

# Investment Under Demand Uncertainty: The Newsboy Problem Revisited

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## *Abstract*

In this article we study the effect of uncertainty on an entrepreneur who must choose the capacity of his business before knowing the demand for his product. The unit profit of operation is known with certainty, but there is no flexibility in our one-period framework. We show how the introduction of global uncertainty reduces the investment of the risk-neutral entrepreneur and, even more, that of the risk-averse one. We also show how marginal increases in risk reduce the optimal capacity of both the risk-neutral and the risk-averse entrepreneur, without any restriction on the concave utility function and with limited restrictions on the definition of a mean preserving spread. These general results are explained by the fact that the newsboy has a piecewise-linear, and concave, monetary payoff with a kink endogenously determined at the level of optimal capacity. Our results are compared with those in the two literatures on price uncertainty and demand uncertainty, and particularly, with the recent contributions of Eeckhoudt, Gollier, and Schlesinger [1991, 1995].

**Key words:** uncertainty, investment, newsboy problem, increase in risk, optimal capacity

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## **1. Introduction**

It is well documented that many political and economic factors may influence the cost of capital and investment decisions. Uncertainty is also an important ingredient. Recently, Dixit and Pindyck ([1994], p. 4) wrote that “real world investment problems seem much less sensitive to interest rate changes and tax policy changes, and much more sensitive to volatility and uncertainty over the economic environment”.

In this article we study in detail the effect of uncertainty on an entrepreneur who must choose the capacity of a business before knowing the demand. This one-period model without any flexibility is often identified as the newsboy problem. We show how the introduction of global uncertainty reduces the investment of the risk-neutral newsboy and, even more, that of the risk-averse one. We also present a necessary and sufficient condition to obtain that a risk-neutral decision maker will reduce his investment when a general marginal increase in risk is introduced. In fact, we show how a mean preserving spread

increases the marginal cost of uncertainty or increases the probability that the production capacity will be underused.

Since the contribution of Rothschild and Stiglitz [1971], it is well known that, under risk aversion, restrictions have to be imposed either on the utility function or the definition of an increase in risk to obtain intuitive comparative static results for a marginal increase in risk. (See Dionne and Gollier [1996] for a detailed comparison of the two sets of restrictions). We show that such restrictions are not necessary for the newsboy problem as long as the optimal level of capacity under the less risky distribution ( $x_{AF}$ ) is lower than any crossing point of the distribution functions, which implies that the marginal cost of uncertainty is always larger under the more risky distribution for all levels of capacity lower than  $x_{AF}$ . This general result is due to the fact that the risk-averse newsboy has not only a strictly concave utility function but also a piecewise-linear and concave monetary payoff with a kink endogenously determined at the optimal level of capacity.

Hymans [1966] considered the risk-averse entrepreneur. He showed the possibility of having a negative supply curve under uncertainty, a result extended to the newsboy problem by Eeckhoudt, Gollier, and Schlesinger [1995], using the notion of partial relative risk aversion. Sandmo [1971] and Baron [1973] also offered pathbreaking contributions to the general problem of the firm under uncertainty. Contrary to the preoccupation of this article, where we study market capacity or inventory problems, Sandmo [1971] was concerned with price uncertainty. He showed that risk aversion is sufficient to obtain that a risk-averse entrepreneur will produce less than a risk-neutral one and will produce even less if the distribution of prices becomes more risky under the additional assumption of decreasing absolute risk aversion. In his framework, the risk-neutral investor is not affected by both global and marginal uncertainty. Here we are concerned with the same questions. We obtain similar results but with different conditions since the payoffs of the two problems are different. In Sandmo's model, the payoff of the entrepreneur is always linear, while, as mentioned above, the newsboy payoff is piecewise-linear with a kink endogenously determined at the optimal plant capacity. We also use different definitions of increases in risk.

Baron [1973] studied problems with piecewise-linear payoffs but did not analyze the effect of changes in risk. Kanbur [1982] obtained specific results for increases in risk in the newsboy problem but only for the special case of the quadratic utility function. Finally, Eeckhoudt, Gollier, and Schlesinger [1995] presented a detailed analysis of the newsboy problem, such as price changes, increased demand risk, and adding background risk. In Section 4 we compare our results on the effect of increasing risk with their results.

## 2. The model

We consider an entrepreneur who must choose the capacity  $x$  before knowing the demand  $y$  ( $y \in [\underline{y}, \bar{y}]$ ) in a one-period investment problem without any flexibility. If the realized demand  $y_0$  is greater than  $x$ , the entrepreneur sells  $x$ . Otherwise he is limited to sell  $y_0$  (for similar presentations of the basic problem, see Baron [1973], Levy-Lambert and Dupuy [1975], Malinvaud [1987], Dionne and Pellerin [1988], and Eeckhoudt, Gollier, and Schlesinger [1995]).

His ex-ante objective function is  $EU[\Pi \cdot \min(y, x) - I(x)]$  where  $\Pi$  is the constant unit profit of operation and  $I(x)$  is the total capacity cost. The explicit maximation problem can be written as

$$\text{Max}_x V(x, F) = \int_y^x U(\Pi y - I(x)) f(y) dy + \int_x^{\bar{y}} U(\Pi x - I(x)) f(y) dy, \quad (1)$$

where  $U(\cdot)$  is the von Neuman Morgenstern utility function of wealth,  $U'(\cdot) > 0$ ,  $U''(\cdot) \leq 0$ ;  $I(x)$  is the total capacity cost function,  $I'(x) > 0$ ,  $I''(x) \geq 0$ . Note that the model differs slightly to that of Eeckhoudt, Gollier, and Schlesinger [1995] since they assume linear costs and allow for a positive salvage value of excess inventory. These differences are not significant for our purpose.

The optimal capacity of the risk-averse entrepreneur ( $x_A$ ) satisfies for an interior solution:

$$\begin{aligned} V'(x_A, F) \equiv & -I'(x_A) \int_y^{x_A} U'(\Pi y - I(x_A)) f(y) dy \\ & + (\Pi - I'(x_A)) U'(\Pi x_A - I(x_A)) (I - F(x_A)) = 0, \end{aligned} \quad (2)$$

which implies  $\Pi > I'(x_A)$ . When  $\Pi \leq I'(x_A)$ ,  $V'(x_A, F) < 0$  and  $x_A = 0$  is optimal. The second-order condition for a maximum follows our assumptions about  $I(x)$  and  $U(\cdot)$ .

By integrating by parts the first term on the left side of (2) we obtain

$$\Pi = I'(x_A) + \Pi F(x_A) - I'(x_A) \Pi \frac{\int_y^{x_A} U''(\Pi y - I(x_A)) F(y) dy}{U'(\Pi x_A - I(x_A))}. \quad (3)$$

For notational convenience, (3) can be written as

$$\Pi = I'(x_A) + \Pi F(x_A) + \Pi I'(x_A) A(x_A), \quad (4)$$

where  $F(x_A)$  is the probability that  $x_A$  will not be sold,  $I'(x_A)$  is the marginal cost of investment,  $\Pi F(x_A)$  is the marginal cost of uncertainty,  $\Pi I'(x_A) A(x_A)$  is the marginal cost of risk aversion, and

$$A(x_A) \equiv \frac{-\int_y^{x_A} U''(\Pi y - I(x_A)) F(y) dy}{U'(\Pi x_A - I(x_A))}.$$

Since  $U''(\cdot) < 0$  implies that  $A(x_A) > 0$ , we verify from (4) that

$$\Pi > I'(x_A) + \Pi F(x_A),$$

while under risk neutrality,

$$\Pi = I'(x_N) + \Pi F(x_N) \quad (5)$$

and, under certainty,

$$\begin{aligned} \Pi &= I'(x_0) \quad \text{when } I''(x) > 0 \quad \text{and} \\ \Pi &> (=, <) I' \quad \text{when } I''(x) = 0, \end{aligned} \tag{6}$$

where the notation  $I'$  is for constant marginal cost of investment.

**Proposition 1:**  $x_A \leq x_N \leq x_0$ .

*Proof.* First note that  $\Pi$  is constant. When  $I''(x) > 0$ , it is straightforward to verify that adding the marginal cost of uncertainty  $\Pi F(x_N)$  to the right side of (5) must decrease the optimal output. The same analysis follows when comparing  $x_A$  to  $x_N$  since  $\Pi I'(x_A)A(x_A) > 0$  and  $F'(x_A) > 0$  for all  $x$ . When  $I'' = 0$ ,  $x_0$  can be undetermined or corresponds to a corner solution either at  $\bar{y}$  or at  $\underline{y}$ . When  $x_0 = \bar{y}$ , it cannot increase and when  $x_0 = \underline{y}$  because  $\Pi < I'$ , the solution cannot change for any type of global mean-preserving increase in risk since  $\Pi$  will remain lower than  $I'$  plus the marginal cost of uncertainty. Finally, when the solution is undetermined, it will go to  $\underline{y}$  since  $\Pi$  will become lower than  $I'$  plus the marginal cost of uncertainty.  $\square$

It is interesting to observe that a risk-neutral entrepreneur produces less under uncertainty than under certainty. When an interior solution prevails, this result is due to the fact that the structure of the problem in (1) introduces a kink in the payoff function, which becomes concave under uncertainty. Moreover, the kink is endogenously determined as shown in figure 1 (see Dionne and Pellerin [1988] for more details).

The endogenous monetary payoff is piecewise-linear contrary to the type of linear payoff studied in Sandmo [1971] and Dionne, Eeckhoudt, and Gollier [1993], where it is linear

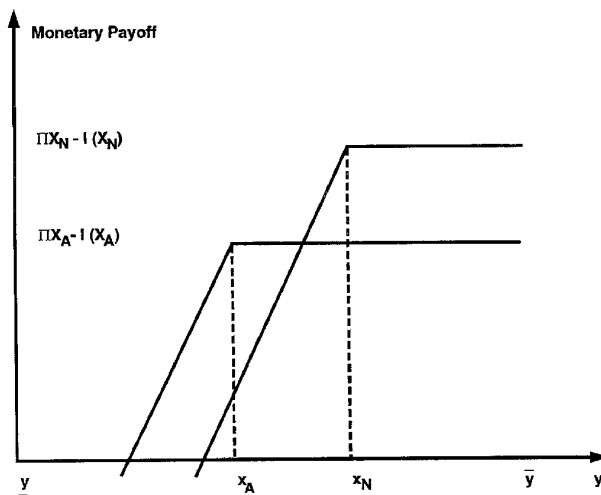


Figure 1. Optimal capacity of the risk-averse ( $x_A$ ) and the risk-neutral ( $x_N$ ) newsboy.

for all values of wealth. This important characteristic implies that the marginal monetary payoff is nil on the right side of the optimal capacity ( $x_A$  or  $x_N$ ) for both the risk-averse and the risk-neutral agent, while it is constant and positive on the left side. The difference between  $x_A$  and  $x_N$  is then explained by the fact that the marginal utility of the risk-neutral agent is constant for all values of  $y < x_N$  while it is strictly decreasing for the risk-averse agent. These characteristics will also play an important role in the analysis of the marginal changes in risk discussed in the next section.

These results contrast with those of Sandmo who obtained that only a risk-averse entrepreneur does produce less under uncertainty. In other words, the output of the risk-neutral entrepreneur is not affected by price uncertainty. As already pointed out these differences are explained by the differences in the payoff functions.

### 3. Marginal changes in risk

The contributions of Rothschild and Stiglitz [1970, 1971] have generated two important general results in the literature on comparative statics under uncertainty with payoffs that are linear in both the random variable and the decision variable (see for example, Cheng, Magill, and Shafer [1987], Dionne, Eeckhoudt, and Gollier [1993], Eeckhoudt and Kimball [1992], Meyer and Ormiston [1985], Sandmo [1971], and Dionne and Gollier [1996]): (1) the risk-neutral decision maker is not affected by mean preserving changes in risk and (2) specific restrictions have to be imposed either to the utility function or the definition of a mean preserving spread to obtain that a risk-averse decision maker will reduce his ex-ante risky position (or capacity here) when facing a more risky situation. In this section we show that such restrictions on the definition of increases in risk are stronger than necessary, for any utility function, to obtain intuitive comparative statics results.

We first consider a risk-neutral decision maker. The optimal interior solution under the distribution  $F(\cdot)$  (from now on  $x_{NF}$ ) solves (5) in the preceding section. Suppose that  $G(\cdot)$  represents a mean preserving increase in risk with respect to  $F(x)$  in the sense of Rothschild and Stiglitz [1971]:

$$\int_{\underline{y}}^{\hat{y}} (G(y) - F(y)) dy \geq 0 \quad \text{for all } \hat{y} \in [\underline{y}, \bar{y}],$$

with a strict inequality for some values of  $y$  and

$$\int_{\underline{y}}^{\bar{y}} (G(y) - F(y)) dy = 0.$$

We now propose a necessary and sufficient condition for the variation of the optimal capacity of the risk-neutral decision maker:

**Proposition 2:** *Let  $x_{NF}$  and  $x_{NG}$  determine the optimal solution for a risk-neutral manager under distribution  $F(\cdot)$  and  $G(\cdot)$ , respectively. Then  $x_{NF} > (=, <)x_{NG}$ , if and only if  $F(x_{NF}) < (=, >)G(x_{NF})$ .*

*Proof.* We have to show that

$$V'(x_{NF}, G) - V'(x_{NF}, F) < (=, >) 0$$

$$\text{as } F(x_{NF}) < (=, >) G(x_{NF}). \quad (7)$$

From (5) and the equivalent first-order condition under distribution  $G(x)$  we can rewrite (7) as

$$-\Pi[G(x_{NF}) - F(x_{NF})],$$

which yields the desired result.  $\square$

Again this result differs from those with linear payoff models where an increase in risk has no effect (see Sandmo [1971] and Dionne, Eeckhoudt, and Gollier [1993]). For the risk-averse individual, contrary to many applications presented in the literature, we do not necessarily restrict our analysis to a simple spread of the two distributions. However, for matter of comparison, we first show that a general result can be obtained for any simple spread at any  $y \geq x_{AF}$ , which is a direct extension of the result presented by Eeckhoudt, Gollier, and Schlesinger [1995]. Then we extend this first result to obtain that the number of crossings does not matter as long as all of them occur at values of  $y \geq x_{AF}$ . In other words, we obtain that following any general mean preserving spread defined by Rothschild and Stiglitz [1970], a risk-averse entrepreneur will always reduce his capacity of production if the optimal level of capacity obtained under the less risky distribution is lower than any crossing point of the distribution functions.

**Proposition 3:** *Suppose that  $x_{AF}$  and  $x_{AG}$  maximize  $V(x, \cdot)$  under  $F(y)$  and  $G(y)$ , respectively, where  $G(y)$  is a single mean preserving spread of  $F(y)$ . Then a sufficient condition for  $x_{AG} < x_{AF}$  is that  $F(x_{AF}) \leq G(x_{AF})$ .*

*Proof.* See appendix.

Observed that when  $F(x_{AF}) = G(x_{AF})$ , the condition isolates the effect of risk aversion since we know from Proposition 2 that the optimal capacity on the risk-neutral decision maker is not altered under that condition. Otherwise, the result follows from a combination of risk aversion and the concavity of the payoff function. But more interesting is the following result, which does introduce a minimal restriction on the nature of the marginal increase in risk when compared with other comparative statics results in the literature (see next section for more details).

**Proposition 4:** *Let  $G(y)$  be a mean preserving spread of  $F(y)$  and  $x_{AG}$  and  $x_{AF}$  the optimal capacity under the two distributions, respectively. Then  $x_{AG} < x_{AF}$ , if for  $\forall y \leq x_{AF}$ ,  $G(y) \geq F(y)$ .*

*Proof.* See appendix.

4. Discussion

Figure 2 compares six results from three different articles. The first column is concerned with two results on deductible insurance in Eeckhoudt, Gollier, and Schlesinger [1991], the second one contains two results related to the newsboy problem in Eeckhoudt, Gollier, and Schlesinger [1995], while the third one represents the results of Propositions 3 and 4 of this article. In all cases distributions  $F(\cdot)$  is less risky than distribution  $G(\cdot)$ .

Figure 2(a) indicates that a risk-averse individual does not change his optimal deductible if a mean-preserving transformation (including an increase in risk as a special case) of the loss distribution is imposed on the right side of the optimal deductible  $D_F^*$  (the optimal deductible under  $F$ ). In this case, the risk-neutral insurer asks for the same insurance premium and takes care of the loss fluctuations above the deductible. However, when the mean-preserving transformation affects only portions of the loss distribution below  $D_F^*$ , then a prudent insured ( $U'''(\cdot) > 0$ ) increases his optimal deductible (reduces his insurance coverage). By reducing the insurance coverage, the prudent insured reduces the premium and increases his precautionary saving to protect himself against the mean-preserving transformation. In this article, the authors did not present results for mean-preserving changes in risk affecting the loss distribution on all the support  $[\underline{y}, \bar{y}]$ .

Results in the middle column contrast with those of the first column since there is no insurance coverage of the fluctuations in wealth above the deductible. In figure 2(c) we observe that a prudent newsboy will increase his optimal capacity since the increase in risk above the

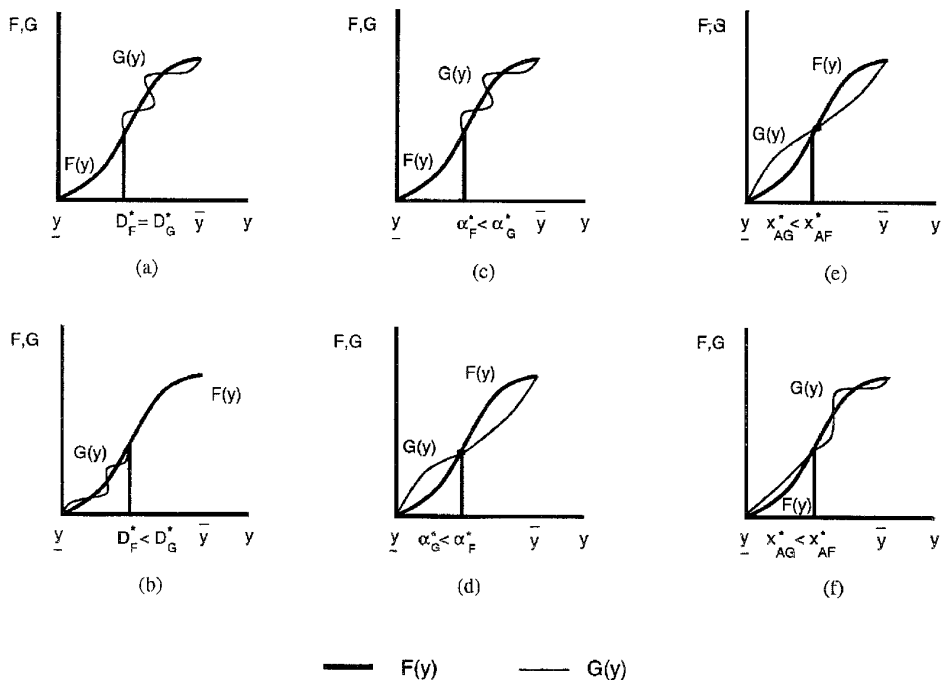


Figure 2. Comparison of different results.

optimal decision variable  $\alpha_F^*$  decreases the marginal cost of production without modifying the marginal benefit. The authors also obtained, under the same assumption, that a prudent newsboy will decrease his capacity under  $G(\cdot)$ , if the increase in risk affects only portions of profit distribution below  $\alpha_F^*$ . Figure 2(d) concerns a simple spread across  $\alpha_F^*$  (see Dionne and Gollier [1996] for details on the notion of simple increase in risk, which is a particular case of a mean-preserving spread). We observe that a risk averse newsboy reduces his optimal capacity. For this particular definition of a mean-preserving spread, risk aversion is sufficient to get the result while it was not sufficient to obtain the previous results of figures 2(b) and 2(c).

Figure 2(e) represents the result of Proposition 3 in this article. It extends the result of figure 2(d) in the sense that the crossing point between the two distribution functions has not to be at the optimal capacity: the same result holds for all single crossing points equal or above the optimal capacity. It is interesting to notice also that the sufficient condition that yields the result of Proposition 3 for a risk-averse newsboy is necessary and sufficient for the risk-neutral newsboy (see Proposition 2). Finally, figure 2(f) shows that we do not have to limit the analysis to single crossing points in order to obtain the desired result for all risk-averse newsboys. Any number of crossing points is admitted as long as the first one is at the optimal capacity or above. The main differences between this result and those of figures 2(a)–(c) is that the change in risk affects all the support of the random variable and not only portions of it either below or above the optimal decision variable, and no condition on  $U(\cdot)$  other than risk aversion is used.

**5. Conclusion**

This article shows how general marginal and global increases in risk reduce the optimal investment of both the risk-neutral and risk-averse newsboy. It emphasizes the role of the piecewise-linear and concave monetary payoff that characterizes this investment problem under demand uncertainty. A natural extension would be to extend our analysis to other applications where such payoff can be observe; debt contracts, deductible insurance, and covered call options are good candidates.

**Appendix**

*Proof of Proposition 3*

Let  $f(\cdot)$  and  $g(\cdot)$  be the density functions of  $F(\cdot)$  and  $G(\cdot)$ , respectively.  $y_1$  and  $y_2 \in [y, \bar{y}]$  are the crossing points of the density functions with  $y_1 < y_2$  and  $y_3$  is the single crossing point of the distribution functions. From (2),  $x_{AG} < x_{AF}$  if and only if

$$\begin{aligned}
 & -I'(x_{AF}) \int_y^{x_{AF}} U'(\Pi y - I(x_{AF}))s(y) dy \\
 & + (\Pi - I'(x_{AF}))U'(\Pi x_{AF} - I(x_{AF})) \int_{x_{AF}}^{\bar{y}} s(y)dy < 0,
 \end{aligned}
 \tag{8}$$

where  $s(y) = g(y) - f(y)$ .



The above inequality is always verified when  $\underline{y} \leq x_{AF} \leq y_1$ . The first term on the left side is always negative since  $s(y)$  is always positive for  $\underline{y} \leq y \leq y_1$ . Moreover the second term is also always negative. Indeed, when  $F(x_{AF}) \leq \bar{G}(x_{AF})$ , this term can be rewritten as

$$(\Pi - I'(x_{AF}))U'(\Pi x_{AF} - I(x_{AF}))(F(x_{AF}) - G(x_{AF})) \leq 0,$$

since  $\Pi - I'(x_{AF}) > 0$  from the first-order condition (2).

When  $y_1 \leq x_{AF} \leq \bar{y}$ ,  $s(y) > 0$  for  $\underline{y} \leq y \leq y_1$ , and  $s(y) < 0$  for  $y_1 < y \leq x_{AF} \leq y_3$ . Consequently, (8) becomes

$$\begin{aligned} -I'(x_{AF}) \left\{ \int_{\underline{y}}^{y_1} U'(\Pi y - I(x_{AF}))s(y) dy + \int_{y_1}^{x_{AF}} U'(\Pi y - I(x_{AF}))s(y) dy \right\} \\ + (\Pi - I'(x_{AF}))U'(\Pi x_{AF} - I(x_{AF})) \int_{x_{AF}}^{\bar{y}} s(y) dy < 0. \end{aligned} \tag{9}$$

From above we know that the third line is negative. It remains to show that the sum of the first two lines is nonpositive.

Since when  $U''(\cdot) < 0$ ,

$$\int_{\underline{y}}^{y_1} U'(\Pi y - I(x_{AF}))s(y) dy > \int_{\underline{y}}^{y_1} U'(\Pi y_1 - I(x_{AF}))s(y) dy$$

because  $s(y) \geq 0, \forall y \in [\underline{y}, y_1]$ , and

$$\int_{y_1}^{x_{AF}} U'(\Pi y - I(x_{AF}))s(y) dy > \int_{y_1}^{x_{AF}} U'(\Pi y_1 - I(x_{AF}))s(y) dy$$

because  $s(y) \leq 0, \forall y \in [y_1, x_{AF}]$ .

It is now sufficient to show that the right side of (10) is nonnegative:

$$\int_{\underline{y}}^{x_{AF}} U'(\Pi y - I(x_{AF}))s(y) dy > U'(\Pi y_1 - I(x_{AF}))(G(x_{AF}) - F(x_{AF})), \tag{10}$$

which is always the case for  $x_{AF} \leq y_3$ . □

*Proof of Proposition 4*

We can write the difference of the first-order conditions evaluated at  $x_{AF}$  under distributions  $G(y)$  and  $F(y)$  as

$$\begin{aligned}
 & -I'(x_{AF}) \int_y^{x_{AF}} U'(\Pi y - I(x_{AF}))(g(y) - f(y)) dy \\
 & - (\Pi - I'(x_{AF}))U'(\Pi x_{AF} - I(x_{AF}))(G(x_{AF}) - F(x_{AF})) \leq 0 \\
 \Leftrightarrow & -I'(x_{AF}) \int_y^{x_{AF}} U'(\Pi y - I(x_{AF})) dS(y) \\
 & - (\Pi - I'(x_{AF}))U'(\Pi x_{AF} - I(x_{AF}))S(x_{AF}) \leq 0 \\
 \Leftrightarrow & I'(x_{AF}) \int_y^{x_{AF}} \frac{U''(\Pi y - I(x_{AF}))}{U'(\Pi x_{AF} - I(x_{AF}))} S(y) dy - S(x_{AF}) \leq 0.
 \end{aligned}$$

which is always verified when  $S(y) \geq 0$  for all  $y \leq x_{AF}$ . □

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**References**

- BARON, D.P. [1973]: "Point Estimation and Risk Preferences," *Journal of the American Statistical Association*, 68, 944–950.
- CHENG, H.G., MAGILL, M.J.P., and SHAFER, W.J. [1987]: "Some Results on Comparative Statics Under Uncertainty," *International Economic Review*, 28, 493–507.
- DIONNE, G., EECKHOUDT, L., and GOLLIER, C. [1993]: "Increases in Risk and Linear Payoffs," *International Economic Review*, 34, 309–319.
- DIONNE, G., and GOLLIER, C. [1996]: "A Model of Comparative Statics for Changes in Stochastic Returns with Dependent Risky Assets," *Journal of Risk and Uncertainty*, 13, 147–162.
- DIONNE, G., and PELLERIN, M. [1988]: "Investissement en incertitude: extension du problème de la taille optimale d'une usine," in *Uncertain et Information*, G. Dionne (Ed.). Economica-G. Vermette, Paris-Montréal, 256–281.
- DIXIT, A., and PINDYCK, R. [1994]: *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ.
- EECKHOUDT, L., GOLLIER, C., and SCHLESINGER, H. [1991]: "Increases in Risk and Deductible Insurance," *Journal of Economic Theory*, 55, 435–440.
- EECKHOUDT, L., GOLLIER, C., and SCHLESINGER, H. [1995]: "The Risk-Averse (and Prudent) Newsboy," *Management Science*, 41, 786–794.
- EECKHOUDT, L., and KIMBALL, M.S. [1992]: "Background Risk, Prudence and the Demand for Insurance," in *Contributions to Insurance Economics*, G. Dionne (Ed.). Kluwer, Boston, MA.
- HYMANS, S.H. [1966]: "The Price Taker: Uncertainty, Utility, and the Supply Functions," *International Economic Review*, 7, 346–356.

- KANBUR, R. [1982]: "Increases in Risk with Kinked Payoff Functions," *Journal of Economic Theory*, 27, 219-228.
- LEVY-LAMBERT, H., and DUPUY, J.P. [1975]: *Les choix économiques dans l'entreprise et dans l'administration*, Dunod, Paris.
- MALINVAUD, E. [1987]: "Capital productif, incertitudes et profitabilité," *Annales d'Économie et de Statistiques*, 5, 1-36.
- MEYER, J., and ORMISTON, M.B. [1985]: "Strong Increases in Risk and Their Comparative Statics," *International Economic Review*, 425-437.
- ROTHSCHILD, M., and STIGLITZ, J. [1970]: "Increasing Risk: A Definition," *Journal of Economic Theory*, 2, 225-243.
- ROTHSCHILD, M., and STIGLITZ, J. [1971]: "Increasing-Risk: Its Economic Consequence," *Journal of Economic Theory*, 3, 66-84.
- SANDMO, A. [1971]: "On the Theory of the Competitive Firm Under Price Uncertainty," *American Economic Review*, 61, 65-73.