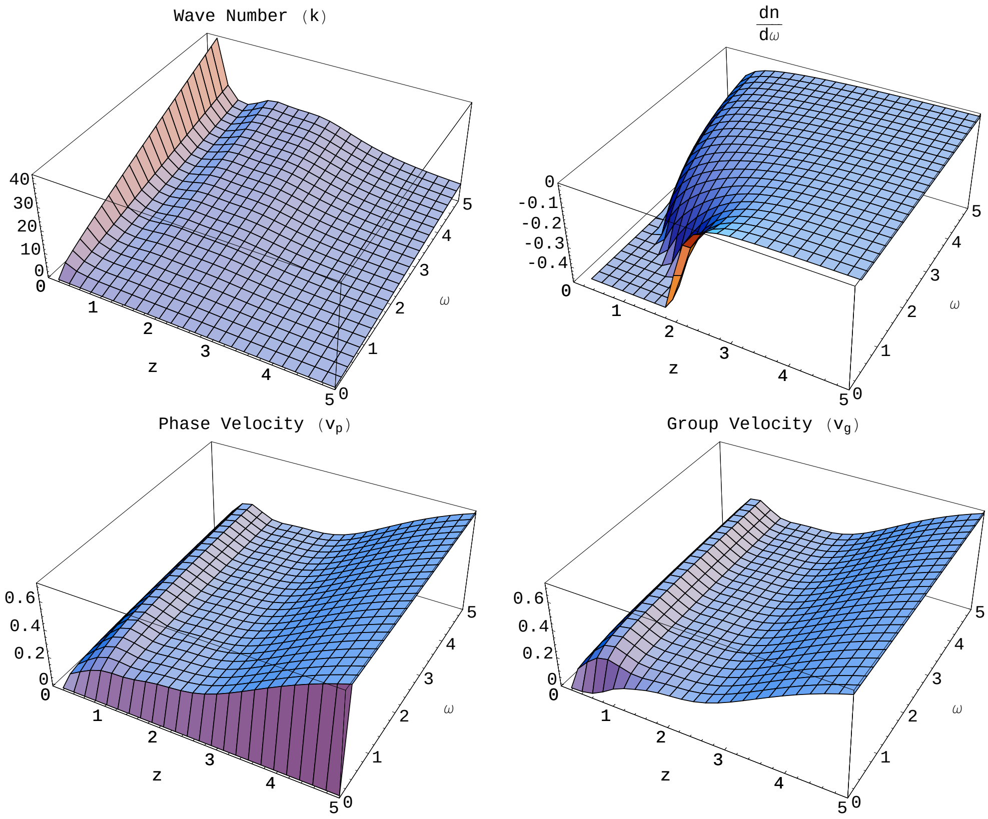

Fig. 3.— The region shows not normal dispersion. Phase and group velocities are same and $\frac{dn}{d\omega} < 0$

It follows from (53) or (54) that c_5 is zero; hence, there is no perturbation occurring in magnetic field of the fluid. Therefore we also get the same equations for non-rotating non-magnetized surroundings. The determinant of the coefficients of c_1, c_2 and c_3 in (55)-(57) we get a dispersion relation of form

$$A_1(z,\omega)k^2 + B_1(z,\omega)k + C_1(z,\omega) + i[A_2(z,\omega)k^3 + B_2(z,\omega)k^2 + C_2(z,\omega)k + D_2(z,\omega)] = 0 \quad (58)$$

Now we analyze the numerical solution mode for this case. We investigate the longitudinal waves propagating parallel to the magnetic field **B**. From the mass conservation law we get $u = 1/\sqrt{1+z^2}$. From the real part of (58) we get only one distinct real value of k which is shown in the Fig. 6. The imaginary part gives only one real of k shown in Fig. 7.

We observe from the Fig. 6 that the wave number is infinite at $z = 0$ and hence no wave exists there. The wave number decreases as we depart from the event horizon. The waves show damping modes for increasing z. The increase in ω increases k. The phase and group velocities have almost same behavior in this region. Since $\frac{dn}{d\omega} < 0$, the region is not of normal dispersion.

Fig. 7 shows that the wave number becomes very large and hence there exists no real wave very close to event horizon. The waves are gaining energy with the increase in ω but losing with the increase of the distance from the event horizon. Here phase velocity is greater than the group velocity and group velocity is negative for some values of z and ω. Since $\frac{dn}{d\omega} < 0$ for most region, the region is not of normal dispersion.

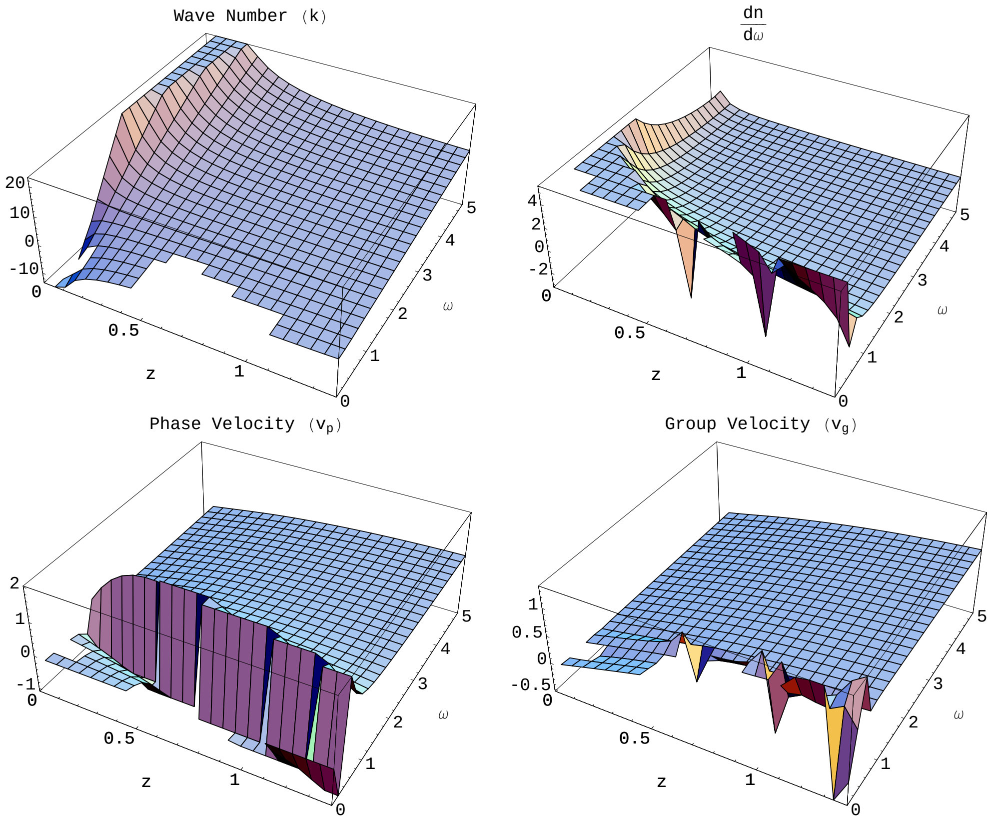

Fig. 4.— The region shows not normal dispersion for $\frac{dn}{d\omega} \leq 0$ and phase velocity is greater than group velocity. k, v_p and v_g are negative for some region

VII. OUTLOOK AND DISCUSSIONS

This study is dedicated completely to analyze wave properties of the isothermal plasma in the dS black-hole's magnetosphere by using the TPM $3+1$ formalism. To do this we derive the GRMHD equations considering linear perturbations in perfect MHD flow with its planar analogue. These equations are then explicitly written in component form and then Fourier analyzed for simple harmonic waves. We considered non-rotating and rotating surroundings (either non-magnetized or magnetized).

From the determinants of the coefficients of Fourier analyzed equations we get wave numbers by solving the complex dispersion relations. The properties of plasma are concluded on the basis of the wave number and the relevant quantities are obtained in graphical form. For all the cases we find out that, the wave number becomes very large at the event horizon and consequently no wave is present there due to immense gravitational field. But when we depart from horizon, the waves lose energy. For most cases the waves are in damping mode as we go away from the horizon and in growing mode as we go close to the horizon.

For rotating magnetized surroundings, we also find region with negative wave number, phase velocity, group velocity, and $\frac{dn}{d\omega}$ which implies that the region has all the characteristics of metamaterials (Fig.1). But Fig. 2 shows anomalous dispersion. In the case of rotating non-magnetized surroundings we find the cases which are not normally dispersive shown in Figs. 3, 4, 5. Our outcomes comply with the result of Mackay et al. (Buzzi, Hines & Treumann, 1995b) according to which negative phase velocity exists for the case of rotating surroundings. For non-rotating surroundings, we observe that the magnetospheric fluid does not disperse normally (Figs. 6, 7). This implies that the surroundings pressure ceases normal dispersion of waves.

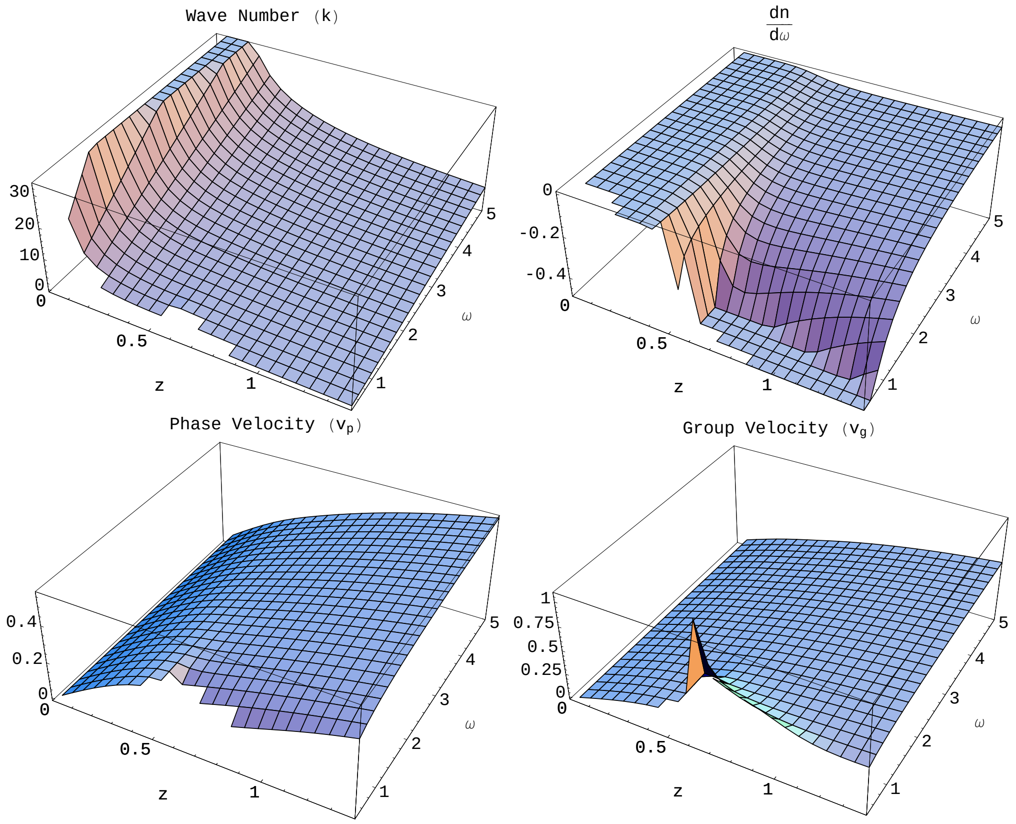

Fig. 5.— The region shows not normal dispersion. Phase and group velocities are same excepts for a few points, $\frac{dn}{d\omega} < 0$

We do not discuss the complex solutions of the dispersion relation since these show little significance compared to those which have been discussed. Our derived dispersion relations are dissimilar with the usual MHD dispersion relations because we use the 3+1 TPM formalism and the factor of acceleration (depend on lapse function and equals to $-g$)

According to recent astronomical observations, it has been suggested that our universe will asymptotically approach a de Sitter space (Sakai & Kawata, 1980). Hence, aspects of the de Sitter space might be of interest in a broader context. We feel that our study of dispersion of isothermal plasma in the de Sitter space is well encouraged.

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REFERENCES

Aharony, O., Gubser, S., Maldacena, J., Ooguri, H., & Oz, Y., 1999, Large N field theories, string theory and gravity, PhR, 323, 183

Anile, A. M., 1989, Relativistic Fluids and Magneto-Fluids with Applications in Astrophysics and Plasma Physics, Cambridge University Press, London

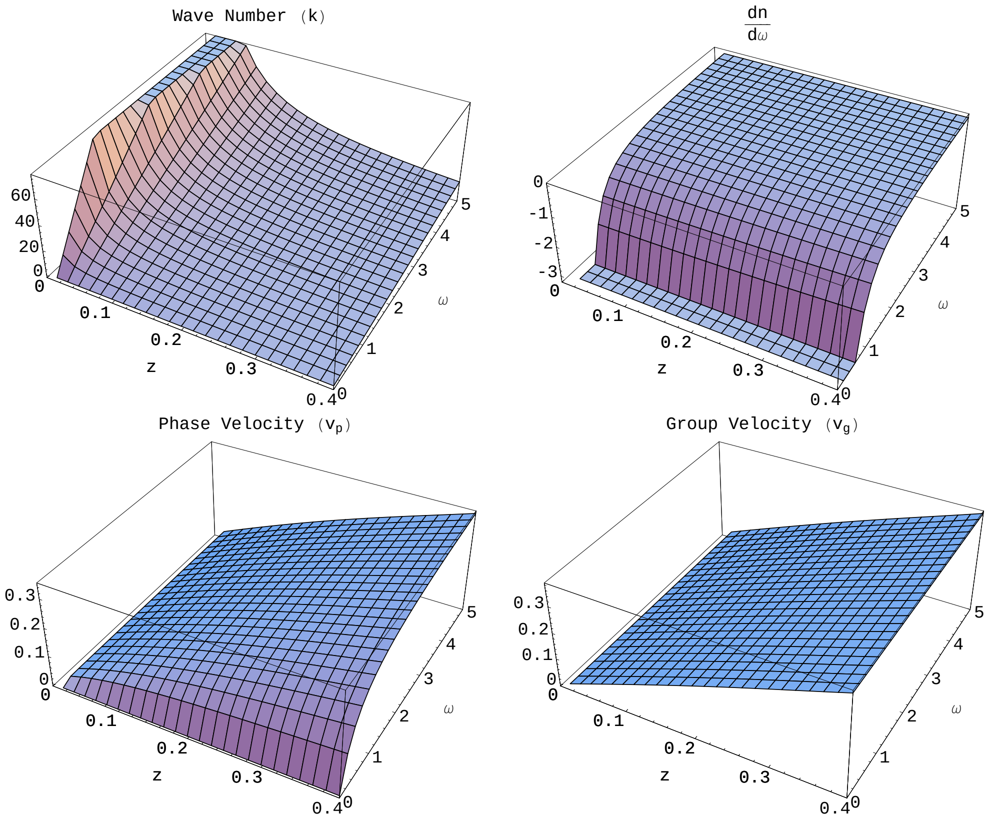

Fig. 6.— The region shows not normal dispersion. Phase and group velocities have almost same pattern and $\frac{dn}{d\omega} \leq 0$

Antón, L., Zanotti, O., Miralles, J. A., Martí, J. A., Ibáñez, J., Font, J. A., & Pons, J. A., 2006, Numerical 3+1 General Relativistic Magnetohydrodynamics: A Local Characteristic Approach, ApJ, 637, 296

Arnowitt, R., Deser, S., & Misner, C. W., 1962, *In*: L. Witten (ed.) Gravitation, An Introduction to Current Research. Wiley, New York

Bahcall, N., Ostriker, J. P., Perlmutter, S., & Steinhardt, P. J., 1999, The Cosmic Triangle: Revealing the State of the Universe, Science, 284, 1481

Bousso, R., Maloney, A., & Strominger, A., 2002, Conformal vacua and entropy in de Sitter space, PhRvD, 65, 104039

Buzzi, V., Hines, K. C., & Treumann, R. A., 1995, Relativistic two fluid plasmas in the vicinity of a Schwarzschild black hole-Local approximation. II, PhRvD, 51, 6677

Buzzi, V., Hines, K. C., & Treumann, R. A., 1995, Relativistic two fluid plasmas in the vicinity of a Schwarzschild black hole-Local approximation. I, PhRvD, 51, 6663

Dettman, C. P., Frankel N. E., & Kowalenko, V., 1993, Plasma electrodynamics in the expanding Universe, PhRvD, 48, 5655

Evans, C. R., Smarr, L. L., & Wilson, J. R., 1986, Norman, M., Winkler, K. H. (eds.) Astrophysical Radiation Hydrodynamics. Reidel, Dordrecht

Gubser, S., Klebanov, I., & Polyakov, A., 1998, Gauge theory correlators from non-critical string theory, PhLB, 428, 105

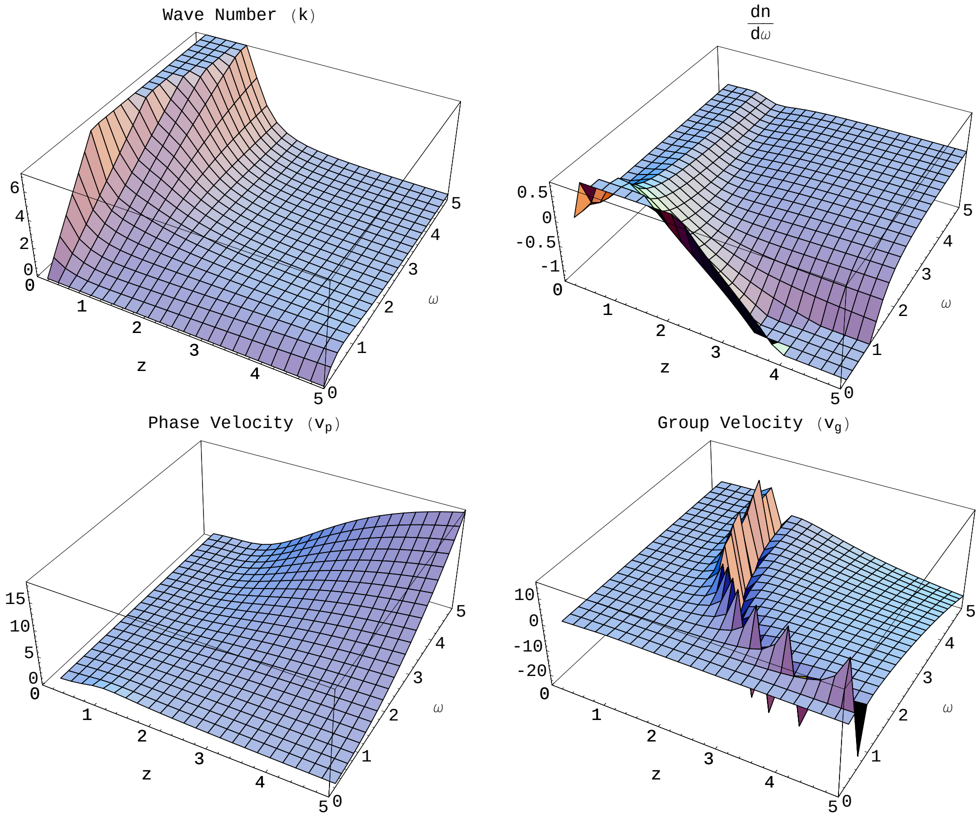

Fig. 7.— The region shows not normal dispersion. Here $v_p > v_g$ and $\frac{dn}{d\omega} < 0$ in most region

Hawking, S., 1972, Black holes in general relativity,CMaPh, 25, 152

Holcom, K. A., 1990, Linear analysis of postrecombination magnetized plasmas, ApJ, 362, 381

Holcomb, K. A. & Tajima, T., 1989, General-relativistic plasma physics in the early Universe, PhRvD, 40, 3809

Hull, C. M., 1998, Timelike T-duality, de Sitter space, large N gauge theories and topological field theory, JHEP, 07, 021

Israel, W., 1968, Event horizons in static electrovac space-times, CMaPh, 8, 245

Israel, W., 1967, Event Horizons in Static Vacuum Space-Times, Phys. Rev., 164, 1776

Khanna, R., 1998, On the magnetohydrodynamic description of a two-component plasma in the Kerr metric, MNRAS, 294, 673

Klemm, D., 2002, Some aspects of the de Sitter/CFT correspondence, NuPhB, 625, 295

Komissarov, S. S. 2002, Time-dependent, force-free, degenerate electrodynamics, MNRAS, 336, 759

Macdonald, D. A. & Thorne, K. S., 1982, Black-hole electrodynamics - an absolute-space/universal-time formulation, MNRAS, 198, 345

Mackay, T. G. , Lakhtakia, A., & Setiawan, S., 2005, Non-classical interference between independent sources, 7, 171

Maldacena, J., 1998, The Large N Limit of Superconformal Field Theories and Supergravity, AdTMP, 2, 231

Medved, A. J., 2002, Radiation via tunneling from a de Sitter cosmological horizon, PhRvD, 66, 124009

Ostriker, J. P. & Steinhardt, P. J., 1995, The observational case for a low-density Universe with a non-zero cosmological constant, 337, 600

Parikh, M., 2002, New coordinates for de Sitter space and de Sitter radiation, PhLB, 546, 189

Park, Mu-In, 1998, Statistical entropy of three-dimensional Kerr-de Sitter space, PhLB, 440, 275

Park, Mu-In, 1999, Symmetry algebras in Chern-Simons theories with boundary: canonical approach, NuPhB, 544, 377

Perlmutter, S. , et al., 1999, Measurements of Omega and Lambda from 42 High-Redshift Supernovae, ApJ, 517,565

Price, R. H. & Thorne, K. S., 1986, Membrane viewpoint on black holes: Properties and evolution of the stretched horizon, PhRvD, 33, 915

Rahman, M. A. & Ali, M. H., 2008, Transverse Wave Propagation in Relativistic Two-fluid Plasmas in de Sitter Space, arxiv: gr-qc/0806.2740

Reiss, A. G., et al., 1998, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, AJ, 116, 1009

Robinson,D., 1974, Classification of black holes with electromagnetic fields, PhRvD, 10, 458

Sakai, J. & Kawata, T., 1980, Waves in an ultra-relativistic electron-positron plasma, 49, 747

Sharif, M. & Sheikh, U., 2007, Effects of Schwarzschild black hole horizon on isothermal plasma wave dispersion, 39, 2095

Strominger, A., 2001, Inflation and the dS/CFT Correspondence, JHEP, 11, 049

Strominger, A., 2001, The dS/CFT correspondence, JHEP, 10, 034

Thorne, K. S. , Price, R. H., & Macdonald, D. A., 1986, Black Holes: The Membrane Paradigm, Yale University Press, New Haven (1986)

Thorne, K. S., & Macdonald, D. A., 1982, Electrodynamics in Curved Spacetime - 3+1 Formulation, 197, 339

Witten, E., 1998, Adv. Theor. Math. Phys., 2, 253

Witten, E., 2001, Quantum gravity in de Sitter space, arXiv:hep-th/0106109

Zhang, X.-H., 1989, 3+1 formulation of general-relativistic perfect magnetohydrodynamics, PhRvD, 39, 2933

Zhang, X.-H., 1989, Interactions of magnetohydrodynamic waves with gravitomagnetic fields, and their possible roles in black-hole magnetospheres, PhRvD, 40, 3858