Investment Incentives in Procurement Auctions*

Leandro Arozamena[†] and Estelle Cantillon[‡]

February 17, 2002

Abstract

This paper investigates firms' incentives to invest in cost reduction in the first price sealed bid auction, a format largely used for procurement. Two central features of the model are that we allow firms to be heterogeneous and that investment is observable. We find that firms will tend to underinvest in cost reduction because they anticipate fiercer head-on competition. Using the second price auction as a benchmark, we also find that the first price auction will elicit less investment from market participants and that this is socially inefficient. These results have implications for market design when investment is important.

Keywords: Asymmetric auctions, endogenous distributions, investment incentives.
JEL codes: C72, D44, L13.

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*We are indebted to Eric Maskin for encouragement and for detailed comments on an earlier draft and to Philippe Aghion, Gian Luigi Albano, Francisco Gomes, Jozsef Molnar, Ariel Pakes, William Thomson, Xavier Vives, Catherine Wolfram and seminar participants at Bonn, CEMA, Di Tella, ECARES (Université Libre de Bruxelles), Harvard, La Plata, LSE, Pompeu Fabra, Michigan and San Andrés for helpful suggestions. We thank the Editor, Mark Armstrong, and two anonymous referees for their many suggestions that helped improve this draft. Huagang Li and John Riley kindly provided us with the source codes for their Bidcomp2 program. During this research, we have benefited from the financial support of a de Fortabat fellowship at Harvard (Arozamena) and the Belgian Fonds National de la Recherche Scientifique (Cantillon).

[†]Universidad Torcuato Di Tella, Buenos Aires, Argentina. Email: larozamena@utdt.edu.

[‡]Harvard Business School and CEPR. Correspondence: Estelle Cantillon, Harvard Business School, Morgan Hall 243, Soldiers Field Road, Boston MA 02163. Phone: (617) 495 6338. Fax: (617) 496 5859. Email: ecantillon@hbs.edu.

1. Introduction

Different market institutions provide different incentives for firms to reduce costs, acquire information, expand, and so on, in short, to undertake any activity that affects their competitive positions. Accordingly, the performance of these market institutions should be judged not only from a static perspective - taking firms' competitive positions as given - but also from a dynamic perspective. Auction markets are a natural competitive environment to study because there is a clear sense in which we can design such markets by choosing appropriate rules for competition.

In this paper, we study how the first price auction, a format commonly used for procurement, affects firms' incentives to invest in cost reduction. Many procurement markets are characterized by a small number of participants, heterogeneity among firms and low rates of turnover. Therefore, it is interesting to investigate to what extent the format used for the auction contributes to these characteristics.

The basic ingredients of our model are as follows. There are N firms and one of them has the opportunity, prior to the auction, to make an investment. We model this investment as an improvement in the ex-ante distribution of costs for this firm. The main question we ask is: What are the firm's incentives to invest given that this investment is observed by its competitors?

Clearly, the benefits of such investment depend on how the firm's competitors react. In other words, to determine the firm's incentive to invest, we need to compare the equilibrium at the procurement stage if it makes this investment and if it does not. In section 3, we derive new comparative statics results for the first price auction. We find that after the investment, the investor's opponents will bid collectively more aggressively. This result holds generally when there are only two bidders or when the investor's opponents are symmetric (proposition 1). Otherwise, a sufficient condition is that investment changes market leadership (proposition 2). In the language of industrial organization, investment has a negative strategic effect in the first price auction. This erodes its benefits. Furthermore, the strategic effect can be quantitatively important: we provide an example where investment hurts the investor even if it comes for free.

In section 4, we investigate how first and second price auctions compare when it comes to pre-auction investment. Under the same condition, i.e, that investment changes market leadership, we find that the first price auction will induce less investment than the second price auction (proposition 3). This result can be tied back to the negative strategic effect identified in section 3 (notice that there is no strategic effect in the second price auction: equilibrium bidding behavior is independent of the distributions of the opponents' costs). Firms' investments always benefit the procurement authority, but the fact that the second price auction generates the socially efficient investment incentives (proposition 4) provides us with a clear normative interpretation of this underinvestment result.

A number of papers examine investment incentives in procurement auctions for the case where firms are ex-ante symmetric and investment is simultaneous and unobservable (e.g. Tan, 1992, Piccione and Tan, 1996, Bag, 1997). What distinguishes our model is that (1) investment is observable, (2) only one firm has the opportunity to invest at a time, and (3) firms may be heterogeneous. Firm heterogeneity is pervasive in procurement markets, so it is important to allow for it in a model. We comment on the other two assumptions in turn.

Investment observability is an attractive assumption in many instances. First, empirical studies have highlighted the role that distance to the market plays in determining firms' competitive advantage (Bajari, 1998 and 1999, Porter and Zona, 1999). It is natural to assume that plant locations are observed by potential bidders. Hence, so are locational investments. Second and in the same vein, capacity is often observed. Finally, investment observability seems to be an appropriate assumption for the case of long term investments such as R&D.[1]

The main motivation for our assumption that investment is sequential is analytical tractability, though that assumption too can be justified on empirical grounds. For example, an important application is the case of repeated auctions when there is some linkage between them (because of capacity constraints, economies of scale or learning) that affects the winner's costs in later auctions. Because there is only one winner in any given auction, only one firm "invests."[2] Alternatively, we can reinterpret investment in our model as sequential entry. The investment might correspond to learning about the procurement competition or developing any specialized capabilities necessary to enter it.

At any rate, understanding investment incentives in the sequential case is a necessary first step towards understanding investment incentives in the simultaneous case. In particular, our finding that investment in the first price auction has a negative strategic effect has implications for the simultaneous investment

[1] Lichtenberg (1988) provides evidence for significant R&D investment prior to procurement in the defense industry.

[2] Notice that investment observability is again natural in this case. Jofre-Bonet and Pesendorfer (2000) provide an empirical study of such effects. von der Fehr and Riis (2000) study sequential second price auctions.

model. A typical finding on simultaneous investment by symmetric firms is that, in equilibrium, firms invest the same amount. As a result, the *post investment* market remains symmetric and a revenue equivalence theorem holds for the first and the second price auctions (Tan, 1992). In other words, the particular market institution does not matter. Our results suggest that the assumption of non-observability of investment that is made in this literature is critical for these findings. Specifically, the assumption that investment is observable introduces an additional strategic element that is absent when investment is unobservable. Equilibrium behavior now requires that firms adapt their behavior in the procurement stage following a "deviation" at the investment stage. Because of the strategic effect associated with investment in the first price auction, ex-ante symmetric firms may not invest the same amount in equilibrium. Hence, the equivalence between the first price and the second price auction can break down even if firms are ex-ante symmetric and investment is simultaneous but observable. We elaborate on this point in section 3.

A large portion of the paper is devoted to deriving comparative statics results for the first price auction. In comparing the equilibrium at the procurement stage before and after the investment, we draw heavily on the recent literature on asymmetric first price auctions. Existence and uniqueness of the equilibrium in the independent private value first price auction have been proved under increasingly general assumptions by Lebrun (1996 and 1999) and Maskin and Riley (1996 and 2000a). Maskin and Riley (2000b) and Li and Riley (1999) provide more precise characterizations of the equilibrium when a stochastic dominance relationship exists among bidders. Riley (1996) and Li and Riley (1999) allow for more than two bidders. Our results in section 3 are closest to Lebrun (1998). Proposition 1 generalizes his result by allowing non common supports for the distribution of costs, risk aversion and endogenous quantities. Our main contribution to this literature however, is proposition 2, which generalizes the result to the case of more than two bidders.[3] When bidders are asymmetric, there is a big difference between two and more than two bidders. Indeed, there is in general no explicit solution to the equilibrium in the first price auction, so we need to work with the system of differential equations that characterizes the equilibrium. However, these equations place much less structure on equilibrium behavior when there are more than two bidders. We introduce two "tricks" to obtain analytical results with more than two bidders: (1) we focus on the *aggregate* behavior of the investor's

[3]In addition, the working paper version provides techniques that allow for a systematic treatment of the case where the supports of the bidding functions may differ across bidders.

opponents rather than on their individual behavior, and (2) we require that the investment changes market leadership.

The paper is organized as follows. In section 2, we present the model and characterize the equilibrium in the first price auction. In section 3, we derive the comparative statics result for the first price auction. Section 4 compares the outcome in the first price and second price auctions. Section 5 gathers the concluding remarks and outlines directions for future research.

2. The model

In this section, we present the model and characterize its equilibrium. A single buyer is in charge of procuring a given good or service. As in Hansen (1988), we allow quantities to be endogenous. Let $D(b)$ be the buyer's demand at price b. We make the following standard assumptions on demand:[4]

Assumption 1: $D(b) \geq 0, D'(b) \leq 0$ and increasing price elasticity $\frac{d}{db}[\frac{D'(b)b}{D(b)}] \leq 0$.

$N \geq 2$ firms take part in a first price, sealed bid auction for the procurement contract. That is, the contract is awarded to the firm offering to provide the good or service at the lowest price, and the winner is paid the per unit price she bid. Ties are resolved by a random draw among the lowest bidders.

Firms' constant marginal costs have support on $[\underline{c}_i, \overline{c}_i]$, where $0 \leq \underline{c}_i < \overline{c}_i$. They are independently distributed according to the twice continuously differentiable cumulative distribution function $F_i(.)$, with a density bounded away from zero on its support. These distributions are assumed to be common knowledge. They can be interpreted as representing the technology available to firms. Firm i's profit when its cost is c_i and it makes a bid b is given by:

$$\pi_i(b, c_i) = \begin{cases} V_i((b - c_i)D(b)) & \text{if it wins} \\ 0 & \text{otherwise} \end{cases}$$

Assumption 2: For all i, $V_i(0) = 0$, $V_i' > 0$ and $V_i'' \leq 0$.

Lemma 1: *Under assumptions 1 and 2, $\pi_i(b, c)$ is strictly log-supermodular in (b, c), i.e. $\frac{\partial}{\partial c}[\frac{\frac{\partial \pi_i}{\partial b}}{\pi_i}] > 0$ over the domain where $\pi_i > 0$.*

Proof. We first claim that, at any equilibrium, $D(b) + (b - c)D'(b) > 0$ for all b such that b is bid by some firm i. $D(b) + (b - c)D'(b) = 0$ corresponds to the first

[4]These guarantee that the complete information monopolist problem is quasiconcave (see, e.g. Caplin and Nalebuff, 1991, proposition 11).

order condition of the monopolist facing demand $D(b)$. It trades off the marginal benefit of increasing prices with the marginal cost of lost trade. In a procurement setting, increasing prices has an additional cost: the potential loss of the whole market. Therefore, $D(b) + (b - c)D'(b)$ must be strictly positive at any bid b submitted in equilibrium by some firm i.

Together with assumptions 1 and 2, this implies that:

$$\begin{aligned}
\frac{\partial}{\partial c}\left[\frac{\frac{\partial \pi_i}{\partial b}}{\pi_i}\right] &= \frac{1}{\pi_i}\frac{\partial^2 \pi_i}{\partial b \partial c} - \frac{1}{\pi_i^2}\frac{\partial \pi_i}{\partial b}\frac{\partial \pi_i}{\partial c} \\
&= \frac{1}{\pi_i}\underbrace{\{-V_i''[D(b) + (b-c)D'(b)]D(b) - V_i'D'(b)\}}_{\text{positive}} \\
&\quad + \frac{1}{\pi_i^2}\underbrace{(V_i')^2 D(b)[D(b) + (b-c)D'(b)]}_{\text{strictly positive}} \quad \|
\end{aligned}$$

The recent results about existence and uniqueness of equilibrium in the first price auction form the basis for our analysis. An equilibrium in this auction is described by an $N-$tuple of bid functions $b_i : [\underline{c}_i, \overline{c}_i] \to \mathbb{R}_+$, $i = 1, ..., N$. For our purposes, it is convenient to look at the inverse bid functions. We denote them by $\phi_i : \mathbb{R}_+ \to [\underline{c}_i, \overline{c}_i]$, $i = 1, ..., N$.

Given its opponents' bidding behavior, firm i's optimization problem when its cost is c_i is

$$\max_b \pi_i(b, c_i) \underbrace{\prod_{j \neq i} (1 - F_j(\phi_j(b))}_{\text{probability of winning}}$$

Maskin and Riley (1996 and 2000a) have shown that there exists a unique equilibrium in this environment.[5] The corresponding equilibrium inverse bid functions solve the first order conditions of bidders' maximization problem:

$$\sum_{j \neq i} \frac{F_j'(\phi_j(b))\phi_j'(b)}{1 - F_j(\phi_j(b))} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i(b))}{\pi_i(b, \phi_i(b))} \qquad i = 1, ..., N \tag{2.1}$$

[5] If one bidder's support is very far to the left of all the other bidders' supports, then the equilibrium is degenerate. We shall ignore this case. For the $N > 2$ case, Maskin and Riley (1996) require an additional condition on the payoff functions to ensure uniqueness. It is satisfied if all bidders are risk neutral or if they have the same CARA or CRRA utility function.

They have support on $[l_i, u]$ with boundary conditions $F_i(\phi_i(l_i)) = 0$, and with u, the maximum equilibrium winning bid, determined uniquely by the following lemma:

Lemma 2 (adapted from Maskin and Riley, 1996): *Suppose that the distributions* $(F_1, ..., F_N)$ *are ordered so that* $\overline{c}_1 \leq \overline{c}_2 \leq ... \leq \overline{c}_{N-1} \leq \overline{c}_N$. *Then, if* $\overline{c}_1 = \overline{c}_2 = \overline{c}$, *then* $u = \overline{c}$. *Otherwise,* u *solves*

$$\min\{\arg\max_b \pi_1(b, \overline{c}_1) \prod_{i \neq 1} (1 - F_i(b))\} \in (\overline{c}_1, \overline{c}_2)$$

If $u < \overline{c}_i$ for some i, we can consider that, for any realization of cost $c_i > u$, firm i bids its own cost (and never wins) or stays out of the auction.

Notice that the lower bounds of the supports of equilibrium bids are endogenously determined by the boundary conditions of the system of differential equations described by (2.1). These minimum bids need not be common to all firms when there are more than two firms and asymmetries among them are important. Finally, the equilibrium inverse bid functions are strictly increasing and twice differentiable on their support. For further details on the structure of the equilibrium, we refer the interested reader to Maskin and Riley (1996).

We assume that, prior to the auction, one firm has the opportunity to make an investment that improves its ex-ante distribution of costs according to the following definition:

Definition 1: Consider two cumulative distribution functions F and $\widetilde{F}$. $\widetilde{F}$ is a *distributional upgrade* of F, denoted $\widetilde{F} \succ F$, if

$$\frac{\widetilde{F}'(c)}{1 - \widetilde{F}(c)} > \frac{F'(c)}{1 - F(c)} \tag{2.2}$$

for all c where these expressions are well defined.[6]

The requirement in (2.2) is one of conditional stochastic dominance. It means that, conditioning on any minimum level of cost, F is more likely to yield a higher cost than $\widetilde{F}$. This condition implies a relation of first-order stochastic dominance

[6] In this paper, investments are zero-one decisions. Alternatively, we could consider a parameterized class of distributions $F(c;\theta)$, where a higher value of θ implies an upgrade. In that context, definition 1 would read $F(c;\widetilde{\theta}) \succ F(c;\theta)$ for $\widetilde{\theta} > \theta$ if $1 - F(c;\theta)$ is strictly log-submodular in (c, θ).

between the distributions: $F(c) < \widetilde{F}(c)$ for all c on the interior of their common support.[7]

Definition 1 has become common in the first price auction literature (see Tan, 1992, Lebrun, 1998, Li and Riley, 1999, or Maskin and Riley, 2000b for instance) and in decision theory under uncertainty (e.g Eeckhoudt and Gollier, 1997 and Athey, 2000). In practice, it is a little bit stronger than needed and a weak inequality in (2.2) would do for our purpose. However, it would also lengthen the proofs without adding any new insight. Comparing (2.2) with (2.1), it should also be clear that this is a natural way to order distributions for the first price auction.

Examples of distributional upgrades. Consider any distribution $H(c)$. The following are examples of distributional upgrades of H: (1) Additional random draws from the same distribution: $F(c;\theta) = 1 - (1 - H(c))^\theta$ for $\theta > 1$.[8] This is a common way of modeling R&D. (2) Shifts of distributions to the left: $F(c;\theta) = H(c+\theta)$ for $\theta > 0$, for distributions with a strictly increasing monotone hazard rate.[9] This can be a convenient way to model a locational investment when transportation costs are important. (3) Distributional contractions with a fixed lower end of the support: $F(c;\theta) = \theta H(c)$ for $\theta > 1$ and $c \in (\underline{c}, H^{-1}(1/\theta))$. This could represent the shift to a more reliable technology. Distributional stretches with a fixed upper end of the support $(1 - F(c;\theta)) = \theta[1 - H(c)]$ for $\theta < 1$) satisfy the weaker requirement of first order stochastic dominance and weakly higher hazard rate.

3. Comparing equilibria

To determine firms' investment incentives when investment is observable, we need to understand how a distributional upgrade by one firm affects the resulting equilibrium in the procurement auction. More precisely, starting from an initial configuration of firms $(F_1, ..., F_N)$, suppose that firm j has the opportunity to upgrade its distribution of costs to $\widetilde{F}_j \succ F_j$. How does the equilibrium in this new auction $(\widetilde{F}_j, F_{-j})$ compare with that of the initial one, (F_j, F_{-j})?

[7] In addition, conditional stochastic dominance is implied by the usual monotone likelihood ratio property.

[8] The fact that $F(c;\theta)$ does not satisfy the assumption of strictly positive density is not important here. See Arozamena and Cantillon (2000) for details.

[9] Distributions that satisfy this condition include the uniform, the normal, the logistic, the extreme value, the exponential and the χ^2 distributions, as well as the Weibull, γ and β distributions for some parameter values (see Bagnoli and Bergstrom, 1989).

We adopt the following notation and terminology. Let (ϕ_j, ϕ_{-j}) and $(\widetilde{\phi}_j, \widetilde{\phi}_{-j})$ be the equilibrium inverse bid functions under (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$ respectively. The lower and upper bounds of the equilibrium bids are denoted by l and u in the original configuration and by $\widetilde{l}$ and $\widetilde{u}$ in the new configuration. We also define $p_i(b) = F_i(\phi_i(b))$, the probability that firm i submits a bid lower than b in configuration (F_j, F_{-j}). $\widetilde{p}_i(b)$ is similarly defined.

We say that firm i *bids more aggressively* if it bids closer to its cost i.e. if $\widetilde{b}_i(c) < b_i(c)$ for all c or equivalently $\widetilde{\phi}_i(b) > \phi_i(b)$ for all b. Firm i *faces a more aggressive environment* if any given bid b by firm i has a lower chance of winning than before. With this terminology, bidding strategies are *strategic complements* if the best response to a more aggressive bidding behavior by one's opponents is to bid more aggressively one-self.

Referring back to (2.1), it is easy to see that the distributional upgrade by firm j shifts its opponents' best response schedules upwards (remember, by lemma 1, the right hand side of (2.1) is increasing in ϕ_i). In other words, their best response is to bid more aggressively to any given bidding behavior of firm j. If bidding strategies in the first price auction were strategic complements, this would be the end of the story. Indeed, the "commitment" of j's opponents to bid more aggressively coupled with strategic complementarity would result in more aggressive bidding behavior by all participants in the "post-upgrade" equilibrium (see the early analyses by Fudenberg and Tirole, 1984 and Bulow, Geanakoplos and Klemperer, 1985, and Vives, 1999 for a general treatment). Unfortunately, bidding strategies are not strategic complements as the following example illustrates:

Example 1: Two risk neutral firms bid for a single object. Firms' costs are distributed independently and uniformly over the interval $[0, 1]$. This is a symmetric first price auction and the equilibrium bidding functions are given by $b_i(c_i) = \frac{1+c_i}{2}$ for $i = 1, 2$ (this means that $\phi(b) = 2b-1$). Now suppose that firm 1 suddenly bids more aggressively: $\widehat{b}_1(c_1) = \sqrt{c_1} < \frac{1+c_1}{2}$ (this corresponds to an inverse bidding function of $\widehat{\phi}_1(b) = b^2$). Firm 2's best response solves $\max_b (b - c_2)(1 - b^2)$. Let $\widehat{\phi}_2(b)$ be the inverse bid function that corresponds to this optimization problem. $\widehat{\phi}_2(b) = \frac{3b^2-1}{2b}$ and has support on $[1/\sqrt{3}, 1]$. Note that $\widehat{\phi}_2(b) < \phi(b)$. In words, though firm 1 is bidding *more aggressively*, firm 2's best response, $\widehat{\phi}_2$, is *less aggressive.* Examples where firm 2 responds to a more aggressive behavior of firm 1 by being more aggressive can similarly be generated.

Example 1 runs a bit counter our intuition about the nature of competition in the private value first price auction. However, on a second thought, it should not be so surprising. Indeed, firm 2's maximization problem is identical to that of the monopolist who faces demand $D(b) = (1 - F_1(\phi_1(b))$ and has constant marginal cost c_2. Firm 2 in example 1 is then analogous to the monopolist who might respond to a decrease in demand by raising prices. Put differently, in the same way as a monopolist cares about the elasticity and not the level of the demand he is facing, bidders in the first price auction care about the hazard rate of their opponents' lowest bid (see (2.1)).

Nevertheless, example 1 is problematic because it rules out the kind of comparative statics exercise based on the properties of best responses. The alternative approach that we take here is to impose the condition of equilibrium, and *compare the equilibria* (prior to and after the upgrade) *directly*. In this section, we show that the kind of comparative statics that did not hold for best responses (more aggressive response to more aggressive behavior) does hold at equilibrium and that the upgrader faces a more aggressive environment in the new equilibrium.[10]

When firms are asymmetric ($F_i \neq F_j$ for some $i \neq j$), there is in general no explicit solution to the equilibrium in the first price auction. Therefore, we resort to firms' first order conditions in order to compare equilibria. With $N = 2$ and a slight abuse of notation, (2.1) becomes:

$$\frac{p_j'(b)}{1 - p_j(b)} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} \qquad i \neq j \tag{3.1}$$

where the term on the right-hand side is increasing in ϕ_i (from lemma 1). For $N > 2$ firms, the equations in (2.1) can be rewritten as:

$$\sum_{j \neq i} \frac{p_j'(b)}{1 - p_j(b)} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} \tag{3.2}$$

Solving for $\frac{p_j'(b)}{1-p_j(b)}$ yields:[11]

$$(N-1)\frac{p_j'(b)}{1 - p_j(b)} = \sum_{i \neq j} \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} - (N-2)\frac{\frac{\partial}{\partial b}\pi_j(b, \phi_j)}{\pi_j(b, \phi_j)} \tag{3.3}$$

[10] This is a strictly weaker statement than strategic complementarity because (1) it applies to the *aggregrate* behavior of the investor's opponents, and (2) it does not apply to the investor's own behavior.

[11] In this section, we assume for simplicity that firms share the same lower bound to equilibrium bids. The working paper deals with the general case.

The key in both (3.1) and (3.2)-(3.3) is that firms' equilibrium bidding behavior is related and that knowing how i's opponents bid (the $\sum_{j\neq i}\frac{p_j'(b)}{1-p_j(b)}$ function), allows us to pin down i's bidding behavior through its first order condition. This property of equilibrium, together with the fact that $\widetilde{F}_j \succ F_j$ for the upgrader in the new configuration, is used in the proofs.

Comparing (3.1) and (3.2) also makes clear that there is a fundamental difference between two firms and more than two firms. With more than two firms, fixing the bidding behavior of firm i in (3.2) only places constraints on the *aggregate* bidding behavior of its opponents (Intuitively, what matters for firm i is its probability of winning). However, it leaves much room for maneuver concerning their *individual* bidding behavior.

Note: The only property of $\frac{\frac{\partial}{\partial b}\pi_i}{\pi_i}$ used in the proofs is that it is increasing in ϕ_i. Therefore, for simplicity, we will write the first order conditons (FOCs) for the single object risk neutral case. So for example (3.3) becomes:

$$(N-1)\frac{p_j'(b)}{1-p_j(b)} = \sum_{i\neq j}\frac{1}{b-\phi_i(b)} - (N-2)\frac{1}{b-\phi_j(b)}$$

Any result proved for this case also holds for the more general case (allowing for risk aversion and endogenous demand).

Our argument proceeds in 3 steps. First, we show that the upper bound to the equilibrium bids must be non increasing i.e. $\widetilde{u} \leq u$ (lemma 3). Second, we show that the lower bound to equilibrium bids is strictly decreasing, $\widetilde{l} < l$ (lemma 5). Finally, we show that the upgrader faces a more aggressive environment after the upgrade when there are two firms (proposition 1) and, under some additional condition on the type of upgrade, for more than two firms (proposition 2).

Lemma 3: *Let $u(F_1, ..., F_N)$ be the upper bound of the equilibrium bids in configuration $(F_1, ..., F_N)$. $u(F_1, ..., F_N)$ is weakly decreasing in its arguments. That is, if $\widetilde{F}_i \succ F_i$, then $u(\widetilde{F}_i, F_{-i}) \leq u(F_i, F_{-i})$.*

Proof. Let $u = u(F_i, F_{-i})$ and $\widetilde{u} = u(\widetilde{F}_i, F_{-i})$. Without loss of generality, $\overline{c}_1 \leq \overline{c}_2 \leq ... \leq \overline{c}_N$. Let $\widetilde{\overline{c}}_i$ be the maximum cost under $\widetilde{F}_i$ ($\widetilde{\overline{c}}_i \leq \overline{c}_i$).
If $\overline{c}_1 = \overline{c}_2$, then $\widetilde{u} \leq u$ follows from lemma 2. If $\overline{c}_1 < \overline{c}_2$, lemma 2 implies that u solves:

$$\min\{\arg\max_b \pi_1(b, \overline{c}_1)\prod_{i\neq 1}(1-F_i(b))\}$$

In particular, u satisfies the FOC:

$$\frac{\frac{\partial}{\partial b}\pi_1(u,\bar{c}_1)}{\pi_1(u,\bar{c}_1)} - \sum_{i\neq 1} \frac{F_i'(u)}{1-F_i(u)} = 0 \tag{3.4}$$

When $u \downarrow \bar{c}_1$, the first term of this expression goes to infinity whereas the second term remains bounded. Therefore, at the solution (remember, u is the smallest value that solves the FOC), the expression in (3.4) crosses the x-axis from above. If firm 1 is the upgrader, lemma 1 implies that (3.4) shifts downward and $\widetilde{u} \leq u$ follows. If the upgrader is not firm 1, then the second term in (3.4) increases. Again, (3.4) shifts downward and $\widetilde{u} \leq u$. $\|$

Lemma 4 is central to the rest of the argument:

Lemma 4: *It cannot be that, at some point, bidding is less aggressive for all bidders after the investment than before. More precisely, $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i and for some $\widehat{b} \in [\max\{l, \widetilde{l}\}, \widetilde{u})$ is impossible.*

Proof. The proof proceeds in two steps.
Claim 1: $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})\ \forall i \Longrightarrow \widetilde{\phi}_i(b) < \phi_i(b)\ \forall i$ and $\forall b > \widehat{b}$.
Proof: Towards a contradiction, suppose that $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i and imagine that $\widetilde{\phi}_i(b) < \phi_i(b)\ \forall i$ does not hold at a later point. Define:

$$b^* = \inf_{b > \widehat{b}} \{b \text{ s.t. } \widetilde{\phi}_j \geq \phi_j \text{ for some } j\}$$

Since $\widetilde{\phi}_j(b^*) = \phi_j(b^*)$, firm j's FOC yields:

$$\frac{1}{b^* - \widetilde{\phi}_j(b^*)} = \sum_{i\neq j} \frac{\widetilde{p}_i'}{1-\widetilde{p}_i} = \sum_{i\neq j} \frac{p_i'}{1-p_i} = \frac{1}{b^* - \phi_j(b^*)} \tag{3.5}$$

In addition, $\widetilde{\phi}_j'(b^*) \geq \phi_j'(b^*)$ must hold hence

$$\frac{\widetilde{p}_j'}{1-\widetilde{p}_j} \geq \frac{p_j'}{1-p_j} \tag{3.6}$$

Finally, for any firm $k \neq j$, $\widetilde{\phi}_k \leq \phi_k$ at b^*, so:

$$\sum_{i\neq k} \frac{\widetilde{p}_i'}{1-\widetilde{p}_i} \leq \sum_{i\neq k} \frac{p_i'}{1-p_i} \qquad \text{for } k \neq j \tag{3.7}$$

We claim that conditions (3.5) to (3.7) are incompatible as soon as any of the inequalities in (3.6) or (3.7) is strict. To see this, suppose first that (3.7) is strict for some k. Adding (3.5) and (3.6) implies that:

$$\sum_i \frac{\widetilde{p}_i'}{1-\widetilde{p}_i} \geq \sum_i \frac{p_i'}{1-p_i}$$

Comparing with (3.7), we conclude that:

$$\frac{\widetilde{p}_i'}{1-\widetilde{p}_i} \geq \frac{p_i'}{1-p_i} \text{ for all } i \neq j, \text{ and strictly so for } k$$

Summing these terms leads to a contradiction with (3.5). The case when (3.6) is strict is analogous.
Next, we show that (3.6) or (3.7) strict must happen at b^*. Suppose not, then $\widetilde{\phi}_i = \phi_i$ for all i and the definition of b^* implies that $\widetilde{\phi}_i' \geq \phi_i'$. In addition, when all three conditions hold with equality, $\frac{\widetilde{p}_i'}{1-\widetilde{p}_i} = \frac{p_i'}{1-p_i}$ for all i. However, one of these firms is the upgrader for which

$$\frac{\widetilde{p}_i'}{1-\widetilde{p}_1} = \frac{\widetilde{F}_1'(\widetilde{\phi}_1)\widetilde{\phi}_1'}{1-\widetilde{F}_1(\widetilde{\phi}_1)} > \frac{F_1'(\widetilde{\phi}_1)\widetilde{\phi}_1'}{1-F_1(\widetilde{\phi}_1)} \geq \frac{F_1'(\phi_1)\phi_1'}{1-F_1(\phi_1)} = \frac{p_i'}{1-p_i}$$

A contradiction.‖

Claim 2: $\widetilde{\phi}_k(\widetilde{u}) < \phi_k(\widetilde{u})$ for k a non-upgrading firm, is impossible.
Proof: By lemma 2, $\widetilde{\phi}_k(\widetilde{u}) = \min\{\overline{c}_k, \widetilde{u}\}$. In addition, lemma 2, lemma 3 together with the fact that equilibrium bidding strategies are increasing imply that $\phi_k(\widetilde{u}) \leq \min\{\overline{c}_k, \widetilde{u}\}$. Hence $\widetilde{\phi}_k(\widetilde{u}) \geq \phi_k(\widetilde{u})$. ‖

Lemma 5: *$l(F_1,...,F_N)$ is strictly decreasing in its arguments. That is, if $\widetilde{F}_j \succ F_j$, then $l(\widetilde{F}_j, F_{-j}) < l(F_j, F_{-j})$.*

Proof. Towards a contradiction, suppose $\widetilde{l} \geq l$. This implies $\widetilde{\phi}_i(\widetilde{l}) \leq \phi_i(\widetilde{l})$ which is impossible by lemma 4.‖

We are now able to prove the main result of this section, that the upgrader faces a more aggressive bidding environment after the investment. We prove this result under two complementary circumstances: when there are two firms, and under an additional assumption, when there are more than two firms. We start with the simplest case: two firms.

Proposition 1: *Let $N = 2$. Then $\widetilde{p}_j(b) > p_j(b)$ for all j and for all b in the interior of their common support.*

Proof. Let 1 be the upgrader. From lemma 5, $\widetilde{p}_j > p_j$ close to l. In addition, as long as $\widetilde{p}_2(b) > p_2(b)$, $\widetilde{\phi}_2(b) > \phi_2(b)$ and so (using firm 2's FOC) $\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} > \frac{p_1'}{1-p_1}$. Therefore, starting from the left, $\widetilde{p}_1 > p_1$ as long as $\widetilde{p}_2 > p_2$.
Now, towards a contradiction, suppose that $\widetilde{p}_2$ and p_2 intersect first at $b_1 < \widetilde{u}$:

$$\phi_2(b_1) = \widetilde{\phi}_2(b_1) \tag{3.8}$$

In addition, we have $\frac{p_2'(b_1)}{1-p_2(b_1)} > \frac{\widetilde{p}_2'(b_1)}{1-\widetilde{p}_2(b_1)}$ so (using firm 1's FOC)

$$\phi_1(b_1) > \widetilde{\phi}_1(b_1) \tag{3.9}$$

By lemma 4, (3.8) and (3.9) together are impossible.‖

Generalizing proposition 1 to more than two firms is harder. However, note that proposition 1 is already stronger than what we need to determine investment incentives. Let $W_1(b)$ and $\widetilde{W}_1(b)$ be the probabilities that the upgrader wins when it submits a bid b before and after the upgrade. For our purpose, comparing $W_1(b)$ and $\widetilde{W}_1(b)$ is enough (that is, in the case of two bidders, $1 - p_2$ and $1 - \widetilde{p}_2$).

Relaxing the claim in this fashion is helpful but not sufficient to get analytical results and we need to impose further conditions. First, we assume that firms have the same payoff functions, V_i for all i. Second, we impose that the upgrader moves from a situation where he is a "laggard" (or one of the laggards) to a situation where he is a "leader" (or one of the leaders). Formally, let $\widetilde{F} \succeq F$ stand for $\widetilde{F} \succ F$ according to definition 1 or $\widetilde{F} = F$, and suppose that firm 1 is the upgrader. We impose that prior to the upgrade, $F_j \succeq F_1$ for all $j \neq 1$, and that $\widetilde{F}_1 \succeq F_j$ for all $j \neq 1$.

These restrictions are useful because it can be shown that if $F_i \succ F_j$ then $\phi_i < \phi_j$ and $p_i > p_j$ at equilibrium (see, e.g., Maskin and Riley, 2000b and for a generalization to $N > 2$ bidders, Li and Riley, 1999).[12]

[12] In addition, firms with identical technologies will bid identically at equilibrium ($F_i = F_j$ implies $\phi_i = \phi_j$). An important consequence is that we can already stretch the interpretation of proposition 1. Suppose there are N firms with firms 2 to N sharing the same payoff function and the same technology F. Firm 1 is the upgrader. Then the claim of proposition 1 also applies to describe the relationship between the equilibria prior to and after the investment (without any additional restriction on the kind of upgrade).

Proposition 2 then covers any patterns of "catching-up" by the investing firm. Arguably, these also represent the relevant cases when one talks about market turnover and leadership changes.

Proposition 2: *Let $\widetilde{F} \succeq F$ stand for $\widetilde{F} \succ F$ according to definition 1 or $\widetilde{F} = F$. Let firm 1 be the upgrader and suppose there are N firms with $\widetilde{F}_1 \succeq F_j \succeq F_1$ for all $j \neq 1$ and $\widetilde{F}_1 \succ F_1$. Define $W_1(b) = \prod_{j\neq 1}(1 - p_j(b))$ i.e. $W_1(b)$ is the probability that firm 1 wins with a bid of b in the original configuration. Define $\widetilde{W_1}(b)$ similarly. Then $\widetilde{W_1}(b) < W_1(b)$ on the interior of their common support.*

Proof. Bidders' FOCs (3.2) can be rewritten as:

$$-\frac{W_j'(b)}{W_j(b)} = \frac{1}{b - \phi_j(b)}$$

Because the distributions are partially ranked, the equilibrium inverse bid functions can also be partially ordered. We have:

$$\begin{aligned} \widetilde{\phi}_1 &\leq \widetilde{\phi}_j \\ \phi_j &\leq \phi_1 \end{aligned} \tag{3.10}$$

for all $j \neq 1$ (with some strict inequalities).

Claim 1: $\widetilde{W_1}(b) < W_1(b)$ close to u.
Proof: This claim is clear when $\widetilde{u} < u$. If $\widetilde{u} = u$, suppose towards a contradiction that $\widetilde{W_1} \geq W_1$ close to $\widetilde{u} = u$. It can be shown (see lemma 6 in the working paper) that this implies that $\frac{\widetilde{W_1}'}{\widetilde{W_1}} \leq \frac{W_1'}{W_1}$ close to u. Therefore, using firm 1's FOC, $\widetilde{\phi}_1 \geq \phi_1$ close to u, and by (3.10), $\widetilde{\phi}_j > \phi_j$ for $j \neq 1$. Hence $\widetilde{p}_j > p_j$ and $\widetilde{W_1} < W_1$ close to u. A contradiction. $\|$

Now, by lemma 5, $\widetilde{l} < l$. Hence, close to l, $\widetilde{p}_j > p_j$ and $\widetilde{W_j} < W_j$ for all j, and $\widetilde{\phi}_j > \phi_j$ for all $j \neq 1$. Together with claim 1, this means that the only way for the claim of proposition 2 to fail is that some of the $\widetilde{p}$ and p must cross over their common supports.

Claim 2: Starting from the left (i.e. from l onwards), the first $\widetilde{p}$ and p to cross cannot be for the upgrader. Moreover, at that first crossing, it must be that $\widetilde{\phi}_1 < \phi_1$.
Proof: Since $\widetilde{p}_1$ would need to cross p_1 from above, we must have

$$\frac{\widetilde{p}_1'}{1 - \widetilde{p}_1} < \frac{p_1'}{1 - p_1} \tag{3.11}$$

At the same time, because $\widetilde{\phi}_j \geq \phi_j$ for all $j \neq 1$ and $\widetilde{\phi}_1 < \phi_1$ (since $\widetilde{F}_1 > F_1$), (3.3) implies:

$$
\begin{aligned}
(N-1)\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} &= \sum_{j\neq 1}\frac{1}{b-\widetilde{\phi}_j} - (N-2)\frac{1}{b-\widetilde{\phi}_1} \\
&> \sum_{j\neq 1}\frac{1}{b-\phi_j} - (N-2)\frac{1}{b-\phi_1} = (N-1)\frac{p_1'}{1-p_1}
\end{aligned}
$$

a contradiction with (3.11). Now, suppose that the first $\widetilde{p}$ and p to cross from the left are for bidder $j \neq 1$. Because, by hypothesis, $\widetilde{\phi}_k \geq \phi_k$ for all $k \neq j, 1,$ we need $\widetilde{\phi}_1 < \phi_1$ to get $\frac{\widetilde{p}_k'}{1-\widetilde{p}_k} < \frac{p_k'}{1-p_k}$ (and this is possible since, by FOSD, $\widetilde{\phi}_1 < \phi_1$ is compatible with $\widetilde{p}_1 > p_1$). $\|$

Claim 3: $\widetilde{W}_1 < W_1$ for all b.
Proof: Suppose firm 2 is the first from the left (say, at b_1) for whom $\widetilde{\phi}_2 = \phi_2$. We have $\widetilde{W}_1 < W_1$ for all $b \leq b_1$, and by claim 2, $\widetilde{\phi}_1(b_1) < \phi_1(b_1)$.
Case 1: $\widetilde{\phi}_1(b) < \phi_1(b)$ for all $b > b_1$.
Using bidder 1's FOC, this implies

$$
-\frac{\widetilde{W}_1'}{\widetilde{W}_1} = \frac{1}{b-\widetilde{\phi}_1} < \frac{1}{b-\phi_1} = -\frac{W_1'}{W_1} \tag{3.12}
$$

for all $b > b_1$. Then using claim 1, we conclude that $\widetilde{W}_1 < W_1$ for all b (since a crossing in $(b_1, \widetilde{u})$ would require $\frac{\widetilde{W}_1'}{\widetilde{W}_1} < \frac{W_1'}{W_1}$ in contradiction with (3.12)).
Case 2: There exists $b_2 > b_1$ such that $\widetilde{\phi}_1(b_2) = \phi_1(b_2)$ (b_2 is the first one from b_1).
Using (3.10), $\widetilde{\phi}_j(b_2) > \phi_j(b_2)$ for all $j \neq 1$, hence $\widetilde{p}_j > p_j$ and $\widetilde{W}_1(b_2) < W_1(b_2)$. Arguing as in case 1, we have $\widetilde{W}_1 < W_1$ for all $b < b_2$. Since $\widetilde{p}_j(b_2) > p_j(b_2)$ for all j, the scenario from b_2 on is identical (the first p and $\widetilde{p}$ cannot be for the upgrader and at that point we must have $\widetilde{\phi}_1 < \phi_1, ...$) and we can replicate the argument.$\|$

Before proceeding, we note the following direct consequence of propositions 1 and 2.

Corollary 1: *Investment lowers procurement costs in the first price auction.*

Proof: By definition, the cumulative distributions of the lowest bid are given by $1-(1-\widetilde{p}_1(b))\widetilde{W}_1(b)$ and $1-(1-p_1(b))W_1(b)$ respectively. For the case of two bidders,

$$1-(1-\widetilde{p}_1(b))\widetilde{W}_1(b) > 1-(1-p_1(b))W_1(b)$$

follows directly from proposition 1, so expected procurement costs are lower under the new configuration. For $N > 2$, we show that $\widetilde{p}_1(b) > p_1(b)$ for all b on the interior of their support. By lemma 5, this inequality is satisfied close to l. Towards a contradiction, suppose that $\widetilde{p}_1$ and p_1 cross at a later point. Then,

$$\frac{p_1^{'}}{1-p_1} > \frac{\widetilde{p}_1^{'}}{1-\widetilde{p}_1} \tag{3.13}$$

Now, by (3.10), $\widetilde{\phi}_1 \leq \widetilde{\phi}_j$ that is, using the FOCs, $\sum_{i\neq 1}\frac{\widetilde{p}_i'}{1-\widetilde{p}_i} \leq \sum_{i\neq j}\frac{\widetilde{p}_i'}{1-\widetilde{p}_i}$, or

$$\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} \geq \frac{\widetilde{p}_j'}{1-\widetilde{p}_j} \tag{3.14}$$

Similarly, $\phi_1 \geq \phi_j$ implies that

$$\frac{p_1'}{1-p_1} \leq \frac{p_j'}{1-p_j} \tag{3.15}$$

Putting (3.13), (3.14) and (3.15) together yields $\frac{p_i^{'}}{1-p_i} > \frac{\widetilde{p}_i^{'}}{1-\widetilde{p}_i}$ for all i so $\sum_{i\neq j}\frac{p_i'}{1-p_i}$ $> \sum_{i\neq j}\frac{\widetilde{p}_i'}{1-\widetilde{p}_i}$. Hence $\phi_j > \widetilde{\phi}_j$ for all j and $W_1 < \widetilde{W}_1$, a contradiction with lemma 4. $\|$[13]

Propositions 1 and 2 allow us to answer our initial question concerning the incentives of firms to upgrade their distributions. When firm i upgrades its distribution, it needs to take two effects on its ex-ante expected payoff into account. First, a *direct* effect through an improvement in the ex-ante distribution of its costs (holding its opponents' strategies fixed) and, second, an *indirect* or *strategic* effect through its opponents' adjustments to the new configuration. Propositions 1 and 2 tell us that, under the new configuration $(\widetilde{F}_i, F_{-i})$, firm i faces a more aggressive environment (collectively, its opponents bid more aggressively). This means that the strategic effect is *negative* for distributional upgrades in the first price auction.

[13] Riley (1996) shows that every non upgrading firm faces a more aggressive environment. Together with our propositions 1 and 2, his result provides an alternative route to proving corollary 1.

At this point, it might be useful to remember that the kind of investment we are considering shifts the best response schedule of the investor's opponents upwards. In other words, holding the bidding strategy of the investor fixed, its opponents prefer to bid more aggressively after the investment than before (refer to (2.1) if needed). One interpretation then, is that at least for the kind of comparative statics that determines investment incentives, the first price auction behaves like a game with strategic complements.

How strong is the strategic effect? Example 2 illustrates that it can be quite strong. There, an inefficient firm is better off avoiding a cost reducing investment, *even if it came at no cost*! Equivalently, in this example, a firm may gain by becoming less efficient.[14]

Example 2: Consider the following initial configuration for firms 1 and 2: F_1 is uniform over [0,10] and F_2 is uniform over [0,5]. Suppose that firm 1 has the possibility to upgrade its distribution to $\widetilde{F}_1 = F_2$. Denote by $\Pi_i(F, \widehat{F})$ firm i's ex-ante payoff when firm 1's distribution is F and firm 2's distribution is $\widehat{F}$. A numerical solution to the first-price auction yields: $\Pi_1(F_1, F_2) = 0.90445$, $\Pi_2(F_1, F_2) = 1.93245$, $\Pi_1(F_2, F_2) = \Pi_2(F_2, F_2) = 0.83333$. The change in its distribution leaves firm 1 worse off.

Propositions 1 and 2 also shed light on the results derived when firms are ex-ante symmetric, investment is simultaneous and unobservable. A typical result in this literature is that there is a unique symmetric equilibrium, that is, ex-ante identical firms facing the same investment opportunities will invest the same amount (Tan, 1992). Our results and example 2 in particular suggest that this equilibrium might not be subgame perfect when investment is observable (and so bidders can react to deviations at the investment stage). Indeed, suppose that the symmetric Nash equilibrium of the investment game with unobservable investment is given by $(\theta^*, ..., \theta^*)$ (where θ represents firms' investment level, indexed according to definition 1). Consider firm 1's incentive to deviate. Since $(\theta^*, ..., \theta^*)$ is a Nash equilibrium, firm 1 has no incentive to deviate *when its opponents' behavior at the procurement stage is held fixed*. Because of the negative strategic effect identified in propositions 1 and 2, investing more than θ^* would not be profitable either if investment is observable. However, investing *less* than θ^* might be a profitable deviation: by choosing a lower investment level, firm 1 induces its opponents to bid less aggressively in the second stage, and this, together with the cost saving involved, might overcome the effect of the lower

[14] Thomas (1997) provides a similar example with discrete types.

probability of winning. In other words, our results suggest that the focus on the symmetric equilibrium of the simultaneous investment game can only be justified if investment is not observable. Otherwise, the simultaneous investment game among symmetric firms might admit asymmetric (subgame perfect) equilibria.

Remark 1: The current model is framed in terms of a two-stage game. However, log-supermodularity of expected payoffs in bids and costs (lemma 1) is all that is needed for the comparative statics result of this section. In particular, consider a repeated auction setting with costs drawn independently each time. In addition, suppose that there is some learning-by-doing so that winning positions a firm better for future auctions (in the context of our model, winning corresponds to an investment). At time t, the optimization problem a typical firm solves is

$$\max_{b_t}\{((b_t - c_t) + \widetilde{V}_{t+1})\mathrm{pr}(\text{win with } b_t) + V_{t+1}\mathrm{pr}(\text{lose with } b_t)\}$$

where $\widetilde{V}_{t+1}$ and V_{t+1} are the continuation values if the firm wins or loses the current auction. This objective function is also log-supermodular in (b, c), hence a winning firm can expect more competition in the next auction.

Remark 2: Sequential entry. Our model is one of investment by an incumbent firm (that is, the investor is already a relevant player prior to the upgrade). However, the proofs of proposition 2 can be straightforwardly adapted to the case of entry, that is, where the investor is irrelevant in the initial configuration. In that case then, the potential entrant can expect more aggressive behavior on the part of the incumbent firms.

4. Investment Incentives in Procurement Auctions

In this section, we turn to our original question of investment incentives and compare the first price auction (FPA) and the second price auction (SPA). Comparing the two auction formats is interesting in two respects. First, both are commonly used auction rules (remember that in our setting the SPA is equivalent to the English auction). Second, the SPA provides an excellent benchmark to analyze the properties of the FPA because the strategic effect identified in the previous section for the FPA is absent in the SPA. Indeed, bidding one's own cost is a dominant strategy in the SPA, irrespective of the distributions of one's opponents. Therefore bidding behavior at the procurement stage is not affected by firms' investment in the first stage. To allow for a comparison between the two auction formats (using the Revenue Equivalence Theorem), we return in this section to the standard assumptions of risk neutrality and a single indivisible object.

To provide intuition for our next result, consider the following two-firm example. Suppose that both firms have originally the same distribution of costs and consider an incremental investment by firm 1. Firm 1's change in payoff can be split into two terms: a direct effect (holding its opponents' behavior fixed) and a strategic effect. By the Revenue Equivalence Theorem, the direct effect for an incremental change is the same under the FPA and the SPA.[15] Moreover, propositions 1 and 2 suggest that the strategic effect is negative for the FPA. Since there is no strategic effect for the SPA, this implies that firm 1 will invest less under the FPA than under the SPA.

Proposition 3 shows that this intuition extends to situations where firms are not ex-ante symmetric and investment is not necessarily incremental: Firms will tend to invest less when the FPA format is used because they anticipate the more aggressive behavior of their opponents. As in proposition 2, proposition 3 requires some level of leadership change for the analytical proof to go through.

Proposition 3 (Underinvestment): *The FPA provides less incentives than the SPA for investments that involve a change of market leadership. Formally, suppose there are $N \geq 2$ firms and let firm 1 be the upgrader. Then, investment incentives are lower in the FPA than in the SPA for investments such that $\widetilde{F}_1 \succeq F_j \succeq F_1$ for all $j \neq 1$ and $\widetilde{F}_1 \succ F_1$.*

We start with the following lemma:

Lemma 6 (Bidders' ranking of the auction formats): *Suppose $N \geq 2$. Denote by $\Pi_i^{FPA}(F_i, F_{-i})$ and $\Pi_i^{SPA}(F_i, F_{-i})$, bidder i's ex-ante expected payoffs in the FPA and SPA respectively. Then:*
(1) If $F_i \succsim F_j$ for all $j \neq i$, then $\Pi_i^{FPA}(F_i, F_{-i}) \leq \Pi_i^{SPA}(F_i, F_{-i})$.
(2) If $F_i \precsim F_j$ for all $j \neq i$, then $\Pi_i^{FPA}(F_i, F_{-i}) \geq \Pi_i^{SPA}(F_i, F_{-i})$.
(the inequalities are strict if $F_i \neq F_j$ for some j).

Proof. Consider the general direct revelation game equivalent of our auction. Let $x_i(c_i)$ and $T(c_i)$, be the expected probability that firm i wins the contract and its

[15] The Revenue Equivalence Theorem (Myerson, 1981) states that, under the assumptions of independent values and risk neutrality, the auctioneer's and bidders' expected payoffs are fully determined by their probabilities of getting the contract and their expected payoff at their worst cost realization (see also equations (4.1) and (4.2) below for an example). Under symmetry, these are the same in the SPA and FPA.

expected payment, when it announces cost c_i. Then, its expected payoff when it (truthfully) announces c_i can be written as $\Pi(c_i; F_i, F_{-i}) = -c_i x_i(c_i) + T_i(c_i)$. By the envelope theorem,

$$\Pi'(c_i; F_i, F_{-i}) = -x_i(c_i)$$

Let x_i^{FPA} and x_i^{SPA} be the probability functions in the FPA and SPA respectively. We have

$$\Pi_i^{FPA}(c_i; F_i, F_{-i}) = \Pi_i^{FPA}(\overline{c}_i; F_i, F_{-i}) + \int_{c_i}^{\overline{c}_i} x_i^{FPA}(c)dc \tag{4.1}$$

$$\Pi_i^{SPA}(c_i; F_i, F_{-i}) = \Pi_i^{SPA}(\overline{c}_i; F_i, F_{-i}) + \int_{c_i}^{\overline{c}_i} x_i^{SPA}(c)dc \tag{4.2}$$

Suppose $F_i \succsim F_j$ for all $j \neq i$. Then firm i bids less aggressively than its competitors in the FPA, $b_i(c) \geq b_j(c)$ for all $j \neq i$ (cfr. equation (3.10)). As a result, its probability of winning the contract is lower than its probability of being the low cost producer, $x_i^{FPA}(c) \leq x_i^{SPA}(c)$ for all c.
Next, bidder i's expected payoff at $\overline{c}_i$ differs from zero only if $\overline{c}_i < \overline{c}_j$ for all $j \neq i$ (lemma 2). Define $G(.) = \prod_{j\neq i}(1 - F_j(.))$. From lemma 2, $\Pi_i^{FPA}(\overline{c}_i; F_i, F_{-i}) = \max_b (b - \overline{c}_i)G(b)$. $\Pi_i^{SPA}(\overline{c}_i; F_i, F_{-i}) = -\int_{\overline{c}_i}^{\min_{j\neq i}\overline{c}_j}(x - \overline{c}_i)dG(x) = \int_{\overline{c}_i}^{\min_{j\neq i}\overline{c}_j} G(x)dx$ using integration by parts. This represents the area under the $G(.)$ curve between $\overline{c}_i$ and $\min_{j\neq i}\overline{c}_j$ and $\max_b(b - \overline{c}_i)G(b)$ is included in it. Hence, $\Pi_i^{FPA}(\overline{c}_i; F_i, F_{-i}) < \Pi_i^{SPA}(\overline{c}_i; F_i, F_{-i})$.
Putting both elements together yields $\Pi_i^{FPA}(c_i) \leq \Pi_i^{SPA}(c_i)$. Claim (2) is proved similarly. $\|$

Lemma 6 generalizes Maskin and Riley (2000b)'s proposition 2.6 to more than two bidders. Intuitively, the FPA favors the weak bidders because their competitors adjust to their presence by bidding less agressively (and the other way round for the case of strong bidders). No such effect is present in the SPA. This property drives our underinvestment result. It generalizes to less restrictive conditions to the extent that lemma 6 captures a more general idea that *relatively* weaker bidders prefer the FPA to the SPA, and *vice versa.*

Proof of proposition 3. The claim follows directly from lemma 6 (1) and (2). $\Pi_1^{FPA}(\widetilde{F}_1, F_{-1}) - \Pi_1^{FPA}(F_1, F_{-1}) < \Pi_1^{SPA}(\widetilde{F}_1, F_{-1}) - \Pi_1^{SPA}(F_1, F_{-1})$. $\|$

It is interesting to compare proposition 3 with the revenue ranking derived by Maskin and Riley (2000b) for exogenous distributions of valuations. Though neither auction format is generally better in their environment, they find that the

FPA performs better for a plausible class of asymmetries. Proposition 3 suggests that allowing for endogenous distributions could overturn this result.

Our next proposition confirms the qualities of the SPA and provides a normative interpretation of the underinvestment result of proposition 3. Not only does it induce higher levels of investment, it also induces the *socially optimal level* of investment.

Proposition 4: *The second price auction provides the socially optimal level of investment incentives.*

Proof. Let F_1 be bidder 1's initial distribution of costs and define by F_2, the distribution of the best cost realization among bidder 1's competitors. Using integration by parts, we can express bidder 1's ex-ante expected payoff as:

$$\begin{aligned}\Pi_1^{SPA}(F_1, F_2) &= \int dF_1(c) \int_c (x - c) dF_2(x) \\ &= \int F_1(c)(1 - F_2(c))dc = -\int cd[F_1(c)(1 - F_2(c))]\end{aligned}$$

Next, social surplus is defined by the minimum cost realization among bidders. Hence, a measure of expected social surplus is the negative of the expected value of the second order statistic, that is,

$$SS(F_1, F_2) = -\int cdS(c)$$

with $S(c) = F_1(c) + F_2(c) - F_1(c)F_2(c) = F_2(c) + F_1(c)(1 - F_2(c))$. That is,

$$SS(F_1, F_2) = \Pi_1^{SPA}(F_1, F_2) - \int cdF_2(c)$$

Hence $SS(\widetilde{F}_1, F_2) - SS(F_1, F_2) = \Pi_1^{SPA}(\widetilde{F}_1, F_2) - \Pi_1^{SPA}(F_1, F_2)$ for all $F_1, \widetilde{F}_1$: Private and social incentives are perfectly aligned.‖

Remark: Proposition 4 illustrates a very general property of the SPA. With one qualification, it extends to situations where firms invest simultaneously: when investment is simultaneous, the socially optimal outcome is always an equilibrium but not necessarily the only one (Stegeman, 1996). In addition, Bergemann and Välimäki (2000) find a similar result for the case of information acquisition in private value environments.[16] In all cases, what drives the result is the fact that

[16]In their setting, bidders' valuations are exogenously given and fixed, but unknown to them. An investment refers to getting a better signal about one's own valuation.

the SPA corresponds to the Vickrey-Clarke-Groves mechanims for which agents' optimization problems are aligned to the social planner's.

5. Conclusions and directions for future research

Asymmetries among bidders are widespread in procurement situations. They are also a source of concern for procurement authorities. However, our understanding of these market situations has been largely limited to date by the lack of explicit solutions for the equilibrium in the asymmetric first price auction.

In this paper, we have provided comparative statics results for a class of investments in cost reduction in the first price auction. In section 3, we showed that, after the investment —with two bidders, and in general when investment changes market leadership— the investor faces a more aggressive environment. In the terminology of industrial organization, investment in the first price auction has a negative strategic effect. In turn, we found in section 4 that this effect leads to lower investment levels in the first price auction than in the second price auction. It is tempting to interpret the low level of competition and of turnover in many procurement markets in light of these results.

Traditionally, auction theory and, in particular, the comparison between market rules, takes the distributions of private information as exogenously given. Our results contribute to an emerging literature that attempts to endogenize the relative strength of market participants. In this paper, we have offered a first comparison between the first price auction and the second price auction when firms are not necessarily symmetric ex-ante, the distributions of costs are endogenous and investment is observable. Our analysis has highlighted two attractive features of the second price auction: (1) it generates higher investment levels than the commonly used first price auction and (2) these investment levels are socially efficient. These results suggest that in markets where investment prior to the auction is deemed important or where there exist positive synergies between auctions, the second price auction is likely to be better at fostering a healthy level of competition.

Finally, we sketch several directions for future research:

Simultaneous investment. Our analysis has assumed that only one firm has the opportunity to invest at a time. In section 3, we have argued that the presence of a negative strategic effect for investment in the first price auction is likely to lead to a breakdown of the equivalence between the first price auction and the second price auction, even when investment is simultaneous and firms are ex-ante symmetric. It would be interesting to investigate whether our underinvestment result continues to hold in that setting (based on several numerical examples along the lines of example 2, we conjecture that this is the case).

Market turnover and the dynamics of competition. Investment by market participants always benefits the procurement authority in our model. Nevertheless, the fact that the second price auction elicits more investment than the first price auction does not in itself imply that it should be favored. First, Maskin and Riley (2000b)'s results suggest that the first price auction may lead to lower expected costs than the second price auction for *given* investment levels. Second, the level of asymmetry among firms is as important as the efficiency levels in determining costs (Cantillon, 2000). Hence, a natural next question is to what extent these auction formats lead to increasing market asymmetries by favoring investment by the current leader. Recently, Athey and Schmutzler (2001) have derived sufficient conditions for leaders in oligopoly markets to invest more than followers (an outcome they term "weak increasing dominance"). In our setting, their conditions reduce to: (1) the return to investment must be decreasing in the initial competitive level of one's opponents, and (2) the higher one's initial competitive level, the higher the return on investment.[17] A complete analysis is outside the scope of this paper, but looking at gross investment incentives (that is, ignoring the costs of investment) is suggestive. For the second price auction, we can use the fact that social and private investment incentives are perfectly aligned (proposition 4) to check that conditions (1) and (2) hold for all the examples of distributional upgrades given in section 2, with the exception of the extra draw for which leaders and laggards have the same gross incentives to invest.[18] Numerical simulations based on Li and Riley (1999)'s Bidcomp^2 program suggest that similar results hold for the first price auction. Obviously, more work needs to be done but a tentative conclusion is that, in most cases, both auction formats are likely to reinforce or at least maintain asymmetries among market participants.[19]

Other auction environments. Our analysis has focused on the independent private value paradigm. If values are private but correlated, the system of differential equations that characterizes the equililibrium is similar to the one we have considered. However, an investment by one firm now also affects the costs

[17]These conditions apply to both simultaneous and sequential investments.

[18]The intuition is that, in this setting, the social returns to an investment depend on the aggregate investment only.

[19]Some of these results can be found in Arozamena and Cantillon (2000). Others are available on our webpage or upon request.

of the other firms (this can be viewed as an investment with externalities). This suggests an even stronger negative strategic effect. By contrast, the nature of strategic interactions is fundamentally different in common values environments. How our results translate to these environments is an open question.[20]

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[20]Persico (2000) studies *covert* information acquisition in a common value setting. Athey (1997) provides numerical examples.

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