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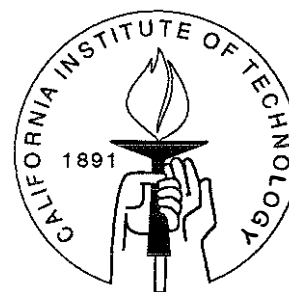
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INVESTMENT AND INSIDER TRADING

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Abstract

Within a dynamic environment, this paper introduces an inside trader to an economy where rational, but uninformed, traders choose between investment projects with different levels of insider trading. When inside information has little value in future investment decisions, insider trading distorts investment towards assets with less private information, imposing net welfare costs on the economy. When an insider's private information is valuable in making future investment decisions, the net social benefit of inside trading can be positive; the resulting increases in investment efficiency due to more informative prices is enough to compensate for the distortion induced by the inside trader.

When insiders receive private information more than once, insiders may trade to reveal their private information at the beginning of their relationship with the firm. This has two effects; i) more information is revealed in equilibrium and ii) there is less chance than an uninformed agent will have to trade with the insider. Both these effects reduce the investment inefficiencies associated with insider trading. As a consequence, uninformed liquidity traders prefer to trade in a market with a long-term insider. This improvement in investment efficiency, leads to a Pareto improvement — both the uninformed traders and the insider are made better off if the insider receives information more than once.

Introduction

One of the more controversial issues in stock market policy is the regulation of insider trading. In this paper, we analyze the costs and benefits of banning insiders in a dynamic, general equilibrium economy. Our analysis allows explicit Pareto comparasons amongst economies with and without insider trading. Insiders' trading reveals private information, and to the extent that this information is correlated to future productive opportunities, insider trading enhances productive efficiency. Since insider trading profits come at the expense of liquidity traders, rational liquidity traders will invest less in firms with substantial insider trading. This will distort investment away from firms with heavy insider trading, resulting in investment inefficiency. We analyze the trade-off amonst these effects.

Suppose the insider receives private information more than once. For example, an oil company most likely develops more than one oil well, and an insider is likely to be privately informed about more than one of those wells. How should the insider trade in this situation? We show that if the insider values future trading profits enough, then in equilibrium, she will trade to reveal her private information at the beginning of her relationship with the firm. This has two effects: i) prices become more informative and ii) there is less chance that uninformed traders will have to trade against an inside trader in the future. Both these effects increase investment efficiency. Thus, insiders with an enduring relationship with the firm can lead to a Pareto improvement, making all participants, including the outsiders, better off.

We view our analysis as particularly relevant to small resource exploration stocks. One can imagine an insider who forsakes short term trading profits from information about a resource discovery for the greater future insider trading profits associated with increased exploration. Such insiders often initially hold large stakes in their firms. Other examples where the analysis seems appropriate include new product development, takeovers and mergers.

To our knowledge, our model is the first dynamic general equilibrium model to incorporate both the benefits and costs of insider trading. The benefits of insider trading evolve from the fact that prices will reflect more information in the presence of insider trading. If this information is socially valuable, permitting insider trading may result in a superior allocation of resources. Manne (1966) was the first to make this argument. George (1988) documents how insider trading may lead to more efficient prices in a rational expectations model. In a static partial equilibrium model, Leland

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(1992) argues that insiders may improve investment decisions. Fishman and Hagerty (1992) point out that allowing insiders to trade may lead to less informative prices. The increased competition from insiders may reduce the incentive of other agents to purchase costly information. Thus, it is possible in equilibrium less information is gathered so that prices are less informative.

The arguments that insider trading is socially costly include Brudney (1979) who claims that insider trading is unfair to uninformed investors. Glosten (1989) develops a model where insider trading leads to imperfect risk-sharing, and if there is too much private information there is a loss of liquidity in the market. This imposes costs on liquidity traders. Leland (1992) points out that insider trade may reduce the liquidity of the market, hurting uninformed traders.

Manove (1989) looks at the investment distortions caused by allowing insider trading. He constructs a static model[1] in which insiders can capture a share of the profits made by investment. This reduces the incentive for firms to make profitable investments. distorting investment.

Our paper is closest in spirit to Ausubel (1990) who also considers a general equilibrium environment. Ausubel introduces both production and preference shocks to his competitive rational expectations economy so that the equilibrium is necessarily partially revealing. He shows that a policy of forcing the insider to reveal his private information before any trading takes place may be Pareto superior. This is generally socially beneficial because when insiders trade, both types of payoff-relevant information are revealed so that investment by the uninformed is better targeted. In his competitive environment, however, insiders do not act strategically. In contrast, our single insider trades in a competitive dealership market which forces her to consider the effects of her trades on the equilibrium price and the information it contains. All other agents must take account of this strategic behaviour. In our dynamic environment, the decisions of whether and how to exploit inside information are closely intertwined with the nature of the insider's relationship with the firm, as well as the firm's past and current performance.

The main results of our analysis are as follows. We first detail how rational, but uninformed. investors are affected by the presence of insiders in some markets. Recognizing that they may have to trade in a market where some agents have private information so that they will not get the "fair" price for their holdings, uninformed investors will over-invest in safe assets. Consequently, the return to the assets with insiders must be greater to compensate the investors.

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[1]His equilibrium concept is nonstandard; those who bid above a certain level are allocated shares, and not to the highest bidders. Further, agents do not optimize.

The presence of the insiders, however, may actually be beneficial to the uninformed investors, *ex ante*. Information that past insiders possessed may be revealed through the trading process. The information in past prices provides useful information about future productive opportunities to the uninformed investors. If past prices are sufficiently correlated with future payoffs, then past insider trade is valuable in targeting the investment of the uninformed. As a consequence, net trading volume predicts future payoffs on production technologies.

The net effects of insider trade on the welfare of the uninformed investors are ambiguous. The uninformed investors prefer to have had insiders trade in the past for the information revealed through past prices, but not in the future when they may have to trade insiders. When there is enough persistence in production shocks, the value of the information content in prices will outweigh the investment distortion costs so that the presence of an insider is socially beneficial.

We also analyze the trading strategy of different types of insiders. Insiders differ with respect to the period of time for which they will be privy to inside information about a firm. Short-term insiders, insiders who only have private information once, care only about maximizing their immediate profits and hence always trade to conceal their information. Long-term insiders, insiders who can exploit private information again in the future, weigh the future investment consequences of their immediate trades. For instance, if they purchase enough to raise current prices, beliefs of uninformed investors will be affected so that more will be invested. This increases future insider trading profits.

We show that an equilibrium to the model always exists and is unique in the sense that expected payoffs are unique. Unless the value of the gain in future profits to an insider seeing good news from increasing investment in the firm strictly exceeds the gain to an insider seeing bad news receiving the same investment, the insider always tries to conceal her private information. As the weight insiders place on future profits increases, the amount an insider trades independent of her information increases, so that the equilibrium volume at which the market maker cannot discern good news from bad increases. This is because an insider who sees bad news also has an incentive to increase purchases or short sell less in order to affect investor beliefs. If insiders value future profits sufficiently, an insider seeing bad news may even purchase a positive amount today in an attempt to convince the uninformed that her private information is good.

Conversely, if an insider seeing good news gains more from increased investment and values

future profits sufficiently, then the insider may select a trading strategy so that initial prices are fully revealing. The insider forsakes immediate insider profits for long term gains. Taking a discernably long position credibly conveys the good news to the investors, information which may not be credibly revealed in other ways. In this instance the uninformed strictly prefer to face long-term insiders to short-term insiders. With long-term insiders, past prices contain more information and there is less chance that the uninformed investors will be exploited by an insider in the future.

The plan of the paper is follows. In Section 1, we develop and analyze a simple general equilibrium, overlapping generations economy with rational, but uninformed "liquidity" traders and a short-term inside trader. Section 2 extends the analysis to consider long-term insiders who will have private information about payoffs in successive periods. Section 3 concludes. All proofs are contained in an appendix.

1 Short-term Insider

In this section we develop the overlapping generations economy. The economy features rational, but uninformed, liquidity traders who live for three periods. In the middle of their lives, some of the liquidity traders will have to liquidate their portfolios. When they are young, they decide how to allocate their investment capital between two assets which pay off two periods hence. One of the assets is risk free, while the other asset has risky returns. There is an insider who next period will acquire advance information about the payoff of the risky asset. Trading is through a competitive, risk-neutral uninformed market maker. The market maker only observes the net order flow, so that he may not be able to distinguish high liquidity shocks and little short selling by an insider seeing bad news from less liquidity demand and large insider purchases by an insider seeing good news. Because the productive shocks are positively autocorrelated, any information revealed through equilibrium prices is useful for the *next generation* of liquidity traders.

Consider an overlapping generations economy with dates $t = -\infty \ldots, -1, 0, 1, \ldots \infty$. There are three types of agents in the economy: uninformed traders, insiders and market makers. All agents are risk neutral. A continuum of measure one of three period-lived uninformed traders is born each period t. Each uninformed investor is endowed with one unit of productive capital, which can be used to invest in the two projects, A and B. These projects pay off in consumption goods two

periods hence. The payoff to project B is random.[2] The payoffs at time $t + 2$ to time t projects are:

$$a\left(K_A^{t,t+2}\right), \quad \theta^{t+2} b\left(K_B^{t,t+2}\right), \tag{1}$$

where $a(\cdot)$ and $b(\cdot)$ are neo-classical, concave production functions, $K_i^{t,t+2}$ is the amount of capital invested in project i at t which comes to fruition in $t + 2$. The first superscript in $K_i^{t,t+2}$ refers to the date at which the investment is made, while the second superscript refers to the date at which the project pays-off. The production functions $a(\cdot)$ and $b(\cdot)$ satisfy:

$$a(0) = b(0) = 0, \ a'(0) > 0, \ b'(0) = \infty, \ a'(1) \geq 1;$$

$$a'(\cdot) > 0, b'(\cdot) > 0, \ b''(\cdot) \leq 0, a''(\cdot) < 0.$$

The conditions on the derivatives of the production technologies ensure that the equilibrium features investment in both projects.

The technology shock, θ^t, follows a two-state Markov process taking on either the value H or L. where $0 < L < H$. Denote the probability transition function for θ^t by:

$$
\begin{aligned}
\pi(\theta^t) &= \ \text{Prob}\left(\theta^{t+1} = H | \theta^t\right), \\
\pi(H) &= \ \text{Prob}\left(\theta^{t+1} = H | \theta^t = H\right), \\
\pi(L) &= \ \text{Prob}\left(\theta^{t+1} = H | \theta^t = L\right).
\end{aligned}
$$

Without loss of generality we rewrite:

$$
\begin{aligned}
\pi(H) &= \ \pi(1 - \gamma) + \gamma. \\
\pi(L) &= \ \pi(1 - \gamma), \tag{2}
\end{aligned}
$$

where π is the unconditional probability that $\theta^t = H$. The parameter γ measures the persistence in the process. We assume that the process is positively autocorrelated, so $0 \leq \gamma < 1$. If $\gamma = 0$ then the technology shock is *i.i.d.* and as γ approaches one, then the technology shocks become perfectly positively correlated over time. The positive persistence in the technology shock implies that $\pi(H) \geq \pi(L)$, so that $E\left[\theta^{t+2} | \theta^t = H\right] \geq E\left[\theta^{t+2} | \theta^t = L\right]$. We will sometimes refer to $\theta^t = H$ as good news.

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[2] Our analysis carries through if project A payoffs are stochastic, as well.

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In period $t+1$, a random measure, $s^{t+1} \in \{l, h\}$ where $0 < l < h < 1$, of the uninformed investors receive an uninsurable liquidity shock. Either measure h or measure l of the uninformed traders receives this liquidity shock which forces them to sell in period $t+1$ all claims to period $t+2$ output. We assume that these shocks are equally likely and are independent of the technology shocks. If the liquidity shock is h, then to a particular uninformed agent, h is the probability that he will receive a liquidity shock, forcing him to sell all his shares. Hence, $1 - \frac{(h+l)}{2}$ is the unconditional probability that he will not receive a liquidity shock, $\frac{h}{2}$ is the unconditional probability that he has to sell in the h state, and $\frac{l}{2}$ is the unconditional probability that he has to sell in the l state. Thus, there are four possible $\{\theta^t, s^t\}$ shock combinations that can occur in the economy, $\{(H, h), (H, l), (L, h), (L, l)\}$.

The economy has two market makers, one for each project. The role of each market maker is to take opposite sides of all trades in the markets for the claims on the projects' outputs at $t + 2$. These market makers stand ready to buy/sell all demands at $t + 1$ for claims to output at $t + 2$. Each market maker sees past prices, past θ's, and current net order flow in his own market, but not that in the other market.[3] As in Admati and Pfleiderer (1988) and Kyle (1985), each competitive specialist pools orders and sets a price in units of the current output good which, conditional on his information, leaves him zero expected profits. The specialist is not liquidity constrained.

Besides the uninformed investors and the market maker, there is an insider. At time t, the insider learns the value of θ^{t+1}. As in Kyle (1985), she uses this information as well as the pricing schedule to determine her asset trades. In this section we assume that this insider has only a temporary relationship with the firm: she receives inside information about the firm only once. Hence she trades to maximize expected current period profits. In Section 2. we analyze the case where the insider has a long-term relationship with the firm.

For simplicity. we assume that each uninformed investor "has his own project." which he sells to the appropriate specialists in the event of a liquidity shock the next period. We could alternatively set up a single project share market. Our results are robust to the project market formulation.

We now define some useful notation. Let $K^{t,t+2} \equiv \{K_A^{t,t+2}, K_B^{t,t+2}\}$ represent aggregate capital invested at time t in projects A, B that pay off at time $t + 2$ and let q^t be the insider's trade in

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[3]This assumption is made to reduce notational clutter. Equilibrium outcomes are unchanged if market makers could observe net order flows in both markets, or equivalently if there were a single market maker. In these instances, the insider must transact in both markets so that order flow in market A contains no information about the realization of liquidity trade and hence cannot reveal information about the θ realization. The insider can do this by making the same transactions in both markets, or by choosing an uninformative stochastic purchase in market A. The assumption that each specialist observes only the net order flow in his market avoids this added complexity.
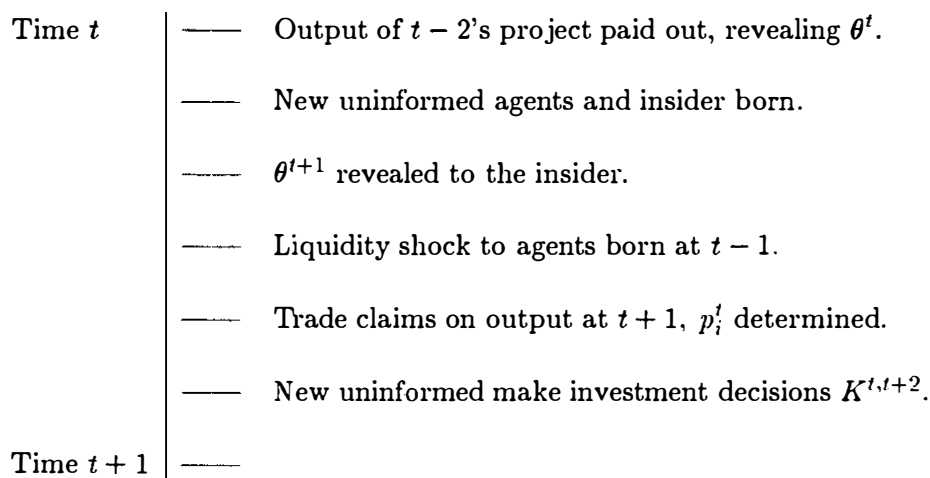
market B in period t. Then,

$$v_B^t = q^t - s^t \tag{3}$$

is the net order flow observed by the market maker in market B. The specialist's pricing function in market $i \in \{A, B\}$, is given by $P_i(v_i^t, K^{t-1,t+1}, \theta^t)$, and realized equilibrium prices are given by p_i^t.

A time line for a representative period t is given by figure 1.

Figure 1 - The Time Line

Time t —— Output of $t - 2$'s project paid out, revealing θ^t.

     —— New uninformed agents and insider born.

     —— θ^{t+1} revealed to the insider.

     —— Liquidity shock to agents born at $t - 1$.

     —— Trade claims on output at $t + 1$, p_i^t determined.

     —— New uninformed make investment decisions $K^{t,t+2}$.

Time $t + 1$ ——

We will consider perfect Bayesian equilibrium of this model. This equilibrium concept requires that at each time and information set, each player is acting optimally. conditional on their beliefs about the technology and liquidity shocks, the distribution of future prices of claims to the technologies, and the strategies of the other players. Further, we require that all agents' beliefs are restricted to the support of the technology and liquidity shocks and satisfy Bayes' rule wherever possible. We now formally define the equilibrium.

Definition: A symmetric[4] perfect Bayesian equilibrium with a short-term insider is a collection of the following at each date t:

1. Investments by each uninformed trader given his conjecture at time t about the price distribution of claims at $t + 1$ in projects A, B, which pay off in two periods, $k^{t,t+2}(p_B^t, \theta^t) \equiv$

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[4]Asymmetric equilibria feature the same aggregate investment levels and prices. For expositional reasons we present the symmetric version of the equilibrium.

$\{k_A^{t,t+2}(p_B^t, \theta^t), k_B^{t,t+2}(p_B^t, \theta^t)\}$, yielding aggregate investment levels, $K^{t,t+2}(p_B^t, \theta^t)$, such that $K^{t,t+2}(p_B^t, \theta^t) = k^{t,t+2}(p_B^t, \theta^t)$.[5]

2. A strategy for the insider in project B which details her purchases as a function of the public information, $(K^{t-1,t+1}, \theta^t)$, and her private information, θ^{t+1}, given her conjectures about the market maker's pricing function: $q(\theta^{t+1}, \theta^t, K^{t-1,t+1}; P_B(\cdot))$.[6]

3. A belief function $\mu : \mathcal{R} \times \mathcal{R} \times \{H, L\} \Rightarrow [0, 1]$ giving the beliefs of the market makers and uninformed investors that the insider observed $\theta^{t+1} = H$ conditional on the current order flow and the current public information $(K^{t-1,t+1}, \theta^t)$.

4. A pricing function for each specialist which, given public information $(K^{t-1,t+1}, \theta^t)$ and the net order flow in the specialist's market, determines the price $P_i(v_i, K^{t-1,t+1}, \theta^t)$, $i = A, B$ at which each specialist buys/sells the asset.

Such that:

1. The insider selects $q(\theta^{t+1}, \theta^t, K^{t-1,t+1}; P_B(\cdot))$ to maximize expected profits, where expectations are taken over the liquidity shocks:

$$\max_q E\left[q\left(\theta^{t+1}b\left(K_B^{t-1,t+1}\right) - P_B\left(q - s^t, K^{t-1,t+1}, \theta^t\right)\right) \mid \theta^{t+1}\right], \tag{4}$$

and her conjecture about the specialist's pricing function satisfies rational expectations.

2. The specialist in each market i sets $P_i(v_i^t, K^{t-1,t+1}, \theta^t)$ to earn zero expected profits, conditional on all information available to him. This implies that for B,

$$P_B\left(v_B^t, K^{t-1,t+1}, \theta^t\right) = E\left[\theta^{t+1} | v_B^t, K^{t-1,t+1}, \theta^t\right]b\left(K_B^{t-1,t+1}\right), \tag{5}$$

and for A,

$$P_A(v_A^t, K^{t-1,t+1}, \theta^t) = a\left(K_A^{t-1,t+1}\right), \tag{6}$$

where expectations are taken using the market maker's beliefs defined above, and the beliefs satisfy Bayes' rule where applicable.

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[5]Without loss of generality all uninformed traders are assumed to make the same investment decisions. The equality follows since there is a measure one of uninformed agents.

[6]We suppress arguments of $q(\theta^{t+1}, \theta^t, K^{t-1,t+1}; P_B(\cdot))$ where they are clear from the context.

3. Each uninformed trader chooses his project allocation to maximize expected lifetime payouts. That is, he solves:

$$\max_{k_A^{t,t+2}, k_B^{t,t+2}} a\left(k_A^{t,t+2}\right) + \left(1 - \frac{h+l}{2}\right) E\left[\theta^{t+2}|\theta^t, p_B^t\right] b\left(k_B^{t,t+2}\right) +$$

$$\frac{h}{2} E\left[P_B(q-h, K^{t,t+2}, \theta^t)|\theta^t, p_B^t\right] + \frac{l}{2} E\left[P_B(q-l, K^{t,t+2}, \theta^t)|\theta^t, p_B^t\right] \qquad (7)$$

$$\text{subject to}: \quad k_A^{t,t+2} + k_B^{t,t+2} \leq 1,$$

where the uninformed's beliefs about θ^{t+1} are consistent with the market maker's and their conjectures about next period's price satisfy rational expectations.

The first term in (7) refers to the uninformeds' investment in the riskless technology. The second term refers to the expected payoff of the risky technology, given the uninformed does not receive a liquidity shock. The third and forth terms refer to the expected payoff in the event of a high and low liquidity shock, respectively. This formulation of the equilibrium incorporates the result that the uninformed will not trade in the market when they are young or middle aged unless required by a liquidity shock to cash in their holdings prematurely: to avoid trading against the insider, they will hold their claims to output if possible.

In the analysis below we first show that the insider must earn strictly positive expected profits. Hence equilibrium prices cannot always perfectly reveal the insider's private information. Were equilibrium prices always revealing, then the equilibrium zero expected profits condition for the market maker conditional on the order flow would require that he set price equal to $\theta^{t+1} b(K_B^{t-1,t+1})$. But then the insider would earn zero profits, a contradiction.

The intuition for why the insider must expect positive profits and hence why prices cannot be perfectly revealing is as follows: the zero expected profits condition for the market maker places a bound on the possible price that can be charged for the asset. However, bounding prices by the possible asset values leaves room for positive expected profits for the insider at one information set. Since she can always trade zero there is no way for the market maker to recoup at the other information set. Thus, no equilibrium exists with revealing prices in all states. Formally stated:

Lemma 1 *In any equilibrium the insider's expected profits are strictly positive. Hence there does not exist an equilibrium in which prices always perfectly reveal the insider's private information.*

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We now strengthen lemma 1 to show how the insider must trade in any equilibrium. To avoid revealing her private information, the insider must trade so that the net order flow is uninformative. A result of this is that the insider's expected profits are the same in any equilibrium, where the expectations are taken over the liquidity and production shocks. Since the losses of the uninformed investors must correspond with the insider's profits, the size of the investment distortion due to insider trading is the same in any equilibrium.

Lemma 2 *In any pure strategy equilibrium, the short-term insider's trading strategy is given by:*

$$q\left(\theta^{t+1} = H, \theta_t, K_B^{t-1,t+1}; P_B\right) = \alpha\left(\theta^t, K^{t-1,t}; P_B\right)(h-l), \tag{8}$$
$$q\left(\theta^{t+1} = L, \theta^t, K_B^{t-1,t+1}; P_B\right) = \left(\alpha\left(\theta^t, K^{t-1,t}; P_B\right) - 1\right)(h-l),$$

where $0 \le \alpha \le 1$.

Further, the insider's expected profits before observing the production shock are constant across all values of α, $0 \le \alpha \le 1$. The insider's expected profits are given by:

$$\frac{(h-l)(H-L)}{2}\pi(\theta^t\left(1 - \pi(\theta^t)\right) b\left(K_B^{t-1,t+1}\right) \equiv IP\left(\pi(\theta^t)\right) b\left(K_B^{t-1,t+1}\right), \tag{9}$$

where $\pi(\theta^t)$ is defined to be $Prob(\theta^{t+1} = H|\theta^t)$, the probability of a high shock in $t+1$ conditional on the previous period's productivity realization.

The zero expected profit condition for the market maker requires that the expected losses of the uninformed investors must equal the insider's expected profits: the investors therefore require the insider premium $IP(\pi(\theta^t))$ to compensate them for the insider's presence in asset B. Since the investment distortion is determined by the size of the insider's expected profits, lemma 2 implies that the size of the distortion is the *same* in any possible equilibrium. Lemma 2 also implies that the only three {net order flow, price} pairs that will be observed in equilibrium are:

$$\left\{\alpha(h-l) - l, \ Hb\left(K_B^{t-1,t+1}\right)\right\},$$
$$\left\{\alpha(h-l) - h, \ \left[\pi(\theta^t)H + (1 - \pi(\theta^t))L\right] b\left(K_B^{t-1,t+1}\right)\right\},$$
$$\left\{(\alpha-1)(h-l) - h, \ Lb\left(K_B^{t-1,t+1}\right)\right\}.$$

We now show that an equilibrium exists and that the equilibrium beliefs of the market maker *uniquely identify* the quantity traded by the insider when he sees θ^{t+1}. *The insider's equilibrium*

trading volumes are unique, as are the set of prices, investment levels and allocations along the equilibrium path. [7]

The intuition for the uniqueness results is as follows. If the insider sees a high realization and hence purchases a positive quantity, this will be revealed when there is little liquidity trade. Conversely, if the insider sees a low realization and hence short sells, this will be revealed when there are high liquidity sales. In equilibrium, the insider seeing a high realization must trade so that when there is a high liquidity shock she is confused by the market maker with an insider seeing a low realization when there is a low liquidity shock. Beliefs must be $\pi(\theta^t)$ in this case. For such a strategy to be optimal, the indifference curves for both types of insiders must be tangent at these beliefs and implied trading volumes in {net order flow, market maker beliefs} space. There is a unique net order flow where such a tangency exists. Figure 2 illustrates a belief schedule that supports the equilibrium.

Proposition 1 *The investment levels of the uninformed and the strategies of the insider are the same across all equilibria; only the market maker's off-equilibrium beliefs differ across equilibria. The following characterizes the set of equilibria that can arise with a short-term insider:*

A. *The insider trades $(1 - \pi(\theta^t))\, (h - l) > 0$ if her private information is H, and $-\pi(\theta^t)(h - l) < 0$ if she observes L.*

B. *The market maker in market A sets price equal to $a\left(K_A^{t-1,t+1}\right)$ for all net order flows. The market maker's pricing function in B satisfies:*

$$P_B(v_B^t, K_B^{t-1,t+1}, \theta^t) = \left(\mu(v_B^t, \theta^t, K_B^{t,t+1})(H - L) + L\right) b\left(K_B^{t-1,t+1}\right), \tag{10}$$

where the market maker's beliefs, $\mu(v_B^t, \cdot, \cdot)$, satisfy Bayes' rule:

$$\mu(v_B^t, \cdot, \cdot) = \begin{cases} 0 & v_B^t \leq -h \\ \pi(\theta^t) & v_B^t = -\pi(\theta^t)(h - l) - l, \\ 1 & v_B^t \geq -l \end{cases} \tag{11}$$

and for $-h < v_B^{t+1} < -l$, satisfy the following bound:

$$\max\left\{0, 1 - \frac{\left(1 - \pi(\theta^t)\right)^2 (h - l)}{v_B^t + h}\right\} \leq \mu\left(v_B^t, \cdot, \cdot\right) \leq \min\left\{1, -\frac{\left(\pi(\theta^t)\right)^2 (h - l)}{v_B^t + l}\right\}.$$

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[7]In a similar vein Rochet and Vila (1992) demonstrate uniqueness of equilibrium for a slightly different class of trading games. There, the key features are that the surplus (the value of the existing project) is constant, independent of the trading decisions of the insider and that the insider knows the value of the liquidity shock before trading.

C. *Investment by each uninformed trader solves:*

$$a'\left(1 - K_B^{t,t+2}\right) = \left(E\left[\theta^{t+2}|\theta^t, p_B^t\right] - E\left[IP(\pi(\theta^{t+1}))|\theta^t, p_B^t\right]\right) b'\left(K_B^{t,t+2}\right), \qquad (12)$$

where

$$E\left[IP(\pi(\theta^{t+1}))|\theta^t, p_B^t\right] = \mu IP(\pi(H)) + [1 - \mu]IP(\pi(L))$$

is the expected insider premium. Note that the expectations reflect the beliefs associated with the price set by the market maker in the current period.

An uninformed trader would never want to trade when middle aged unless forced to by a liquidity shock: his expected consumption and, hence his utility since he is risk neutral, would be reduced by the size of the insider premium. The condition $a'(1) \geq 1$ ensures that the uninformed want to invest all of their productive capital when young and do not want to trade with the old.

Since the production functions are strictly concave, there exists a unique efficient level of investment which equates

$$a'\left(1 - \hat{K}_B^{t,t+2}\right) = E\left[\theta^{t+2}|\theta^t, p_B^t\right] b'\left(\hat{K}_B^{t,t+2}\right). \qquad (13)$$

A comparison of equations (12) and (13) immediately reveals that the effect of insider trading is to induce discretionary, but uninformed, investors to invest disproportionately in asset A where there is no adverse selection. More formally,

Proposition 2 *The equilibrium level of investment is inefficiently distorted toward the safe asset A and away from asset B in the sense that $K_B^{t,t+2} < \hat{K}_B^{t,t+2}$, where $\hat{K}_B^{t,t+2}$ is the efficient level of investment in project B conditional on current public information.*

More generally, this investment distortion exists in any economy characterized by *differing* degrees of insider trade (inducing differing degrees of adverse selection) and discretionary, rational investment by the uninformed. These welfare costs are, however, offset by the fact that insider trading also helps the uninformed to make better investment decisions through the conditioning of their forecasts of θ^{t+2} on the information contained in the current price, p_B^t. The presence of the insider leads to a finer public information set. We now document the tradeoffs numerically.

1.1 Example

Suppose that the unconditional probability of a production shock of H is given by 0.3, the state-space for the productivity shocks is $\{0.1, 2.0\}$, the liquidity shocks are given by: $h = 0.75$, $l = 0.25$, and the production functions are given by:

$$a(k_A) = 1.05 k_A, \quad b(k_B) = \sqrt{k_B}.$$

We interpret each "generation" as a year. This parameter for the riskless technology gives a risk-free rate of 5% per year. The premium for holding the risky techology is between 5% and 10 % per year. In figure 3, we graph the change in unconditional expected output due to introducing an insider in the economy for different levels of the persistence parameter, γ. Our measure of welfare is expected output (expectations are taken before any trading on the current market occurs). Since all agents are risk neutral, this seems a reasonable welfare measure. In fact, this measure corresponds to the Pareto criterion. An alternative measure would be to focus solely on uninformed investors and examine their expected consumption. In either instance, the measure of the net value of the insider is given by the difference in the measure (e.g. expected output) between the economy with and without the insider.

At $\gamma = 0$, the productivity shock is *i.i.d.*, so the insider's private information has no value in future production decisions. At this point, the uninformed investors pay the insider premium without getting any useful information. Thus, expected output is lower in the economy with the insider. At γ equal to one, there is no private information in the economy. As γ increases from zero, the expected productive value of the insider's private information increases, and reaches an interior maximum for a value of γ strictly between zero and one. The expected insider premium changes with γ, reaching an interior maximum at a value between zero and one. Together, the relations imply that the change in expected output due to introducing the insider takes on the humped shape shown in figure 3.

Increasing the difference between the liquidity shocks $h - l$ increases the trade size that the insider can successfully conceal. This increases her profits and hence the distortionary effects of the insider without changing the value of information released by the insider. Increasing $H - L$ has two effects on the welfare measure. Firstly, it increases the insider premium, increasing the investment distortions. However, it also increases the expected value of the insider's information. The net welfare change depends on which effect dominates.

2 Long-Term Insider

We now introduce a long-term insider to project B, one who will have insider information in the future as well as the present. Specifically, the long-term insider is assumed to have inside information about two consecutive projects. This allows us to capture the qualitative effects of a long-term insider in a simple fashion.

We believe this model captures important elements of insider trading in small resource exploration stocks. Such insiders may forsake short term trading profits from information about a resource discovery for the greater future insider trading profits associated with increased exploration. Other relevant examples include new product development, risky high technology R&D, takeovers and mergers.

For expositional ease, we introduce particular functional forms for the two production technologies:

$$a\left(k_A\right) = \Gamma k_A, \quad \Gamma \geq 1, \quad b\left(k_B\right) = \sqrt{k_B}. \tag{14}$$

The benefit of making these assumptions is that it implies that the payoff of project B is linear in the beliefs μ of the uninformed that the current technology shock is high.[8] We analyze perfect Bayesian equilibria of this model.

Since the information in her trade size conveyed through the equilibrium price affects future investment levels and hence future insider profits, when young, a long-term insider will consider the effects of her trades on the information content of prices. In the last period in which she has inside information, her horizon is identical to that of a one-time insider, so that their behavior will be identical. Consequently, the equilibrium pricing function for the short-term insider will clear the market when the long-term insider is old. We use the subscript and superscript o to index states and functions associated with this "old insider" who is in the last period of her life. Similarly, we index states and functions associated with the first period of the insider's life by y. To simplify notation, we drop the date t subscripts, replacing them with y or o where appropriate.

The "young insider" chooses her trade size to maximize discounted expected profits. She weighs the current profits from insider trading against the change in the present value of future insider profits due to the effect of the release of news on investment. We use $\rho > 0$ to denote the weight placed on second project insider trading profits. We allow the possibility that $\rho > 1$. This captures

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[8]The results of this section can be generalized to different production technologies.

the idea that fund-raising/investment may take place only at the beginning of an insider's affiliation with a firm, but that the insider will have many subsequent opportunities to trade on future information without affecting investment.

Following the analysis of the short-term insider we know that given beliefs μ_y that the current technology shock is H, that the uninformed investors' first order conditions for investment which pays off when the insider is old are

$$\Gamma = (E\left[\theta^o | \theta, p_B^y\right] - E\left[IP(\pi(\theta^y)) \mid \theta, p_B^y\right]) \frac{1}{2\sqrt{k_B^o}}, \tag{15}$$

where θ^o refers to the unknown technology shock two periods hence, which the insider will trade on when old, θ^y is the young insider's current private information, and θ refers to last period's publicly observed technology shock. Here, K_B^o refers to the aggregate capital invested in the risky project when the insider is young about which she will have private information when old, and K_B^y represents the existing aggregate investment in the project that pays off next period about which the young insider currently has private information. We use k to denote the investment of individual uninformed traders. $IP(\pi(n))$ is the insider premium that the young insider will receive when she is old, given that her private information when young is $n \in \{L, H\}$.

Using the uninformeds' beliefs μ_y that the current technology shock is H, we can solve for the current new aggregate investment in project B. Substituting from (15) yields:

$$\sqrt{K_B^o} = C + D\mu_y, \tag{16}$$

where

$$C = \frac{1}{2\Gamma}\left[\pi(L)H + [1 - \pi(L)]L + \frac{(h - l)(H - L)}{2}\pi(L)[1 - \pi(L)]\right], \tag{17}$$

$$D = \frac{(H - L)}{2\Gamma}[\pi(H) - \pi(L)]\left[1 - \left(\frac{(h - l)}{2}\right)[1 - \pi(H) - \pi(L)]\right]. \tag{18}$$

The above reveals that $C > 0$. Our assumption that the productivity process exhibits positive persistence implies that $\pi(H) \geq \pi(L)$, so that $D \geq 0$: the more promising investors believe the current project, the more they invest. If the technology shock is *i.i.d.*, then $\pi(H) = \pi(L)$, $D = 0$ and $IP(\pi(H)) = IP(\pi(L))$. In the case of a symmetric transition matrix for the technology shock, where $\pi(H) = 1 - \pi(L)$, so that the unconditional probability of a high shock is equal to $\frac{1}{2}$, again $IP(\pi(L)) = IP(\pi(H))$. If $\pi(H) > \pi(L)$, then as the probability that $\theta^o = H$ increases, so does the expected value of θ^o. Of course, the expected value of the insider premium also depends on the

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probability that $\theta^o = H$. However, the effect of increasing the expected return on capital in the risky project outweighs any effect on the insider premium, and so investment in the risky technology increases.

Given the uninformed investors' beliefs μ_y and the young insider's private information, θ^y, the expected second project insider profits are given by:

$$\rho IP\left(\pi(\theta^y)\right)[C + D\mu_y]. \tag{19}$$

This confirms that expected future profits are linearly increasing in the beliefs of the uninformed investors that $\theta^y = H$.

In the first period of her life, the long-term insider's problem is to choose a trade quantity $q^y(\theta^y, \theta, K_B^y; \mu_y)$ to maximize lifetime expected profits, where she explicitly takes into account the possible effects of her trades on the beliefs of uninformed investors. Her problem can be written as:

$$\max_{q^y} E\left[\left.\begin{array}{c} q^y[\theta^y - (\mu_y(q^y - s^y, \cdot)(H - L) + L)]\sqrt{K_B^y} \\ + \rho IP\left(\pi(\theta^y)\right)(C + D\mu_y(q^y - s^y, \cdot)) \end{array}\right| \theta^y \right]. \tag{20}$$

Expectations are taken over values of the liquidity shock. The insider's conjecture about the specialist's belief function satisfies rational expectations. The above objective function for the long-term insider reflects the condition that the specialist's beliefs correspond to those of the current uninformed investors both on and off the equilibrium path. The first term in the objective function refers to the long-term insider's first period insider trading profits. The second term gives the present value of future insider trading profits. This is affected by the insider's first period trade through the uninformeds' beliefs, as a function of the first period price.

Inspection of the long-term insider's objective function reveals that the insider gets no benefit from revealing her private information when young if either D or $\rho = 0$. If $\rho = 0$, the long-term insider does not care about future profits, so she maximizes one period profits. When $\pi(H) = \pi(L)$, the technology shock is *i.i.d.* so that the current technology realization does not help to predict future realizations and hence the uninformed investors' expectations of future productivity shocks are unaffected by the insider's private information. In this case, $D = 0$, and the insider's second period profits do not depend on the uninformed investors' beliefs. Consequently, if either $\rho = 0$ or $\pi(H) = \pi(L)$ then the long-term insider maximizes one-period profits:

Lemma 3 *Necessary conditions for the equilibrium with the long-term insider to differ from the equilibrium with the short-term insider are:*

$$\rho > 0, \tag{21}$$

$$\pi(H) > \pi(L). \tag{22}$$

We now analyze equilibria in which the above necessary conditions are satisfied. We start by considering the trading game that occurs when the insider is young. Intuitively, two conditions must be satisfied for an insider seeing good news to prefer to trade in such a way as to reveal her information. First, it must be the case that she gains more from positively influencing the beliefs of uninformed investors and hence increasing investment than an insider seeing bad news gains. That is, it must be true that next period's insider premium is greater for an insider who currently observes a good realization: $IP(\pi(H)) > IP(\pi(L))$. Otherwise, the insider seeing bad news gains more from convincing the uninformed investors that the realization was in fact a good one – because both her current profits as well as her future profits would be greater. But, for the insider to see bad news and then convince the uninformed that the realization was a good one would be inconsistent with equilibrium. That is, the bad insider's non-mimicry condition would be violated.

The second condition which must be satisfied is that the future insider profit generated by revealing good news must be sufficiently high relative to the cost of forgoing current insider profits. This is an implicit condition on the size of the current capital stock which will determine profits from concealing information relative to ρ, the weight the insider places on future profits.

Suppose now that the young insider inherits a capital stock $\bar{K}$, $0 < \bar{K} \leq 1$. We first provide necessary bounds on $\bar{K}$ for the young insider to completely reveal her private information in the equilibrium of the resulting trading game. Then, we determine conditions on $\bar{K}$ for the equilibrium to feature concealment. Finally, we determine which capital stocks the young insider will inherit in equilibrium. At this investment level, the equilibrium price distribution that the uninformed expect to receive next period, when trading against a new young insider, satisfies rational expectations.

The following proposition provides the conditions on the capital stock when the insider is young for a fully revealing equilibrium to exist.

Proposition 3 *A fully revealing equilibrium exists in which the young insider reveals her private*

information if and only if *the capital stock that the young insider inherits, $\dot{K}$, satisfies,*

$$0 < \sqrt{\dot{K}} \le \rho\Psi, \tag{23}$$

where

$$\Psi \equiv \frac{1}{(H-L)(h-l)}D\left[IP\left(\pi(H)\right) - IP\left(\pi(L)\right)\right] \tag{24}$$

If (23) is satisfied, then the following describes the equilibrium: Let

$$\bar{v} \in \left[\frac{\rho IP(\pi(L))D}{(H-L)\sqrt{\dot{K}}} - l, \frac{\rho IP(\pi(H))D}{(H-L)\sqrt{\dot{K}}} - h\right]. \tag{25}$$

A. *The insider seeing good news trades $q_H > \bar{v}+h$, and the insider seeing bad news trades $q_L < \bar{v}+l$.*

B. *The beliefs of the market makers and uninformed investors that $\theta^y = H$ are a function of the trading volume v that they observe, and are given by*

$$\mu_y(v) = \begin{cases} 0, & v < \bar{v} \\ 1, & v \ge \bar{v}. \end{cases} \tag{26}$$

C. *The investment of the uninformed is given by:*

$$K_B^o = \min\left\{1, (C + D\mu_y)^2\right\}. \tag{27}$$

This result is intuitive: algebraic manipulation or equation (24) reveals that the necessary and sufficient conditions for Ψ to be positive so that the insider ever contemplates completely revealing her private information are

$$\pi(H) > \pi(L) \quad (\text{since } \pi(H) \ge \pi(L))$$

and

$$\pi(H) + \pi(L) < 1.$$

But these are the necessary conditions for next period's insider premium to be greater for an insider who currently observes a good realization: $IP(\pi(H)) > IP(\pi(L))$. That is, these are the necessary conditions for the insider who sees good news to gain more from convincing the uninformed investors that the realization was, in fact, a good one than the insider who sees bad news. Solving the second inequality in terms of the unconditional probability that $\theta' = H$ yields the following:

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Corollary 1 *A necessary condition on π, the unconditional probability that $\theta^t = H$, for a fully revealing equilibrium to occur when the long-term insider is young is given by*

$$\pi < \frac{1}{2}. \tag{28}$$

If this inequality holds and $\pi(H) > \pi(L)$, then $IP(\pi(H)) > IP(\pi(L))$, so that $\Psi > 0$. A good shock must be enough of a surprise for investment to change enough for it to be worthwhile for the insider to reveal her private information. From now on, we will assume that $\pi \le \frac{1}{2}$, so that $IP(\pi(H)) \ge IP(\pi(L))$.

Even if Ψ is positive, then $\bar{K}$ must still be small enough that the future benefits obtained by the insider who sees good news by revealing her private information outweigh the trading profits she could achieve by trading to conceal her information today: equation (24) provides the specific bound. If $\bar{K}$ is low enough that the insider who see good news wants to reveal the news, then she must trade enough that the bad insider has no incentive to mimic her. The condition that $q_H \ge \bar{v} + h$ describes the necessary minimum bound on the trade size. Similarly, the insider seeing bad news must sell sufficient quantities to ensure that the good insider has no incentive to trade in such a way that her trade might be confused with that of the insider with bad information in order to earn short-term profits. If the insider short-sells, the uninformed believe that the productivity realization was a bad one so she cannot earn positive insider profits.

The largest that $\bar{K}$ can be is one. If (24) can be satisfied for $\bar{K} = 1$, then in equilibrium the young insider always reveals her private information. The right hand side of (24) is strictly increasing in ρ when the necessary conditions for complete revelation to occur are satisfied. In these cases there is a ρ large enough for such a full revealing equilibrium to exist. Solving for this ρ yields the following corollary.

Corollary 2 *If $\Psi > 0$. and if ρ satisfies the following bound, then there exists an equilibrium where the young insider always fully reveals her private information at all equilibrium information sets.*

$$\rho > \left(\frac{1}{\Psi} \right). \tag{29}$$

If the young insider cares enough about her future trading profits, then she will fully reveal her private information when young. As Ψ decreases, the required ρ increases. We now show that if the young insider inherits a large enough capital stock, then equilibrium in the trading game involves the insider concealing her private information.

Proposition 4 *In the trading game when the long-term insider is young, an equilibrium involving the insider trading such that in states $\{(H,l),\ (L,h)\}$ trading volume is uninformative and in states $\{(H,h),\ (L,l)\}$ trading volume is fully informative exists if and only if the capital stock that the young insider inherits, $\bar{K}$ is large enough:*

$$\sqrt{\bar{K}} \geq \rho\Psi, \tag{30}$$

where $\rho\Psi$ is defined as in (24). If $\sqrt{\bar{K}} > \rho\Psi$, is satisfied, the strategy of the insider is unique*: only the off-equilibrium beliefs of the uninformed investors and market makers differ across equilibria. The following describes an equilibrium with a piece-wise linear pricing schedule.*

A. *The insider's trades are given by*

$$H: \quad q_H^c = \frac{\rho D}{2\Gamma(H-L)}\{\pi(\theta)IP(\pi(H)) + [1 - \pi(\theta)]IP(\pi(L))\} + (1 - \pi(\theta))(h - l), \tag{31}$$

$$L: \quad q_L^c = \frac{\rho D}{2\Gamma(H-L)}\{\pi(\theta)IP(\pi(H)) + [1 - \pi(\theta)]IP(\pi(L))\} - \pi(\theta)(h - l),$$

where $\pi(\theta)$ denotes the prior probability that the insider's information will be H.

B. *The beliefs of the market maker and uninformed are given by:*

$$\mu_y(v) = \begin{cases} 0. & v \leq q_H^c - h - \pi(h - l) \\ 1, & v \geq q_H^c - l \\ \pi(\theta) + \Delta\,(v - q_H^c - h) & else, \end{cases} \tag{32}$$

where

$$\Delta = \frac{(H - L)(1 - \pi(\theta))}{(H - L)q_H^c - \frac{\rho IP(\pi(H))D}{2\Gamma\sqrt{\bar{K}}}}.$$

C. *The investment of the uninformed agents is given by*

$$K_B^o = \min\left\{1,\ (C + D\mu_y)^2\right\}. \tag{33}$$

Note from (31) that the long-term insider's trade is strictly greater than the short-term insider's trade in this equilibrium. As either the weight insiders place on future profits, ρ, increases, or amount that investment changes by changing investor beliefs, D, increases, the equilibrium insider trades increase, so that the equilibrium volume at which the market maker cannot discern good news from bad increases. This is because an insider who sees bad news also has an incentive to increase purchases or short sell less in order to influence investor beliefs. If insiders value future

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profits sufficiently, an insider seeing bad news will even purchase a positive amount in an attempt to conceal her news from investors. That is, she incurs a short-term loss in order to increase investment. Note also that as $\rho \to 0$, or $D \to 0$, the long-term insider's trades converge to the short-term insider's.

It is straightforward to show that when $\Psi > 0$ (the necessary conditions for any revelation to occur are satisfied) no pure pooling equilibrium exists in which the young insider trades the same quantity irrespective of her private information. This follows because there is no volume so that both types receive payoffs of at least their full-information values. If $\Psi = 0$, the equilibrium with the long-term insider is the same as the equilibrium with the short-term insider. Together with Propositions 1, 3 and 4, the above implies that for $\tilde{K} \neq \rho\Psi$, the pure strategy equilibrium uniquely identifies the young insider's payoffs and the investment by the uninformed born that period. In the pure strategy equilibrium, the insider always trades to reveal the current productivity shock if (1) the insider premium associated with good news exceeds that associated with bad news (and D is positive) *and* (2) $\frac{\rho}{\sqrt{\tilde{K}}}$ is sufficiently large. Otherwise, the insider always tries to conceal her information so that the equilibrium is partially concealing.

When $\tilde{K} = \rho\Psi$, then both fully revealing and partially concealing equilibria exist. In this case, we assume that the market maker randomizes over the pricing functions that support the fully revealing and partial pooling equilibria, with probability ω, $0 \leq \omega \leq 1$ of choosing the partially concealing pricing function. Since the market maker earns zero expected profits in both equilibria, he is indifferent between the equilibria, and so will mix. Thus, if $\tilde{K} = \rho\Psi$, to the young insider, a fully revealing equilibrium occurs with probability $1 - \omega$. In general equilibrium, the investment by uninformed investors must be consistent with the subsequent actions of insiders. Since the payoffs to the uninformed are discontinuous in whether the insider successfully conceals her information, the general equilibrium may require this mixing to smooth the payoffs of the uninformed investors. The mixing probability chosen by the market maker is such that the resulting equilibrium price distribution satisfies the rational expectations of the uninformed who trade against the young insider, and hence is consistent with their investment. In "ρ" space, holding the other parameters of the economy constant, for a given public information state, there is generally a range of ρ's for which general equilibrium would mandate this mixing. The probability that the market maker selects the revealing equilibrium price schedule, $\omega(\rho)$ is increasing in ρ in such a way that $\frac{\rho}{\sqrt{\tilde{K}(\omega(\rho))}}$ is constant, where $\tilde{K}(\omega(\rho))$ is the equilibrium investment level by uninformed investors who correctly

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anticipate this mixing probability (detailed below).

Let $\pi(\theta^o)$ be the probability that $\theta^y = H$, before the young insider receives her private information. Following the discussion above, we write the young insider's expected first period trading profits as a function of $\check{K}$, ω, and $\pi(\theta^o)$ as

$$\phi\left(\check{K},\omega,\pi(\theta^o)\right)\sqrt{\check{K}}, \tag{34}$$

where

$$\phi\left(\check{K},\omega,\pi(\theta^o)\right) = \begin{cases} 0, & \check{K} < \rho\Psi \\ \omega IP(\pi(\theta^o)), & \check{K} = \rho\Psi \\ IP(\pi(\theta^o)), & \check{K} > \rho\Psi. \end{cases} \tag{35}$$

We are now ready to determine the investment strategies of the uninformed who will trade against the young insider so that the resulting equilibrium prices satisfy rational expectations. Since the uninformed act competitively, their first order conditions are given by:

$$\Gamma = \left(E\left[\theta^y|\theta, p_B^o\right] - E\left[\phi(K_B^y, \omega, \pi(\theta^o)) \mid \theta, p_B^o\right]\right)\frac{1}{2\sqrt{k_B^y}}, \tag{36}$$

where again K_B^y denotes the aggregate investment of the uninformed for the project that pays off when the new insider is young. Also. $\pi(\theta^o)$ denotes the probability that $\theta^y = H$ conditional on the current old insiders private productivity shock. In equilibrium, we have $k_B^y = K_B^y$, and so the equilibrium $\{K_B^y(\theta, p_B^o), \omega(\theta, p_B^o)\}$ solves:

$$\sqrt{K_B^y(\theta, p_B^o)} = \frac{E\left[\theta^y|\theta, p_B^o\right] - E\left[\phi\left(K_B^y(\theta, p_B^o), \omega(\theta, p_B^o), \pi(\theta^o)\right) \mid \theta, p_B^o\right]}{2\Gamma}. \tag{37}$$

In the next lemma, we give the unique solution to the above equation.

Lemma 4 *The unique solution to equation (37), and thus the investment of the uninformed born when the insider is old, is given by:*

$$\sqrt{K_B^y(\theta, p_B^o)} = \begin{cases} \frac{E\left[\theta^y|\theta, p_B^o\right]}{2\Gamma}, & \frac{E\left[\theta^y|\theta, p_B^o\right]}{2\Gamma} < \rho\Psi \\ \frac{E\left[\theta^y|\theta, p_B^o\right] - E\left[IP(\pi(\theta^o))|\theta, p_B^o\right]}{2\Gamma}, & \frac{E\left[\theta^y|\theta, p_B^o\right] - E\left[IP(\pi(\theta^o))|\theta, p_B^o\right]}{2\Gamma} > \rho\Psi \\ \rho\Psi & otherwise \end{cases} \tag{38}$$

When $\sqrt{K_B^y(\theta, p_B^o)} = \rho\Psi$, the mixing probability chosen by the market maker, $\omega(\theta, p_B^o)$, is consistent with the investment by the uninformed. so that

$$\omega(\theta, p_B^o) = \frac{E\left[\theta^y|\theta, p_B^o\right] - 2\Gamma\rho\Psi}{E\left[IP(\pi(\theta^o))|\theta, p_B^o\right]}. \tag{39}$$

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Thus, we have:

Proposition 5 *An equilibrium to the long-term insider game exists. Further, the expected profits of all market participants are unique across all possible equilibria.*

Since $E[\theta^y|H] \geq E[\theta^y|L]$, then if $E[\theta^y|H] \leq 2\Gamma\rho\Psi$, the young insider will always reveal reveal her private information at all information sets in equilibrium. If $IP(\pi(H)) > IP(\pi(L))$, so that $\Psi > 0$, then there will always be a weight on future profits, ρ^* large enough such that the young insider will reveal her information in all states. The critical ρ^* is given by:

$$\rho^* = \frac{E[\theta^y|H]}{2\Gamma\Psi}.$$

If $\Psi > 0$ and the insider values her future profits enough, the young insider will always trade to reveal her private information in equilibrium.

Before proceeding with a numerical example, we present an important welfare implication of the model. If the long-term insider ever chooses with positive probability to reveal her information, then the uninformed prefer to trade against a long-term insider than against a short-term insider. Since investment improves in this case, even the insiders are better off, leading to a strict Pareto improvment over the short-term insider case.

Proposition 6 *The uninformed always weakly prefer to trade against a long-term insider. If*

$$\max_{n\in\{L,H\}} (E[\theta^y|n] - E[IP(\pi(\theta^o)|n)]) < 2\Gamma\rho\Psi, \tag{40}$$

then the uninformed strictly prefer trading in an economy with a long-term insider to trading in an economy with a short-term insider. When the above inequality is satisfied, then any equilibrium with a long-term insider ex ante strictly Pareto dominates all equilibria with a short-term insider.

Long-term insiders do not always create adverse selection problems and more information is released through prices. The above inequality insures that there is a positive probability that the young insider will trade to completely reveal her information. An uninformed investor is aided by more informative prices in the past (no adverse selection) because they help target current investment, and are helped by less adverse selection in the future because they do not have to pay an insider premium in those instances.

2.1 Example

We continue with our previous example. Figure 4 plots the change in unconditional expected output due to introducing an insider into an economy with the following parameters: $\pi = 0.3$, $h - l = 0.5$, $H = 2$, $L = 0.1$, $\Gamma = 1.05$. The bottom curve shows changes in expected output in the case where ρ, the weight that the insider places on future profits, is equal to zero. In this case, the equilibrium with the long-term insider is the same as that with the short-term insider. The top curve shows changes in expected output for $\rho = 6$. In this case, the insider will reveal her private information with probability one when last period's productivity shock was L for all values of γ above 0.35. In this case, the long-term insider strictly Pareto dominates the short-term insider. The set of persistence parameters where the economy with the longer-term insider is Pareto dominated by the no-insider economy shrinks as ρ increases. With a large enough persistence parameter, the young insider will always reveal her information. In these cases, expected output with the long-term insider exceeds expected output with the short-term insider. This surplus is split between the long-term insider and the uninformed investors.

The middle curve shows the change in expected output for $\rho = 5$. In this case, the young insider reveals her private information for all values of the persistence parameter above 0.55.

3   Conclusions

This paper introduces inside traders into a dynamic general equilibrium economy in which uninformed, but *rational*, traders choose investment levels across assets with different levels of adverse selection. The investments of these liquidity traders depend crucially on the information held by the insider which is revealed through the equilibrium price. We distinguish insiders by the length of time over which they will have access to inside information. This enables us to examine the consequences for their incentives either to conceal their information so as to obtain immediate profits or to trade in such a way that they reveal their information and thereby influence investment. When inside information has little predictive power for future payoffs, introducing an inside trader to the economy causes welfare losses because all types of insiders seek to profit by concealing their information. Further, any slight benefits of informative prices fail to counterbalance the direct costs of trading in a market with an insider. Insider trading distorts investment so that the marginal costs of capital are unequal across otherwise identical investment opportunities.

However, when an insider's information has high value in investment decisions, the insider's presence is socially beneficial. This is particularly true when the insider has a long-term relationship with a firm and, hence a stake in future investment. To the extent that she influences future investment decisions through her inside trading, the insider may want to trade in such a way as to reveal her information. Consequently, prices are more informative and uninformed investors are less likely to receive "unfair" prices if they have to liquidate their holdings prematurely. For the long-term insider to convince investors that she has observed good news, or to conceal her bad news, it may be necessary for her to hold a portfolio featuring a large stake in her firm.

These features, we believe, may closely characterize small exploration or R&D firms. There is often more "private information/insider trading" in such firms. Significant investments by insiders are frequently, if sometimes fraudulently, trumpeted. The theory also predicts that such firms should have greater and more variable gross returns than others, generating a "liquidity premium" in their stock returns.
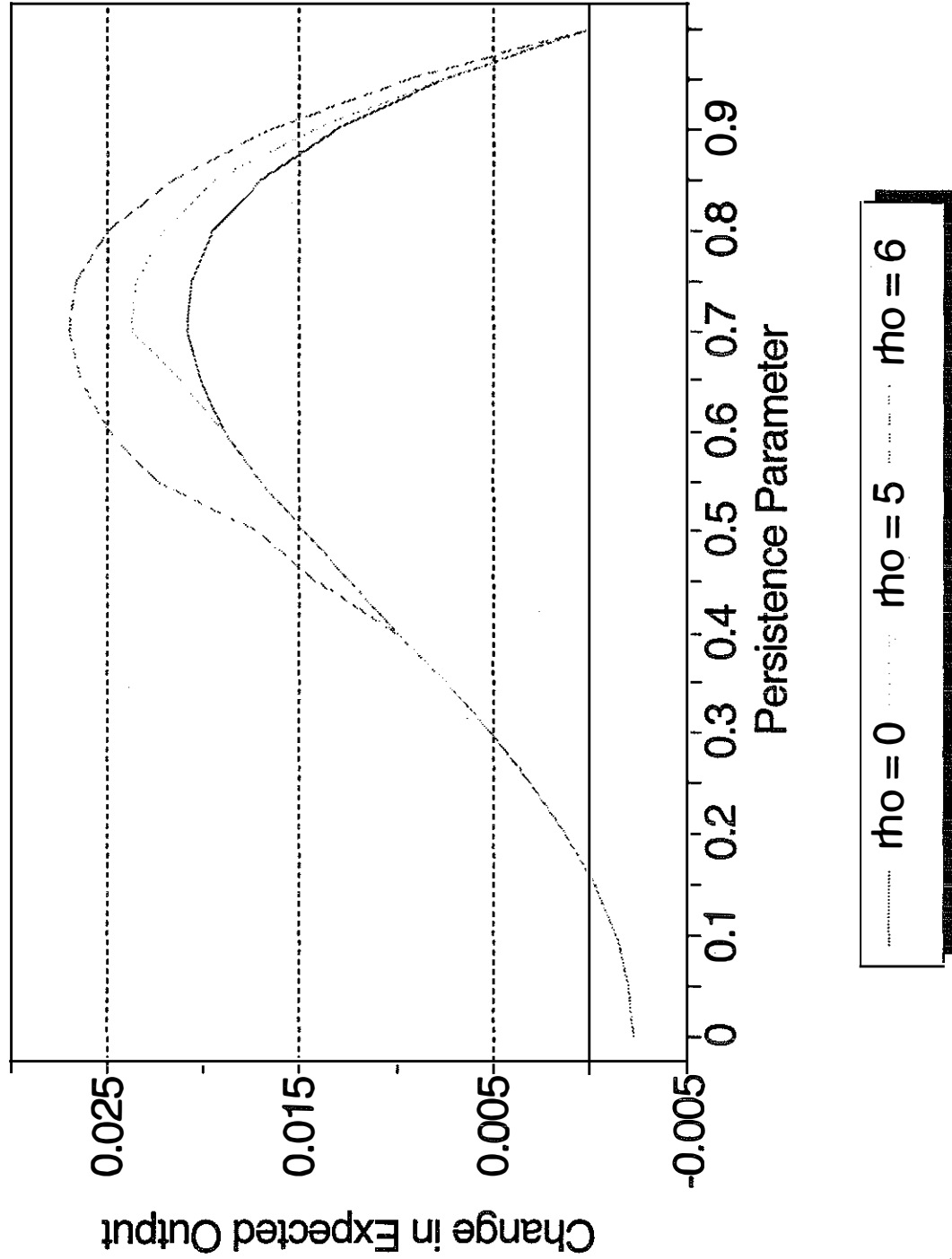
Figure 2 - Equilibrium Beliefs

slope $= \dfrac{1}{h-l}$

$\pi(\theta^t)$

1

$-h$

$\pi(\theta^t)(h-l)-h$

$-l$

0
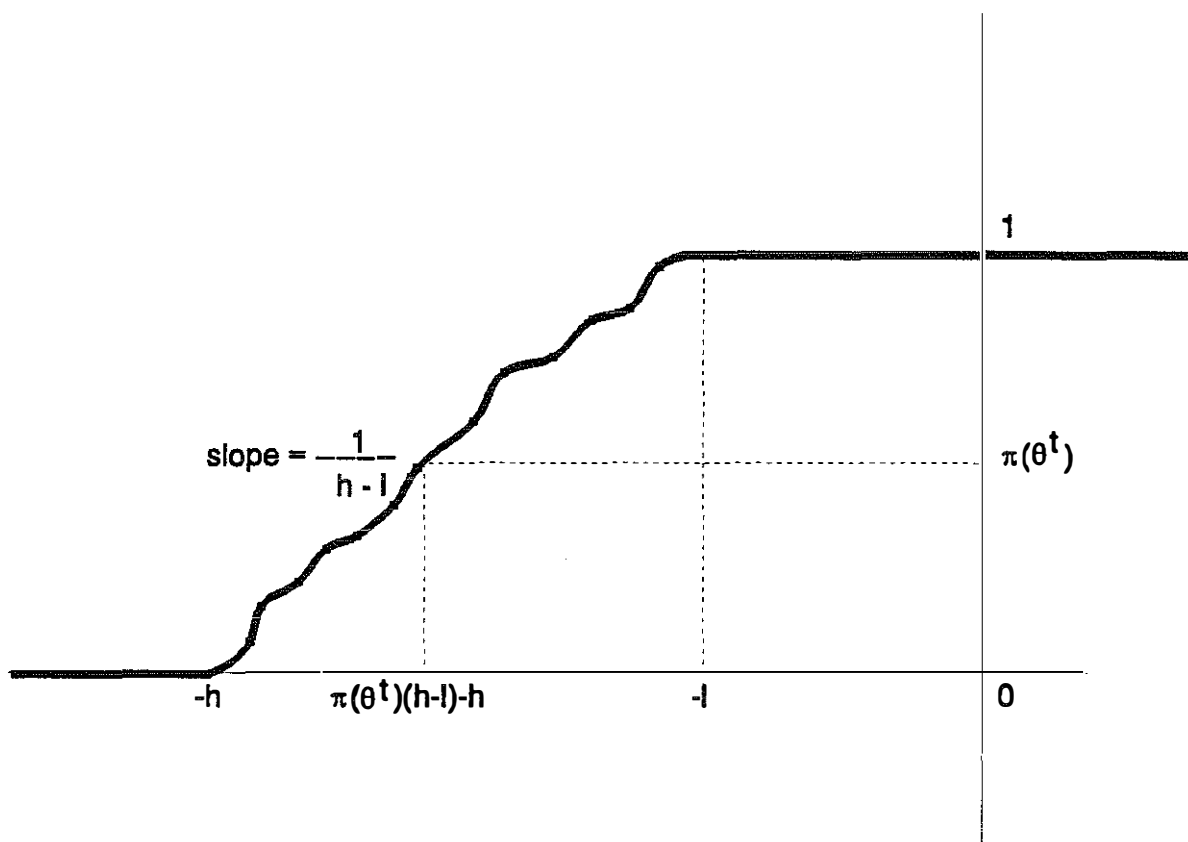
Figure 3 - Change in Output
Short-Lived Insider
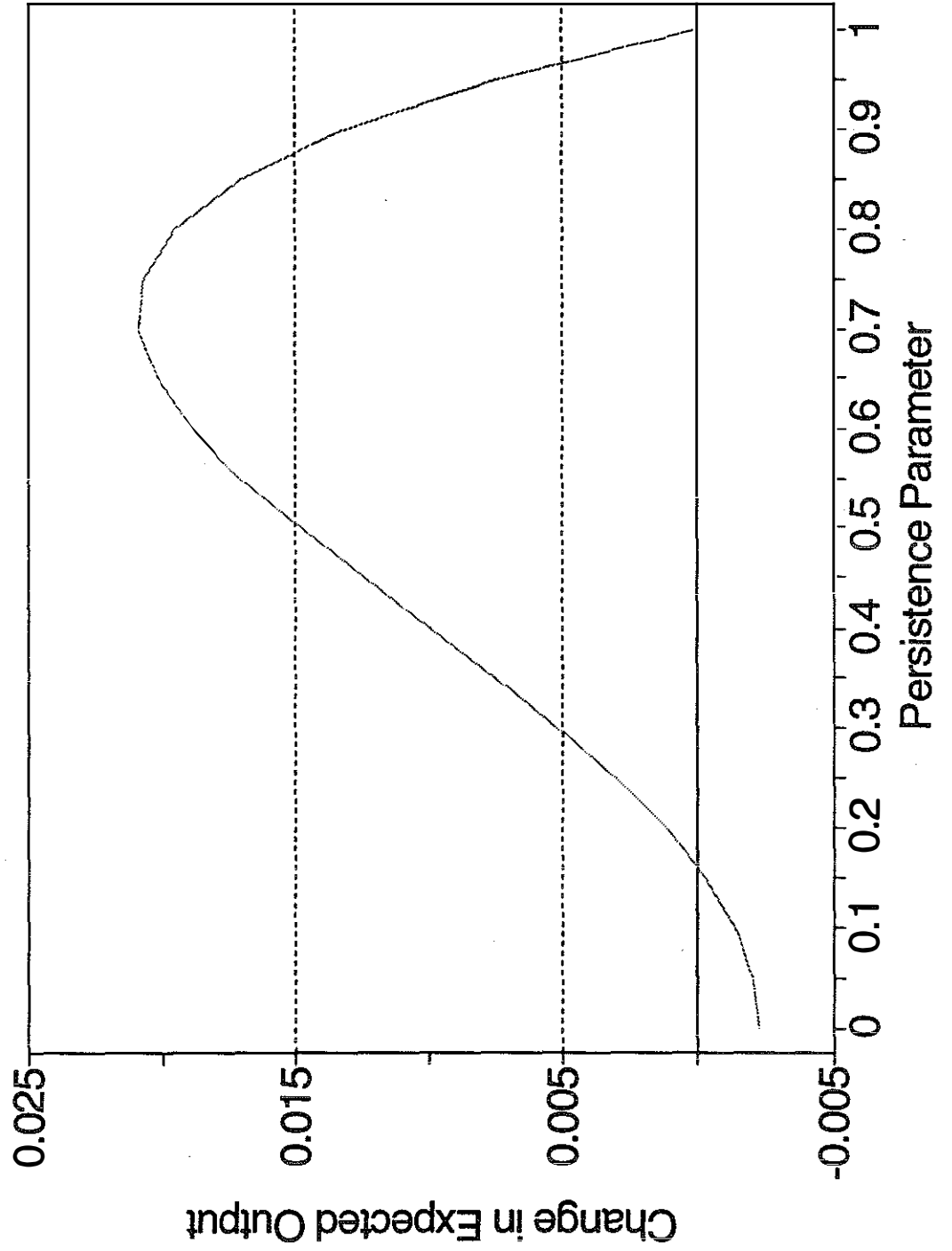
Figure 4 - Change in Output

Long-Lived Insider

Change in Expected Output

Persistence Parameter

rho = 0   rho = 5   rho = 6

Appendix - Proofs

Proof of Lemma 1

Proof: Since the market maker earns zero expected profits for any given net order flow then

$$Lb\left(K_B^{t-1,t+1}\right) \le P_B\left(v_B^t, K^{t-1,t+1}, \theta^t\right) \le Hb\left(K_B^{t-1,t+1}\right).$$

This follows because were the price outside these bounds, the market maker would buy/sell the asset at a price greater or less than payoffs in any possible state, violating the competitive zero expected profits condition. Note also that in any equilibrium without adverse selection, the insider's expected profits must be zero state by state, since the price must equal the expected value of the asset *conditional* on the insider's private information should there be no adverse selection in equilibrium. So suppose that for any net order flow prices are between $Lb\left(K^{t-1,t+1}\right)$ and $Hb\left(K^{t-1,t+1}\right)$. We show that this implies that the insider can earn positive expected profits in at least one state so that the equilibrium must feature adverse selection. In state H, for any order submitted by the insider, q, her expected profits are:

$$q\left[Hb\left(K_B^{t-1,t+1}\right) - \left(\frac{1}{2}\right)\left(P_B\left(q - h, K^{t-1,t+1}, \theta^t\right) + P_B\left(q - l, K^{t-1,t+1}, \theta^t\right)\right)\right]. \tag{A1}$$

For this to be non-positive, it must be that for $v_B^t > -h$,

$$\left(\frac{1}{2}\right)\left(P_B\left(v_B^t, K^{t-1,t+1}, \theta^t\right) + P_B\left(v_B^t + h - l, K^{t-1,t+1}, \theta^t\right)\right) \ge Hb\left(K_B^{t-1,t+1}\right). \tag{A2}$$

Similarly, it must follow that for $v_b^t < -l$,

$$\left(\frac{1}{2}\right)\left(P_B\left(v_B^t, K^{t-1,t+1}, \theta^t\right) + P_B\left(v_B^t + l - h, K^{t-1,t+1}, \theta^t\right)\right) \le Lb\left(K_B^{t-1,t+1}\right). \tag{A3}$$

Let $-h < v_B^t < -l$. To satisfy (A2), we must have $P_B\left(v_B^t, \cdot, \cdot\right) = Hb\left(K_B^{t-1,t+1}\right)$. But to satisfy (A3), $P_B\left(v_B^t, \cdot, \cdot\right) = Lb\left(K_B^{t-1,t+1}\right).$ ∎

Proof of Lemma 2

Proof: Suppose the insider sets

$$q(H, K_B^{t-1,t+1}) - h = q(L, K_B^{t-1,t+1}) - l,$$

so that the market maker cannot determine the production shock in the states $\{H, h\}$ and $\{L, l\}$. Substituting the market maker's zero expected profit condition in for *equilibrium* prices and using Bayes' rule, we see that the insider's *equilibrium* expected profits in states H and L are given by:

$$
\begin{aligned}
H &: \quad q\left(H, K_B^{t-1,t+1}\right) \tfrac{1}{2}\left(H - L\right)\left(1 - \pi(\theta^t)\right) b\left(K_B^{t-1,t+1}\right) \\
L &: \quad -\left[q\left(H, K_B^{t-1,t+1}\right) + (h - l)\right] \tfrac{1}{2}\left(H - L\right)\pi(\theta^t) b\left(K_B^{t-1,t+1}\right)
\end{aligned}
\tag{A4}
$$

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respectively. These are both non-negative if and only if,

$$0 \le q\left(H, K_B^{t-1,t+1}\right) \le (h-l).$$

Inspection reveals that the only other ways to induce adverse selection are either to set

$$q(H, K_B^{t-1,t+1}) = q(L, K_B^{t-1,t+1}),$$

or to set

$$q(H, K_B^{t-1,t+1}) - l = q(L, K_B^{t-1,t+1}) - h.$$

But, these trades cannot make non-negative expected profits in both states H and L. So, set the insider's trades to $\alpha(h-l)$ in state H and $(\alpha-1)(h-l)$ in state L. Substituting these expressions into the insider's expected payoffs yields:

$$\frac{1}{2}(h-l)(H-L)\pi(\theta^t)\left(1 - \pi(\theta^t)\right) b\left(K_B^{t-1,t+1}\right) \equiv IP\left(\pi(\theta^t)\right) b\left(K_B^{t-1,t+1}\right), \qquad (A5)$$

which does not depend on α. ■

Proof of Proposition 1
Proof: *Verification that (A), (B), (C) describe an equilibrium*
Statement (C) follows directly from lemma 2, since the liquidity traders know that the expected loss to the insider trader is given by:

$$\frac{(h-l)(H-L)}{2}\left\{\mu\pi(H)[1-\pi(H)] + (1-\mu)\pi(L)[1-\pi(L)]\right\} b\left(K^{t,t+2}\right).$$

If the insider's private information is H (L), any volume less (greater) than zero results in negative expected profits for the insider, as long as the market maker's pricing function lies between $Lb\left(K_B^{t-1,t+1}\right)$ and $Hb\left(K_B^{t-1,t+1}\right)$. Therefore, we only need to verify that the insider does not wish to trade any other positive (negative) volume if she observes H (L). Using the above pricing function, the insider's expected profits from following the equilibrium strategies are given by:

$$H: \quad \left(\tfrac{1}{2}\right)(1-\pi(\theta^t))^2 (h-l)(H-L)b\left(K_B^{t-1,t+1}\right),$$
$$L: \quad \left(\tfrac{1}{2}\right)(\pi(\theta^t))^2 (h-l)(H-L)b\left(K_B^{t-1,t+1}\right). \qquad (A6)$$

The insider's expected profits for an order of size $(h-l) > q > 0$ in state H are given by:

$$q\left[H - \left(\tfrac{1}{2}\right)(H + \mu(q-h,\cdot,\cdot)(H-L) + L)\right] b\left(K_B^{t-1,t+1}\right)$$
$$= q\left(\tfrac{1}{2}\right)[1 - \mu(q-h,\cdot,\cdot)](H-L)b\left(K_B^{t-1,t+1}\right)$$
$$\le q\left(\tfrac{1}{2}\right)\left[1 - \max\left\{0, 1 - \frac{(1-\pi(\theta^t))^2 (h-l)}{q}\right\}\right](H-L)b\left(K_B^{t-1,t+1}\right), \text{ (since } q-h = v_B^t)$$
$$\le \left(\tfrac{1}{2}\right)(1-\pi(\theta^t))^2 (h-l)(H-L)b\left(K_B^{t-1,t+1}\right),$$

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using the left-hand side of the inequality in (11). But

$$\left(\frac{1}{2}\right)\left(1-\pi(\theta^t)\right)^2(h-l)(H-L)b\left(K_B^{t-1,t+1}\right)$$

represents the insider's expected profits in state H from submitting the proposed equilibrium volume of

$$\left(1-\pi(\theta^t)\right)(h-l).$$

If the insider submits an order $q \geq (h-l)$, beliefs will be given by 1 irrespective of the level of the liquidity shock. In this case, her profits will be zero. Therefore, the proposed equilibrium volume is optimal given the above pricing function. Similar logic shows that volume $-\pi(\theta^t)(h-l)$ is optimal in state L given the beliefs in (B). Hence, given the beliefs in (B) the strategies presented in (A) are indeed optimal.

In this proposed equilibrium, the market maker only observes volumes of:

$$\left(1-\pi(\theta^t)\right)(h-l)-h, \ or \ \left(1-\pi(\theta^t)\right)(h-l)-l \ in \ state \ H;$$

$$\left(1-\pi(\theta^t)\right)(h-l)-l, \ or \ \left(-\pi(\theta^t)\right)(h-l)-l \ in \ state \ L.$$

Thus, for observed equilibrium volumes beliefs are considered with Bayes' rule. For off-equilibrium volumes beliefs satisfying (11) lie between 0 and 1. Therefore, given the insider's strategies in (A), the beliefs in (B) are consistent with equilibrium. So, (A), (B), (C) describe an equilibrium of the game.

Uniqueness

By lemma 2, we know that in any equilibrium the insider must trade $\alpha(h-l)$ in state H and $(\alpha-1)(h-l)$ in state L, where $0 \leq \alpha \leq 1$. So, suppose

$$\{\alpha(h-l), \ (\alpha-1)(h-l)\}$$

describes equilibrium strategies of the game. Let $m(v)$ be the beliefs of the market maker that support this equilibrium. The proof that α is unique is divided into two claims.

Claim 1: If $m(v)$ supports an equilibrium α, so does the revised set of beliefs:

$$m^*(v) = \begin{cases} m(v) & -h < v < -l \\ 1 & v \geq -l \\ 0 & v \leq -h \end{cases} \tag{A7}$$

Proof: In state H, for $(h-l) > q > 0$, for α to be an equilibrium, $m(v)$ must satisfy:

$$\frac{\alpha(h-l)(H-L)\left(1-\pi(\theta^t)\right)}{2} \geq q(H-L)\left(1-\frac{[(m(q-h)+m(q-l)]}{2}\right)$$

$$\geq q(H-L)\left(1-\frac{[m(q-h)+1]}{2}\right)$$

$$= q(H-L)\left(1-\frac{[m^*(q-h)+m^*(q-l)]}{2}\right)$$

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since $q > 0$.

If the insider trades $d \geq (h - l)$, then the beliefs will be H under $m^*(\cdot)$, leaving the insider zero expected profits for this trade. Therefore, beliefs $m^*(\cdot)$ will still make $\alpha(h - l)$ an optimal choice in H if beliefs $m(\cdot)$ do. Similar reasoning holds for state L. $\square$

Claim 2: In any equilibrium, $\alpha = 1 - \pi(\theta^t)$.

Proof: By claim 1, we can take beliefs such that

$$m(v) = 1, \quad v > -l, \quad m(v) = 0, \quad v < -h.$$

For a trade $(h - l) > q > 0$:

$$m(q - h) = 1.$$

For $\alpha(h - l)$ to be the optimal trade in state H,

$$\frac{\alpha(h - l)(H - L)(1 - \pi(\theta^t))}{2} \geq q(H - L)\left(1 - \frac{[m(q - h) + 1]}{2}\right),$$

$$\alpha(h - l)\left(1 - \pi(\theta^t)\right) \geq q(1 - m(q - h)). \tag{A8}$$

Since $v = d - h$, (A8) implies for $-h < v < l$,

$$m(v) \leq 1 - \alpha(h - l)\frac{1 - \pi(\theta^t)}{v + h}. \tag{A9}$$

For a trade $(l - h) < q < 0$,

$$m(q - l) = 0.$$

For $(\alpha - 1)(h - l)$ to be an optimal trade in state L,

$$\frac{(1 - \alpha)(h - l)(H - L)\pi(\theta^t)}{2} \geq \frac{-q(H - L)m(q - l)}{2} \tag{A10}$$

Since $v = d - l$, (A10) implies that for $-h < v < -l$,

$$m(v) \leq (\alpha - 1)(h - l)\frac{\pi(\theta^t)}{(v + l)} \tag{A11}$$

Combining (A9) and (A11) yields for $-h < v < -l$,

$$1 - \alpha(h - l)\frac{(1 - \pi(\theta^t))}{v + h} \leq m(v) \leq (\alpha - 1)\frac{(h - l)\pi(\theta^t)}{v + l}. \tag{A12}$$

For the above to lead to beliefs between zero and one, it must be that for $-h < v < -l$,

$$(\alpha - 1)\frac{(h - l)\pi(\theta^t)}{(v + l)} \geq 1 - \alpha\frac{(h - l)(1 - \pi(\theta^t))}{(v + h)}.$$

Solving for α,

$$\alpha(v + l + \pi(\theta^t)(h - l)) \leq \frac{(v + h)}{(h - l)}(v + l + \pi^t(h - l)). \tag{A13}$$

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Let

$$v = -l - \pi(\theta^t)(h - l) + \epsilon(h - l), \quad \epsilon > 0,$$

so that

$$v + h = \left(1 - \pi(\theta^t) + \epsilon\right)(h - l).$$

This is certainly feasible for $-h < v < -l$ and ϵ small. Then we have:

$$\alpha \leq 1 - \pi(\theta^t) + \epsilon. \tag{A14}$$

Let

$$v = -l - \pi(\theta^t)(h - l) - \epsilon(h - l) \quad \epsilon > 0.$$

Again, this is feasible. This gives

$$\alpha \geq 1 - \pi(\theta^t) - \epsilon. \tag{A15}$$

Combining (A14) and (A15), for $\epsilon > 0$

$$1 - \pi(\theta^t) - \epsilon \leq \alpha \leq 1 - \pi(\theta^t) + \epsilon.$$

Thus, in order for (A12) to be satisfied, $\alpha = 1 - \pi(\theta^t)$. This proves claim 2. $\square$

Therefore,

$$\left\{\left(1 - \pi(\theta^t)\right)(h - l), \quad -\pi(\theta^t)(h - l)\right\}$$

are the only strategies by the informed that can be supported in equilibrium. This α implies that the belief function of the market maker and the investment strategies of the uninformed are as presented in (B) and (C). ■

Proof of Proposition 2

Proof: Together. (12) and (13) imply that:

$$\frac{a'\left(1 - \hat{K}_B^{t,t+2}\right)}{b'\left(\hat{K}_B^{t,t+2}\right)} > \frac{a'\left(1 - K_B^{t,t+2}\right)}{b'\left(K_B^{t,t+2}\right)}. \tag{A16}$$

The strict concavity of $b(\cdot)$ and concavity of $a(\cdot)$ implies that $\frac{a'(1 - K_B)}{b'(K_B)}$ is strictly increasing in K_B.
■

Proof of Lemma 3

Proof: If $\rho = 0$, then the long-term insider's objective functions are the same in every possible node of the game as the short-term insider's objective function. Thus, the game is the same as the game with the short-term insider and consequently, the equilibria must be the same. If $\pi(H) = \pi(L)$ then using (18), D = 0, and $IP(\pi(H)) = IP(\pi(L))$. So, the objective function of

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the young long-term insider differ from that of the short-term insider by a constant that does not depend on beliefs of the uninformed agents nor the private information of the long-term insider. Thus, the game with the long-term insider is isomorphic to the game with the short-term insider state by state. ∎

Proof of Proposition 3
Proof: If the insider reveals her private information in equilibrium, then her equilibrium payoffs are given by:

$$H : \quad \rho I P(\pi(H))[C + D]$$
$$L : \quad \rho I P(\pi(L))C$$

Let $\mu(v)$ be the equilibrium belief function. For notational ease, define:

$$\tilde{\mu}(q) \equiv \frac{1}{2}\left[\mu(q-h) + \mu(q-l)\right],$$

the beliefs that the insider expects to receive given that she trades a quantity q. Suppose that the insider trades a quantity q in state H. Then, her payoffs would be given by:

$$q\left(1 - \tilde{\mu}(q)\right)\left(H - L\right)\sqrt{\tilde{K}} + \rho I P(\pi(H))[C + D\tilde{\mu}(q)]. \tag{A17}$$

Optimality of the revealing equilibrium payoffs when the private information is H requires:

$$\rho I P(\pi(H))[C + D] \geq q\left(1 - \tilde{\mu}(q)\right)\left(H - L\right)\sqrt{\tilde{K}} + \rho I P(\pi(H))[C + D\tilde{\mu}(q)],$$

or,

$$0 \geq \left(1 - \tilde{\mu}(q)\right)\left[q(H - L)\sqrt{\tilde{K}} - \rho I P(\pi(H))D\right]. \tag{A18}$$

To satisfy (A18), beliefs must satisfy

$$\tilde{\mu}(q) = 1, \quad q > \frac{\rho I P(\pi(H))D}{(H - L)\sqrt{\tilde{K}}},$$

or, translating to beliefs, $\mu(v)$,

$$\mu(v) = 1, \quad v > \frac{\rho I P(\pi(H))D}{(H - L)\sqrt{\tilde{K}}} - h. \tag{A19}$$

Suppose that the insider trades deviates from the equilibrium by trading a quantity q when her private information is L. Then, her expected profits are given by

$$-q\tilde{\mu}(q)(H - L)\sqrt{\tilde{K}} + \rho I P(\pi(L))[C + D\tilde{\mu}(q)]. \tag{A20}$$

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Optimality of the revealing payoffs in state L requires

$$\rho IP(\pi(L))C \geq -q\tilde{\mu}(q)(H-L)\sqrt{\tilde{K}} + \rho IP(\pi(L))[C + D\tilde{\mu}(q)],$$

or,

$$0 \geq \tilde{\mu}(q)\left[-q(H-L)\sqrt{\tilde{K}} + \rho IP(\pi(L))D\right]. \tag{A21}$$

To satisfy (A21),

$$\tilde{\mu}(q) = 0, \quad q < \frac{\rho I P(\pi(L))D}{(H-L)\sqrt{\tilde{K}}},$$

or,

$$\mu(v) = 0 \quad v < \frac{\rho IP(\pi(L))D}{(H-L)\sqrt{\tilde{K}}} - l. \tag{A22}$$

A belief function, $\mu(v)$ exists which satisfies (A19) and (A22) *if and only if:*

$$0 < \sqrt{\tilde{K}} \leq \frac{\rho D}{(H-L)(h-l)}[IP(\pi(H)) - IP(\pi(L))].$$

Verification that (A), (B) and (C) describe an equilibrium follows from the above arguments since the pricing function satisfies (A19) and (A22). ∎

Proof of Corollary 1
Proof: Using (2), the condition that $\pi(H) + \pi(L) < 1$ can be written as:

$$2\pi(1-\gamma) + \gamma < 1.$$

Since $0 \leq \gamma < 1$, the result follows. ∎

Proof of Corollary 2
Proof: This follows from solving the equation $\rho\Psi \geq 1$ for ρ. ∎

Proof of Proposition 4
Proof: *Necessity:* In the equilibrium, the insider with good news trades q_H and the insider with a technology shock of L must trade q_L so that

$$q_L - l = q_H - h.$$

The insider's expected profits are given by:

$$H: \quad q_H\frac{(1-\pi(\theta))}{2}(H-L)\sqrt{\tilde{K}} + \rho IP(\pi(H))[C + D\frac{(1+\pi(\theta))}{2}], \tag{A23}$$

$$L: \quad -[q_H - (h-l)]\frac{\pi(\theta)}{2}(H-L)\sqrt{\tilde{K}} + \rho IP(\pi(L))[C + D\frac{(\pi(\theta))}{2}],$$

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where $\pi(\theta)$ denotes the probability that the insider's private information is H, conditional on last period's publicly observed technology shock. Since the insider seeing good news can trade an unbounded amount, her profits would become unbounded if beliefs of the market maker did converge to one as the volume gets arbitrarily large. Similar reasoning applies to an insider seeing bad news, eventually beliefs must converge to 0 for a small enough volume. But then, in equilibrium, there are strategies for insiders with both types of information such that their separating profits can be obtained. Since the trades q_H and q_L must be optimal, the insider's profits evaluated at these trades must be at least as large as their separating payoffs. This implies the following inequality for the insider with a productivity shock of H,

$$q_H \frac{(1 - \pi(\theta))}{2}(H - L)\sqrt{\tilde{K}} + \rho IP(\pi(H))[C + D\frac{(1 + \pi(\theta))}{2}] \geq \rho IP(\pi(H))[C + D],$$

or,

$$q_H \geq \frac{\rho IP(\pi(H))D}{(H - L)\sqrt{\tilde{K}}}. \tag{A24}$$

Similarly, for the insider with shock L,

$$-[q_H - (h - l)]\frac{\pi(\theta)}{2}(H - L)\sqrt{\tilde{K}} + \rho IP(\pi(L))[C + D\frac{(\pi(\theta))}{2}] \geq \rho IP(\pi(L))C,$$

or,

$$q_H \leq \frac{\rho IP(\pi(L))D}{(H - L)\sqrt{\tilde{K}}} + (h - l). \tag{A25}$$

A q_H exists which solves (A24) and (A25) *if and only if*

$$\sqrt{\tilde{K}} \geq \frac{\rho}{(H - L)(h - l)}D\left[IP(\pi(H)) - IP(\pi(L))\right],$$

thus proving necessity.

Sufficiency: To prove sufficiency, we show that the strategies and beliefs given in the proposition give an equilibrium when the inequality in (30) is satisfied. Statement (B) defines beliefs which are consistent on the equilibrium path, and between zero and one off the equilibrium path. Statement C follows from the uninformeds' first order conditions. It is straightforward to verify that maximizing the insider's expected payoffs subject to the belief function yields the trades given in statement A, when (30) is satisfied. This proves sufficiency.

Uniqueness: The proof that the insider's trades are unique follows the same logic as in the uniqueness proof for the short-term insider. The only way that we can find a set of consistent off-equilibrium beliefs is when the insider seeing shock H trades a quantity such that

$$q_H^c = \frac{\rho D}{2\Gamma(H - L)}\{\pi(\theta)IP(\pi(H)) + [1 - \pi(\theta)]IP(\pi(L))\} + (1 - \pi(\theta))(h - l).$$

■

Proof of Lemma 4

Proof: The result follows by inspection. ∎

Proof of Proposition 5

Proof: Together, propositions 3 and 4 imply that for any capital stock that could be inherited by the young insider, an equilibrium to the trading game exists. Further, the *ex-ante* payoffs to the insider, before seeing her private information are unique as long as the capital stock inherited does not equal $\rho\Psi$. Lemma 4 implies that there is only 1 investment that the uninformed who will trade against the young insider will make at every information set. Further, the mixing probability for the market maker is unique and so if the young insider inherits a capital stock equal to $\rho\Psi$, her expected profits are unique.

In the last period of the insider's life the game is isomorphic to the game with a short term insider. So, the expected insider premium for the uninformed who trade against the old insider is unique, and thus the uninformeds' investment decision is unique.

Thus, there is one and only one investment decision that can be make at any information set in the game such that rational expectations are satisfied about the distribution of future prices. This set of investment decisions clearly gives us an equilibrium. Since there is only one equilibrium investment decision at every node, and expected payoffs of the insider, market maker and uninformed agents are unique in the investment levels, the equilibrium payoffs of the agents are unique. ∎

Proof of Proposition 6

Proof: An uninformed investor who will trade against the old insider always has a weakly finer information partition with a long-term insider than a short-term insider; he is weakly more likely to know θ^y. Ceteris paribus, the expected profits of an uninformed investor are greater if he knows θ^y, since he can trivially make the same investment independent of θ^y. Uninformed investors who trade against the young insider are also better off because they are less likely to face adverse selection in the next period with a long-term insider and their information sets are the same whether or not the insider is a long-term or short-term insider. Therefore, the uninformed weakly prefer to trade against the long term insider.

If inequality (40) is satisfied, then there is a positive probability that the young insider will trade to completely reveal her information in equilibrium. By the arguments given above, the uninformed investors strictly prefer this to trading against the short-term insider. In this case, the investment of the uninformed is closer to the efficient level than with a short-term insider; there will be less under-investment in the risky technology. Therefore, expected output will be higher, leading to a strict *ex ante* Pareto improvement since all agents are risk neutral. ∎

Appendix—9

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