Investment Options with Debt Financing Constraints

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Abstract

Building on the Mauer and Sarker (2005) model that captures both investment flexibility and optimal capital structure and risky debt, we study the impact of debt financing constraints on firm value, the optimal timing of investment and other important variables like the credit spreads. The importance of debt financing constraints on firm value and investment policy depends largely on the relative importance of investment timing flexibility and debt financing gains. In cases where investment flexibility has high relative importance the firm can mitigate the effects of debt financing constraints by adjusting its investment policy. We show that these adjustments are non-monotonic and may create a U shape of the investment trigger as a function of the degree that debt is constrained. We show that in a reduced investment horizon, constraints have a more significant impact on firm value. We also consider managerial pre-investment risky growth options (e.g. R&D, or pilot projects). We see that they reduce the maturity effect, and (in contrast to the Brownian volatility) they tend to reduce expected credit spreads.

1. Introduction

The main purpose of this study is to investigate the effect and importance of debt financing constraints on firm's timing of investment decision, firm value and some other important variables like the credit spreads. The study of these issues are also important since some parameters like the tax rate, the risk-free rate, but also the level of debt constraints themselves, can be potentially (directly or indirectly) be controlled by policy makers.

We build on the contingent claim approach to investigate these issues. Since the initial contingent claims approach of valuing equity and debt was set by Merton (1974), several papers generalized and extended this idea into new dimensions including coupon payments, the tax benefits of debt and bankruptcy costs (for example, Kane et al., 1984, and 1985). Leland (1994) uses a perpetual horizon assumption and derives closed form expressions for the value of levered equity, debt and the firm in the presence of taxes and bankruptcy costs. Security values are contingent on the uncertain unlevered value of the firm. He abstracts from the investment decision and he analyzes equity holders optimal trigger point of default (unprotected debt case). Leland and Toft (1996) extend Leland (1994) to allow the firm to choose the optimal maturity of the debt, and debt level.

The above papers do not incorporate equity holders investment option decisions. Brennan and Schwartz (1984) present a finite horizon model for the valuation of the levered firm when equity holders optimally choose *both* the investment and financial policy continuously over time. Bankruptcy is triggered by bond covenant provisions when the value of the firm is less than the face value of debt that matures at the end of the time horizon. Mauer and Triantis (1994) analyze interactions of investment and financing decisions. The model allows for dynamic change in capital structure and default is triggered through a positive net worth bond covenant restriction. Gamba et al. (2005) analyze investment options with exogenous debt policy and both corporate and personal taxes. Mauer and Sarkar (2005) include optimal capital structure, optimal default and the investment option of the firm and discuss agency issues[1].

We adopt the contingent claims framework of Mauer and Sarkar (2005) and we study debt financing constraints which may exist due to exogenous regulatory restrictions set to financial institutions[2]. In addition, the suppliers of credit may engage in credit rationing or reduce their stakes in a firm trying to diversify their risk by "investing" only partially in some projects (see Fazzari et al., 1987, Stiglitz and Weiss, 1981 and Pawlina and Renneboog, 2005, for discussions on financing constraints issues). The setting we employ here allows for the optimal investment timing, optimal capital structure decisions and optimal/endogenous default on risky debt by equity holders. In the Mauer and Sarkar

[1] Fries et al (1997) explore the valuation of corporate securities (debt and equity) incorporating the tax benefits, bankruptcy costs and the agency costs of debt in a competitive industry with entry and exit decisions. Valuation of corporate securities in a duopoly with entry and exit decisions has been studied by Lambrecht (2001).

[2] Such restrictions may arise due to compliance to the Basel Accord. Debt holders may also wish to reduce their stakes in a firm due to asymmetric information or moral hazard; see Myers and Majluf, 1984, for discussion of agency issues between existing and new shareholders.

model (which extends Leland, 1994, to include the investment timing option) we introduce and study the impact of debt financing constraints. In contrast to Boyle and Guthrie (2003) our model does not focus on liquidity/cash constraints but on constraints on the level of debt financing. Furthermore, we explore the effect of debt constraints in a model that allows endogenous capital structure decisions and valuation of risky debt, an issue not considered explicitly in that paper[3].In our model investment can be launched with sufficient equity and debt funds, the latter being constrained, even in the absence of available internal financing. This situation might be particularly relevant in closely own private firms or where the information asymmetries on the equity side are of less importance. Other related work is that of Uhrig-Homburg (2004) that explores costly equity issue that can lead to a cash-flow shortage restriction. In relation to Mauer and Triantis (1994) the model we use here (prior to imposing the constraints) captures optimal default decisions rather than default based on bond covenant restrictions. Since our focus is on the effect of financing constraints we however avoid issues of recapitalization (financing flexibility) like they do. Gamba and Triantis (2005) consider personal and corporate taxes, capital issuance costs and liquidity constraints in a dynamic model, without the endogenous (optimal) default determination in the analytic framework of Leland and Mauer and Sarkar that we use.

We study the effect of debt financing constraints in respect to the risk-free rate, dividend yield (competitive erosion), volatility of the value of unlevered assets, bankruptcy costs and taxes. The investment trigger often exhibits a U shape with respect to the level of financing constraint. The importance of financing constraints under different parametarizations of the model depends on the relative importance of investment flexibility versus the net benefits of debt. Further insights are provided through a comparison of the Mauer and Sarkar model with Leland (1994) and the McDonald and Siegel (1986). Leland's (1994) model includes only the financing decision (with no investment timing) while McDonald and Siegel (1986) is an all-equity model that focuses on the investment option decision. Using this comparison we clearly demonstrate the trade-off between investment timing and the net benefits of debt and explain the importance of debt financing constraints under different parameter values. In the numerical sensitivity we also show the effect of financing constraints on equity value, the bankruptcy triggers, the optimal leverage, and the credit spreads. Additionally, we implement the models with finite maturity horizon for the investment option using a numerical lattice scheme and investigate the effect of financing constraints depending on the maturity of the investment option.

Finally, we introduce at the pre-investment stage the (growth) option to enhance the value of the unlevered asset, but in our setting the exercise of this option has random outcome. This assumption is similar to Martzoukos (2000) (see also Martzoukos, 2003 for the special case with analytic solution) where an all-equity framework was used. Koussis,

[3] Boyle and Guthrie (2003) modelling approach of external financing constraints does not distinguish between debt or equity financing. Effectively in this way they ignore the issues involved with respect to optimal capital structure, the tax benefits of debt, and endogenous default decisions that lead to risky debt. Furthermore their model implies immediate repayment as opposed to coupon paying debt that is explicitly modelled here.

Martzoukos and Trigeorgis (2005) have extended it to include path-dependency between actions, and optimal timing of the exercise of growth options. Our assumption of growth options that when exercised have a random outcome differs from the growth option component of Childs, Mauer and Ott (2005) and Mauer and Ott (2000) in that the potential exercise of the equity financed pre-investment growth option affects the distribution of project value before investment is made and uncertainty reverts to "normal" once the full investment is in place. This situation is particularly relevant for risky start-up ventures. Leland (1998) investigates alternative modes of riskiness of the project but he uses this to investigate equity holders ability to engage in "asset substitution" i.e. engage in riskier strategies ex-post to debt agreement thus transferring wealth from bond holders to equity holders. Equity holders in that model can switch between low risk and high risk strategies. Our emphasis is on the study of the interaction between these pre-investment managerial actions and investment options and financing decisions with borrowing constraints. We find that a managerial decision to exercise these growth option increases firm value, mostly by increasing the value of the option on the unlevered assets; their effect on the expected net benefits of debt is of lesser importance. We also find that exercise of these growth options decrease leverage ratios and expected credit spreads in the presence of constraints, in contrast to the case of no constraints where managerial actions have no effect on leverage ratios and expected credit spreads.

In the next section, we present the theoretical framework of Leland (1994) and its extension based on Mauer and Sarkar (2005) and we then introduce the borrowing constraints. We also implement the model with finite investment horizon using a numerical binomial tree approach to study the effect of investment horizon. In section 3 we study numerically and discuss the model with investment option and optimal capital structure, and the impact of the financing constraints on firm value, the optimal threshold to invest, and other interesting variables like credit spreads. In section 4 we consider pre-investment managerial growth actions with random outcome and their interaction with borrowing constraints.

2. The Leland and Mauer and Sarkar model with financing constraints

In this section, we review the theoretical framework of Leland (1994) that allows for optimal default policy and optimal capital structure and its extension by Mauer and Sarkar (2005) that also incorporates the optimal investment timing decision. Then we incorporate and discuss the debt financing constraints (studied numerically in section 3). The control-growth option will be added in the model and its numerical investigation will be discussed separately in section 4.

We assume that the firm's unlevered assets follow a Geometric Brownian Motion

$$\frac{dV}{V} = \mu dt + \sigma dZ \tag{1}$$

where μ denotes the capital gains of this asset, σ denotes its volatility, dZ is an increment of a standard Weiner process.

We also consider a dividend-like payout rate in the form of opportunity cost of waiting to invest δ that can be used to model coupon payments on debt and may also have the interpretation of a competitive erosion on the value of assets (e.g., Childs and Triantis (1999), Trigeorgis (1996) ch.9, and Trigeorgis (1991)). Similarly to Leland (1994) we avoid using the first interpretation and we assume that V is unaffected by the firm's capital structure: any coupon payments on debt are financed by new equity leaving the value of unlevered assets unaffected. Leland (1994) has shown that liquidation of assets to meet debt coupon obligation is inefficient (reduces firm value) compared to equity financed payments. Using either a replication argument of Black and Scholes-Merton or the risk-neutral valuation as established in Constantinides (1978) we know that any contingent claim f on V should satisfy the following PDE:

$$\frac{1}{2}\sigma^2 V^2 f_{VV} + (r-\delta)V f_V - f_t - rf = 0 .$$

Figure 1 shows the sequence of decisions in our model. Working backwards and in the absence of a control, or after the control has been activated, we refer to $F(V)$ as the value of the firm. $F(V)$ is the value of an option (see figure 1) to invest capital I (potentially with borrowing) at the optimal time t_I and acquire a levered position $E(V)$. The money the firm actually needs to pay (the equity financing, not to be confused with equity value) equals $I - D(V)$. Thus the firm has the option on $\max(E(V) - (I - D(V)),0)$ which is equivalent to $\max(E(V) + D(V) - I,0)$. This demonstrates that optimal exercise of the investment option is by using the first best approach to maximize the total value of the levered firm. The maturity T of the investment option can be either finite (in which case a binomial lattice will be implemented) or infinite (in which case the analytic solution of the following equation 2 holds). The investment option is exercised when V hits the optimal investment trigger V_I which is determined by simultaneously finding optimal capital structure (through coupon payment R) and the optimal default trigger V_B. To retain an analytic component for the values of $E(V)$ and $D(V)$, default can be triggered after t_I and at any time up to infinity (following Leland, 1994).

[Insert figure 1]

When both the investment and the default horizons are infinite we use Mauer and Sarkar (2005) to get the following equation which is a variant of their model[4] more consistent

[4] In their model the underlying asset equals the present value of a stochastic yearly revenue flow minus the present value of constant costs. We make an assumption consistent with Leland (and McD&S) that the underlying unlevered asset does not have a fixed component and follows a geometric Brownian motion. Because of the absence of the fixed yearly costs, the abandonment option treated in Mauer and Sarkar (2005) is meaningless in our version of the model.

with Leland and a focus on the value of unlevered assets (see Appendix for a review of the steps followed):

$$F(V) = \left[E(V_I) + D(V_I) - I\right]\left(\frac{V}{V_I}\right)^a \text{ where}$$

$$E(V_I) = V_I - (1-\tau)\frac{R}{r} + \left[(1-\tau)\frac{R}{r} - V_B\right]\left[\frac{V_I}{V_B}\right]^\beta,$$

$$D(V_I) = \frac{R}{r} + \left[(1-b)V_B - \frac{R}{r}\right]\left[\frac{V_I}{V_B}\right]^\beta,$$

$$V_B = \frac{-\beta}{(1-\beta)}(1-\tau)\frac{R}{r}, \tag{2}$$

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

$$a = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

By I, τ, R, r we denote the investment cost, tax rate, coupon, and the risk free rate respectively. The term b denotes proportional (to *V*) bankruptcy costs and V_B the bankruptcy trigger point that will be optimally selected by equity holders in order to maximize equity value $E(V_I)$. $E(V_I)$ is equity holders position once investment is initiated which can be re-written in the form

$$E(V_I) = V_I - V_B\left[\frac{V_I}{V_B}\right]^\beta - \frac{R}{r} + \left[\frac{R}{r}\right]\left[\frac{V_I}{V_B}\right]^\beta + \tau\frac{R}{r} - \left[\tau\frac{R}{r}\right]\left[\frac{V_I}{V_B}\right]^\beta$$

and has the following interpretation: conditional on investment, equity holders will obtain the value of unlevered assets V_I minus the expected value of unlevered assets at bankruptcy (second term) minus a perpetual stream of coupon payments (third term) that is netted with the payments that will not be made after bankruptcy (fourth term) plus the tax benefits (fifth term) also netted in the event of bankruptcy (sixth term). At the investment trigger, debt can also be re-written as

$$D(V_I) = \frac{R}{r} - \left[\frac{R}{r}\right]\left[\frac{V_I}{V_B}\right]^\beta + (1-b)V_B\left[\frac{V_I}{V_B}\right]^\beta$$

which equals a perpetual stream of coupons received (first term) netted with the expected coupon payments not received after bankruptcy trigger (second term) plus the expected value of the firm received at the bankruptcy trigger netted for the potential bankruptcy costs (third term). The derivations of the above formulas are discussed in Leland (1994).

For the optimal investment threshold we use a "first best" rule throughout the paper numerical results where V_I is selected to maximize the levered value of the firm (equity plus debt) as opposed to the "second best" of equity maximization. The first order condition is (see the appendix):

$$1+\beta\left((1-\tau)\frac{R}{r}-V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right)+ \beta\left((1-b)V_B-\frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right)-\alpha\left(\frac{1}{V_I}\right)\left(E(V_I)+D(V_I)-I\right)=0 \tag{3}$$

Equation (6) is solved numerically by simultaneously searching for optimal *R*.

Effectively, the model presented here so far is a special case of Mauer and Sarkar (2005) and we will call it the extended-Leland/MS model. It includes Leland (1994) and McDonald and Siegel (1986) (McD&S thereon) as special cases. Leland's model can be obtained by setting $V=V_I$ in equation (2) (immediate development with no investment timing). McD&S model can also be obtained by setting coupons R equal to zero (all-equity firm with an investment option), effectively imposing a zero debt restriction and that the firm never defaults ($V_B=0$). Furthermore, applying $R = 0$ in equation 3 we get the McD&S investment trigger that equals $V_I=\frac{a}{(a-1)}I$.

Replacing for $E(V_I)$ and $D(V_I)$ into $F(V)$ (see equation 2) the firm value can also be written as:

$$F(V)=(V_I-I)\left(\frac{V}{V_I}\right)^{a}+\frac{\tau R}{r}\left(1-\left(\frac{V_I}{V_B}\right)^{\beta}\right)\left(\frac{V}{V_I}\right)^{a}-bV_B\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{V}{V_I}\right)^{a}= \\ =E(V-I)+E(TB)-E(BC) \tag{4}$$

where E in the last line now reads "expected value". The last line effectively shows that the value of the firm can be written as the expected value of the unlevered assets (option on unlevered assets) plus the expected value of tax benefits minus the expected value of bankruptcy costs (as in Mauer and Sarkar, 2005, but with emphasis on the value of the unlevered assets). The net benefits of debt are defined as the difference between the expected tax benefits and the expected bankruptcy costs i.e. $NB=E(TB)-E(BC)$. As we will show in the next section, this decomposition proves useful since it is shown that optimal coupon and investment trigger selection involves a trade-off between obtaining higher option on unlevered assets (the investment flexibility that the McD&S model studies) versus higher *NB* of debt (debt financing gains that the Leland model studies).

Before moving to the discussion of financing constraints that is our main issue of analysis we show how $F(V)$ in the extended-Leland/MS model in finite investment option horizon can be obtained by implementing a numerical lattice scheme. With N lattice steps we have that up and down lattice moves and the probabilities of up and down equal:

$$u = \exp(\sigma\sqrt{\frac{T}{N}}), \qquad d = 1/u$$
$$p_u = \frac{\exp\left((r-\delta)T\right)-d}{u-d}, \quad p_d = 1-p_u \tag{5}$$

For optimal coupon selection for a given V value we apply the condition $\frac{\partial V^L(V)}{\partial R} = 0$ which gives:

$$\frac{\tau}{r}\left(1-\left(\frac{V}{V_B}\right)^{\beta}+\beta\frac{1}{r}\left(\frac{V}{V_B}\right)^{\beta}\right)+\alpha\frac{\beta}{(1-\beta)}\frac{(1-\tau)}{r}\left(\frac{V}{V_B}\right)^{\beta}+\beta\alpha V_B\left(\frac{V}{V_B}\right)^{\beta}\frac{1}{R}=0 \tag{6}$$

with V_B given in equation (2).

We apply equation 6 at *each* node of the lattice and we additionally allow for the early exercise of the investment option. At exercise, option value equals $E(V)+D(V)-I$ with $E(V)$ and $D(V)$ as in equation (2).

We now make the above framework more realistic by adding financing constraints that may exist for example due to asymmetric information, moral hazard or even by internal or regulatory constraints set to the banks. Debt financing constraints set a cap $D^{\max}$ to the level of debt financing so that $D(V_I) \le D^{\max}$. Without the constraint, $D(V_I)$ could even be higher than the required level of investment, which is rather unrealistic in practical applications. Furthermore, we could have percentage constraints i.e. $D(V) \le cV^L(V)$, $V^L(V) = E(V) + D(V)$ which can be interpreted as a cap on the maximum allowable leverage ratio (e.g. imposed by debt holders). In this paper we discuss the effects of the constant value $D^{\max}$. We now effectively face a constrained maximization problem. When we use the analytic solution of equation 5 we impose the constraint by running a numerical search for the coupon that satisfies the first order condition of the investment trigger *and* at the same time satisfies that debt does not exceed $D^{\max}$. Our approach is consistent with the "first-best" strategy for the firm value maximization. In the cases where the lattice framework is used the constraint is applied and must be satisfied at each lattice node. In the following section we discuss how the firm will adjust its investment and optimal default strategies in the face of financial constraints and control-growth options.

3. Numerical results and discussion

In this section we provide numerical results for the extended-Leland/MS model described earlier. In subsection 3.1 we provide insights on the trade-off between investment timing flexibility and the net benefits of debt that will be useful in the subsequent discussion of financing constraints. The effects of financing constraints will be discussed in subsection 3.2.

3.1. Insights on the trade-off between investment timing flexibility and the net benefits of debt

In order to illustrate the trade-off between investment flexibility and debt financing gains, we first use the decomposition of firm value from equation (7). Figures 2 and 3 use arbitrary (not optimal) values for the investment trigger. Figure 2 shows that the net benefits of debt, are decreasing in the investment threshold, while there is an optimal coupon at immediate exercise that maximizes firm value. It can be seen in figure 3, that the option on unlevered assets is invariant to the coupon and there is an investment trigger higher than the current value of unlevered assets that maximizes option value. It is thus expected that optimal investment trigger and coupon decisions involve a trade off between investment option benefits and the net benefits of debt financing.

[Insert figure 2 and 3]

This tradeoff can be further seen in Table 1, where we compare the extended-Leland/MS model (that has both investment and financing options), with the McDonald and Siegel (1986) model (with the investment only option) and the Leland (1994) model (with the financing only option). It provides the firm values, and then the (%) net gain that has the following decomposition in (%) gain of investment flexibility and (%) gain in net benefits of debt:

$$\%\text{ Net Gain} = \frac{F(V) - F^i(V)}{F^i(V)} = \frac{\left[E(V-I) - E^i(V-I)\right]}{F^i(V)} + \frac{\left[NB - NB^i\right]}{F^i(V)} \qquad (7)$$

where i = {McD&S, Leland}. We keep the base case of parameters of Leland (1994) plus a positive opportunity cost δ of 6%. Other parameters are as follows: value of unlevered assets V =100, risk-free rate r = 0.06, investment cost I =100. For the extended-Leland/MS and the Leland models bankruptcy costs b = 0.5 and tax rate τ = 0.35. The table provides sensitivity analysis for the risk-free rate r, the opportunity cost δ, the volatility of unlevered assets σ, the bankruptcy costs b, and the tax rate τ and the investment cost *I*. Note that the different components for Leland's model are found by applying $V_I = V$ in equation (2). When we compare the extended-Leland/MS model with the McD&S, we see that the net gain is due to the net benefits of debt only (at a loss in investment flexibility). When comparing it to the Leland model, the net gain is due to investment flexibility only (at a loss in the net benefits of debt).

[Table 1]

The comparison will provide insights on the effect of financing constraints that is studied in detail in the next subsection. It is expected that when debt financing gains are relatively more important, the effect of financing constraints will be more severe. First note that, as expected, the firm value in the extended model is higher than in both other models. The (%) differences between the extended and the McD&S (Leland) models are at a maximum (minimum) at higher opportunity cost δ, higher risk-free rate r, lower volatility σ, lower bankruptcy costs b, and higher tax rate τ. In absolute values, this relation is reversed for the comparison with the Leland model in the case of the risk-free rate r, and for the comparison with the McD&S model in the case of the opportunity cost δ.

Another interesting observation is the effect on firm value of changes in the above parameters. Higher risk-free rate r and lower opportunity cost δ increase firm value in all models (both investment flexibility and net benefits of debt are affected positively). Taxes and bankruptcy costs affect the extended model only through the effect on net benefits similarly with the Leland model. A most important observation is on the effect of volatility. An increase in volatility increases the firm value in the McD&S model (investment flexibility increases) but it decreases firm value in the Leland model (net benefits of debt decrease). In the extended-Leland/MS model, these opposite forces result in a U-shape in firm value (a result not reported in Mauer and Sarkar, 2005). Finally, at higher investment cost *I*, firm value decreases in all models. Investment flexibility to delay is relatively more important than the net benefits of debt and thus the differences between the extended-Leland/MS and the Leland model are increasing. At high investment costs it is possible (i.e., I =120) that immediate investment is not feasible since firm value will be negative (so in the Leland model firm value equals zero).

Table 2 shows additional information with respect to the three models. The investment triggers and the bankruptcy triggers are reported first. The other columns show for all models, equity and debt values, optimal coupon and credit spreads, reported at the optimal investment trigger (note that for the standard Leland model, investment takes place immediately at time zero). We first see that the investment triggers in the extended model are in all cases lower than in the McD&S model. This result is driven by the existence in the extended model of the benefits of debt which are decreasing in the investment trigger (see discussion in the previous subsection and figure 2). Note that the comparison is for two extreme cases, the extended model at optimal debt, and the

McD&S which is effectively a model constrained to zero debt. As we will see in the next subsection, for in-between cases (with arbitrary levels of debt constraint) this relationship is not monotonic. We also note that the bankruptcy triggers in the extended-Leland/MS model are higher than in the Leland model. It can be seen that the optimal coupon that is actually paid is higher in the extended model than in the Leland model. The optimal leverage and the credit spreads are the same in the extended-Leland/MS and in the Leland model, despite the differences in the equity and debt values and in the optimal coupon.

[Insert table 2]

The sensitivity results for the Leland model are consistent with the analysis in Leland (1994). For the extended model, we can see that the bankruptcy trigger and the debt at the investment trigger may exhibit a U-shape with respect to the volatility. Also, as we know from Leland, the optimal capital structure is invariant to the level of unlevered assets V, and the same holds for the extended model.

3.2. The effect of financing constraints

In this section we explore the effect of financing constraints on firm and equity value, bankruptcy and investment thresholds, and on leverage and the credit spreads. In the following figures, firm values are reported at time zero. All other information about equity values, etc. is for a value of V equal to the optimal investment trigger V_I. Figures 4 and 4A show the implications of financing constraints on firm and equity value, the investment and bankruptcy triggers, leverage and the credit spread at different levels of risk-free rate, opportunity cost δ and volatility. Our discussion will concentrate on realistic levels of debt equal to the total required investment (= 100) and below. We compare the base case with ones reflecting *lower* parameter values. As can be seen from the figures, the truly unconstrained case often leads to unrealistic debt levels above 100% of the required investment capital, with an unrealistically high firm value. This is an important observation that shows the significance of our constrained borrowing approach, since to even remain at 100% debt, we need to apply the constraints. Similarly unrealistic is the high investment and bankruptcy trigger values for the truly unconstrained case.

[Insert figures 4 – 4A]

In figure 4 as expected we see that financing constraints decrease firm values and increase equity values. An interesting observation is that they often produce a U-shape in the investment trigger. This result differs from Boyle and Guthrie (2003) since they effectively focus on constraints on cash balances and we focus on constraints on debt. We interpret this U-shape as follows: when the firm is unconstrained, it will use debt at a maximum. As constraints start to become binding, the firm will adjust its investment policy by lowering the investment trigger so as to capture net benefits of debt (as we have discussed in the previous subsection, the net benefits of debt are decreasing in the investment trigger). When constraints become much more binding, the effect of net benefits of debt becomes less important, and the firm gives priority to the exploitation of its investment timing flexibility by increasing the investment trigger. After careful inspection, we also see that a small dividend yield results in a less pronounced (%) decrease in firm value (due to the higher importance of investment flexibility at lower δ discussed in subsection 3.1). A small volatility results in a more pronounced (%) decrease in firm value (reducing thus the larger financial flexibility benefits of low volatility discussed in subsection 3.1).

In figure 4A we see that bankruptcy trigger and leverage ratios are decreasing. The fact that lines on the figures may cross, shows that firms with different characteristics (i.e., different parameter values) will adjust optimal leverage differently in respect to imposed constraints. The last figure shows the impact of constraints on credit spreads, which is far from linear. Compared to the base case, for lower δ credit spreads are lower (see table 2 of subsection 3.1). This in general reflects lower bankruptcy risk, since investment trigger is higher, the bankruptcy trigger is lower, and the (risk-neutral) drift is higher. With stricter constraints, the difference between the levels of the bankruptcy and the investment triggers is larger, thus the credit spreads are further reduced. Again compared to the base case, for lower interest rates credit spreads are higher (see table 2 of subsection 3.1). This now reflects higher bankruptcy risk, since although both the investment and the bankruptcy trigger are somewhat lower, the (risk-neutral) drift is lower. With stricter constraints, the investment trigger goes up and the bankruptcy trigger goes down, thus further decreasing bankruptcy risk and credit spreads. The case of volatility is more complex. Lower volatility reduces the gap between the two triggers, which would increase bankruptcy risk, but with lower volatility the probability of hitting the bankruptcy trigger may be reduced and apparently this latter effect is more important.

In figures 5 and 5A we similarly see the implications of financing constraints on firm and equity value, the investment and bankruptcy triggers, leverage and the credit spread at different levels of bankruptcy costs and tax rates. In figure 5 and to the left, all values for zero debt converge to the same point which corresponds to the McD&S case, since the bankruptcy costs and tax rates affect the net benefits of debt only.

[Insert figures 5 – 5A]

We observe that for low taxes, stricter constraints have a small effect on firm value and the investment trigger since for low taxes the net benefits of debt are low. In figure 5A we see that leverage and more importantly credit spreads tend to converge in the constrained region, whereas in the unconstrained region there can be significant differences for different levels of bankruptcy costs and tax rates. In the constrained region the optimal bankruptcy trigger for low tax rates is higher than in the base case. In both figures we see that reducing bankruptcy costs in the constrained region has a negligible effect.

We have also implemented a numerical lattice with 2 steps per year (figures not shown for brevity). The lattice captures a finite investment horizon. We have observed that for stricter constraints, the decrease in firm value is more pronounced when option maturity is lower. For looser constraints, the decrease in option value is rather insensitive to option maturity.

4. The effect of managerial control/growth options with random outcome

In this section we use the previous models and we introduce managerial control/growth options that exist prior to the exercise of the investment option (see Martzoukos, 2000). These controls may be costly, they have an instantaneous (impulse) and random outcome and they are assumed to be equity financed (a reasonable assumption for start-up growth firms). Control characteristics are their volatility, expected impact and cost. Such actions may represent managerial growth options to engage in R&D, product redesign, advertisement, marketing, or any other actions that are targeted towards an increase in value, albeit have a random outcome. We wish to study the effect of such actions on firm value and its components (option on unlevered assets and the net benefits of debt), and on the expected at development optimal leverage, equity and debt value, and credit spreads. Of particular interest is the effect of the volatility of such actions on the aforementioned variables in contrast to the effect of Brownian volatility. Changes in the Brownian volatility that we discussed in the previous section hold both before and after the investment decision; they thus affect both the investment timing option, and the risk of debt and debt capacity of the firm. The effect (increase) of uncertainty due to the control/growth actions is action-specific and thus affects volatility before the investment decision and not after[5].

We assume that the control can be activated at time zero at a cost I_C. Its effect on the unlevered asset will have a random outcome $(1+k)$ where:

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[5] Merton (1974) uses the Black and Scholes model to value equity and risky debt. In that model, increases in volatility create the so-called asset-substitution effect by transferring value from debt holders to equity holders. The assumption is that the investment and the level of debt have been already decided upon, and then there is a change in volatility. In the Leland (1994) model asset substitution can be studied by first deciding on the coupon level, and then changing volatility given the coupon level decision (see also Leland, 1998). In section 3 we discussed the sensitivity to volatility for the Leland and the extended Leland model. In our implementation of both models the new volatility level holds both before and after the investment decision. In our implementation thus of the Leland model, optimal coupon is decided given the new level of volatility. In section 4, the action-specific volatility has a direct effect on uncertainty before the investment decision and not after.

$$\ln(1+k) \sim N\left(\gamma - \frac{1}{2}\sigma_C^2, \sigma_C^2\right). \tag{8}$$

The assumption of a lognormal distribution is convenient since we retain the lognormality of the asset values when controls are activated. The expected multiplicative impact of control on V is $1+\bar{k} = \exp(\gamma)$ with a variance $\exp(\gamma)\left(\exp(\sigma_C^2)-1\right)^{0.5}$ (from lognormal distribution). We assume that an equilibrium continuous-time CAPM (see Merton, 1973) holds and that controls have firm-specific risks which are uncorrelated with the market portfolio and are thus not priced.

In general we may have optimal timing of controls and issues of path dependency (see Koussis, Martzoukos, and Trigeorgis, 2005, for an all-equity model with control/growth actions). For simplicity here we assume that controls are available only at $t = 0$, although optimal timing of those actions could be added in the present capital structure framework but at a significant expense of computational intensity and without offering any important additional insights.

Optimal firm value, $F^*(V)$ is calculated as the option to invest capital I_C in control-growth action at time zero that will potentially enhance V but has a random outcome. This gives the investment option $F(V)$ to pay capital cost I and acquire a potentially levered position $V^L(V) = E(V) + D(V)$. Note that $E(V)$ and $D(V)$ denote the stochastic values of equity and debt respectively (see section 2). Optimal firm value at $t = 0$ can be defined as follows:

$$F^*(V) = \max_{\varphi_{I_C}}\{E^C[F(V)] - I_C, F(V)] \tag{9}$$

where φ_{I_C} = {exercise of growth option, no exercise of growth option} and $E^C[.]$ is expectation under the managerial control distribution. For the evaluation of the expectation under control activation we use a Markov chain implementation. Effectively, we create a grid of V values with attached probabilities consistent with the distribution of control-growth option activation described in equation (8). In the Tables 3 and 4 that follow, all the values reported are expected due to the presence of control uncertainty, since we report them conditional on control activation. They are calculated with the use of the Markov-Chain that approximates the lognormal distribution of the multiplicative effect of the control as discussed earlier. In the extended model where delay is possible, the values are the expected ones given control activation of the expected values at the investment trigger given the uncertainty coming from the Wiener process.

Martzoukos (2000) and Koussis, Martzoukos and Trigeorgis (2005) have shown that these managerial control actions increase investment option value for an all-equity firm. Here we investigate their effect with both investment timing flexibility and net benefits of debt. Table 3 shows numerical results for the effect of controls on firm value and its two components, the expected value of unlevered assets and the expected net benefits of debt. In the same table we explore the effect of control actions with random outcome in the presence of financing constraints on debt. We assume that the cost of the control is zero to concentrate on the effect of the control distributional characteristics. Effectively, the control can be activated if its cost is less than the increase in added value it provides. For example, the firm value in the extended Leland model equals 35.42 without any control activated, and 55.18 when a control with volatility 0.60 and mean impact 0.10 is activated. Thus, an equity-financed cost up to 55.18 – 35.42 = 19.76 could be paid for this control action. Concentrating on the first panel (the case with no constraints) we see that in all models firm values are increasing in both the volatility of control and the expected impact. This is in contrast to the effect of an increase in the Brownian volatility (see discussion on Table 1) that decreases firm value in the Leland model (and creates a U-shape in the extended model). In both the extended Leland and the Leland models, an increase in the mean impact has a positive effect on both the option on unlevered asset and the net benefits of debt. An increase in volatility though, increases the option on unlevered assets, but may decrease the net benefits of debt. The net effect though of an increased control volatility is still positive, since the effect of higher volatility on the option on unlevered assets is strong enough to counterbalance a negative effect on the net benefits of debt.

[Insert Tables 3 and 4]

The second and third panel of table 3 show the effect of different levels of financing constraints on firm value and its components. For a given debt constraint, the effect of controls is like before. Comparing the panels with increasingly strict debt constraints, we still see (as expected) a decrease in firm values. The driver of the decrease in firm value is mostly due to the decrease in the net benefits of debt. But now, we do not necessarily observe a decrease in expected option on unlevered assets. This is because of the often observed U-shape of the investment trigger (see discussion on figure 4) where the firm adjusts its investment policy to stricter constraints.

Table 4 presents more information for the expected optimal capital structure (expected leverage) and the expected credit spread. Note that firm values (of Table 3) are equal to expected equity plus expected debt minus the expected investment cost. We see that (in both the unconstrained and the constrained cases) expected equity is increasing in both control volatility and control mean impact in the extended model, while in Leland´s model it is only increasing in the mean impact (but may be decreasing in control volatility). In the unconstrained case, expected leverage and expected credit spreads stay unchanged in the presence of controls (and expected debt is affected positively in the impact and volatility of the control). With the simultaneous presence of controls and stricter debt constraints we see a decrease in expected optimal leverage and an accompanying decrease in expected credit spreads. This is to be contrasted with the case of an increase in Brownian volatility that would increase credit spreads.

Expected costs reflect the probability of development. We see that, in both the unconstrained and the constrained cases, an increase in control volatility decreases expected cost while an increase in its mean impact increases expected cost.

[Insert Table 5]

In the results for the numerical implementation shown in table 5, we see the impact on the firm value in the extended Leland model of investment maturity, constraints and controls. Note that with very high maturities (T = 50) the numerical solution approximates the analytic model (see first column of table 3). Reduced maturity obviously results in a decreased firm value. This result appears in both constrained and unconstrained case, and both in the presence and in the absence of controls. An interesting observation is that in the presence of controls, the effect of maturity on firm values is lessened.

5. Summary

We use the Mauer and Sarkar (2005) contingent claims model of firm value with the option for optimal investment timing and net benefits of risky debt (that allows for optimal capital structure and endogenously determined optimal bankruptcy), with an adaptation so that it is consistent with Leland (1994). We make the interesting observation that in this extended model firm value exhibits a U-shape in volatility (not reported in previous research).

To this (extended Leland/MS) model we add financing constraints, and with the use of a Markov-Chain method we also accommodate the presence of pre-investment control/growth options with random outcome. Beyond the analytic solution for a perpetual horizon, we also implement the investment option in a finite horizon on a binomial lattice, while maintaining the analytic structure for the capital structure decisions. The scope is to study the effect of capital constraints on firm, equity and debt value, optimal investment and bankruptcy trigger, leverage and credit spreads.

A comparison of the extended model with the McD&S model that does not include a debt financing option and the Leland (1994) model that does not include an investment option provides insights on the trade-off between investment timing flexibility and the net benefits of debt. We show that financing constraints have a more significant relative impact on firm values at higher opportunity cost (dividend yield), riskless rate of interest and taxes, and lower volatility and bankruptcy costs. The effect of financing constraints is more severe when investment option maturity is lower. Financing constraints also reduce leverage and credit spreads in a nonlinear fashion. An important observation is a U-shape of the investment trigger as a function of the constraint. This result is driven by the trade-off between investment timing flexibility and the net benefits of debt.

Exercise of pre-investment managerial growth options increase firm value, although they may decrease expected net benefits of debt. In contrast to the Brownian volatility, the volatility of the managerial growth options does not create a U-shape on the firm value. This action-specific volatility affects uncertainty prior to the investment decision and has no effect in the absence of constraints (and a very small reduction effect in the presence of constraints) on expected credit spreads after development. The probability of investment increases in the mean impact and decreases in the volatility of the growth option; however, firm value always increases in the mean and the volatility of the growth options. Reduced maturity results in a decreased firm value, with and without constraints. In the presence of controls, this maturity effect on firm value tends to disappear.

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Appendix:

In this appendix, we show the derivation of the analytic solution for the extended Leland/MS model (see equations 2 and 3) with the embedded investment option. Although the model is a special case of Mauer and Sarkar (2005), we retain the derivation in order to demonstrate the exact form of the first order condition we use in the paper. Similarly with Leland (1994), and conditional on investment, the optimal default point V_B is found by solving for the following smooth-pasting condition:

$$\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0 \tag{A1}$$

which is equivalent to maximizing $E(V_I)$ at $V = V_I$. The optimal bankruptcy trigger is:

$$V_B = \frac{-\beta}{(1-\beta)}(1-\tau)\frac{R}{r}$$
$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{A2}$$

Equation (A2), compared to the one in Leland, includes dividend-like competitive erosion (included in term β). Since $\beta < 0$, this means that $V_B > 0$ for any positive level of coupons *R*.

The general solution of the option to invest $F(V)$ can be written as:

$$F(V) = A_1 V^a + A_2 V^\beta \tag{A3}$$

The option also satisfies the usual ordinary deferential equation (since the investment horizon is perpetual):

$$rF = \frac{1}{2}\sigma^2 V^2 F_{VV} + (r-\delta) V F_V$$

(A4)

By applying the general solution (A3) to the differential equation we find the solution for parameters a to be:

$$a = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{A5}$$

Consistently with Mauer and Sarkar (2005) we apply three boundary conditions to obtain the values for A_1, A_2 and the investment threshold V_I. In particular we have the following boundary conditions:

$$F(0) = 0 \tag{A6}$$

$$F(V_I) = E(V_I) + D(V_I) - I \tag{A7}$$

$$\left.\frac{\partial F}{\partial V}\right|_{V=V_I} = \left.\frac{\partial E}{\partial V}\right|_{V=V_I} \text{ (Second best) or } \left.\frac{\partial F}{\partial V}\right|_{V=V_I} = \left.\frac{\partial V^L}{\partial V}\right|_{V=V_I} \text{ (First best)} \tag{A8}$$

where $E(.)$ and $D(.)$ functional forms are given in equation 5 (derived in Leland, 1994) and are evaluated at V_I and $V^L(V_I) = E(V_I) + D(V_I)$ is the value of the levered firm at the investment trigger. Using (A6) we find that $A_2 = 0$ (since $\beta < 0$). With (A7) we find $A_1 = \left[E(V_1) + D(V_I) - I\right]\left(\frac{1}{V_I}\right)^a$ so replacing into (A3) we find equation (5) for the firm value:

$$F(V) = \left[E(V_I) + D(V_I) - I\right]\left(\frac{V}{V_I}\right)^{\alpha} \tag{A9}$$

Finally, we use (A8) to find the investment threshold. If the second best (equity maximization) approach is used we arrive at the following non-linear first order condition that can be solved (numerically) for V_I:

$$\begin{aligned} &\left[1 + \beta\left((1-\tau)\frac{R}{r} - V_B\right)\right]\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right) \\ &- \alpha\left(\frac{1}{V_I}\right)\left[V_I - (1-\tau)\frac{R}{r} + \left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta} + D(V_I) - I\right] = 0 \end{aligned} \tag{A10}$$

Alternatively, if the first best (firm value maximization) approach is used we have the first order condition:

$$\begin{aligned} &1 + \beta\left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right) \\ &+ \beta\left((1-b)V_B - \frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right) - \alpha\left(\frac{1}{V_I}\right)\left(E(V_I) + D(V_I) - I\right) = 0 \end{aligned} \tag{A11}$$

For optimal capital structure, when coupon is also a choice variable, we solve the first-order condition for the investment trigger by simultaneously searching for the optimal coupon R. In this paper, we use the first best approach and we implement equation A11.

Figure 1: Extended Leland model with growth option, investment option, and debt financing constraints

Time 0: **Control decision ($F^*(V)$)**

- Exercise of growth options, or
- exercise investment option, or
- wait

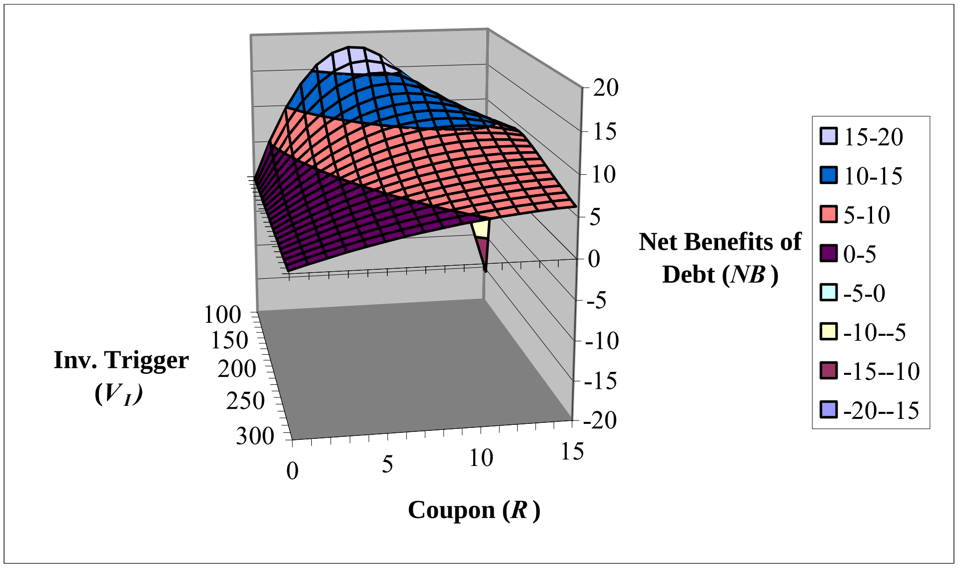
Time $t \in [0, T]$ (T is infinite in the analytic solution case): **Investment and capital structure decision ($F(V)$)**

- Wait, or
- exercise investment option at t_I when V hits optimal investment trigger V_I; determine optimal coupon subject to financing constraints, and optimal default trigger V_B

Time $t > t_I$ until ∞: **Default decision ($E(V)$)**

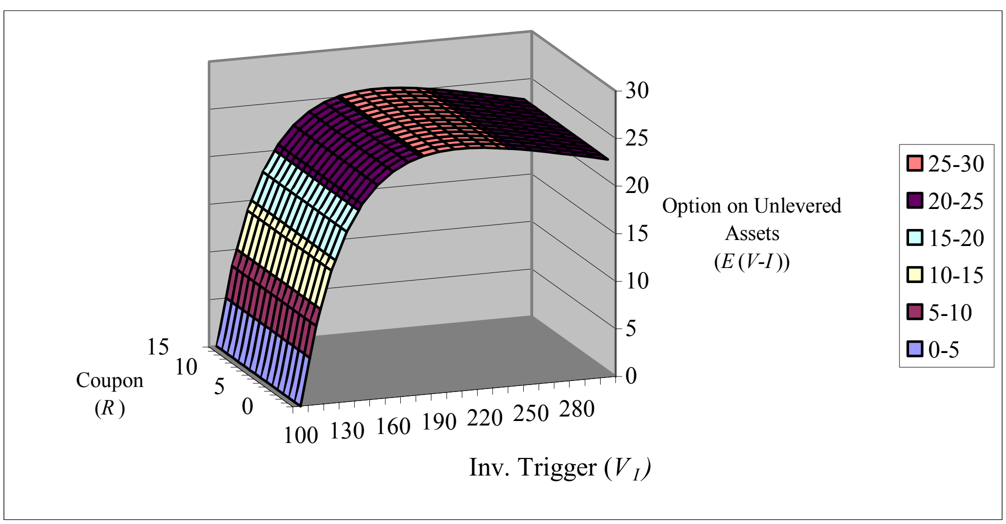
- Default if $V \leq V_B$

Figure 2: Net benefits of debt as a function of the coupon and investment trigger



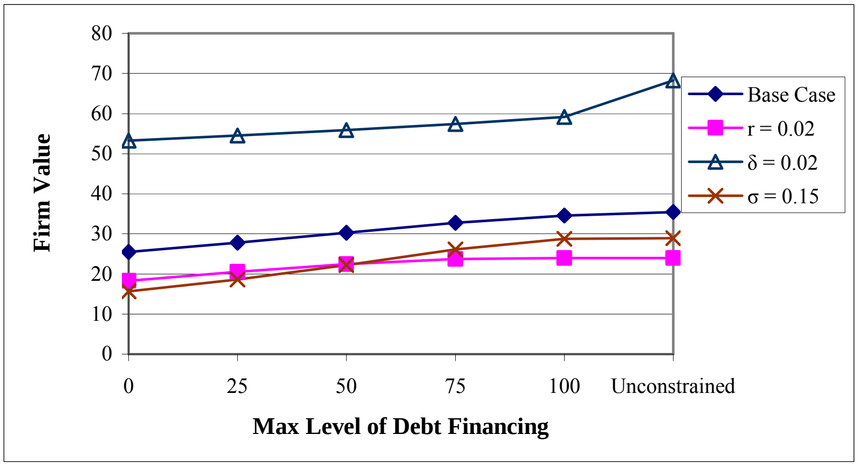
Notes: Net benefits of debt (*NB*) are defined as the tax benefits minus bankruptcy costs (see equation 7 of the main text). We use a value of unlevered assets V =100, a risk-free rate r = 0.06, an opportunity cost δ = 0.06, an investment cost I = 100, a volatility of unlevered assets σ = 0.25, a tax rate τ = 0.35 and a bankruptcy costs level of b =0.5.

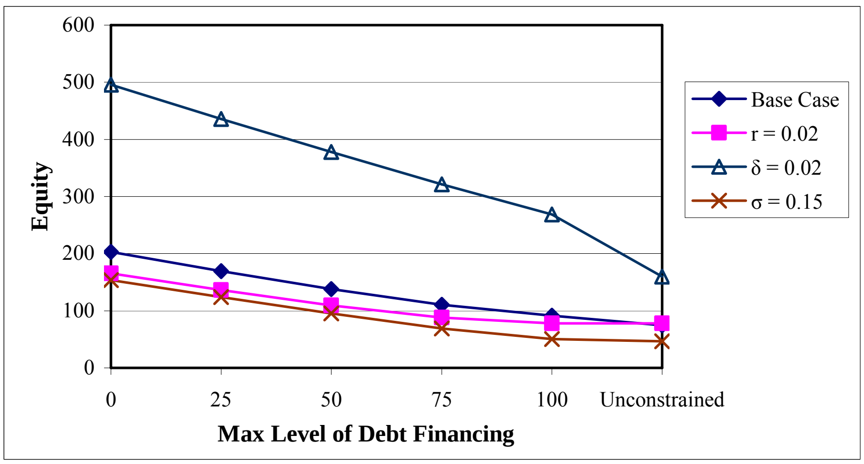
Figure 3: Option on Unlevered Assets as a function of the coupon and investment trigger

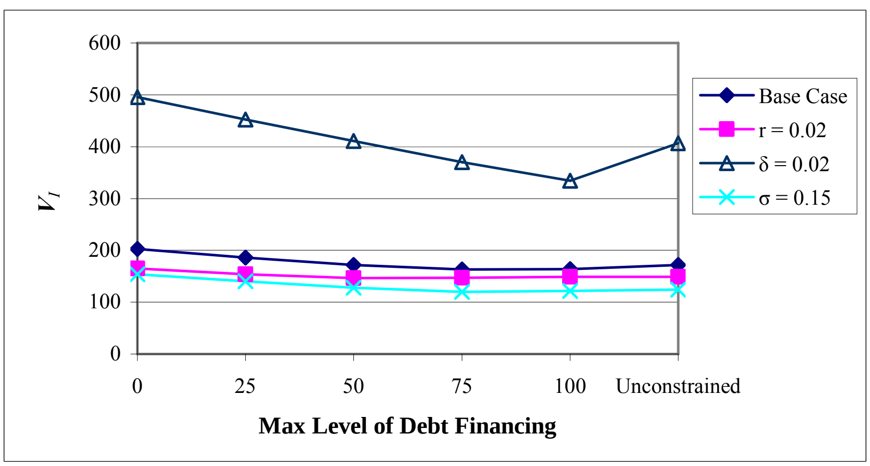


Notes: Option on unlevered assets is defined as the option to pay I and get V (see equation 7 of the main text). We use a value of unlevered assets V =100, a risk-free rate r = 0.06, an opportunity cost δ = 0.06, an investment cost I = 100, a volatility of unlevered assets σ = 0.25, a tax rate τ = 0.35 and a bankruptcy costs level of b=0.5.

Figure 4: Firm value, equity values, and investment trigger as a function of maximum levels of debt: Sensitivity with respect to *r*, *δ* and *σ*.

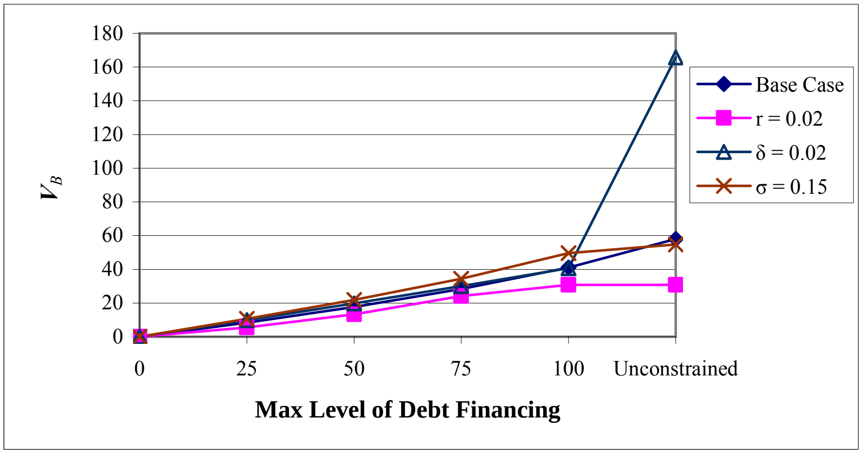


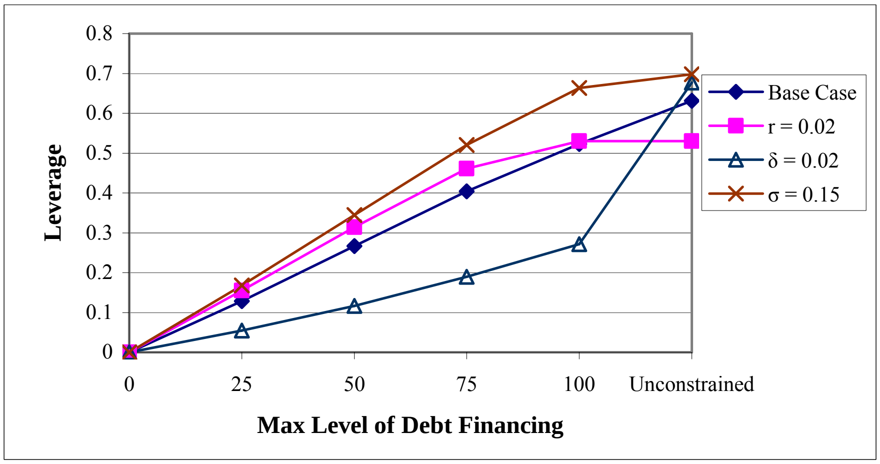


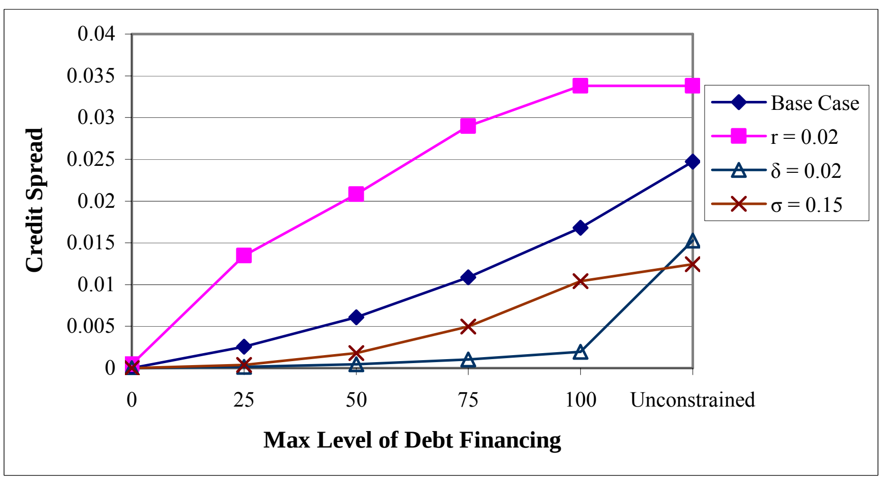


Notes: Base case used: Value of unlevered assets V =100, risk-free rate r =0.06, oopportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5. Sensitivity with respect to the risk free rate r, opportunity cost δ, and volatility σ.

Figure 4A: Bankruptcy trigger, leverage and credit spreads as a function of maximum levels of debt: Sensitivity with respect to *r*, *δ* and *σ*.

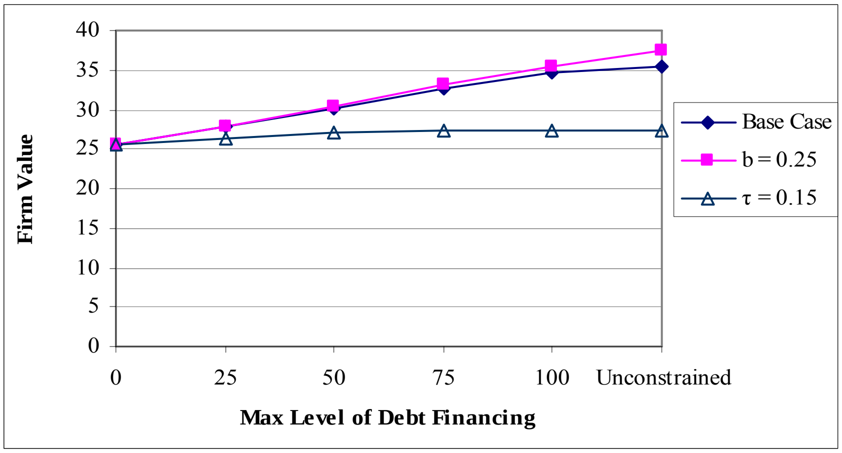


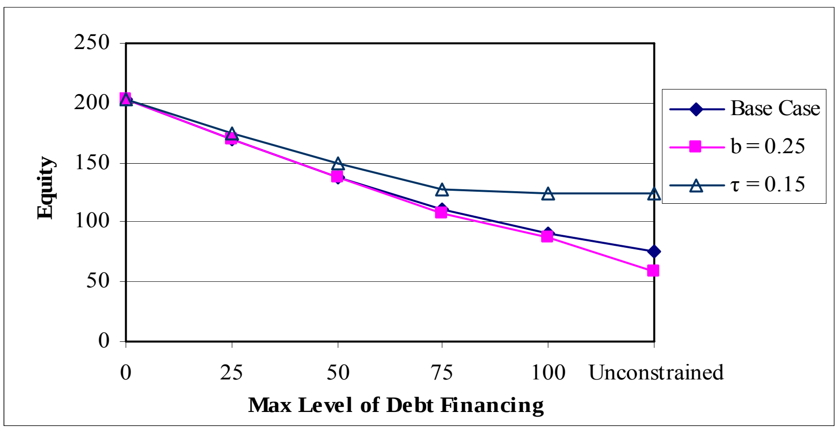


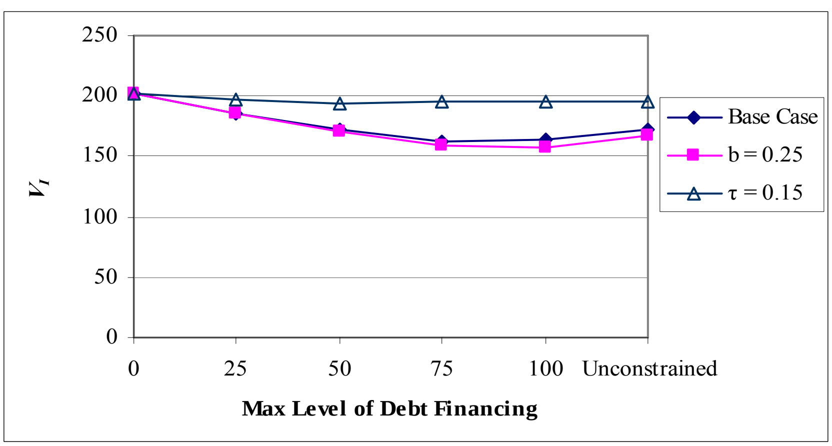


Notes: Base case used: Value of unlevered assets V =100, risk-free rate r =0.06, oopportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5. Sensitivity with respect to the risk free rate r, opportunity cost δ, and volatility σ.

Figure 5: Firm value, equity values, and investment trigger as a function of maximum levels of debt: Sensitivity with respect to *τ* and *b*.

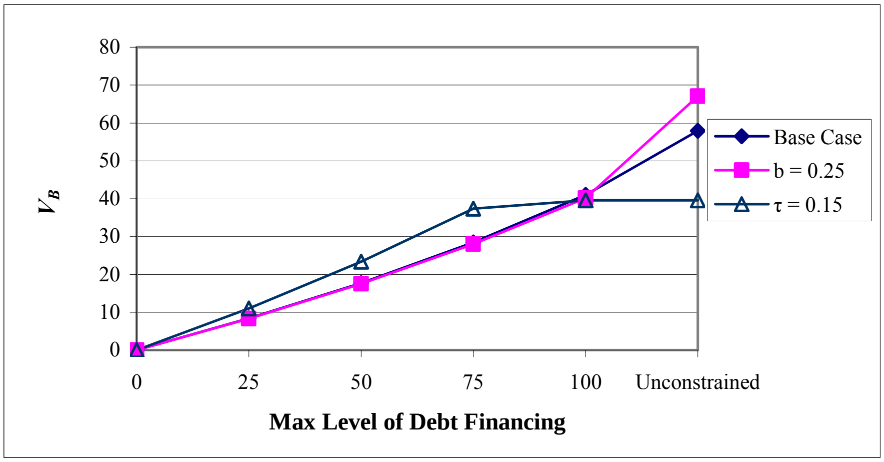


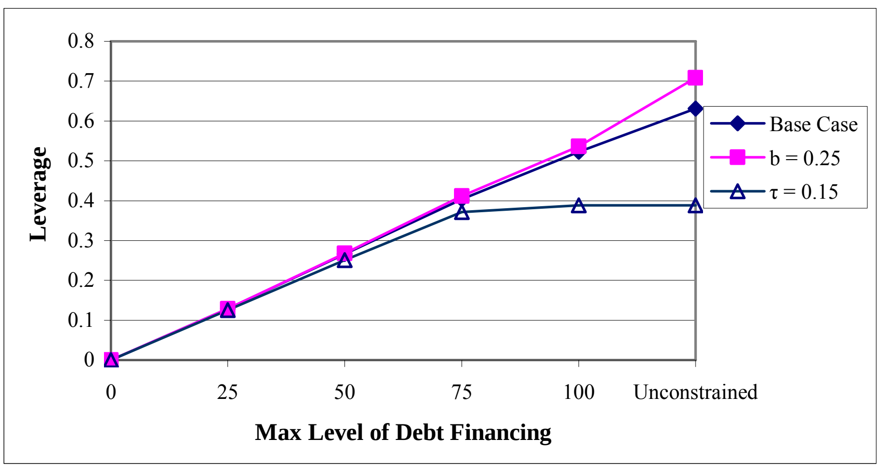


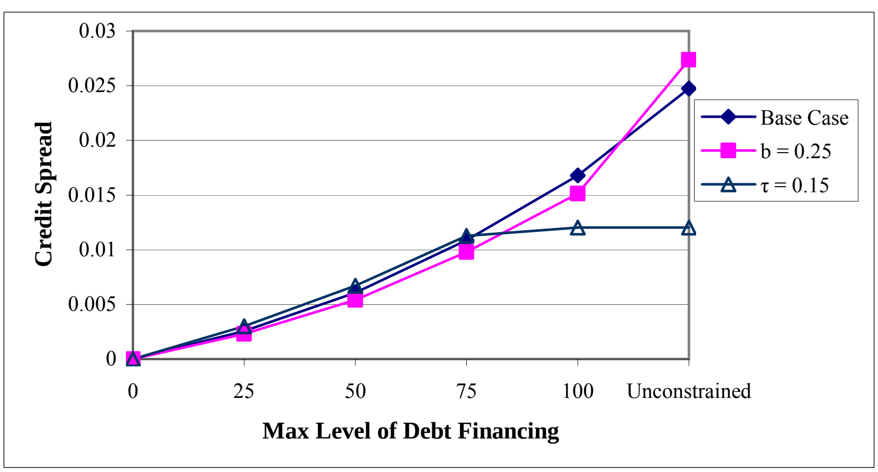


Notes: Base case parameters used: Value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, investment cost I=100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b=0.5. Sensitivity with respect to bankruptcy cost b and tax rate

Figure 5A: Bankruptcy trigger, leverage and the credit spread as a function of maximum levels of debt: Sensitivity with respect to *τ* and *b*.







Notes: Base case parameters used: Value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b=0.5. Sensitivity with respect to bankruptcy cost b and tax rate τ.

Table 1: Comparison of three models with various levels of flexibility: firm value and investment and debt financing gains analysis

	Firm Value			Ext.-Leland/MS vs McD&S			Ext.-Leland/MS vs Leland		
	Ext.-Leland/MS	McD&S	Leland	% Gain *E*(*V*-*I*)	% Gain *NB*	% Net Gain	% Gain *E*(*V*-*I*)	% Gain *NB*	% Net Gain
Base	35.42	25.48	18.18	-0.03	0.42	0.39	1.36	-0.41	0.95
$r = 0.02$	23.92	18.28	11.19	-0.03	0.33	0.31	1.59	-0.46	1.14
$r = 0.04$	29.48	21.74	14.73	-0.03	0.39	0.36	1.43	-0.43	1.00
$r = 0.08$	41.38	29.27	21.34	-0.03	0.45	0.41	1.33	-0.39	0.94
$\delta = 0.02$	68.30	53.27	21.95	-0.01	0.29	0.28	2.41	-0.30	2.11
$\delta = 0.04$	47.29	35.49	19.96	-0.02	0.35	0.33	1.75	-0.38	1.37
$\delta = 0.08$	28.05	19.28	16.68	-0.05	0.51	0.45	1.10	-0.42	0.68
$\sigma = 0.05$	35.99	5.30	35.99	-1.00	6.79	5.79	0.00	0.00	0.00
$\sigma = 0.15$	28.88	15.69	23.76	-0.17	1.01	0.84	0.55	-0.33	0.22
$\sigma = 0.35$	43.09	34.40	15.04	-0.01	0.26	0.25	2.26	-0.40	1.87
$b = 0.05$	39.93	25.48	25.58	-0.06	0.63	0.57	0.93	-0.37	0.56
$b = 0.25$	37.51	25.48	21.67	-0.04	0.52	0.47	1.12	-0.39	0.73
$b = 0.75$	33.94	25.48	15.65	-0.02	0.36	0.33	1.59	-0.42	1.17
$\tau = 0.15$	27.30	25.48	3.57	0.00	0.07	0.07	7.12	-0.48	6.64
$\tau = 0.25$	30.41	25.48	9.38	-0.01	0.20	0.19	2.69	-0.45	2.24
$\tau = 0.45$	43.43	25.48	31.04	-0.09	0.80	0.70	0.75	-0.35	0.40
$I = 60$	58.23	41.88	58.18	-0.03	0.42	0.39	0.01	-0.01	0.00
$I = 80$	44.01	31.65	38.18	-0.03	0.42	0.39	0.28	-0.13	0.15
$I = 120$	29.66	21.33	0.00	-0.03	0.42	0.39	-	-	-

Notes: "Ext.-Leland/MS" refers to the main model used with investment and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext.-Leland/MS and the Leland model we use bankruptcy costs b = 0.5, tax rate τ = 0.35. The notation "% gain $E(V\text{-}I)$" refers to the % change in value of the option on unlevered assets and "% gain NB" refers to the % change in the net benefits of debt relative to the other two models. Sensitivity analysis is with respect to the risk-free rate r, opportunity cost δ, volatility of unlevered assets σ, bankruptcy costs b, and the tax rate τ, investment cost I.

Table 2: Comparison of three alternative with various levels of flexibility: Investment and bankruptcy triggers, optimal leverage, optimal coupons and credit spreads

	Inv. Trigger (V_I)		Bankr. Trigger (V_B)		Optimal Capital Structure at Investment Trigger V_I: Equity		Debt		Optimal Leverage		Optimal Coupon		Credit Spread	
	Ext. – Leland/MS	McD&S	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland
Base	171.57	202.77	57.92	33.76	74.82	43.60	127.94	74.57	0.63	0.63	10.84	6.32	0.0247	0.0247
$r = 0.02$	148.61	165.24	30.88	20.78	77.69	52.27	87.55	58.92	0.53	0.53	4.71	3.17	0.0338	0.0338
$r = 0.04$	158.75	182.15	43.42	27.36	75.71	47.68	106.43	67.05	0.58	0.58	7.30	4.60	0.0286	0.0286
$r = 0.08$	186.71	226.57	73.97	39.62	74.78	40.04	151.77	81.29	0.67	0.67	15.47	8.29	0.0219	0.0219
$\delta = 0.02$	406.51	495.73	165.73	40.77	159.98	39.36	335.77	82.60	0.68	0.68	25.28	6.22	0.0153	0.0153
$\delta = 0.04$	227.75	273.23	84.39	37.06	94.73	41.59	178.47	78.37	0.65	0.65	14.19	6.23	0.0195	0.0195
$\delta = 0.08$	145.64	169.93	45.14	30.98	66.01	45.34	103.92	71.34	0.61	0.61	9.44	6.48	0.0308	0.0308
$\sigma = 0.05$	100.00	115.51	66.83	66.83	23.57	23.57	112.42	112.42	0.83	0.83	7.13	7.13	0.0034	0.0034
$\sigma = 0.15$	124.17	153.68	54.77	44.12	46.40	37.36	107.27	86.40	0.70	0.70	7.77	6.26	0.0124	0.0124
$\sigma = 0.35$	229.71	264.24	64.16	27.93	108.65	47.30	155.61	67.73	0.59	0.59	15.65	6.81	0.0406	0.0406
$b = 0.05$	161.48	202.77	76.72	47.50	44.13	27.34	158.65	98.24	0.78	0.78	14.36	8.89	0.0305	0.0305
$b = 0.25$	166.65	202.77	67.05	40.24	59.10	35.46	143.66	86.21	0.71	0.71	12.55	7.53	0.0274	0.0274
$b = 0.75$	175.34	202.77	50.97	29.06	87.73	50.05	115.05	65.60	0.57	0.57	9.54	5.44	0.0229	0.0229
$\tau = 0.15$	195.76	202.77	39.61	20.25	124.03	63.34	78.72	40.24	0.39	0.39	5.67	2.90	0.0120	0.0120
$\tau = 0.25$	185.38	202.77	52.22	28.16	95.15	51.34	107.63	58.04	0.53	0.53	8.47	4.57	0.0187	0.0187
$\tau = 0.45$	154.75	202.77	58.73	37.94	59.18	38.25	143.61	92.79	0.71	0.71	12.99	8.39	0.0305	0.0304
$I = 60$	102.96	121.66	34.78	33.76	44.87	43.60	76.81	74.57	0.63	0.63	6.51	6.32	0.0248	0.0247
$I = 80$	137.25	162.21	46.32	33.76	59.86	43.60	102.33	74.57	0.63	0.63	8.67	6.32	0.0247	0.0247
$I = 120$	205.89	243.32	69.51	33.76	89.78	43.60	153.54	74.57	0.63	0.63	13.01	6.32	0.0247	0.0247

Notes: "Ext.-Leland/MS" refers to the model developed with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35.Equity, debt, optimal leverage, optimal coupons and the credit spread are calculated at the investment trigger. Sensitivity analysis with respect to the risk-free rate r, opportunity cost δ, volatility of unlevered assets σ, bankruptcy costs b, and the tax rate τ, investment cost I.

Table 3: The effect of managerial control actions and financing constraints on firm value and its components (option on unlevered assets and expected net benefits of debt)

	Firm value			Option on Unlevered Assets E(V-I)		Net Benefits of Debt (NB)	
	Ext.-Leland/MS	McD&S	Leland	Ext.-Leland/MS	Leland	Ext.-Leland/MS	Leland
No constraints							
No control	35.42	25.48	18.18	24.67	0.00	10.75	18.18
$\gamma = 0.10$							
$\sigma_C = 0.2$	44.81	32.24	31.56	31.23	13.37	13.58	18.18
$\sigma_C = 0.4$	49.34	35.86	37.50	35.02	21.23	14.32	16.26
$\sigma_C = 0.6$	55.18	41.01	44.94	40.35	30.29	14.83	14.65
$\sigma_C = 0.2$							
$\Gamma = 0.1$	44.81	32.24	31.56	31.23	13.37	13.58	18.18
$\Gamma = 0.3$	66.25	47.74	59.60	46.41	35.30	19.84	24.30
$\Gamma = 0.5$	96.90	70.46	94.85	69.17	64.88	27.73	29.96
Max Debt = 75							
No control	32.70	25.48	18.18	24.02	0.00	8.68	18.18
$\gamma = 0.10$							
$\sigma_C = 0.2$	41.36	32.24	30.41	30.45	13.37	10.92	17.04
$\sigma_C = 0.4$	45.24	35.86	35.07	34.41	21.23	10.84	13.84
$\sigma_C = 0.6$	50.06	41.01	41.10	39.88	30.29	10.18	10.81
$\sigma_C = 0.2$							
$\Gamma = 0.1$	41.36	32.24	30.41	30.45	13.37	10.92	17.04
$\Gamma = 0.3$	61.02	47.74	56.16	45.46	35.30	15.57	20.86
$\Gamma = 0.5$	88.66	70.46	87.40	68.43	64.88	20.22	22.52
Max Debt = 50							
No control	30.25	25.48	14.87	24.67	0.00	5.59	14.87
$\gamma = 0.10$							
$\sigma_C = 0.2$	38.28	32.24	26.58	31.23	13.37	7.05	13.21
$\sigma_C = 0.4$	42.13	35.86	31.76	35.01	21.23	7.11	10.53
$\sigma_C = 0.6$	47.08	41.01	38.23	40.35	30.29	6.74	7.94
$\sigma_C = 0.2$							
$\gamma = 0.1$	38.28	32.24	26.58	31.23	13.37	7.05	13.21
$\gamma = 0.3$	56.58	47.74	50.71	46.40	35.30	10.18	15.40
$\gamma = 0.5$	82.74	70.46	80.93	69.16	64.88	13.58	16.05

Notes: "Ext.-Leland/MS" refers to the model with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35. Managerial control parameters have expected impact γ and volatility σ_C and are implemented using a Markov-chain with N =50 states. Max. Debt refers to constraints on the total amount of debt that can be issued.

Table 4: The effect of managerial control actions and financing constraints on optimal capital structure, expected costs, expected leverage ratio and on expected credit spreads.

	Optimal capital structure: Expected Equity		Expected Debt		Expected Cost		Expected Leverage		Expected Credit Spread	
	Ext. - Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland
No constraints										
No control	25.79	43.60	44.10	74.57	34.47	100.00	0.63	0.63	0.0247	0.0247
$\gamma = 0.10$										
$\sigma_C = 0.2$	32.58	43.61	55.71	74.58	43.47	86.63	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.4$	34.35	39.01	58.74	66.71	43.75	68.23	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.6$	35.58	35.14	60.84	60.09	41.24	50.29	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.2$										
$\gamma = 0.1$	32.58	43.61	55.71	74.58	43.47	86.63	0.63	0.63	0.0247	0.0247
$\gamma = 0.3$	47.59	58.28	81.38	99.68	62.73	98.35	0.63	0.63	0.0247	0.0247
$\gamma = 0.5$	66.53	71.87	113.76	122.91	83.39	99.93	0.63	0.63	0.0247	0.0247
Max Debt = 75										
No control	42.24	43.60	28.61	74.57	38.15	100.00	0.40	0.63	0.0109	0.0247
$\gamma = 0.10$										
$\sigma_C = 0.2$	53.35	53.11	35.96	63.93	47.95	86.63	0.40	0.55	0.0108	0.0194
$\sigma_C = 0.4$	57.05	53.07	35.43	50.23	47.24	68.23	0.38	0.49	0.0103	0.0173
$\sigma_C = 0.6$	61.04	53.72	32.96	37.67	43.95	50.29	0.35	0.41	0.0096	0.0147
$\sigma_C = 0.2$										
$\gamma = 0.1$	53.35	53.11	35.96	63.93	47.95	86.63	0.40	0.55	0.0108	0.0194
$\gamma = 0.3$	78.07	81.02	51.12	73.50	68.17	98.35	0.40	0.48	0.0106	0.0154
$\gamma = 0.5$	110.56	112.41	65.71	74.93	87.61	99.93	0.37	0.40	0.0098	0.0115
Max Debt = 50										
No control	47.50	64.87	17.25	50.00	34.50	100.00	0.27	0.44	0.0061	0.0122
$\gamma = 0.10$										
$\sigma_C = 0.2$	60.03	69.90	21.75	43.32	43.50	86.63	0.27	0.38	0.0061	0.0105
$\sigma_C = 0.4$	64.01	65.88	21.89	34.11	43.77	68.23	0.25	0.34	0.0058	0.0096
$\sigma_C = 0.6$	67.71	63.37	20.63	25.15	41.26	50.29	0.23	0.28	0.0055	0.0080
$\sigma_C = 0.2$										
$\gamma = 0.1$	60.03	69.90	21.75	43.32	43.50	86.63	0.27	0.38	0.0061	0.0105
$\gamma = 0.3$	87.96	99.88	31.38	49.18	62.77	98.35	0.26	0.33	0.0060	0.0086
$\gamma = 0.5$	124.45	130.90	41.71	49.97	83.42	99.93	0.25	0.28	0.0057	0.0067

Notes: "Ext.-Leland/MS" refers to the model with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35. Managerial control parameters have expected impact γ and volatility σ_C and are implemented using a Markov-chain with N =50 states. All values reported are time zero expected values. Max. Debt refers to constraints on the total amount of debt that can be issued.

Table 5: The effect of controls and financing constraints with finite investment option maturity

	Firm value				
	$T = 2$	$T = 5$	$T = 10$	$T = 20$	$T = 50$
No constraints					
No control	24.83	29.06	32.17	34.34	35.22
$\gamma = 0.10$					
$\sigma_C = 0.2$	36.02	39.38	41.99	43.79	44.52
$\sigma_C = 0.4$	41.38	44.33	46.71	48.38	49.08
$\sigma_C = 0.6$	48.05	50.54	52.70	54.32	55.03
$\sigma_C = 0.2$					
$\gamma = 0.1$	36.02	39.38	41.99	43.79	44.52
$\gamma = 0.3$	61.08	62.83	64.38	65.51	65.97
$\gamma = 0.5$	95.07	95.55	96.09	96.53	96.71
Max Debt = 50					
No control	21.03	24.74	27.44	29.33	30.08
$\gamma = 0.10$					
$\sigma_C = 0.2$	30.62	33.57	35.84	37.39	38.03
$\sigma_C = 0.4$	35.22	37.79	39.86	41.30	41.91
$\sigma_C = 0.6$	40.90	43.07	44.95	46.34	46.96
$\sigma_C = 0.2$					
$\gamma = 0.1$	30.62	33.57	35.84	37.39	38.03
$\gamma = 0.3$	52.07	53.62	54.97	55.95	56.34
$\gamma = 0.5$	81.14	81.57	82.04	82.42	82.57

Notes: Base case used models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100, bankruptcy cost b = 0.5 and tax rate τ = 0.35. Firm values are calculated using a Markov-chain implementation with N =50 states for the controls (with average impact γ and volatility σ_C) and a numerical lattice scheme for the investment option with dt = 0.5 years. Max. Debt refers to constraints on the total amount of debt that can be issued.