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ABSTRACT

We document a new business cycle fact: the cross-sectional standard deviation of firm-level investment (investment dispersion) is robustly and significantly procyclical. This makes investment dispersion different from the dispersion of productivity and output growth, which is countercyclical. Investment dispersion is more procyclical in the goods-producing sectors, for smaller firms and for structures. We show that a heterogeneous-firm real business cycle model with countercyclical idiosyncratic firm risk and non-convex adjustment costs calibrated to match moments of the long-run investment rate distribution, produces a time series correlation coefficient between investment dispersion and aggregate output of 0.58, close to the 0.45 in the data. We argue, more generally, that cross-sectional business cycle dynamics impose tight empirical restrictions on the physical environments and the structural parameters of heterogeneous-firm models.

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1 Introduction

Investment at the micro level is lumpy and infrequent (Doms and Dunne, 1998, Cooper and Haltiwanger, 2006). The distribution of investment rates is positively skewed and has excess kurtosis (Caballero et al., 1995). Motivated by these facts about the *long-run* cross-sectional distribution of *micro*-level investment, researchers have studied models with non-convex capital adjustment costs and asked whether "getting the micro facts right" matters for *aggregate* (investment) dynamics.[1]

In this paper we show that lumpy investment models have important and thus far unexplored implications for the *dynamics of the cross-section* of investment rates. Conversely, we show that the joint dynamics of the cross-section of firm-level investment, output growth and productivity growth provide important parameter restrictions for heterogeneous firm models.

We start with the data and establish a new business cycle fact: *the cross-sectional standard deviation of firm-level investment rates is procyclical.*[2] Figure 1 and Table 1 on the next page illustrate this new cross-sectional business cycle fact. The scatter plot reveals a positive correlation between the cross-sectional investment rate dispersion and the cyclical component of aggregate output.

Relatedly, a recent literature has documented that, across different countries and data sets, the dispersion of changes in firm- (or plant-) level output, productivity and prices is countercyclical.[3] We find the same negative association with the cycle also for the dispersion of firm-level employment growth. Table 1 shows that the signs of the correlation between aggregate output and the cross-sectional dispersion of core economic variables are statistically significant at conventional levels (see Appendix A.2 for extensive robustness checks and a time series graph of the investment rate dispersion).

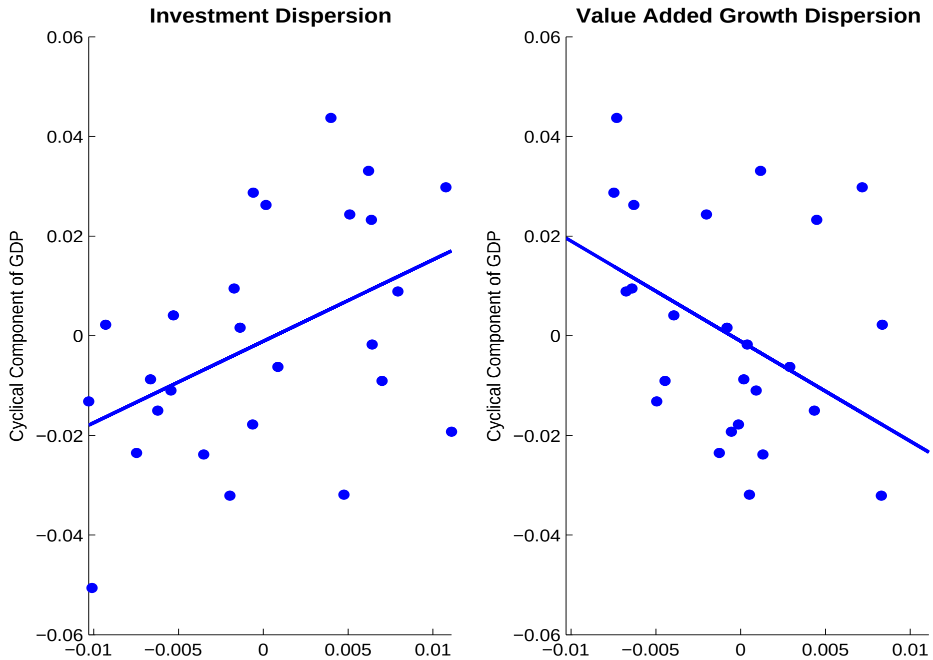
These findings together are incompatible with a simple frictionless model of the firm. Such a physical environment implies that the distributions of firms' decision variables comove over the cycle. We propose a heterogenous-firm real business cycle model with *fixed capital adjustment costs* to explain qualitatively and quantitatively the procyclicality of investment dispersion, even when the dispersion of firm-level productivity growth is countercyclical.

[1] There is an ongoing debate in the literature about this issue: Gourio and Kashyap (2007) and Bachmann et al. (2010) answer in the affirmative, Khan and Thomas (2008) negatively.

[2] Our main data source is the *Deutsche Bundesbank* balance sheet data base of German firms, USTAN. The literature has documented several related facts: Doms and Dunne (1998) show that between 1973 and 1988 the Herfindahl index of U.S. plant-level manufacturing investment is positively correlated with aggregate investment. Beaudry et al. (2001) show that cross-sectional investment dispersion between 1970 and 1990 in an unbalanced panel of roughly 1,000 U.K. manufacturing plants is negatively correlated with conditional inflation volatility. Eisfeldt and Rampini (2006) document that capital reallocation in U.S. Compustat data is procyclical.

[3] See Bachmann and Bayer (2011), Bloom et al. (2010), Doepke et al. (2005), Doepke and Weber (2006), Gourio (2008), Higson et al. (2002, 2004) as well as Kehrig (2010) for output and/or productivity, Berger and Vavra (2010) for prices.

Figure 1: Cyclicality of Cross-sectional Dispersions



Notes: the left panel shows a scatter plot of the cross-sectional standard deviation, linearly detrended, of the investment rate (firm fixed and 2-digit industry-year effects removed) against the cyclical component of the aggregate real gross value added of the nonfinancial private business sector, which is detrended using an HP(100)-filter. The right panel shows the same for the cross-sectional standard deviation of the log-change in firm-level real gross value added.

Table 1: CYCLICALITY OF CROSS-SECTIONAL DISPERSIONS

Cross-sectional Moment	$Correl(\cdot, HP(100) - Y)$	5%	95%
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.451	0.070	0.737
$std(\Delta \log y_{i,t})$	-0.451	-0.675	-0.196
$std(\Delta \log \epsilon_{i,t})$	-0.481	-0.678	-0.306
$std(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})})$	-0.499	-0.718	-0.259

Notes: the rows refer to the cross-sectional standard deviations, linearly detrended, of, respectively, the investment rate, the log-change of real gross value added, the log-change of Solow residuals and the net employment change rate, all at the firm level. We have removed firm fixed and 2-digit industry-year effects from each variable. The first column shows the time series correlation coefficient with the cyclical component of the aggregate real gross value added of the nonfinancial private business sector, detrended by an HP(100)-filter. The columns '5%' and '95%' refer to the top and bottom 5-percentiles in a parametric bootstrap of the correlation coefficient, using an unrestricted VAR with one year lag (a nonparametric overlapping block bootstrap gives similar results).

Fixed capital adjustment costs lead to nonlinear, two-step investment rules at the firm-level. First, firms have a discrete decision whether to adjust or not to adjust (extensive margin). Second, conditional on adjustment, firms decide by how much (intensive margin). The cross-sectional investment dispersion is in general a complicated nonlinear function of both steps. To fix ideas, eliminate the intensive margin decision and consider the case where firms can only increase their capital stock by a given percentage or let it depreciate. In this case, both the cross-sectional average and dispersion of investment are solely determined by how many firms adjust. Average investment is increasing in the fraction of firms adjusting. And so is the dispersion of investment if less than half of the firms invest. Hence, as long as average investment is procyclical, the cross-sectional investment dispersion is procyclical, too.

Of course, in reality and in realistic quantitative models both the extensive and the intensive margins of capital adjustment are operative and multiple shocks hit the economy. We calibrate our model (i) to match the deviations from normality in the *steady-state* investment rate distribution, and (ii) to the observed joint stochastic process for aggregate Solow residuals and firm-level productivity growth dispersions, and show in Section 5 that such a model can quantitatively match the procyclicality of investment dispersion in the data.

This important result provides a new validation of the lumpy investment model. More generally, we argue that a fully fledged business cycle theory from the bottom up has to and can - as we show - successfully speak to the dynamics of more than just the cross-sectional means of the distributions underlying macroeconomic aggregates. We view this paper as a step towards such a research program.

Why is this direction of research important? Heterogenous-firm models have seen increased use in the macroeconomic and international finance literature. This paper argues (see Section 5) that cross-sectional dynamics impose tight restrictions on physical environments and structural parameters in these models. For instance, we show that procyclical investment dispersion – generated by a procyclical extensive margin effect – requires curvature in the firm's revenue function, for this procyclical extensive margin effect to be quantitatively relevant. Only with substantial curvature rely firms mostly on the extensive margin for capital adjustment, as large investments would put the firm too far off its optimal scale of operation (see Gourio and Kashyap (2007) for a related observation). We also argue that matching jointly the cyclicality of the dispersion of investment and output growth restricts cyclical fluctuations in firm-level productivity risk, which have been recently studied in a variety of models by Arellano et al. (2010), Bloom et al. (2010), Chugh (2009), Gilchrist, Sim and Zakrajsek (2009) as well as Schaal (2010). We finally show that general equilibrium price movements are crucial to match quantitatively cross-sectional firm dynamics, a conjecture by Khan and Thomas (2008).

2 The Facts

When cross-sectional dispersion is concerned, Davis et al. (2006) show that studying only publicly traded firms (Compustat) can lead to wrong conclusions. Therefore, we use the *Deutsche Bundesbank* balance sheet data base of German firms, USTAN. USTAN is a private sector, annual, firm-level data set that allows us to make use of 26 years of data (1973-1998), with cross-sections that have, on average, over 30,000 firms per year. USTAN has a broader ownership, firm size and industry coverage than the available comparable U.S. data sets from Compustat and the Annual Survey of Manufacturers. This allows us to study industry and size differences in the behavior of the cross-section of firms over the business cycle.[4]

The evidence presented in the Introduction derived from the entire *nonfinancial private business sector*, which includes firms that are in one of the following six 1-digit industries: agriculture, mining and energy, manufacturing, construction, trade, transportation and communication. This Section provides additional evidence that is highly suggestive of the mechanism we exploit in this paper. 1) Across 2-digit industries there is a positive association between the cyclicality of the extensive margin of investment and the cyclicality of the investment rate dispersion. 2) In the goods-producing sectors, like manufacturing and construction, where we would expect non-convex capital adjustment to be most important, the investment rate dispersion is particularly procyclical. 3) The procyclicality of investment dispersion is declining in firm size, consistent with the view that larger firms can partially outgrow adjustment costs. 4) Investment dispersion is less procyclical for equipment than it is for structures, which often constitute by their very nature large and indivisible investment projects.

For firm-level employment adjustment rates we use the symmetric adjustment rate definition proposed in Davis et al. (1996), $\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}$; and, analogously, for firm-level investment rates: $\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}$. To compute firm-level Solow residuals, we start, in accordance with our model in Section 3, from the following Cobb-Douglas production function:

$$y_{i,t} = z_t \epsilon_{i,t} k_{i,t}^{\theta} n_{i,t}^{\nu},$$

where $\epsilon_{i,t}$ is firm-specific and z_t aggregate productivity. We assume that labor input $n_{i,t}$ is immediately productive, whereas capital $k_{i,t}$ is pre-determined and inherited from last period. This difference is reflected in the different timing convention in the definitions of the investment and employment adjustment rates. We estimate the output elasticities of the production factors, ν and θ, as median factor expenditures shares over gross value added within each industry.

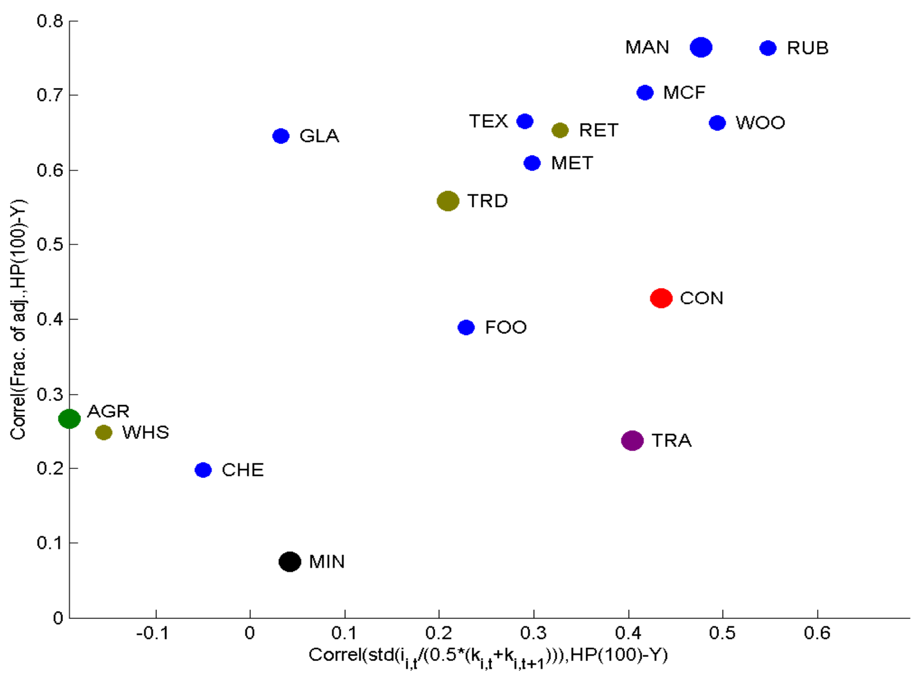
[4]For details on the data set and the sample selection see Appendix A.1 as well as Bachmann and Bayer (2011). See Appendix A.2 for evidence on the cyclical behavior of investment dispersion from the UK DTI and the U.S. Compustat data bases.

We remove firm fixed and 2-digit industry-year effects from all four first-difference variables in Table 1 to focus on idiosyncratic changes. We thus eliminate differences in industry-specific responses to aggregate shocks as well as permanent ex-ante and predictable heterogeneity between firms.

We combine these firm-level data with annual national accounting data from Germany (*VGR*) on gross value added, investment, capital and employment from two-digit industries. We use these same *VGR* data to compute aggregate and industry-level Solow residuals.

We start by showing that across 2-digit industries there is a positive association between the procyclicality of investment dispersion and the procyclicality of the extensive margin of investment. Figure 2 shows on the x-axis the correlation coefficients of the investment rate dispersion with the cyclical component of own-industry output.

Figure 2: Two-Digit Industry Variation in the Procyclicality of the Extensive Margin and of the Investment Rate Dispersion



Notes: the x-axis displays the correlation coefficient between the investment rate dispersion (linearly detrended) and the business cycle frequency component of 2-digit industry real gross value added. The y-axis displays the same correlation coefficient for the extensive margin (linearly detrended), i.e. the fraction of adjusters ('Frac. of adj.') with an investment rate of $|\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})}| > 0.01$ (see Cooper and Haltiwanger (2006) for this convention). Larger dots represent 1-digit industries, smaller ones 2-digit industries. The acronyms mean: 'AGR' (agriculture), 'MIN' (mining & energy), 'MAN' (manufacturing), 'CHE' (chemical industry & oil), 'RUB' (plastics & rubber), 'GLA' (glass & ceramics), 'MET' (metals), 'MCF' (machinery, cars & furniture), 'WOO' (wood, paper & printing), 'TEX' (textiles & leather), 'FOO' (food & tobacco), 'CON' (construction), 'TRD' (trade), 'WHS' (wholesale), 'RET' (retail & cars), 'TRA' (transportation & communication).

On the y-axis Figure 2 displays the correlation coefficients of the fraction of adjusters with the same output measure. Using the convention in Cooper and Haltiwanger (2006), we define "adjusters" as those firms with annual investment rates of absolute value larger than 1%.[5]

Table 2 provides numbers on how the dispersion of investment and Solow residual growth as well as the fraction of ("lumpy") adjusters comove with output across one-digit industries. It shows that investment dispersion and the extensive margin of investment are strongly procyclical in the goods-producing sectors, manufacturing in particular. Trade exhibits lower procyclicality, while in the primary sectors the cross-section of firm-level investment is by and large acyclical. To put these findings in perspective, we also display the cyclicality of the cross-sectional Solow residual growth dispersion, which is strongly countercyclical in manufacturing and trade. Manufacturing, an industry the literature has focused on to find evidence for non-convex adjustment technologies (Doms and Dunne, 1998, Caballero and Engel, 1999, and Cooper and Haltiwanger, 2006), exhibits the most countercyclically disperse productivity growth as well as the most procyclical investment dispersion.

Table 2: CYCLICALITY OF CROSS-SECTIONAL DISPERSIONS AND EXTENSIVE MARGINS - ONE-DIGIT INDUSTRIES

Industry	$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	$std(\Delta \log \epsilon_{i,t})$	'Frac. of adj.'	'Frac. of lumpy adj.'
Aggregate	0.451	-0.481	0.727	0.614
AGR	-0.192	-0.283	0.267	0.10
MIN	0.042	0.107	0.075	-0.123
MAN	0.477	-0.397	0.765	0.646
CON	0.435	0.037	0.428	0.153
TRD	0.209	-0.387	0.559	0.388
TRA	0.404	0.034	0.237	0.263

Notes: see notes to Table 1 and Figure 2. The table displays correlation coefficients of four cross-sectional variables (in columns 2-5) with the cyclical component of aggregate real gross value added of the nonfinancial private business sector in the first row, thereafter with the real gross value added of the corresponding 1-digit industry. The fraction of lumpy adjusters ('Frac. of lumpy adj.') is defined as the fraction of firms with an investment rate of: $|\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})}| > 0.2$ (see Cooper and Haltiwanger (2006) for this convention). The latter is linearly detrended.

Table 3 shows the cyclicality of investment dispersion by firm size, according to three different size criteria: employment, value added and capital. Just as for the aggregate numbers in Table 1 we use the cyclical component of the aggregate output of the private nonfinancial

[5] This positive association is robust to using the fraction of lumpy investors – with investment rates larger than 20% or 10% in absolute value – or the fraction of lumpy investors with positive investment rates. It is also robust to using the nonfinancial private business sector's aggregate output as a cyclical indicator, although owing to industry-specific components in the industry cycles this association is somewhat weaker.

business sector as the cyclical indicator. We find procyclical investment dispersion mainly for the smaller firms, especially when size is measured by employment or value added. The very large firms, in contrast, have an almost acyclical investment dispersion. This distinction is statistically significant in the sense that if size is measured in terms of employment or value added, neither the point estimate for the smallest size class lies in the $[5\%, 95\%]-$bands of the largest size class nor vice versa.

Table 3: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - FIRM SIZE

Size Class / Criterion	Employment	Value Added	Capital
Smallest 25%	0.583	0.601	0.391
25% to 50%	0.456	0.468	0.422
50% to 75%	0.366	0.330	0.387
Largest 25%	0.188	0.215	0.399
Largest 5%	0.050	0.048	0.184

Notes: see notes to Table 1. The table displays correlation coefficients of the investment rate dispersion by firm size with the cyclical component of the aggregate real gross value added of the private nonfinancial business sector.

This is at least consistent with the view that larger firms can smooth the effects of non-convex capital adjustment costs and the extensive margin over several production units. Tables 2 and 3 also show that, while there is considerable variation in cross-sectional dynamics across industries and firm sizes, our main empirical result that investment dispersion is procyclical is not driven by one large industry or the very large firms. We also find no evidence that a large industry or the large firms have procyclical Solow residual growth dispersions, in which case they could generate procyclical investment dispersion without any frictions (Bachmann and Bayer (2011) documents this in more detail).

Table 4: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - TYPE OF CAPITAL GOOD

Aggregate	Equipment	Structures
0.792	0.725	0.925

Notes: see notes to Table 1. The table displays correlation coefficients of the investment rate dispersion by type of capital good with the linearly detrended cross-sectional average investment rate by type of capital good: $mean(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$.

Finally, Table 4 shows the cyclicality of investment dispersion by type of capital good. Since the average investment rate for structures is not very correlated with the overall business cycle, we now use the linearly detrended cross-sectional average investment rate for equipment and structures, respectively, as cyclical indicator. We find that the investment dispersion of structures is almost perfectly correlated with the cycle in aggregate structural investment. We interpret this as evidence that the extensive margin effect on cross-sectional investment dispersion is larger for structures, consistent with the available evidence that investment in structures is subject to higher fixed adjustment costs (see Caballero and Engel, 1999).

Altogether, we view the evidence gathered in this section as at least suggestive that non-convex capital adjustment costs have a role to play in explaining procyclical investment dispersion. We show next that quantitatively realistic fixed capital adjustment costs can indeed generate procyclical investment dispersion, in the magnitude observed in the data.

3 The Model

Our model follows closely the real business cycle models in Khan and Thomas (2008) as well as Bachmann et al. (2010). The main departure from either paper is that we introduce a second aggregate shock, namely to the the standard deviation of idiosyncratic productivity shocks. Such a shock is a convenient way to generate the observed countercyclicality of the dispersion of firm value added growth. Moreover, it is required by the quantitative nature of our exercise, as we will show that without it, even for very small fixed costs of capital adjustment, the extensive margin effect is so strong that the business cycle and investment dispersion are almost perfectly correlated.

3.1 Firms

The economy consists of a unit mass of small firms. There is one commodity in the economy that can be consumed or invested. Each firm produces this commodity, employing its predetermined capital stock (k) and labor (n), according to the following Cobb-Douglas decreasing-returns-to-scale production function ($\theta > 0$, $\nu > 0$, $\theta + \nu < 1$):

$$y = z\epsilon k^{\theta} n^{\nu}, \tag{1}$$

where z and ϵ denote aggregate and idiosyncratic revenue productivity, respectively.

The idiosyncratic log productivity process is first-order Markov with autocorrelation ρ_ϵ and a time-varying conditional standard deviation, $\sigma(\epsilon)$. The trend deviation of the natural logarithm of aggregate productivity and $\sigma(\epsilon)$ evolve jointly according to an unrestricted VAR(1) process, with normal innovations υ that have zero mean and covariance Ω:

$$\begin{pmatrix} \log z' \\ \sigma(\epsilon'') - \bar{\sigma}(\epsilon) \end{pmatrix} = \varrho_A \begin{pmatrix} \log z \\ \sigma(\epsilon') - \bar{\sigma}(\epsilon) \end{pmatrix} + \upsilon, \tag{2}$$

where $\bar{\sigma}(\epsilon)$ denotes the steady state standard deviation of idiosyncratic productivity shocks. We make a timing assumption that gives shocks to $\sigma(\epsilon)$ the interpretation of uncertainty shocks: firms today observe the standard deviation of idiosyncratic productivity shocks tomorrow.[6] The shocks to the exogenous aggregate states, ν, and idiosyncratic productivity shocks are independent. Idiosyncratic productivity shocks are independent across productive units. In contrast, we do not impose any restrictions on Ω or $\varrho_A \in \mathbb{R}^{2\times 2}$.

We denote the trend growth rate of aggregate productivity by $(1-\theta)(\gamma-1)$, so that aggregate output and capital grow at rate $\gamma - 1$ along the balanced growth path. From now on we work with k and y (and later aggregate consumption, C) in efficiency units.

Each period a firm draws its current cost of capital adjustment, $0 \leq \xi \leq \bar{\xi}$, which is denominated in units of labor, from a time-invariant distribution, G. G is a uniform distribution on $[0, \bar{\xi}]$, common to all firms. Draws are independent across firms and over time, and employment is freely adjustable.

Upon investment, i, the firm incurs a fixed cost of $\omega\xi$, where ω is the current real wage. Capital depreciates at rate δ. We can then summarize the evolution of the firm's capital stock (in efficiency units) between two consecutive periods, from k to k', as follows:

	Fixed cost paid	$\gamma k'$
$i \neq 0$:	$\omega\xi$	$(1-\delta)k + i$
$i = 0$:	0	$(1-\delta)k$

Given the i.i.d. nature of the adjustment costs, it is sufficient to describe differences across firms and their evolution by the distribution of firms over (ϵ, k). We denote this distribution by μ. Thus, $\left(z, \sigma(\epsilon'), \mu\right)$ constitutes the current aggregate state and μ evolves according to the law of motion $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$, which firms take as given.

To summarize: at the beginning of a period, a firm is characterized by its pre-determined capital stock, its idiosyncratic productivity, and its capital adjustment cost. Given the aggregate state, it decides its employment level, n, production and depreciation occurs, workers are paid, and investment decisions are made. Then the period ends.

Next we describe the dynamic programming problem of a firm. We will take two shortcuts (details can be found in Khan and Thomas, 2008). We state the problem in terms of utils of the representative household (rather than physical units), and denote the marginal utility of consumption by $p = p\left(z, \sigma(\epsilon'), \mu\right)$. Also, given the i.i.d. nature of the adjustment costs, continuation values can be expressed without future adjustment costs.

[6] We have experimented with the other timing assumption, where $\sigma(\epsilon)$ and z are jointly observed. The quantitative differences are small. Investment dispersion tends to be slightly more procyclical under this alternative timing assumption.

Let $V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$ denote the expected discounted value - in utils - of a firm that is in idiosyncratic state (ϵ, k, ξ), given the aggregate state $(z, \sigma(\epsilon'), \mu)$. Then the firm's expected value prior to the realization of the adjustment cost draw is given by:

$$V^0(\epsilon, k; z, \sigma(\epsilon'), \mu) = \int_0^{\bar{\xi}} V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) G(d\xi). \tag{3}$$

With this notation the dynamic programming problem becomes:

$$V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) = \max_n \{\mathrm{CF} + \max(V_{\text{no adj}}, \max_{k'}[-\mathrm{AC} + V_{\text{adj}}])\}, \tag{4}$$

where CF denotes the firm's flow value, $V_{\text{no adj}}$ the firm's continuation value if it chooses inaction and does not adjust, and V_{adj} the continuation value, net of adjustment costs AC, if the firm adjusts its capital stock. That is:

$$\mathrm{CF} = [z\epsilon k^\theta n^\nu - \omega(z, \sigma(\epsilon'), \mu) n] p(z, \sigma(\epsilon'), \mu), \tag{5a}$$

$$V_{\text{no adj}} = \beta \mathrm{E}[V^0(\epsilon', (1-\delta)k/\gamma; z', \sigma(\epsilon''), \mu')], \tag{5b}$$

$$AC = \xi \omega(z, \sigma(\epsilon'), \mu) p(z, \sigma(\epsilon'), \mu), \tag{5c}$$

$$V_{\text{adj}} = -i\, p(z, \sigma(\epsilon'), \mu) + \beta \mathrm{E}[V^0(\epsilon', k'; z', \sigma(\epsilon''), \mu')], \tag{5d}$$

where both expectation operators average over next period's realizations of the aggregate and idiosyncratic shocks, conditional on this period's values, and we recall that $i = \gamma k' - (1-\delta)k$. The discount factor, β, reflects the time preferences of the representative household.

Taking as given $\omega(z, \sigma(\epsilon'), \mu)$ and $p(z, \sigma(\epsilon'), \mu)$, and the law of motion $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$, the firm chooses optimally labor demand, whether to adjust its capital stock at the end of the period, and the optimal capital stock, conditional on adjustment. This leads to policy functions: $N = N(\epsilon, k; z, \sigma(\epsilon'), \mu)$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$. Since capital is pre-determined, the optimal employment decision is independent of the current adjustment cost draw.

3.2 Households

We assume a continuum of identical households that have access to a complete set of state-contingent claims. Hence, there is no heterogeneity across households. They own shares in the firms and are paid dividends. We do not need to model the household side in detail (see Khan and Thomas (2008) for that), we just use the first-order conditions that determine the equilibrium wage and the marginal utility of consumption.

Households have a standard felicity function in consumption and labor:[7]

$$U(C, N^h) = \log C - AN^h, \tag{6}$$

where C denotes consumption and N^h the household's labor supply. Households maximize the expected present discounted value of the above felicity function. By definition we have:

$$p(z, \sigma(\epsilon'), \mu) \equiv U_C(C, N^h) = \frac{1}{C(z, \sigma(\epsilon'), \mu)}, \tag{7}$$

and from the intratemporal first-order condition:

$$\omega(z, \sigma(\epsilon'), \mu) = -\frac{U_N(C, N^h)}{p(z, \sigma(\epsilon'), \mu)} = \frac{A}{p(z, \sigma(\epsilon'), \mu)}. \tag{8}$$

3.3 Recursive Equilibrium

A *recursive competitive equilibrium* for this economy is a set of functions

$$\left(\omega, p, V^1, N, K, C, N^h, \Gamma\right),$$

that satisfy

1. *Firm optimality*: Taking ω, p and Γ as given, $V^1(\epsilon, k; z, \sigma(\epsilon'), \mu)$ solves (4) and the corresponding policy functions are $N(\epsilon, k; z, \sigma(\epsilon'), \mu)$ and $K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$.

2. *Household optimality*: Taking ω and p as given, the household's consumption and labor supply satisfy (7) and (8).

3. *Commodity market clearing*:

$$C(z, \sigma(\epsilon'), \mu) = \int z\epsilon k^\theta N(\epsilon, k; z, \sigma(\epsilon'), \mu)^\nu d\mu - \int\int_0^{\bar{\xi}} [\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) - (1-\delta)k] dG d\mu.$$

4. *Labor market clearing*:

$$N^h(z, \sigma(\epsilon'), \mu) = \int N(\epsilon, k; z, \sigma(\epsilon'), \mu) d\mu + \int\int_0^{\bar{\xi}} \xi \mathscr{J}\left(\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) - (1-\delta)k\right) dG d\mu,$$

where $\mathscr{J}(x) = 0$, if $x = 0$ and 1, otherwise.

[7] We have experimented with a CRRA of 3 without much impact on our results.

5. *Model consistent dynamics*: The evolution of the cross-section that characterizes the economy, $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$, is induced by $K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$ and the exogenous processes for z, $\sigma(\epsilon')$ as well as ϵ.

Conditions 1, 2, 3 and 4 define an equilibrium given Γ, while step 5 specifies the equilibrium condition for Γ.

3.4 Solution

It is is well-known that (4) is not computable, because μ is infinite dimensional. We follow Krusell and Smith (1997, 1998) and approximate the distribution, μ, by its first moment over capital, $\bar{k}$, and its evolution, Γ, by a simple log-linear rule. In the same vein, we approximate the equilibrium pricing function by a log-linear rule, discrete aggregate state by discrete aggregate state:

$$\log \bar{k}' = a_k\big(z, \sigma(\epsilon')\big) + b_k\big(z, \sigma(\epsilon')\big) \log \bar{k}, \tag{9a}$$

$$\log p = a_p\big(z, \sigma(\epsilon')\big) + b_p\big(z, \sigma(\epsilon')\big) \log \bar{k}, \tag{9b}$$

Given (8), we do not have to specify an equilibrium rule for the real wage. As usual with the Krusell and Smith procedure, we posit the log-linear forms (9a)–(9b) and check that in equilibrium they yield a good fit to the actual law of motion.

Substituting $\bar{k}$ for μ into (4) and using (9a)–(9b), (4) becomes a computable dynamic programming problem with policy functions $N = N(\epsilon, k; z, \sigma(\epsilon'), \bar{k})$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \bar{k})$. We solve this problem by value function iteration on V^0. We do so by applying multivariate spline techniques that allow for a continuous choice of capital when the firm adjusts.

With these policy functions, we can then simulate a model economy *without* imposing the equilibrium pricing rule (9b). Rather, we impose market-clearing conditions and solve for the pricing kernel at every point in time of the simulation. We simulate the model economy for a large number of time periods. This generates a time series of $\{p_t\}$ and $\{\bar{k}_t\}$ endogenously, on which the assumed rules (9a)–(9b) can be updated with a simple OLS regression. The procedure stops when the updated coefficients $a_k\big(z, \sigma(\epsilon')\big)$ to $b_p\big(z, \sigma(\epsilon')\big)$ are sufficiently close to the previous ones.

4 Calibration

The model period is a year. This corresponds to the data frequency in USTAN. Most firm-level data sets that are based on balance sheet data are of that frequency. The following parameters then have standard values: $\beta = 0.98$ and $\delta = 0.094$, which we compute from German national accounting data for the nonfinancial private business sector. Given this depreciation rate, we pick $\gamma = 1.014$, in order to match the time-average aggregate investment rate in the nonfinancial private business sector: 0.108. $\gamma = 1.014$ is also consistent with German long-run growth rates. The disutility of work parameter, A, is chosen to generate an average time spent at work of 0.33: $A = 2$. We set the output elasticities of labor and capital to $\nu = 0.5565$ and $\theta = 0.2075$, respectively, which correspond to the measured median labor and capital shares in manufacturing in the USTAN data base.[8]

We measure the steady state standard deviation of idiosyncratic productivity shocks as $\bar{\sigma}(\epsilon) = 0.1201$. Since idiosyncratic productivity shocks in the data also exhibit above-Gaussian kurtosis - 4.4480 on average -, and since the fixed adjustment costs parameters will be identified by the kurtosis of the firm-level investment rate (together with its skewness), we want to avoid attributing excess kurtosis in the firm-level investment rate to lumpy investment, when the idiosyncratic driving force itself has excess kurtosis. We incorporate the measured excess kurtosis into the discretization process for the idiosyncratic productivity state by using a mixture of two Gaussian distributions: $N(0, 0.0777)$ and $N(0, 0.1625)$ - the standard deviations are 0.1201 ± 0.0424, with a weight of 0.4118 on the first distribution. Finally, we set $\rho_\epsilon = 0.95$. This process is discretized on a 19−state-grid, using Tauchen's (1986) procedure with mixed Gaussian normals. Heteroskedasticity in the idiosyncratic productivity process is modeled with time-varying transition matrices between idiosyncratic productivity states, where the matrices correspond to different values of $\sigma(\epsilon')$.

To calibrate the parameters of the two-state aggregate shock process we estimate a bivariate, unrestricted VAR with the linearly detrended natural logarithm of the aggregate Solow residual[9] and the linearly detrended $\sigma(\epsilon')$-process from the USTAN data. The parameters of this VAR are:[10]

$$\varrho_A = \begin{pmatrix} 0.4474 & -3.7808 \\ 0.0574 & 0.6952 \end{pmatrix} \quad \Omega = \begin{pmatrix} 0.0146 & 0.1617 \\ 0.1617 & 0.0023 \end{pmatrix} \tag{10}$$

This process is discretized on a $[5 \times 5]$−state grid, using a bivariate analog of Tauchen's procedure.

[8] If one views the DRTS assumption as a mere stand-in for a CRTS production function with monopolistic competition, than these choices would correspond to an employment elasticity of the underlying production function of 0.7284 and a markup of $\frac{1}{\theta+\nu} = 1.31$. The implied capital elasticity of the revenue function, $\frac{\theta}{1-\nu}$ is 0.47.

[9] We use $\nu = 0.5565$ and $\theta = 0.2075$ in these calculations.

[10] With a slight abuse of notation, but for the sake of readability, Ω has standard deviations on the main diagonal and correlations on the off diagonal.

Given the aforementioned set of parameters $\left(\beta, \delta, \gamma, A, \nu, \theta, \varrho_A, \Omega, \bar{\sigma}(\epsilon), \rho_\epsilon\right)$, we calibrate the adjustment costs parameter, $\bar{\xi}$, to minimize a quadratic form in the normalized differences between the time-average firm-level investment rate skewness produced by the model and the data, as well as the time-average firm-level investment rate kurtosis:[11]

$$\min_{\bar{\xi}} \Psi(\bar{\xi}) \equiv 0.5 \cdot \Bigg[\Big(\Big(\frac{1}{T} \sum_t skewness(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})(\bar{\xi}) - 2.1920 \Big) / 0.6956 \Big)^2 + \Big(\Big(\frac{1}{T} \sum_t kurtosis(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})(\bar{\xi}) - 20.0355 \Big/ 5.5064) \Big)^2 \Bigg]. \quad (11)$$

As can be seen from (11), the distribution of firm-level investment rates exhibits both substantial positive skewness – 2.1920 – as well as excess kurtosis – 20.0355. Caballero et al. (1995) document a similar fact for U.S. manufacturing plants. They also argue that non-convex capital adjustment costs are an important ingredient to explain such a strongly non-Gaussian distribution, given a close-to-Gaussian shock process. With fixed adjustment costs, firms have an incentive to lump their investment activity together over time in order to economize on these adjustment costs. Therefore, typical capital adjustments are large, which creates excess kurtosis. Making use of depreciation, firms can adjust their capital stock downward without paying adjustment costs. This makes negative investments less likely and hence leads to positive skewness in firm-level investment rates. We therefore use the skewness and kurtosis of firm-level investment rates to identify $\bar{\xi}$.

Table 5: CALIBRATION OF ADJUSTMENT COSTS - $\bar{\xi}$

$\bar{\xi}$	Skewness	Kurtosis	$\Psi(\bar{\xi})$	Adj. costs/ Unit of Output
0	-0.0117	3.2893	19.2865	0%
0.01	0.7840	5.0383	11.5154	1.5%
0.1	1.9329	9.3329	3.9167	6.8%
0.25 (BL)	2.5590	12.1591	2.3244	13.3%
0.5	3.0683	14.7695	2.5016	23.3%
0.75	3.3738	16.4950	3.2996	33.2%
1	3.5927	17.8153	4.2171	43.3%
5	4.8536	27.1159	16.2937	263.01%

Notes: 'BL' denotes the baseline calibration. Skewness and kurtosis refer to the time-average of the corresponding cross-sectional moments of firm-level investment rates. The fourth column displays the value of Ψ in (11). The last column shows the average adjustment costs conditional on adjustment as a fraction of the firm's annual output.

[11]The normalization constants in (11) are, respectively, the time series standard deviation of the cross-sectional investment rate skewness and the time series standard deviation of the cross-sectional investment rate kurtosis in the data.

Table 5 shows that $\bar{\xi}$ is indeed identified in this calibration strategy, as cross-sectional skewness and kurtosis of the firm-level investment rates are monotonically increasing in $\bar{\xi}$. The minimum of Ψ is achieved for $\bar{\xi} = 0.25$, which constitutes our baseline case. This implies average costs conditional on adjustment equivalent to roughly 13% of annual firm-level value added, which is in line with estimates from the U.S. (see Bloom (2009), Table IV, for an overview).

5 Results

5.1 Baseline Results

Can a dynamic stochastic general equilibrium model with persistent idiosyncratic productivity shocks, countercyclical aggregate shocks to their dispersion and fixed capital adjustment costs, calibrated to the *long-run* non-Gaussianity of the investment rate distribution, reproduce the cyclicality of important cross-sectional dispersion measures? Table 6 says "yes". The model matches the cyclicality of the dispersion of firm-level investment rates, of value added growth and of employment growth as well as the cyclicality of the extensive margin reasonably well.[12]

Table 6: CYCLICALITY OF CROSS-SECTIONAL DISPERSIONS AND THE EXTENSIVE MARGIN OF INVESTMENT - BASELINE MODEL

	$Correl(\cdot, HP(100) - Y)$	
Cross-sectional Moment	Model	Data
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.521	0.451
$std(\Delta \log y_{i,t})$	-0.417	-0.451
$std(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})})$	-0.455	-0.499
Fraction of adjusters	0.425	0.727
Fraction of lumpy adjusters	0.381	0.614

Notes: see notes to Tables 1 and 2. The table displays correlation coefficients with HP(100)-filtered aggregate output. The column 'Model' refers to the correlation coefficients from a simulation of the baseline model.

A basic, albeit only partial intuition why lumpy capital adjustment is a suitable candidate to explain procyclical investment dispersion, can be gleaned from a simple Ss-model as in Caplin and Spulber (1987):

[12]Figure 4 in Appendix A.2 shows a simulated time path of investment dispersion from the baseline model that displays positive comovement with aggregate output.

Proposition:

In a one-sided Ss-model with a fixed optimal adjustment policy (conditional on adjustment), $S-s$, and where the fraction of adjusters is given by $\frac{\Delta k}{S-s}$, the variance of adjustments increases with average adjustment if and only if the fraction of adjusters is smaller than 0.5.

Proof: Average adjustment in this environment is obviously Δk. From this it follows that the variance of adjustment is given by: $(0-\Delta k)^2\left(1-\frac{\Delta k}{S-s}\right)+\left((S-s)-\Delta k\right)^2\left(\frac{\Delta k}{S-s}\right)=\Delta k(S-s-\Delta k)$, which is increasing in Δk if and only if $\frac{\Delta k}{S-s}<0.5$.

This example shows that with sufficient inertia procyclical investment dispersion is generated by a procyclical extensive margin, as in this simple model all aggregate dynamics are driven by the extensive margin. The intensive margin of adjustment, $S-s$, is fixed by assumption. Of course, this example is rather stylized and our quantitative model has both an extensive and an intensive margin of investment operating and two aggregate shocks hitting the economy.

To investigate the mechanism quantitatively, Table 7 displays the cyclicality of the investment rate and output growth dispersions that our model generates for various levels of adjustment costs, both with and without shocks to the dispersion of productivity growth.

Table 7: ADJUSTMENT COSTS AND THE CYCLICALITY OF CROSS-SECTIONAL DISPERSIONS

$\bar{\xi}$	$std(\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})})$		$std(\Delta \log y_{i,t})$	
	2nd moment shocks	No 2nd moment shocks	2nd moment shocks	No 2nd moment shocks
0	-0.553	-	-0.571	-
0.0001	-0.546	-0.181	-0.570	0.008
0.001	-0.469	0.674	-0.564	0.017
0.005	-0.365	0.810	-0.544	0.057
0.01	-0.298	0.832	-0.532	0.057
0.05	-0.035	0.855	-0.493	0.070
0.1	0.172	0.861	-0.465	0.111
0.25 (BL)	0.521	0.874	-0.417	0.138
0.5	0.734	0.883	-0.375	0.137
0.75	0.821	0.891	-0.326	0.107
1	0.865	0.896	-0.290	0.120
5	0.956	0.917	-0.106	0.137

Notes: see notes to Table 6. 'BL' denotes the baseline calibration. '2nd moment shocks' refers to a simulation with aggregate productivity shocks and shocks to the dispersion of firm-level Solow residuals, as specified in equation (10). 'No 2nd moment shocks' refers to a simulation with only aggregate productivity shocks, where $\rho_A=0.5259$ and $\Omega=0.0182$. Note that in this case with $\bar{\xi}=0$ $std(\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})})$ and $std(\Delta \log y_{i,t})$ are constant, which means that their correlation coefficients with output are not defined.

Two findings are important: the third and last column of Table 7 show that without second moment shocks neither the procyclicality of the investment dispersion nor the countercyclicality of the output growth dispersion can be quantitatively replicated. Already a very small non-convex capital adjustment cost generates procyclical investment dispersion – the gradient of procyclicality in the adjustment cost factor, $\bar{\xi}$, is steep. The model overshoots the number in the data considerably. Also, without countercyclical second moment shocks the dispersion of value added growth is essentially acyclial. Countercyclical second moment shocks are thus an important part in understanding cross-sectional firm dynamics.

Secondly, in the presence of countercyclical second moment shocks, the procyclicality of investment dispersion is a gradually and monotonically increasing function of the adjustment cost parameter. What is perhaps surprising is that the same level of adjustment costs that best matches the time average of the cross-sectional skewness and kurtosis of firm-level investment rates - two statistics closely related to the extent of non-convexities at the micro-level as we have explained in Section 4 - also makes the model match almost exactly the extent of procyclicality of investment dispersion in the data, as measured by its correlation coefficient with the cyclical component of output.

By contrast, in the frictionless case, $\bar{\xi} = 0$, the dispersions of investment and output growth merely mirror the countercyclicality of the dispersion of the idiosyncratic driving force, namely idiosyncratic productivity. As the output growth dispersion is slightly too countercyclical for $\bar{\xi} = 0$, incidentally, non-convex capital adjustment costs help to align model and data also in this dimension.

We view these results as a good validation of our mechanism and an example of the larger premise of this paper that cross-sectional dynamics are an important aspect of the data that heterogeneous firm models should address. In the presence of quantitatively realistic countercyclicality in the dispersion of firm-level Solow residual shocks, it is one level of adjustment costs that jointly matches the *time-average* skewness and kurtosis of the investment rate distribution and the *time series* correlation between the standard deviation of investment rates and aggregate output. Moreover, such a model replicates the cyclicality of the cross-sectional dispersions of other important firm-level quantities. Table 7 shows that this identification is rather tight.

Table 8 illustrates how the procyclicality of the investment dispersion and the procyclicality of the extensive margin interact with the curvature of the firm's revenue function, $\frac{\theta}{1-\nu}$. The results in columns three and four refer to setups with factor elasticities $\nu = 0.5333$, $\theta = 0.2667$ and $\nu = 0.5556$, $\theta = 0.2778$, respectively, compared to $\nu = 0.5565$, $\theta = 0.2075$ in the baseline scenario.[13] Larger revenue elasticities in capital imply lower procyclicality of the extensive margin

[13] In a monopolistic competition framework, column two implies a scenario with a CRTS-one-third-two-third

Table 8: FACTOR ELASTICITIES AND CYCLICALITY OF INVESTMENT DISPERSION / EXTENSIVE MARGIN

Cross-sectional Moment	Baseline (Rev. Ela.=0.47)	Rev. Ela.=0.57	Rev. Ela.=0.63
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.521	-0.112	-0.497
Fraction of Adjusters	0.425	-0.312	-0.612

Notes: see notes to Table 6. 'Rev. Ela.' stands for the revenue elasticity of capital in a reduced form revenue function, where labor has been maximized out. It is given by $\frac{\theta}{1-\nu}$.

and of the investment rate dispersion. With larger revenue elasticities or "closer to" linear production functions, the intensive margin of investment is more flexible and thus more used to adjust the firm-level capital stock (see Gourio and Kashyap, 2007). In long model simulations, the range of the target levels of capital that firms adjust to is the wider the closer the revenue elasticity of capital is to unity. In the baseline scenario the target levels of capital range from 0.0241 to 42.0646. They span, for the third column, 0.0180 to 99.7068, and 0.0072 to 177.4022 for the last column. Note that all three cases have the same process for idiosyncratic productivity.

Table 9: GENERAL EQUILIBRIUM AND CYCLICALITY OF INVESTMENT DISPERSION / EXTENSIVE MARGIN

Cross-sectional Moment	Baseline - GE	PE	Data
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.521	-0.156	0.451
Fraction of Adjusters	0.425	-0.0001	0.727

Notes: see notes to Tables 6. 'GE' stands for general equilibrium and refers to a model simulation with market clearing real wages and interest rates. 'PE' stands for partial equilibrium and refers to a model simulation, where the real wage and interest rate are held constant at the average level in the 'GE'-simulation.

Table 9 shows the effect of general equilibrium on the procyclicality of the extensive margin and the cross-sectional investment rate dispersion. It is real wage and interest rate movements that lead to coordination of micro-level adjustment and therefore, to the extent that this is done mainly through the extensive margin, to procyclicality of the fraction of adjusters. A more procyclical extensive margin increases the cyclical comovement of the investment rate dispersion. Since both cross-sectional moments are strongly procyclical in the data, we thus provide a first confirmation of the conjecture in Khan and Thomas (2008) that general equi-

production function and a markup of 1.25, column three a markup of 1.20. In each case, we recompute firm-level and aggregate Solow residuals, estimate a new aggregate shock process (2) and re-calibrate the adjustment cost parameter $\bar{\xi}$ to minimize $\Psi(\bar{\xi})$ in (11). For the third column this leads to $\bar{\xi} = 0.45$, and $\bar{\xi} = 0.5$ for the last.

librium price movements are important to quantitatively account for cross-sectional business cycle dynamics.

Table 10: VOLATILITY OF SECOND MOMENT SHOCKS AND CYCLICALITY OF INVESTMENT DISPERSION/EXTENSIVE MARGIN

Cross-sectional Moment	Baseline	Double Volatility of $std(\Delta \log \epsilon_{i,t})$	Quadruple Volatility of $std(\Delta \log \epsilon_{i,t})$
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.521	-0.070	-0.314
Fraction of Adjusters	0.425	-0.157	-0.335

Notes: see notes to Table 6. For the 'Double Volatility of $std(\Delta \log \epsilon_{i,t})$'-case, we use the data time series of $std(\Delta \log \epsilon_{i,t})$ multiplied by a factor 2 to reestimate the aggregate shock process (2) and recompute the equilibrium with an otherwise unaltered model specification. For the 'Quadruple Volatility of $std(\Delta \log \epsilon_{i,t})$'-case, we multiply by a factor of 4.

With the final Table 10 we argue that it is important to match the cyclicality of the investment and the output growth dispersions *jointly*, and that the procyclicality of investment dispersion places tight bounds on the volatility of the second-moment shocks. If we increase this volatility and make second moment shocks "more important", the procyclicality of investment dispersion is eliminated rather quickly.

In summary, both the procyclicality of investment dispersion as well as the countercyclicality of the dispersion of firm-level value added growth imposes rather tight restrictions on important structural parameters, such as adjustment costs, factor elasticities in the production function and the volatility of second moment shocks. This makes the study of cross-sectional business cycle dynamics important for the calibration of heterogenous-firm models. We also show that the nature of the aggregate environment - market clearing with flexible prices or not - and cross-sectional business cycle dynamics are linked and should be jointly used to discipline models.

5.2 Results from Industry Calibrations

We next use the one-digit industry variation in the procyclicality of investment dispersion and the extensive margin depicted in Figure 2 in Section 2 and check whether the model can generate a similar variation. Instead of calibrating and computing a six-industry general equilibrium model of the entire German economy, which would be computationally too burdensome, we run six variants of our baseline model, where we adjust the important parameters to industry-specific moments, but otherwise treat the corresponding industry as if it was the aggregate economy. With this caveat in mind, we view this exercise as a useful additional test for the extensive margin mechanism and its effect on cross-sectional dynamics.

Specifically, we calibrate the factor elasticities in the production function to the median industry income shares in USTAN and use the industry cuts from USTAN to compute the long-run average and the time series process of the standard deviations of the firm-level Solow residual shocks. We calibrate the depreciation rates to match the industry-specific long-run aggregate investment rate from German national accounting data. The industry national accounting data also allow us to compute industry-specific Solow residuals and then reestimate the VAR in equation (2). Finally, just as in the baseline calibration, we calibrate the adjustment costs parameter, $\bar{\xi}$, to minimize a quadratic form in industry-specific skewness and kurtosis of the firm-level investment rates, analogous to (11).[14]

Table 11: RESULTS FROM INDUSTRY CALIBRATIONS

Industry	$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$		Fraction of Adjusters	
	Model	Data	Model	Data
Aggregate	0.521	0.451	0.425	0.727
MIN	0.071	0.042	0.075	0.075
MAN	0.776	0.477	0.760	0.765
CON	0.698	0.435	0.580	0.428
TRD	0.912	0.209	0.925	0.559
TRA	0.281	0.404	0.299	0.237

Notes: see notes to Tables 2 and 6. See Figure 2 for the industry acronyms. The table displays correlation coefficients with HP(100)-filtered output. The column 'Model' refers to the correlation coefficients from model simulations. The column 'Data' uses aggregate real gross value added of the nonfinancial private business sector in the first row, thereafter the real gross value added of the corresponding 1-digit industry.

Table 11 displays the results for the correlations of investment dispersion and the extensive margin with own-industry output.[15] The correlation coefficients between model simulations and data are 0.471 for the investment rate dispersion and 0.901 for the extensive margin. The corresponding rank correlations are 0.4 and 0.9, respectively. The model captures the overall variation rather well, with mining and energy not displaying any cyclicality of either investment rate dispersion or the extensive margin, transportation and communication taking up a middle position and manufacturing and construction having the strongest procyclicality. The biggest exception is trade, where the model produces an almost perfect procyclicality of investment

[14] Table 18 in Appendix B collects the main parameters for each industry.

[15] For computational reasons we leave out agriculture, for which we estimate a revenue elasticity of capital of 0.935. This would require extremely fine and large grids for firm-level capital. Given this low curvature of the firm's revenue function and our results in Table 8 from Section 5.1, the slightly negative correlation of investment dispersion in agriculture supports the extensive margin mechanism ex negativo, making the cumbersome computation unnecessary. For similar reasons, in the computation for the mining and energy sector we scale down the measured factor elasticities by a factor of 0.9.

rate dispersion and the extensive margin, which is inconsistent with the data. We estimate the revenue elasticity of capital for trade to be 0.403, the lowest value for any of the six one-digit industries. This low curvature facilitates the extensive margin mechanism to an extent that is obviously at odds with the data. We nevertheless view the industry-specific results overall as additional support for the extensive margin mechanism and its relation to cross-sectional dynamics, although, clearly, other things are going on, too.

6 Final Remarks

This paper establishes a new business cycle fact: the cross-sectional standard deviation of firm-level investment is robustly and significantly procyclical. Investment dispersion is more procyclical in the goods-producing sectors, for smaller firms and for structures. This paper also shows that important structural parameters such as capital adjustment costs, the curvature of the firms' revenue function or the heteroskedasticity of the firm-level productivity process have important implications for the dynamics of the cross-section of firms. This means, conversely, that cross-sectional dynamics should be used in the calibration and evaluation of heterogeneous-firm models. In such models the nature of the aggregate environment - market clearing with flexible prices or not - and cross-sectional business cycle dynamics are tightly linked and should be jointly used as a disciplining device.

We view this paper as just the beginning of a new research program that attempts to understand more comprehensively the time-series behavior of the entire cross-section of firms, not merely the cyclicality of second moments. This will, hopefully, lead to a better microfoundation and identification of structural heterogeneous-firm models and contribute to making them better suitable for policy analysis.

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A Appendix - Data

A.1 Description of the Sample

Our firm-level data source is USTAN (*Unternehmensbilanzstatistik*) of *Deutsche Bundesbank*, which is a large annual firm-level balance sheet data base. It provides annual firm level data from 1971 to 1998 from the balance sheets and the profit and loss accounts of over 60,000 firms per year. It originated as a by-product of the Bundesbank's rediscounting and lending activities. Bundesbank law required the Bundesbank to assess the creditworthiness of all parties backing a commercial bill put up for discounting. It implemented this regulation by requiring balance sheet data of all parties involved. These balance sheet data were then archived and collected into a database (see Stoess (2001) and von Kalckreuth (2003) for details).

Although the sampling design – one's commercial bill being put up for discounting – does not lead to a perfectly representative selection of firms in a statistical sense, the coverage of the sample is very broad. USTAN covers incorporated firms as well as privately-owned companies. Its industry coverage – while still somewhat biased towards manufacturing firms – includes the construction, the service as well as the primary sectors. The following Table 12 displays the industry coverage of our final baseline sample.

Table 12: INDUSTRY COVERAGE

One-digit Industry	Firm-year observations	Percentage
Agriculture	12,291	1.44
Mining & Energy	4,165	0.49
Manufacturing	405,787	47.50
Construction	54,569	6.39
Trade (Retail & Wholesale)	355,208	41.59
Transportation & Communication	22,085	2.59

While there remains a bias towards larger and financially healthier firms, the size coverage is still fairly broad: 31% of all firm-year observations in our final baseline sample have less than 20 employees and 57% have less than 50 employees. In terms of ownership structure, only 2% of firm-year observations are from publicly traded firms, just under 60% from limited liability companies and just under 40% from private firms with fully liable partners. Finally, the Bundesbank itself frequently uses the USTAN data for its macroeconomic analyses and for cross-checking national accounting data. We take this as an indication that the bank considers the data as sufficiently representative and of high quality. This makes the USTAN data a suitable data source for the study of cross-sectional business cycle dynamics.

From the original USTAN data, we select only firms that report information on payroll, gross value added (before depreciation) and capital stocks. Moreover, we drop observations from East German firms to avoid a break of the series in 1990. We deflate all but the capital and investment data by the implicit deflator for gross value added from the German national accounts.

Capital is deflated with one-digit industry- and capital-good specific investment good price deflators within a perpetual inventory method. Similarly, we recover the amount of labor inputs from wage bills (we calculate an average wage for cells of firms described by industry, year, firm-size, and region and then divide the payroll by this average), as information on the number of employees is only updated infrequently for some companies. Finally, the firm-level Solow residual is calculated from data on real gross value added and factor inputs.

We remove outliers according to the following procedure: we calculate log changes in real gross value added, the Solow residual, real capital and employment, as well as the firm-level investment rate and drop all observations where a change falls outside a three standard deviations interval around the year-specific mean. We also drop those firms for which we do not have at least five observations in first differences. This leaves us with a sample of 854,105 firm-year observations, which corresponds to observations on 72,853 firms, i.e. the average observation length of a firm in the sample is 11.7 years. The average number of firms in the cross-section of any given year is 32,850. Details on the implementation as well as the representativeness of the resulting sample can be found in Bachmann and Bayer (2011).

A.2 Robustness

We start by showing in Table 13 that the cross-sectional *averages* of investment as well as output, productivity and employment growth, computed from USTAN, are strongly positively correlated with the cyclical component of the real gross value added of the nonfinancial private business sector. This means that USTAN represents well the cyclical behavior of the sectoral aggregate it is meant to represent.

Table 13: CYCLICALITY OF CROSS-SECTIONAL AVERAGES

Cross-sectional Moment	$Correl(\cdot, HP(100) - Y)$
$mean(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.756
$mean(\Delta \log y_{i,t})$	0.663
$mean(\Delta \log \epsilon_{i,t})$	0.592
$mean(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})})$	0.602

Notes: the table shows the time series correlation coefficient of cross-sectional averages, linearly detrended, with the cyclical component of the aggregate real gross value added of the nonfinancial private business sector, which we detrend using an HP(100)-filter. The rows refer, respectively, to the investment rate, the log-change of real gross value added, the log-change of Solow residuals and the net employment change rate. We have removed firm fixed and 2-digit industry-year effects from each variable.

The next two tables provide robustness checks for our main empirical result that the cross-sectional investment rate dispersion is procyclical. We start by varying the cyclical indicator in Table 14, while Table 15 deals with robustness related to the choice and treatment of the cross-sectional dispersion measure as well as robustness with respect to general data treatment.

Table 14: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - ROBUSTNESS TO CYCLICAL INDICATOR

Cyclical Indicator	$Correl(std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), \cdot)$
Baseline	0.451
HP(6.25)-Y	0.370
Log-diff-Y	0.351
$mean(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.792
HP(100)-I	0.719
HP(100)-N	0.485
HP(100)-Solow Residual	0.387

Notes: see notes to Table 1. Y refers to aggregate real gross value added of the nonfinancial private business sector. I refers to aggregate real gross fixed investment and N to aggregate employment of the same sector. 'HP(6.25)' refers to an HP-filter with smoothing parameter 6.25 (see Ravn and Uhlig, 2002). 'Log-diff-Y' stands for the year-over-year difference of the natural logarithm of Y. For $mean(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$, see notes to Table 13.

Table 15: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - MORE ROBUSTNESS

Cross-sectional Moment	$Correl(\cdot, HP(100) - Y)$
Baseline	0.451
$IQR(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.567
$std(\Delta \log k_{i,t})$	0.442
Raw data - no fixed effects	0.451
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$quadratic detrending	0.555
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$cubic detrending	0.599
$std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$HP(100) detrending	0.618
Pre-reunification: 1973-1990	0.297
Without the oil shock in 1975: 1977-1998	0.359
Uniform price index for investment	0.427
Stricter outlier removal (> 2.5 std)	0.452
Looser outlier removal (> 5 std)	0.422
Very loose outlier removal (> 10 std)	0.427
Outlier removal - 1% largest	0.485
Outlier removal - 5% largest	0.592
Outlier > 3 std means merger	0.416
Shorter in sample (2 obs.)	0.439
Longer in sample (20 obs.)	0.392
Selection correction	0.382

Notes: see notes to Tables 1 and 14. $IQR(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$ refers to the cross-sectional interquartile range. $std(\Delta \log k_{i,t})$ refers to the cross-sectional standard deviation of the firm-level capital growth rate. 'Raw data - no fixed effects' uses the standard deviation of the raw firm-level investment rates, no fixed effects removed. The next three rows show results, where we detrend $std(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$ not with a linear trend, but, respectively, with a quadratic, cubic trend and an HP(100)-filter. 'Uniform price index for investment' refers to a scenario, where we deflate firm-level investment and capital with an aggregate price deflator for investment goods, not with one-digit industry- and capital-good specific ones. 'Stricter outlier removal (> 2.5 std)' refers to a scenario, where we remove all firms with 2.5 (instead of 3) standard deviations above or below the year-specific mean in either Solow residual growth, value added growth, investment rates or employment change rates. The next two rows explore two more liberal outlier removal criteria. 'Outlier removal - 1% largest' refers to a scenario, where observations are removed if they belong to the smallest or to the largest 1% of the observations in a given year. The next row applies a 5% criterion. 'Outlier > 3 std means merger' refers to a scenario, where we treat an observation of 3 standard deviations above or below the year-specific mean as indicating a merger and mark the firm henceforth as a new one. 'Shorter in sample (2 obs.)' refers to a scenario, where we require firms to have two observations in first differences (instead of five) to be in the sample. The next row requires 20 observations. 'Selection correction' refers to a scenario where we estimate a simple selection model, where lagged firm-level Solow residuals determine selection and the firm-level investment rate is modeled as a mean regression. We use the maximum likelihood estimator by Heckman (1976) to infer the selection-corrected variance of the residual in the firm-level investment rate equation.

Perhaps most importantly, Table 15 shows in the last and next to last row that the cyclical effects we find are not due to cyclical variations in the sample composition. In the scenario 'Selection correction' we control for sample selection in the following way: we estimate a simple selection model, where lagged firm-level Solow residuals determine selection and the firm-level investment rate is modeled as a mean regression. We use the maximum likelihood estimator by Heckman (1976) to infer the selection-corrected variance of the residual in the firm-level investment rate equation. The latter is very close to the sample variance of firm-level investment rates, indicating that our results are not influenced by systematic sample drop-outs. Interestingly, the correlation we receive from this procedure is almost identical to the one we obtain when we restrict the analysis to firms that are almost always in the sample.

Table 16 shows that the ownership structure matters for cross-sectional results (focussing on publicly traded firms in Germany would eliminate the procyclicality of investment dispersion), making it important to use broader data sets for the study of cross-sectional facts (see Davis et al. (2006), for a similar point).

Table 16: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - LEGAL FORM

Aggregate	Publicly Traded	Limited Liability Companies	Fully Liable Partnerships
0.451	0.010	0.322	0.640

Notes: see notes to Table 1. The table displays correlation coefficients of the investment rate dispersion by legal form with the cyclical component of the aggregate real gross value added of the nonfinancial private business sector, detrended by an HP(100)-filter. 'Publicly Traded' means the German legal forms of *AG* and *KGaA*. 'Limited Liability Companies' means the German legal forms of *GmbH* and *GmbH& Co KG*. 'Fully Liable Partnerships' means the German legal forms of *GBR*, *OHG* and *KG*.

Other Data Sources - Cross Country Evidence

To asses whether procyclical investment dispersion is specific to Germany, we compare our findings with results obtained from the UK DTI-database and from Compustat data in the U.S. The UK data covers the period 1977-1990 and stems from a sample of firms in the manufacturing and some selected non-financial service sectors in Britain. It oversamples large firms, but is otherwise meant to be representative. For the U.S. we use Compustat annual accounts from 1968-2006.

We apply the same data treatment criteria as for USTAN. Then the UK data comprises 10,966 firm-year observations after removal of outliers and constraining the sample to firms with at least 5 observations in first differences. The same procedure yields for the U.S. a final sample of 67,394 firm-year observations.

Table 17 shows that investment dispersion is robustly procyclical in UK and U.S. data sets, very much in line with our findings for the German USTAN data. For the Compustat sample, we find a slightly mitigated (though positive) cyclicality of the investment rate dispersion. This reflects that, in contrast to both the DTI data base and USTAN, Compustat covers only publicly traded and mostly large companies. The firms in the Compustat sample are typically larger than even the top 5% largest firms in the USTAN data. That they nevertheless display a procyclical investment rate dispersion and the fact that in Germany publicly traded and very large firms have an investment dispersion that is basically acyclical (see Tables 3 and 16), suggests that in the U.S. investment dispersion may overall be even more procyclical than in Germany.

Table 17: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - EVIDENCE FROM THE UK AND THE U.S.

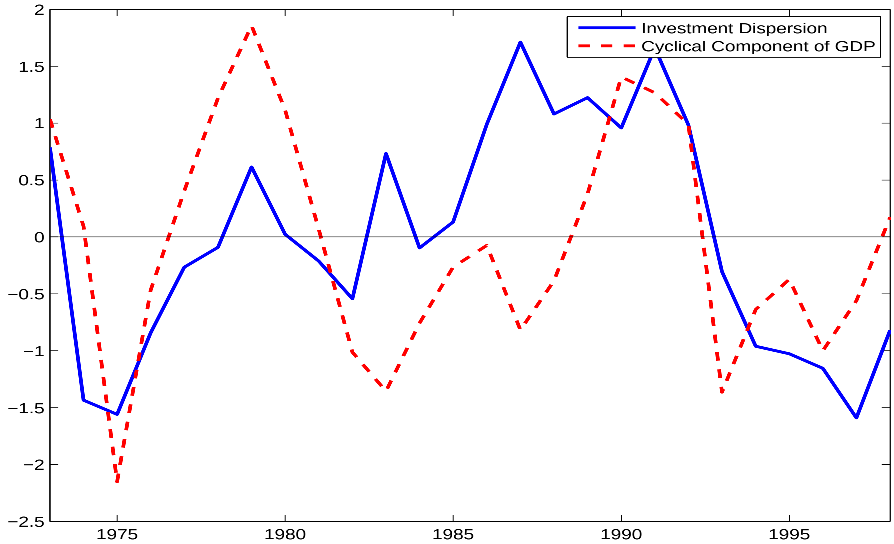
Cyclical Indicator	$Correl\left(std\left(\frac{i_{it}}{0.5(k_{i,t-1}+k_{i,t})}\right),\cdot\right)$	$Correl\left(IQR\left(\frac{i_{it}}{0.5(k_{i,t-1}+k_{i,t})}\right),\cdot\right)$
UK: Cambridge DTI, 1977 - 1990		
HP(100)-Y	0.506	0.687
HP(6.25)-Y	0.488	0.749
Log-diff-Y	0.653	0.263
U.S.: Compustat 1969 - 2006		
HP(100)-Y	0.326	0.649
HP(6.25)-Y	0.334	0.628
Log-diff-Y	0.259	0.421

Notes: aggregate output data, Y, for the U.S. refers to real gross value added in the nonfinancial private business sector. For the UK we use aggregate real gross value added instead, as the corresponding sectoral data is not publicly available for the relevant time period. Dispersion measures are linearly detrended.

The following Figures 3 and 4 depict the time series of the cross-sectional investment rate dispersion and cyclical aggregate output, respectively, for the data and from a simulation of the baseline model.

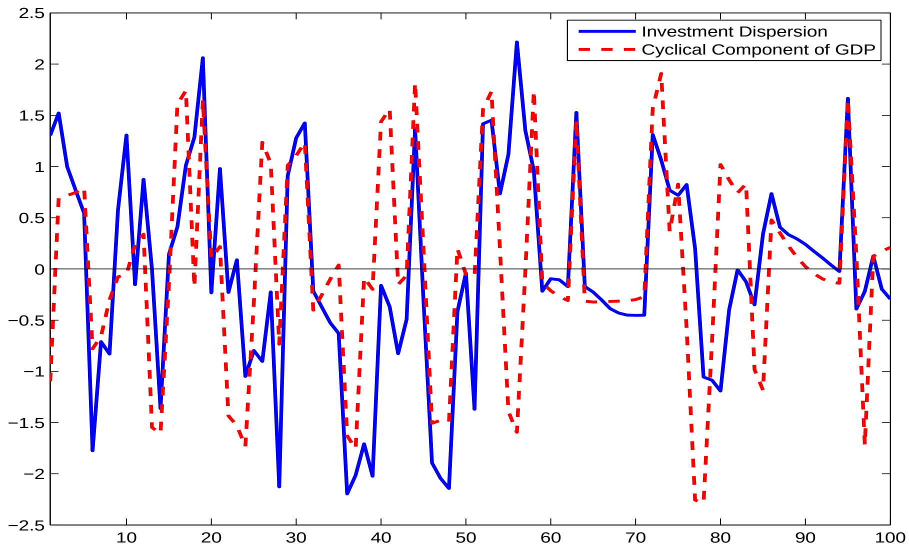
Two Additional Graphs

Figure 3: Cyclicality of Cross-sectional Investment Dispersion - Data



Notes: see notes to Figure 1. Both time series have been normalized by their standard deviation.

Figure 4: Cyclicality of Cross-sectional Investment Dispersion - Baseline Model



Notes: see notes to Figure **??**. Both time series have been normalized by their standard deviation.

B Industry Calibrations

Table 18: INDUSTRY CALIBRATIONS - PARAMETERS

Industry	$\bar{\xi}$	$\bar{\sigma}(\epsilon)$	$skew(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	$kurt(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	θ	ν	δ
Aggregate	0.25	0.1201	2.1920	20.0355	0.2075	0.5565	0.094
MIN	0.5	0.1156	1.3355	15.8334	0.4942	0.3201	0.093
MAN	0.25	0.1147	2.2511	21.4518	0.2075	0.5565	0.119
CON	0.25	0.1117	1.7684	20.9611	0.1771	0.6552	0.153
TRD	0.4	0.1244	2.1091	17.6077	0.2204	0.4536	0.123
TRA	0.07	0.1356	1.3315	10.6363	0.2896	0.4205	0.112

Notes: see Figure 2 for the industry acronyms. $\bar{\xi}$ is the calibrated adjustment cost parameter. $\bar{\sigma}(\epsilon)$ denotes the steady state standard deviation of the shocks to the firm-level Solow residual. θ and ν are the capital and employment, respectively, elasticity in the production function. For the mining and energy sector we scaled down the measured median income shares by a factor of 0.9 (reported here). δ are industry-specific depreciation rates.