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INVESTIGATIONS OF THE K–STAGE ERLANGIAN SOFTWARE RELIABILITY GROWTH MODEL

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Abstract: The Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal K–stage Erlangian growth curve based on the Khoshgoftaar's [1] software reliability model is investigated and an expression for the error of the best approximation is obtained in this paper. The results of numerical examples confirm theoretical conclusions and they are obtained using programming environment Mathematica. We give real examples with dataset proposed in [8] using Khoshgoftaar's model.

AMS Subject Classification: 68N30, 41A46
Key Words: K–stage Khoshgoftaar's model, shifted Heaviside function $h_{t_0}(t)$, Hausdorff approximation, upper and lower bounds

1. Introduction

In this article we study the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal cumulative function based on the Khoshgoftaar's model.

We give a software modules within the programming environment CAS Mathematica for illustrating the results.

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Some results for Erlang distributed moments of impulses are given by Agarwal, Hristova, O'Regan, Kopanov in [2].

For the hyper–Erlang distribution model and its applications in wireless network and mobile computing system, see [3].

For the hypoexponential distribution, or the generalized Erlang distribution, see [4].

The Erlang distribution is now used in the fields of statistic processes, teletraffic engineering, biomathematics, applied insurance mathematics (see, for instance [5]), etc.

Definition 1. Based on the Erlang cumulative distribution function Khoshgoftaar [1] developed the following software reliability model:

$$M(t) = a\left(1 - \sum_{n=0}^{k-1} \frac{e^{-\lambda t}(\lambda t)^n}{n!}\right); \quad t, \lambda \geq 0. \tag{1}$$

The K–stage model (1) is a two parameter model with parameters a and λ being the total number of errors and the error detection rate, respectively.

The model at $k = 3$; $k = 4$ is widely used in practice.

Definition 2. The generalized Erlangian software reliability growth model proposed by Khoshgoftar and Woodcock [6] with n types of defects will have the following function (a is the total number of defects):

$$M_1(t) = \sum_{i=1}^{n} a_i \left(1 - \sum_{j=0}^{i-1} \frac{e^{-\lambda_i t}(\lambda_j t)^n}{n!}\right); \quad \sum_{i=1}^{n} a_i = a. \tag{2}$$

Some investigations of the model (2) can be found in [7].

Definition 3. The shifted Heaviside function is defined as:

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0 \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{3}$$

We will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation of the function $h_{t_0}(t)$ by cumulative functions of type (1), or (2) - the subject of study in the present paper.

Definition 4. [9] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f,g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

2. Main Results

2.1. A note on the Khoshgoftaar's software reliability model (1) [1]

Without loosing of generality we will look at the following ”cumulative sigmoid” with $a = 1$:

$$M^*(t) = 1 - \sum_{n=0}^{k-1} \frac{e^{-\lambda t}(\lambda t)^n}{n!}. \tag{4}$$

Let t_0 is the unique positive solution of the nonlinear equation $M^*(t) - \frac{1}{2} - 0$, i.e. $M^*(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid (4) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{5}$$

Let

$$A = -\frac{Gamma[k, t_0\lambda]}{Gamma[k]},$$

$$B = 1 + \frac{e^{-t_0\lambda}\lambda(t_0\lambda)^{k-1}}{Gamma[k]},$$

where

$$Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt; \quad Gamma(s,x) = \int_x^\infty t^{s-1}e^{-t}\,dt.$$

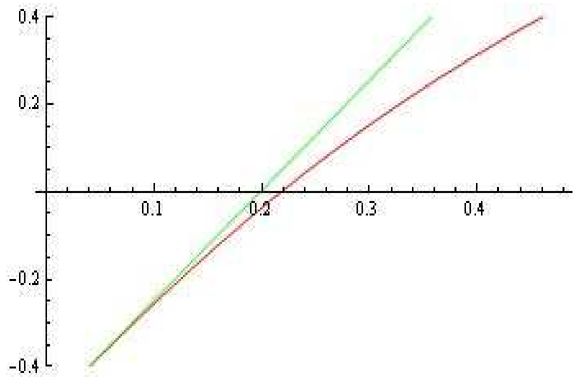

Figure 1: The functions $F(d)$ and $G(d)$.

are the gamma function and incomplete gamma function, respectively.

The following theorem gives upper and lower bounds for d

Theorem. For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and the sigmoid (4) the following inequalities hold for: $2.1B > e^{1.05}$

$$d_l = \frac{1}{2.1B} < d < \frac{\ln(2.1B)}{2.1B} = d_r. \tag{6}$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 + d) - 1 + d. \tag{7}$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = A + Bd. \tag{8}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $2.1B > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

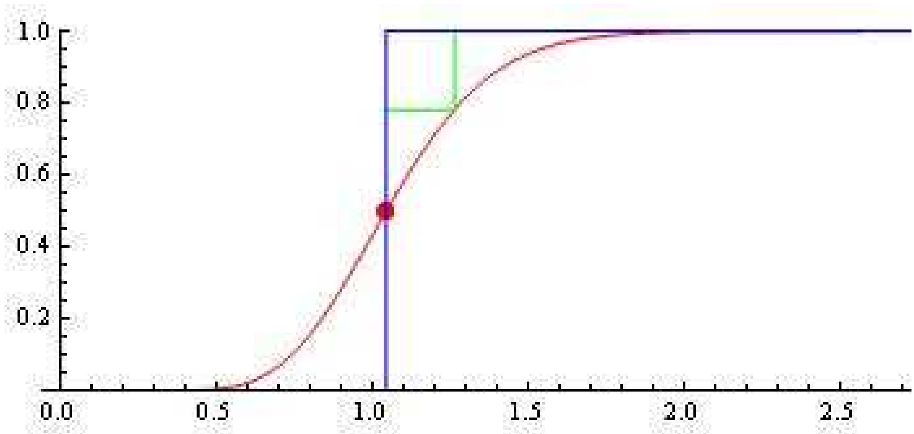

Figure 2: The model (4) for $\lambda = 15$, $a = 1$, $k = 16$, $t_0 = 1.04453$; H–distance $d = 0.218431$, $d_l = 0.189378$, $d_r = 0.315127$.

The model

$$\begin{aligned} M^*(t) &= 1 - e^{-15t} - 15e^{-15t}t - \frac{225}{2}e^{-15t}t^2 - \frac{1125}{2}e^{-15t}t^3 - \frac{16875}{8}e^{-15t}t^4 \\ &\quad - \frac{50625}{8}e^{-15t}t^5 - \frac{253125}{16}e^{-15t}t^6 - \frac{3796875}{112}e^{-15t}t^7 - \frac{56953125}{896}e^{-15t}t^8 \\ &\quad - \frac{94921875}{896}e^{-15t}t^9 - \frac{2847656250}{1792}e^{-15t}t^{10} - \frac{4271484375}{19712}e^{-15t}t^{11} \\ &\quad - \frac{21357421875}{78848}e^{-15t}t^{12} - \frac{320361328125}{1025024}e^{-15t}t^{13} - \frac{4805419921875}{14350336}e^{-15t}t^{14} \\ &\quad - \frac{4805419921875}{14350336}e^{-15t}t^{15} \end{aligned}$$

for $\lambda = 15$, $a = 1$, $k = 16$, $t_0 = 1.04453$ is visualized on Fig. 2.

From the nonlinear equation (5) and inequalities (6) we have: $d = 0.218431$, $d_l = 0.189378$, $d_r = 0.315127$.

2.2. Numerical example.

We examine the following data presented in Table 1, was proposed in [8]. The week index is from 1 week to 18 weeks, and there are 176 cumulative failures at 18 weeks in Dataset.

The fitted model $M(t)$ based on the data of Table 1 for the estimated parameters: $a = 176$; $\lambda = 1.93382$ is plotted on Fig. 3.

$Week\,Index$	$Failures$	$Cumulative\,failures$
1	28	28
2	1	29
3	0	29
4	0	29
5	0	29
6	8	37
7	26	63
8	29	92
9	24	116
10	9	125
11	14	139
12	13	152
13	12	164
14	0	164
15	1	165
16	3	168
17	2	170
18	6	176

Table 1: Dataset [8]

The example results show a good fit to the presented model.

Obviously, studying of phenomenon "super saturation" is mandatory element along with other important components - "confidence bounds" and "confidence intervals" when dealing with questions from Software Reliability Models domain.

For some software reliability models, see [10]–[42].

We hope that the results will be useful for specialists in this scientific area.

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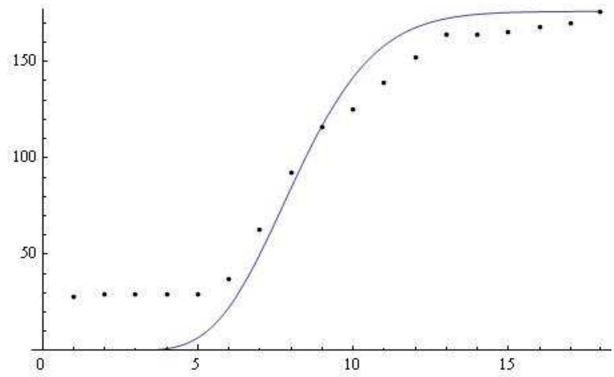

Figure 3: The fitted model $M(t)$ with $a = 176$; $\lambda = 1.93382$.

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