Investments and network competition*

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Abstract

This paper analyzes the incentives that operators have to invest in facilities with different levels of quality. A network of better quality is more expensive but may give an important edge to an operator when competing against a rival. We extend the framework of Armstrong-Laffont-Rey-Tirole by introducing an investment stage, prior to price competition. We show that the incentives to invest are influenced by the way termination charges are set. In particular, when the quality of a network has an impact on all calls initiated by own customers (destined both on-net and off-net), we obtain a result of "tacit collusion" even in a symmetric model with two-part pricing. Firms tend to underinvest in quality, and this would be exacerbated if they can negotiate reciprocal termination charges above cost. We also show that when the quality of off-net calls depends on the interaction between the quality of the two networks, there is another serious problem, namely that no network has an incentive to jump ahead of the rival.

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1. Introduction

In this paper we address the question of network competition between telecommunications operators when they have to invest in their own facilities. This situation should be described as one of two-way access, in the sense that each operator needs access to the rival's network in order to terminate calls originated by its own customers but destined to subscribers belonging to the other network. Recent literature has found that regulatory concerns under two-way network competition are reduced compared to one-way competition. Since network A wants its customers to be able to complete their calls destined to customers belonging to network B, and *vice versa*, A and B have a "double coincidence of wants" that makes the interconnection terms less problematic. The foreclosure problem typical of one-way access disappears. This does not necessarily imply that regulation is not needed any more in such an environment, since there may be other concerns. For instance, access prices could used as an instrument of tacit collusion because of a raise-each-other's cost effect (Armstrong, 1998; Laffont *et al.*, 1998 - hereafter ALRT - see Armstrong, 2002, for a survey). However, this problem is drastically reduced when operators compete over non-linear prices (e.g. two-part tariffs) to attract consumers.

The model that we propose builds on the framework of ALRT but challenges the result of access charges and "profit neutrality". In standard symmetric models, it is generally found that profit is a constant that does not depend on the interconnection fee when firms compete in two-part tariffs. The intuition for this result is that when the termination charge is increased, firms will increase call prices but, at the same time, they will reduce the fixed component to keep market shares. This is because higher termination revenues make firms more willing to compete for customers. These effects cancel out and firms are indifferent over the level of (reciprocal) termination charges.

Profit neutrality in models of network competition has recently been addressed by other authors. Dessein (2001) introduces customers' heterogeneity in volume demand (some customers are light users and some customers are heavy users). Operators cannot distinguish between the various users and use second-degree price discrimination. He shows that, under some conditions, profit neutrality still holds. Only when subscription demand is elastic, then firms would have strict preferences for a particular termination charge.[1] When participation is not an issue (i.e. everybody subscribes), then it seems that symmetry matters for profit neutrality. Carter and Wright (2001) allow for some exogenous brand loyalty that generates asymmetric market shares. Providing for this particular type of asymmetry implies that the larger work prefers the access charge to be set at the marginal cost of termination.

This paper analyzes the incentives that operators have to invest in facilities with different levels of quality. A network of better quality is more expensive but may give an important edge to an operator when competing against a rival. We believe this is an interesting and important question that has not been addressed by the literature yet. This paper makes two contributions to the literature. We study the role of access charges with asymmetric firms, where the asymmetry derives from quality choices that affect the amount of calls that customers are willing to make at a given price. In particular we study two versions of a basic model, one where a network's quality influences all the calls made both on-net and off-net, and another one where quality has a "bottleneck" feature, i.e. the off-net quality depends on the minimum level of the qualities chosen by the two competing networks. Our second main contribution concerns the incentives to invest in networks of differing quality, when the level of quality chosen is the source of asymmetries in later stages of competition among network operators. While there are some recent papers that have also studied the role of access charges with asymmetric firms in different settings,[2] there is much less literature related to our second contribution.[3]

We extend the framework of ALRT by introducing an investment stage, prior to price competition. We show that the incentives to invest are influenced by the way termination charges are set. In particular, when the quality of a network has an impact on all calls initiated by own customers (destined both on-net and off-net), we obtain a result of "tacit collusion" even in a symmetric model with two-part pricing. Firms tend to underinvest in quality, and this would be exacerbated if they can negotiate reciprocal termination charges above cost.

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[1] In particular, they may prefer an access charge below marginal cost. See also Schiff (2002). This result would also arise with full participation and termination-based price discrimination (Gans and King, 2001).

[2] This work is similar in spirit to Carter and Wright (2001) but allows for asymmetries to arise from endogenous choices, namely from the quality a firm chooses for its network. See also Armstrong (2001) and Peitz (2002).

[3] Gans (2001) analyzes a model where the access pricing regime may give different incentives to invest; however there is no two-way interconnection feature in his model. DeGraba (2001) also discusses how termination charges can have important effects on the investment decisions of carriers, but his main interest there is on efficient interconnection arrangements when customers derive some utility from receiving calls.

Intuitively, when termination charges are above cost, an increase of own quality relative to the rival creates an access deficit that makes an operator reluctant to invest. This has a positive impact on operators' profits since they avoid a costly battle over investments. We also show that when the quality of off-net calls depends on the interaction between the quality of the two networks, there is another problem, namely that no network has an incentive to jump ahead of the rival. Also in this case, access charges above cost can be used as a collusive instrument.

The remainder of the paper is organized as follows. Section 2 describes the basic model when quality affects both on- and off-net calls. The last stage of the game (price competition) is solved in Section 3, where we also conduct comparative static exercises. Section 4 considers the investment stage. Section 5 solves a different game where quality has a bottleneck feature, i.e. the quality of calls between networks is given by the quality of the lower-quality network. Section 6 concludes.

2. The model

2.1 Demand structure

We imagine a situation with two operators that may decide to enter a telecommunications market. As in LRT, the two networks are differentiated à la Hotelling. A unit mass of consumers is uniformly located on the segment [0,1] while the network operators are located at the two extremities. We denote by 1 (respectively 2) the firm located at the origin (respectively at the end) of the line.

Network operators can use non-linear tariffs. Since the customers' demand function is known, network i cannot do better than offering a two-part tariff:

$$T_i(q) = F_i + p_i q,\ i = 1, 2$$

where the fixed fee F_i can be interpreted as a subscriber line charge and p_i as the marginal price for a call or usage fee.

When a consumer located at x buys from firm i located at x_i, he enjoys a utility given by:

$$y + v_0 - |x - x_i|/(2\sigma) + v_i(p) - F_i$$

where y is the income, v_0 is a fixed surplus component from subscribing ("option value", assumed to be large enough so that all customers always choose to be connected to a network) and $v_i(p)$ is the indirect utility derived from making calls at a price p. The parameter σ represents an index of substitutability between the networks, and its function is to regulate the intensity of price competition.[4]

In order to discuss the important notion that there may be a benefit from having heavy investments in infrastructure, despite the associated costs, we introduce a parameter $k \geq \underline{k} > 0$ that is increasing in the operator's investments and affects quantities and utilities. $\underline{k}$ is some minimum quality level that operators have to supply. We assume that both quantity and indirect utility are increasing in k; in particular we assume that they can be expressed in a multiplicative form:

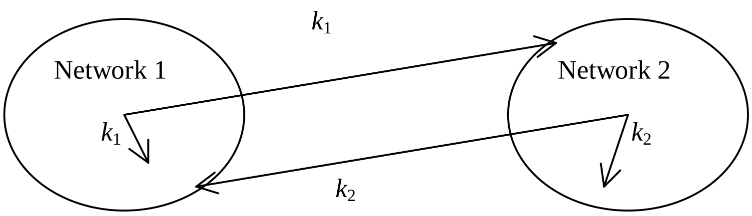
$$q_i(p) = k_i q(p),$$
$$v_i(p) = k_i v(p).$$

This notion captures the idea that investment is a "quality" parameter that consumers enjoy, ceteris paribus, since they will have easier access, faster delivery, less congestion problems, and so on. For instance, an utility function that satisfies our assumption is the following functional form: $u_i(q) = k_i (q/k_i)^{1-1/\eta}/(1-1/\eta)$. This represents the utility from making q calls from a network of quality k_i, and it generates a constant elasticity demand for calls, $q_i(p) = k_i q(p)$ where $q(p) = p^{-\eta}$ and η is the demand elasticity. Moreover, the consumers' indirect utility from making calls when buying from firm i is $v_i(p) = \max_q \{u_i(q) - pq\} = k_i p^{-(\eta-1)}/(\eta - 1) = k_i v(p)$ where $v(p) = p^{-(\eta-1)}/(\eta-1)$.[5]

[4] σ is related to the inverse of transport cost, i.e. the notional costs that consumers pay when they purchase a variety distant from their ideal.

[5] This particular specification is the same as the one in Dessein (2001), but in his paper the author uses the exogenous parameter k in order to differentiate customers according to their calling patters (i.e. heavy users and low users). Here the endogenous parameter k stands for a proxy of the investment made by the networks in setting (or upgrading) the quality of their infrastructures. We should also stress that our results do not depend on constant elasticity, rather on the multiplicative form.

In the first part of this paper we assume that the supplied quality for on-net and off-net services is the same. The structure of the model is then depicted in the following picture:



This assumption describes a situation where the rival's quality does not affect a consumer behavior, once the subscription decision has already occurred. For instance, the number of files sent or downloaded depends more on the bandwidth supported by the subscriber's final link. This could also apply to video-on-demand. Another relevant interpretation is where k represents a certain number of functionalities, such as number of terminals, ease of access etc., that would induce the subscriber to call more, independently of what happens at the terminating end. On the other hand, if one wants to place an emphasis on the simultaneity of a conversation, then this assumption does not seem particularly satisfactory. In Section 5 we will introduce an alternative model where off-net quality depends on the interaction with the rival's quality.

2.2 Market shares

The consumer indifferent between the two networks determines the market share of the two firms. In particular firm i's share is α_i where:

$$\alpha_i = \alpha(p_1, p_2, F_1, F_2) = \frac{1}{2} + \sigma(w_i - w_j) \text{ or } \alpha_i = \alpha(w_1, w_2) = \frac{1}{2} + \sigma(w_i - w_j) \quad (1)$$

where:

$$w_i = v_i(p_i) - F_i = k_i v(p_i) - F_i \,, \; i = 1, 2.$$

is the net surplus for customers connected to network i.

2.3 Cost structure

Both networks have full coverage. Coverage is defined as the fraction of customers who can but are not necessarily served by the network. Serving a customer involves a fixed cost f of connection and billing. Each call has to be originated and terminated. The marginal cost is c per call at the originating end and t at the terminating end. The total marginal cost for a call is thus $c + t$. Networks pay each other an exogenous negotiated or regulated two-way access charge, denoted by a, for terminating each others calls.

Finally, each network incurs a fixed cost $I(k)$ convex in k to provide a service of quality k, $I'(\cdot) > 0$ and $I''(\cdot) > 0$.

The timing of the game is as follows. First the operators invest in quality and interconnection terms are set. Then the operators compete in prices.

3. Solution of stage II

3.1 Price competition

We showed above that market shares α_i are determined directly by net surplus w_i and it is analytically convenient to imagine firms compete in p_i and w_i rather than in p_i and F_i, with $\partial\alpha_i / \partial w_i = \sigma$. In the last stage investments are fixed, hence network 1 has to solve:

$$\max_{w_1, p_1} \Pi_1 = \max_{w_1, p_1} \pi_1 - I(k_1)$$

$$\begin{aligned}\pi_1 = {} & \alpha(w_1, w_2)\{[p_1 - c - t - (1-\alpha(w_1, w_2))(a-t)]k_1 q(p_1) + v_1(p_1) - w_1 - f\} + \\ & + \alpha(w_1, w_2)(1-\alpha(w_1, w_2))(a-t)k_2 q(p_2)\end{aligned}$$

Rearranging, we have:

$$\begin{aligned}\pi_1 = {} & \alpha(w_1, w_2)\{[(p_1 - c - t)k_1 q(p_1)] + v_1(p_1) - w_1 - f\} + \\ & + \alpha(w_1, w_2)(1-\alpha(w_1, w_2))(a-t)[k_2 q(p_2) - k_1 q(p_1)]\end{aligned}$$

Thus, it results:

$$\frac{\partial \pi_1}{\partial p_1} = \alpha_1 k_1 \left[q(p_1) + q'(p_1)(p_1 - c - t) - q(p_1) \right] - \alpha_1 (1 - \alpha_1)(a - t) k_1 q'(p_1) = 0$$

and so we have at equilibrium:

$$p_1^* = c + t + (a - t)(1 - \alpha_1^*), \qquad (2)$$

i.e. the usage fee is equal to the perceived marginal cost. This is a typical result when firms compete in two-part prices. From eq. (2) it is immediate to observe that, when access charges are set above cost, then both firms would charge above marginal cost. In addition, the firm with a market share above 50% would charge less than the rival. This is because, being the larger firm, it would terminate more calls on-net than the rival, hence the perceived marginal cost for the larger firm would be smaller.

The FOC w.r.t. w_1 is:

$$\frac{\partial \pi_1}{\partial w_1} = \sigma \left[(p_1 - c - t) k_1 q(p_1) + v_1(p_1) - f - w_1 \right] - \alpha_1 + \sigma (1 - 2\alpha_1)(a - t) \left(k_2 q(p_2) - k_1 q(p_1) \right) = 0 .$$

Substituting condition (2) and resolving for F_1, we have at equilibrium:

$$F_1^* = \frac{\alpha_1^*}{\sigma} + f - (p_1^* - c - t) k_1 q(p_1^*) - (1 - 2\alpha_1^*)(a - t) \left(k_2 q(p_2^*) - k_1 q(p_1^*) \right) \qquad (3)$$

Similar expressions can be obtained for network 2. After substitution into eq. (1) and rearranging, we have the following expression for the market share of firm 1 at equilibrium:

$$\alpha_1^* = \frac{1}{2} + \frac{\sigma}{3} \left[k_1 v(p_1^*) - k_2 v(p_2^*) + (p_1^* - c - t) k_1 q(p_1^*) - (p_2^* - c - t) k_2 q(p_2^*) \right]. \qquad (4)$$

Finally, in equilibrium, the profit (gross of investment) of network operator 1 is given by:

$$\pi_1^* = \frac{(\alpha_1^*)^2}{\sigma} - (\alpha_1^*)^2 (a - t) \left(k_1 q(p_1^*) - k_2 q(p_2^*) \right). \qquad (5)$$

We can now state the following proposition.[6]

Proposition 1. One of the following conditions is sufficient for the firm with the higher investment to always have more than 1/2 of the market, independently from the way access charges are set:

- Access charges are sufficiently close to termination costs;
- Products are sufficiently differentiated;
- Demand is sufficiently rigid.

Proof. Eq. (4) can be rewritten as $\alpha_1^* = 1/2 + \sigma\left(k_1 W(p_1^*) - k_2 W(p_2^*)\right)/3$ where $W(p) = v(p) + (p - c - t)q(p)$ is total welfare generated by a network of unit quality. Clearly, W is maximized when $p = c + t$. Without loss of generality, let firm 1 be the larger firm, $k_1 > k_2$. Imagine first $a > t$. If we conjecture that firm 1 has more than 50% of the market share, then we know from eq. (2) that both firms charge a mark up on costs and that firm 1 is the cheaper firm. Hence unit welfare $W(p_1^*) > W(p_2^*)$ and the bracket in the expression for the equilibrium market share is unambiguously positive. Similarly, if $a < t$, then both prices are charged below costs, and firm 1 is relatively more expensive, i.e. it is closer to marginal costs. Hence unit welfare for firm 1 is still greater than for firm 2 and the bracket is still positive. On the other hand, imagine $a > t$ and let us conjecture that firm 1 is the smaller firm (a similar argument would apply if $a < t$). Then firm 1 is the more expensive firm and both firms charge a mark up above costs. However, since $p_1 - p_2 = (a - t)(1 - 2\alpha_1)$, if a is close to t, then the price differential is small, hence $W(p_1^*) \approx W(p_2^*)$, implying that the bracket in the expression for market share will be positive, violating our initial conjecture. Similarly, if products are sufficiently differentiated, then market shares will be close to 1/2, making the price differential close to zero, and the same violation would occur. Finally, unit welfare would not change with price differences if demand is perfectly rigid. Hence, if demand is sufficiently rigid, then it cannot be the case that the bigger firm has less than 50% of the market. *QED*

An immediate implication of Proposition 1 is that, if firm 1 is the larger firm, then the extra term in eq. (5) is negative if access is regulated above cost, while the opposite would be true for the smaller firm. Before turning the analysis to the impact of the access charge, we briefly characterize what happens when access is set at cost.

3.1 Access is regulated at cost

Suppose that the access charge is fixed by regulator (or negotiated by the parties) at its marginal cost: $a = t$. From (2), we obtain $p_1^* = p_2^* = c + t$, since the perceived marginal cost is equal to the true marginal cost for both firms, independently from their market shares. Identical costs do not imply identical shares. In fact, we have:

[6] The proof of existence and uniqueness would follow the lines of a similar proof in Laffont *et al*. (1998).

$$\alpha_1^* = \frac{1}{2} + \frac{\sigma}{3} v(c+t)(k_1 - k_2) \ . \qquad (6)$$

If the quality of the two networks differs, market share is not equally shared between the two operators, in line with Proposition 1:

- If $k_1 > k_2$, then $\alpha_1^* > ½ > \alpha_2^*$;
- If $k_1 = k_2$, then $\alpha_1^* = ½ = \alpha_2^*$;
- If $k_1 < k_2$, then $\alpha_1^* < ½ < \alpha_2^*$.

3.2 Comparative statics: effects of access charges and quality on equilibrium shares and prices

Cost-based regulation is quite a typical regulatory benchmark. Our interest here is to consider the impact of stage-I parameters on stage-II equilibrium when access charges are slightly increased above (or decreased below) termination costs. We are able to prove the following results:

Proposition 2. A small variation of the access charge above the marginal cost of termination on market share has:

- a positive impact on prices:

$$\left.\frac{\partial p_i^*}{\partial a}\right|_{a=t} = 1 - \alpha_i^* = \alpha_j^* \ ; \qquad (7)$$

- no local effect on market shares:

$$\left.\frac{\partial \alpha_i^*}{\partial a}\right|_{a=t} = 0 \ ; \qquad (8)$$

- in particular if $\alpha_1^* > (<) 1/\left(1 + \sqrt{k_2 / k_1}\right)$, then the market share of the bigger firm reaches a minimum (maximum) at $a = t$. The reverse is true for the smaller firm.

When access is regulated at cost, prices are not affected by quality, while market shares are:

- $$\left.\frac{\partial p_i^*}{\partial k_i}\right|_{a=t} = \left.\frac{\partial p_i^*}{\partial k_j}\right|_{a=t} = 0 \ ; \qquad (9)$$

- $\left.\frac{\partial \alpha_i^*}{\partial k_i}\right|_{a=t} = \sigma v(c+t)/3 = -\left.\frac{\partial \alpha_i^*}{\partial k_j}\right|_{a=t}$. (10)

Proof. See the Appendix.

A small increase in the termination charge implies that the perceived marginal cost increases, which explains eq. (7), and also why the effect is magnified for the smaller firm (since it terminates relatively more calls off-net than the rival). As far as the market share is concerned, the smaller impact on price for the bigger firm has to be multiplied by its higher quality, while the bigger impact on the price of the smaller firm is multiplied by its smaller quality. These two effects exactly offset each other, giving the result summarized by eq. (8). When access is set at cost, the perceived marginal cost is equal to the true marginal cost, independently from the firm's share of the market, which explains eq. (9). On the other hand, quality has an impact on indirect utility of customers and a higher quality allows a firm to obtain a bigger share of the market (eq. (10)) when access is regulated at cost.

3.3 Effects of the access charge on gross profits

We can state the following result:

Proposition 3. In a neighborhood of $a = t$, a small increase in the access charge increases the profit of the lower quality (smaller) network and decreases the profit of the higher quality (larger) network.

Proof. From eq. (5) we have:

$$\frac{\partial \pi_1^*}{\partial a} = 2\frac{\alpha_1^*}{\sigma}\frac{\partial \alpha_1^*}{\partial a} - (\alpha_1^*)^2\left(k_1 q(p_1^*) - k_2 q(p_2^*)\right) - \\ -(a-t)\left[2\alpha_1^*\frac{\partial \alpha_1^*}{\partial a}\left(k_1 q(p_1^*) - k_2 q(p_2^*)\right) - (\alpha_1^*)^2\left(k_1\frac{\partial q(p_1^*)}{\partial p_1^*}\frac{\partial p_1^*}{\partial a} - k_2\frac{\partial q(p_2^*)}{\partial p_2^*}\frac{\partial p_2^*}{\partial a}\right)\right]$$

Using eq. (7) and (8) when evaluating the previous expression in $a = t$, it results:

$$\left.\frac{\partial \pi_1^*}{\partial a}\right|_{a=t} = -(\alpha_1^*)^2\left(k_1 q(p_1^*) - k_2 q(p_2^*)\right) = -(\alpha_1^*)^2(k_1 - k_2)q(c+t) < 0 \text{ for } k_1 > k_2$$

since $p_1^* = p_2^* = c + t$. *QED*

The reason for this result is that, when the access charge is slightly increased above cost, the effect on market shares is very small. On the other hand, the bigger firm becomes relatively cheaper than the rival, hence relatively more calls will be placed on the rival network than received from it. This creates a net loss on access that negatively affects the larger firm. On the other hand, the smaller firm terminates more calls, earning a small margin on them.

This result complements the "profit neutrality" result that has been found in the seminal papers of ALRT on this topic. It is has already been shown by Dessein (2001) that the profit neutrality may disappear when call flows are unbalanced or when customers may perceive the substitutability of the networks in a different way. Carter and Wright (2001) also find, like in this paper, that asymmetries matter for profit neutrality. The type of asymmetries they consider (brand loyalty) are given exogenously, while in this paper asymmetries may arise from the endogenous decision of firms to invest in quality. Moreover, our results differ from Carter and Wright (2001) since our quality variable affects customers' quantities and associated benefits. This implies that when the access charge is increased above cost there is a first-order effect that does not arise in their paper where brand loyalty has only an impact on the decision to subscribe but no impact on calling behavior. This is why in this paper the effects of the access charge are always pushing in opposite directions for the larger and for the smaller firm, while in Carter and Wright the analysis has to rely on second-order effects.

In the context of our model, we have then shown that for given investment levels, a small increase in the access charge would give benefits to the smaller firms. But does this imply that the smaller firm will have more incentives to invest? This is what we turn to study next.

4. Investment decision

4.1 Access is regulated at cost

Suppose that $a = t$. In stage I network 1 has to solve the following problem:

$$\max_{k_1} \Pi_1 = \pi_1^* - I(k_1) \text{ where } \pi_1^* = \frac{(\alpha_1^*)^2}{\sigma},$$

where α_1* is given by condition (4), i.e. $\alpha_1^*\big|_{a=t} = 1/2 + \sigma v(c+t)(k_1 - k_2)/3$. We fix the investment level of firm 2 and consider the best reaction of firm 1. It results:

$$\frac{\partial \Pi_1}{\partial k_1} = 2\alpha_1^* v(c+t)/3 - I'(k_1) = v(c+t)/3 + 2\sigma v(c+t)^2 (k_1 - k_2)/9 - I'(k_1) = 0 \qquad (11)$$

with SOC $\partial^2 \Pi_1 / \partial k_1^2 = 2\sigma v(c+t)^2 / 9 - I''(k_1) < 0$ that we assume to be satisfied at equilibrium. Implicitly differentiating eq. (11) we have the following slope of the best reply:

$$\frac{\partial k_1}{\partial k_2} = \frac{1}{1 - 9I''/(2\sigma v^2)} < 0\,,$$

i.e. a network would decrease its quality when the rival invests more.

If firms choose their investments simultaneously, then eq. (11) simplifies to $v(c+t)/3 - I'(k_i) = 0$ at the symmetric equilibrium. It is easy to see that, although equilibrium profits are affected by the substitutability parameter, investments are not. On the other hand, investment levels would increase with indirect utility *v* (hence they would decrease in *c* and *t*), having a negative impact on net profits since market shares would be fixed anyway at 1/2 at equilibrium.

4.2 Access is not regulated at cost

In stage I, the FOC w.r.t. quality of firm 1 takes this expression:

$$\frac{\partial \Pi_1}{\partial k_1} = 2\frac{\alpha_1^*}{\sigma}\frac{\partial \alpha_1^*}{\partial k_1} - (a-t)\frac{\partial \Omega}{\partial k_1} - I'(k_1) = 0 \text{ where } \Omega = (\alpha_1^*)^2 \left(k_1 q(p_1^*) - k_2 q(p_2^*)\right). \qquad (12)$$

When $a = t$, the FOC reduces to eq. (11). As in Section 3.2, we are interested to see if, starting at $a = t$, a firm would increase or decrease quality for a given rival's quality when the access charge is slightly increased above cost. In the Appendix we are able to obtain this result:

Proposition 4. Imagine the access charge is slightly increased above cost. Then the optimal investment level of firm *i* changes in the following way:

$$\left.\frac{\mathrm{d}k_i}{\mathrm{d}a}\right|_{a=t} = \frac{\alpha_i^* \left[1/2 + \sigma v(c+t)(k_i - k_j)\right] q(c+t)}{\frac{2}{9}\sigma v(c+t)^2 - I''(k_i)}\,. \qquad (13)$$

Proof. See the Appendix.

An immediate implication of Proposition 4 is the following:

Corollary 5. If the access charge is set slightly above cost, a firm larger that the rival would always decrease its investment level. This would also be true if a firm is smaller than the rival and product are sufficiently differentiated.

Proof. The denominator of eq. (13) is negative by SOC. Imagine first $k_1 > k_2$ (i.e. we know from Proposition 1 that firm 1 is the larger network). From eq. (13) it is then immediate that k_1 would decrease with $a > t$. Now imagine $k_1 < k_2$ (i.e. firm 1 is the smaller network). Using eq. (11) we can rewrite eq. (13) as:

$$sign\left(\left.\frac{\mathrm{d}k_1}{\mathrm{d}a}\right|_{a=t}\right) = -sign(I'(k_1) - 2v(c+t)/9).$$

Substituting the FOC w.r.t. k_1 into the expression for market shares gives $\alpha_1^* = 1/2 + (3I'(k_1) - v(c+t))/(2v(c+t))$. This places a restriction in order to have an interior solution: $0 < I'(k_1) < v(c+t)2/3$. When products are sufficiently differentiated ($\sigma \to 0$) then from eq. (11): $I'(k_1) \to v(c+t)/3$, implying that the overall sign is negative. *QED*

This result can be applied to a situation where investments occur sequentially. With a slight abuse of terminology, we can call firm 2 the "incumbent" and firm 1 the "entrant". Then Corollary 5 states that if the incumbent has a low quality network and the entrant decides to come into the market with a higher-quality network, then the results of Proposition 3 would translate into similar incentives to invest, i.e. an access charge <u>below</u> cost would give higher profits and higher incentives to invest. However, this would not be true with an entrant that has to challenge an incumbent with an existing network of high quality: in this case an access charge below cost would induce higher investments but would diminish the overall profit for the entrant. This result can be rephrased by noting that a policy designed to help a small entrant (i.e. by setting $a > t$) would <u>not</u> translate into higher incentives for the entrant to enter on a bigger scale with higher investments.

Imagine now investment levels are chosen simultaneously. We know from the previous section that when access is regulated around its cost, then the reaction function is downward sloping. What happens in a symmetric situation if firms are left to negotiate termination charges?

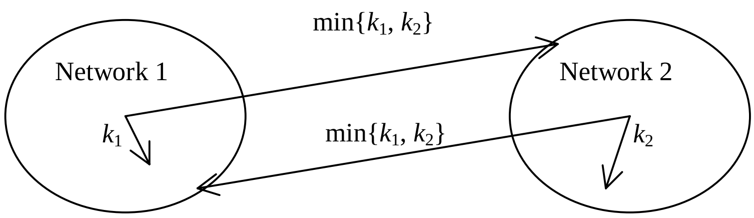
Proposition 6. In a symmetric equilibrium, firms have always an incentive to negotiate access charges above cost. This is detrimental to social welfare.

Proof. In a symmetric equilibrium net profits are $\Pi_i = 1/(4\sigma) - I(k_i^*)$. From eq. (13) we know that in a symmetric equilibrium a small increase in the access charge will induce firms to invest less, which has a positive impact on profits. From eq. (11) this implies that $k_i^* < I'^{-1}(v/3)$. When firms are located at the opposite ends, the socially optimal investment for each firm would be $k_i^{**} = I'^{-1}(v/2) > k_i^*$. Hence, by increasing the charge above costs, firms would depart even further away from the socially optimal levels. *QED*

The result is intuitive since in a symmetric equilibrium market shares would be split equally, and, given two-part pricing, the profit neutrality result would apply to gross profits. What is new here is that there is no neutrality with respect to investments: an access charge above cost would induce both firms to invest less. Firms like that because they would be saving on investment costs without losing anything from gross profits. In other words, we can restore a "tacit collusion" story even in a symmetric equilibrium under two-part pricing.

5. Bottlenecks and interacting qualities

In this section we consider a specification where off-net calls depend on the interaction between the quality of the two networks. In particular we imagine that the quality for services supplied off-net depends on the minimum quality provided by the two networks. The structure of the model is then as follows:



This assumption describes a typical situation in the telecommunications sector where the simultaneity of a conversation plays a central role.[7] While a network is in control of the on-

[7] This assumptions is used, for instance, by Economides (1999) to describe a long distance phone call that requires the use of long distance lines as well as local lines at both ends. In his example, the fidelity of sound of a call would then be the minimum of the qualities of the three services used. An interesting potential extension of

net behavior of its customers as in the previous model, now off-net calls depend on the minimum quality available on the two networks that choose quality non-cooperatively. The presence of a bottleneck facility, i.e. the access to the rival network, influences the demand of own consumers even after subscription decisions have occurred.

5.1 Market shares

W.l.o.g. suppose that network 1 provides the higher-quality network, i.e. $\min\{k_1, k_2\} = k_2$. The consumer indifferent between the two networks determines the market share of the two firms. In particular firm i's share is α_i where:

$$\alpha_i = \alpha(p_1, p_2, F_1, F_2) = \frac{1}{2} + \sigma(w_i - w_j) \text{ or } \alpha_i = \alpha(w_1, w_2) = \frac{1}{2} + \sigma(w_i - w_j) \qquad (14)$$

where:

$$\begin{aligned} w_1 &= v_1(p_1) - F_1 = \alpha_1 k_1 v(p_1) + (1-\alpha_1) k_2 v(p_1) - F_1 \\ w_2 &= v_2(p_2) - F_2 = (1-\alpha_1) k_2 v(p_2) + \alpha_1 k_2 v(p_2) - F_2 = k_2 v(p_2) - F_2 \end{aligned}$$

is the net surplus for customers connected to network i. Note that the net surplus for consumers connected to network 2 does not change with respect to the previous case, since on-net and off-net calls have the same quality; this is not true for net surplus of customers connected to network 1 which depends also on the lower quality of interconnection that decreases the volume of off-net calls.

5.2 Stage II competition

Imagine firms compete in p_i and w_i. In the last stage k_i's are fixed, hence network 1 solves:

$$\max_{w_1, p_1} \Pi_1 = \max_{w_1, p_1} \pi_1 - I(k_1)$$

our model could be to compare a situation of facility based competition (FBC) - where operators have their own facilities - with local loop unbundling (LLU) - where one firm (the incumbent, say firm 1) has full coverage, while the other firm (the entrant) leases the local loop from the incumbent. FBC corresponds to the model analyzed here (each firm can choose its own k_i that is applicable to on-net calls, while off-net there would be an interaction between the two investments). On the other hand, under LLU, it is only the incumbent that invests in access facilities, hence there would be only k_1 in all the expressions.

$$\pi_1 = \alpha(w_1,w_2)\{(p_1 - c - t)q(p_1)[\alpha(w_1,w_2)k_1 + (1-\alpha(w_1,w_2))k_2] + v_1(p_1) - w_1 - f\} +$$
$$+ \alpha(w_1,w_2)(1-\alpha(w_1,w_2))(a-t)k_2(q(p_2) - q(p_1))$$

For network 2, since the quality for on-net and off-net calls remains the same, profit is:

$$\max_{w_2,p_2} \Pi_2 = \max_{w_2,p_2} \pi_2 - I(k_2)$$
$$\pi_2 = (1-\alpha(w_1,w_2))[(p_2 - c - t)k_2 q(p_2) + v_2(p_2) - w_2 - f] +$$
$$+ \alpha(w_1,w_2)(1-\alpha(w_1,w_2))(a-t)k_2(q(p_1) - q(p_2))$$

We should note that both expressions for profits are different from those obtained in Section 3.1. Firm 1's expression differs for the reasons described above (which influence both indirect utility and quantities). Firm 2's expression differ only for the quantity of incoming calls received from the rival network, which are now reduced compared to the previous model.

In the Appendix we characterize the equilibrium.[8] We report here only the expressions for call prices (since the expressions for fixed fees are long):

$$p_1^* = c + t + (a-t)(1-\alpha_1^*)\frac{k_2}{\bar{k}}, \text{ with } \bar{k} = \alpha_1^* k_1 + (1-\alpha_1^*)k_2, \tag{15}$$

$$p_2^* = c + t + \alpha_1^*(a-t). \tag{16}$$

where α_1^* is the market share of network 1 at the equilibrium. Call prices are set to maximize joint surplus between a network and its customers (and fixed fees are then set to split such surplus created, depending on the intensity of price competition). Since own quality affects joint surplus for firm 2 in a multiplicative way, this explains why it still charges the perceived marginal cost, as in Section 3. On the other hand, joint surplus for firm 1 is weighted down for off-net calls by the lower rival's quality, which results in the additional factor (less than 1) in eq. (15). In particular, when $a = t$, we have $p_1^* = p_2^* = c + t$ and:

$$\alpha_1^*\Big|_{a=t} = \frac{1/2 - \sigma v(c+t)(k_1 - k_2)/3}{1 - \sigma v(c+t)(k_1 - k_2)} \tag{17}$$

In order for an interior equilibrium to exist, it is needed that products are sufficiently differentiated (or that quality differences are not too big):

$$\sigma v(c+t)(k_1 - k_2) < 3/4 \,. \tag{18}$$

It is straightforward to show that, whenever $k_1 > k_2$, it is always $\alpha_1^*\big|_{a=t} > 1/2$. We are able to show that this result is more general and we obtain a result parallel to Proposition 1:

Proposition 7. One of the following conditions is sufficient for the firm with the higher investment to always have more than 1/2 of the market, independently from the way access charges are set:

- Access charges are sufficiently close to termination costs;
- Products are sufficiently differentiated;
- Demand is sufficiently rigid.

Proof. See the Appendix.

Despite the larger network is being penalized by the quality of the bottleneck, it still gets more than 50% of the market, as in Section 3. Comparative statics are also similar to Section 3 (Propositions 2 and 3) with two notable differences:

Proposition 8. A small variation of the access charge above the marginal cost of termination on market share has:

- a positive impact on prices:

$$\left.\frac{\partial p_1^*}{\partial a}\right|_{a=t} = (1-\alpha_1^*)\frac{k_2}{\bar{k}} > 0 \,;$$

$$\left.\frac{\partial p_2^*}{\partial a}\right|_{a=t} = \alpha_1^* > 0 \,;$$

- a <u>positive</u> impact on the market share of the larger firm:

$$\left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t} > 0 \,.$$

When access is regulated at cost, prices are not affected by investments, while market shares are:

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[8] As before, we refer to Laffont *et al*. (1998) for the proof of existence and uniqueness of the equilibrium.

- $\left.\frac{\partial p_i^*}{\partial k_i}\right|_{a=t} = \left.\frac{\partial p_i^*}{\partial k_j}\right|_{a=t} = 0$;

- $\left.\frac{\partial \alpha_i^*}{\partial k_i}\right|_{a=t} = \frac{\frac{\sigma}{6} v(c+t)}{\left[1-\sigma v(c+t)(k_1-k_2)\right]^2} = -\left.\frac{\partial \alpha_i^*}{\partial k_j}\right|_{a=t}$. (19)

In a neighborhood of $a = t$, a small increase in the access charge increases the gross profit of <u>both</u> networks:

- $\left.\frac{\partial \pi_i}{\partial a}\right|_{a=t} > 0, i = 1,2.$

Proof. See the Appendix.

The key novelty here is that an increase in the access charge above cost has an impact on market shares. The reason is that now the impact of a higher termination charge is to increase both prices, but the impact is reduced for firm 1 - compared to the previous case - by the "quality-adjusted" factor in eq. (15). This gives an edge to firm 1 that is able to capture more customers. This is central to understand why both firms would be better off with termination charges above cost, for given investment levels. For firm 1, the market expansion effect dominates on the increased access deficit. For firm 2, despite the loss of market share, the traffic imbalance of off-net calls still generates a net positive effect.

5.3 Investment decisions

Imagine $a = t$. It matters if a firm is smaller or larger than the rival. The net profit of firm 1 is:

$$\Pi_1 = \frac{\alpha_1^{*2}}{\sigma}(1/2 + M_2^q) - I(k_1) \text{ where}$$

$$\begin{cases} \alpha_1^* = \dfrac{1/2 - \sigma v(c+t)(k_1-k_2)/3}{1-\sigma v(c+t)(k_1-k_2)} & \text{if } k_1 > k_2 \\ M_2^q = 1/2 - \sigma v(c+t)(k_1-k_2) & \end{cases}$$

$$\begin{cases} \alpha_1^* = \dfrac{1/2 - \sigma v(c+t)(k_2-k_1)2/3}{1-\sigma v(c+t)(k_2-k_1)} & \text{if } k_1 < k_2. \\ M_2^q = 1/2 - \sigma v(c+t)(k_2-k_1) \end{cases}$$

Consider first the best reply from above ($k_1 > k_2$). The expression for profit is re-written as:

$$\Pi_1 = \alpha_1^* [1/2 - \sigma v(c+t)(k_1-k_2)/3]/\sigma - I(k_1).$$

Using eq. (19), the derivative w.r.t. k_1 is:

$$\frac{\partial \Pi_1}{\partial k_1} = -\frac{\alpha_1^* v(c+t)}{3}\left[\alpha_1^* - \frac{2}{3}\frac{\sigma v(c+t)(k_1-k_2)}{1-\sigma v(c+t)(k_1-k_2)}\right] - I'(k_1). \tag{20}$$

As a result, profit is decreasing in own quality (a sufficient condition is that products are sufficiently differentiated, even without taking into account investment costs).[9] Hence the best reply from above is to match the rival's quality. The best reply from below is obtained from maximizing:

$$\Pi_1 = \alpha_1^* [1/2 - \sigma v(c+t)(k_2-k_1)2/3]/\sigma - I(k_1)$$

$$\frac{\partial \Pi_1}{\partial k_1} = \frac{5\alpha_1^* v(c+t)}{6}\frac{1-\frac{4}{5}\sigma v(c+t)(k_2-k_1)}{1-\sigma v(c+t)(k_2-k_1)} - I'(k_1) = 0. \tag{21}$$

Since for an interior equilibrium to exist $\sigma v(c+t)(k_2-k_1) < 3/4$, the first term in the previous FOC is always positive. Hence there are two cases. Either the solution to eq. (21) (denoted as $\hat{k}_1(k_2)$) lies to the left of the rival's quality, or – if it does not – the best reply from below is to match the rival's quality. Putting all this together, brings to the following best response (an example of a typical reaction function is presented in figure 1):

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[9] Recall that an interior equilibrium exists when $0 < k_1 - k_2 < 3\sigma v/4$. From eq. (20) it can be seen that the first term is negative when $0 < k_1 - k_2 < \sigma v/2$. When $\sigma v/2 < k_1 - k_2 < 3\sigma v/4$, then the first term in eq. (20) is positive and increasing in k_1. So we have to check that it does not pay for firm 1 to try to get the whole market. If firm 1 mimics firm 2 it obtains $1/(4\sigma) - I(k_2)$. On the other hand, if it tries to get the whole market by setting $k_1 = k_2 + 3\sigma v/4$, it would obtain $1/(4\sigma) - I(k_1)$ which is strictly lower than the previous value.

$k_1^* = \min\{k_2, \hat{k}_1(k_2)\}$.

If we imagine investments occur sequentially and we imagine firm 1 is the entrant, this result is saying that an entrant will never want to jump ahead of the incumbent. Either it will match the rival, or it will invest strictly less.

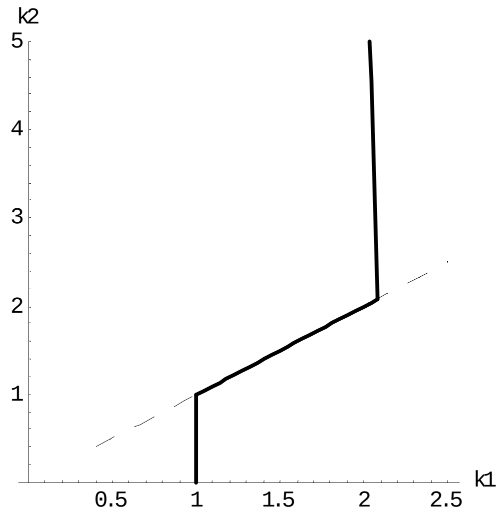

Figure 1: Firm 1's reaction function [$I(k) = k^2/2$, $\sigma = 0.01$, $v(\cdot) = 5$, $\underline{k} = 1$][10]

If investments occur simultaneously, there is a continuum of symmetric equilibria, where the highest investment occurs when eq. (21) is satisfied, $5v(c+t)/12 - I'(k_1) = 0$:

$$k_i^* \in [\underline{k}, I'^{-1}(5v(c+t)/12)].$$

where $\underline{k}$ is some minimum quality level that operators have to supply. This result is quite worrying. Not only firms are investing less than the efficient level (however, they <u>may</u> invest more that in Section 4, ceteris paribus),[11] but they also have a serious coordination problem: if a rival invests very little, also the other firm will do the same. The preferred equilibria, from the firms' point of view, are those corresponding to low quality levels.

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[10] In setting these values, we also made sure that the equilibrium exists and it is unique. In particular, we checked that it never pays to try to monopolize the market.

[11] Recall that the efficient investment level is $k_i^{**} = I'^{-1}(v/2)$, while the investment chosen in the previous section in a symmetric equilibrium was $k_i^* = I'^{-1}(v/3)$.

The very last question that we address is whether the access charge could still be used as a device of “tacit” collusion. We focus only on the case of simultaneous investments since it is cumbersome to characterize the best reply of a firm for any level of investments of the rival. In a symmetric equilibrium, we can prove that “tacit collusion” under two-part pricing holds also in this case of “bottleneck” quality:

Proposition 9. In a symmetric equilibrium, firms decrease their investments when access charges are set slightly above cost:

$$\left.\frac{dk_i}{da}\right|_{\substack{a=t\\ k_i=k_j}} = -\frac{-q(c+t)/24}{\frac{1}{18}\sigma v(c+t)^2 - I''(k_i)} < 0 .$$

Firms then have an incentive to negotiate access charges above cost, which would be detrimental to social welfare.

Proof. See the Appendix.

The multiplicity of equilibria that we have described with $a = t$, would still be present with $a > t$. However, by agreeing to do so, firms reduce the upper range of equilibrium investment levels, getting rid of those equilibria that are “good” from the point of view of social welfare but “bad” from the firms’ point of view.

6. Conclusions

This paper has analyzed the incentives that network operators have to invest in facilities with different levels of quality. We have extended the framework of ALRT by introducing an investment stage, prior to price competition. Quality is costly but beneficial to customers that would be calling more. We have shown that the level of termination charges has important implications in terms of incentives to invest. In particular, the standard result of profit neutrality with respect to access charges does not hold any more. When the quality of a network influences both on-net and off-net calls, we have shown that:

- For given investment levels, a small firm would benefit from a charge above cost for while the opposite would hold for a large firm;
- Once investment is endogenized, the larger firm would always react to higher termination charges by reducing investment; this would also be the case for the smaller firm as long as products are sufficiently differentiated;

- Since in a symmetric equilibrium net profits are simply $1/(4\sigma)$ - $I(k)$, this implies that firms would negotiate an access charge above cost;
- In other words, we can restore a "tacit collusion" story *à la* ALRT even in a symmetric equilibrium with two-part pricing. Tacit collusion arises not because of a "raise each other's cost" effect, but because of an effect that we may describe as "diminish each other's incentives to invest". Although gross profits are fixed at the Hotelling level, an access charge will be used in order to induce the firms to invest less, i.e., to save on costs. This is detrimental to social welfare.
- When the quality of off-net calls depends on the interaction between the quality of the two networks (i.e. quality is a "bottleneck"), there is another serious problem, namely that no network has an incentive to jump ahead of the rival.
- "Tacit collusion" with two-part pricing in symmetric models holds also when quality has a bottleneck feature.[12]

To conclude, this paper sheds some light on the normative question on how to regulate termination charges. In particular, our results suggest that private negotiations over reciprocal access charges would not be efficient, and that firms would strictly prefer to set access charges above termination costs. On the contrary, in order to induce firms to invest in an efficient manner, access charges should be set below costs. Hence our results show that the current understanding of interconnection may not be right in a dynamic perspective. Many countries (including the US since the Telecommunications Act of 1996) have put in practice regulations where the calling party's network should pay the called party's network a termination fee based on the Long Run Incremental Cost (LRIC) of the traffic sensitive facilities of the receiving network used to terminate the call. Our model suggests that LRIC may not be a good benchmark when the incentives to invest are taken into consideration. Of course, setting the "right" level of access charges below cost would be a very delicate job in practice, given that regulators may not be able to observe the cost structure of a network and that it is difficult to give an operator an incentive to report it truthfully. All in all, it then look

[12] We checked the robustness of our results when firms offer two-part discriminatory prices (i.e. the on-net and the off-net call prices can differ). Also in that case firms would invest less in symmetric models when the access charge is increased slightly above cost. This is important since previous literature has found that – without taking into account asymmetries or investment choices – firms that offer two-part discriminatory pricing would select access charges below cost (Gans and King, 2001). The investment effect that we have discussed can well prevail over the result of Gans and King (in particular, when the investment function is not too convex); hence tacit collusion over high termination charges in order not to invest "too" much would still be a serious concern. This would hold true for both specifications of the model we studied in this paper.

interesting a regime based on "bill and keep" arrangements, i.e. a zero interconnection rate, not because the optimal interconnection need be zero, but because it would be simple to put in practice and it would give higher incentives to invest. The Federal Communications Commission (FCC) has recently initiated a proceeding to seek comments on implementing a regime based on "bill and keep". Such systems have been advocated as a way of sharing efficiently the value created by a call when both callers and receivers benefit from it (DeGraba, 2000). Our results, based on incentives to invest, reinforce their good properties.

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Appendix

Proof of Proposition 2.

Denote with s_1 and s_2 the gross utility minus the cost of calls provided by network 1 and network 2. Then, we have for firm 1 (similar expressions are obtained for firm 2):

$$s_1 = k_1 v(p_1^*) + (p_1^* - c - t) k_1 q(p_1^*) .$$

The market share can be rewritten $\alpha_1^* = 1/2 + \sigma(s_1 - s_2)/3$. We now want to analyze the effect of a variation of a on market share, in a neighborhood of $a = t$, i.e.:

$$\left. \frac{\partial \alpha_1^*}{\partial a} \right|_{a=t} = \frac{\sigma}{3} \left(\frac{\partial s_1}{\partial a} - \frac{\partial s_2}{\partial a} \right)_{a=t} . \qquad \text{(a1)}$$

We have:

$$\frac{\partial s_i}{\partial a} = \frac{\partial s_i}{\partial p_i} \frac{\partial p_i^*}{\partial a}, \; i = 1,2 ,$$

where

$$\frac{\partial p_1^*}{\partial a} = (1 - \alpha_1^*) - \frac{\partial \alpha_1^*}{\partial a}(a - t) \; \text{ and } \; \left. \frac{\partial p_1^*}{\partial a} \right|_{a=t} = 1 - \alpha_1^* .$$

Thus,

$$\frac{\partial s_1}{\partial a} = \left[- k_1 q(p_1^*) + k_1 q(p_1^*) + (p_1^* - c - t) k_1 q'(p_1^*) \right] \frac{\partial p_1^*}{\partial a} = (p_1^* - c - t) k_1 q'(p_1^*) \frac{\partial p_1^*}{\partial a} . \qquad \text{(a2)}$$

Valuing the above condition at $a = t$, since $p_1* = c + t$, we have $\left. \frac{\partial s_1}{\partial a} \right|_{a=t} = 0$. In the same way, it results $\left. \frac{\partial s_2}{\partial a} \right|_{a=t} = 0$. So, we obtain:

$$\left. \frac{\partial \alpha_1^*}{\partial a} \right|_{a=t} = \frac{\sigma}{3} \left(\frac{\partial s_1}{\partial a} - \frac{\partial s_2}{\partial a} \right)_{a=t} = 0 . \qquad \text{(a3)}$$

Market share reaches an optimum in $a = t$. We do not know yet if this optimum is a maximum or a minimum. Let us consider the second derivatives. From (a2) we obtain:

$$\frac{\partial^2 s_1}{\partial a^2} = k_1 q'(p_1^*) \left(\frac{\partial p_1^*}{\partial a} \right)^2 + (p_1^* - c - t) k_1 \left[q''(p_1^*) \left(\frac{\partial p_1^*}{\partial a} \right)^2 + q'(p_1^*) \frac{\partial^2 p_1^*}{\partial a^2} \right] \qquad \text{(a4)}$$

where:

$$\frac{\partial^2 p_1^*}{\partial a^2} = -2\frac{\partial \alpha_1^*}{\partial a} - \frac{\partial^2 \alpha_1^*}{\partial a^2}(a-t)\text{, and } \left.\frac{\partial^2 p_1^*}{\partial a^2}\right|_{a=t} = -2\left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t} = 0.$$

Condition (a4), evaluated in $a = t$, thus becomes:

$$\left.\frac{\partial^2 s_1}{\partial a^2}\right|_{a=t} = k_1 q'(p_1^*)\left(\left.\frac{\partial p_1^*}{\partial a}\right|_{a=t}\right)^2 = (1-\alpha_1^*)^2 k_1 q'(c+t).$$

In the same way, we obtain:

$$\left.\frac{\partial^2 s_2}{\partial a^2}\right|_{a=t} = (\alpha_1^*)^2 k_2 q'(c+t)$$

where α_1* is given by eq. (4) in the main text. If $\left.\frac{\partial^2 s_1}{\partial a^2}\right|_{a=t} > \left.\frac{\partial^2 s_2}{\partial a^2}\right|_{a=t}$, then the optimal solution in (a3) is a minimum and thus a small increase in the access charge above the marginal cost of termination yields ad increase in the market share of firm 1 (and a decrease for firm 2). It results:

$$(1-\alpha_1^*)^2 k_1 q'(c+t) > (\alpha_1^*)^2 k_2 q'(c+t),$$

rearranging, since $q' < 0$, we obtain:

$$(1-\alpha_1*)^2 k_1 < (\alpha_1*)^2 k_2,$$

which is satisfied when:

$$\alpha_1^* > \frac{1}{1+\sqrt{k_2/k_1}}.$$

We now turn to the impact of stage-I investment levels on equilibrium in stage II:

$$\frac{\partial \alpha_1^*}{\partial k_1} = \frac{\sigma}{3}\left(\frac{\partial s_1}{\partial k_1} - \frac{\partial s_2}{\partial k_1}\right) \tag{a5}$$

where

$$\begin{aligned} \frac{\partial s_1}{\partial k_1} &= v(p_1^*) + (p_1^* - c - t)[q(p_1^*) + k_1 q'(p_1^*)\frac{\partial p_1^*}{\partial k_1}] \\ \frac{\partial s_2}{\partial k_1} &= (p_2^* - c - t)k_2 q'(p_2^*)\frac{\partial p_2^*}{\partial k_1} \end{aligned} \tag{a6}$$

while the effects of the own and rival quality on the equilibrium prices are the following:

$$\frac{\partial p_1^*}{\partial k_1} = -\frac{\partial \alpha_1^*}{\partial k_1}(a-t),\ \ \frac{\partial p_1^*}{\partial k_2} = -\frac{\partial \alpha_1^*}{\partial k_2}(a-t) \text{ with } \left.\frac{\partial p_1^*}{\partial k_1}\right|_{a=t} = \left.\frac{\partial p_1^*}{\partial k_2}\right|_{a=t} = 0\,.$$

As a result, (a5) simplifies to $\left.\frac{\partial \alpha_1^*}{\partial k_1}\right|_{a=t} = \sigma v(c+t)/3$. Similarly, $\left.\frac{\partial \alpha_1^*}{\partial k_2}\right|_{a=t} = -\sigma v(c+t)/3$.

Proof of Proposition 4.
We start by implicitly differentiating the FOC given by eq. (12):

$$\begin{aligned}&\frac{2}{\sigma}\left[\left(\frac{\partial \alpha_1^*}{\partial k_1}\right)^2 \mathrm{d}k_1 + \alpha_1^* \frac{\partial^2 \alpha_1^*}{\partial k_1^2}\mathrm{d}k_1 + \frac{\partial \alpha_1^*}{\partial k_1}\frac{\partial \alpha_1^*}{\partial a}\mathrm{d}a + \alpha_1^*\frac{\partial^2 \alpha_1^*}{\partial k_1 \partial a}\mathrm{d}a\right] - \frac{\partial \Omega}{\partial k_1}\mathrm{d}a - \\ &-(a-t)\left(\frac{\partial^2 \Omega}{\partial k_1^2}\mathrm{d}k_1 + \alpha_1^*\frac{\partial^2 \Omega}{\partial k_1 \partial a}\mathrm{d}a\right) - I''(k_1)\mathrm{d}k_1 = 0\end{aligned} \tag{a7}$$

Since we want to calculate all the previous expressions at $a = t$, we can use the results in Proposition 2 to simplify calculations:

$$\frac{\partial \Omega}{\partial k_1} = 2\alpha_1^* \frac{\partial \alpha_1^*}{\partial k_1}\left(k_1 q(p_1^*) - k_2 q(p_2^*)\right) + (\alpha_1^*)^2\left(q(p_1^*) + k_1 q'(p_1^*)\frac{\partial p_1^*}{\partial k_1} - k_2 q'(p_2^*)\frac{\partial p_2^*}{\partial k_1}\right)$$

$$\left.\frac{\partial \Omega}{\partial k_1}\right|_{a=t} = 2\alpha_1^* \sigma v(c+t)(k_1 - k_2)q(c+t)/3 + (\alpha_1^*)^2 q(c+t) = \alpha_1^*\left[1/2 + \sigma v(c+t)(k_1 - k_2)\right]q(c+t)$$

We still have to compute two terms in (a7) (the second derivatives) From (a5):

$$\left.\frac{\partial^2 \alpha_1^*}{\partial a \partial k_1}\right|_{a=t} = \frac{\sigma}{3}\left.\left(\frac{\partial^2 s_1}{\partial a \partial k_1} - \frac{\partial^2 s_2}{\partial a \partial k_1}\right)\right|_{a=t}.$$

In particular, from eq. (a6) we get:

$$\frac{\partial^2 s_1}{\partial a \partial k_1} = k_1 q'(p_1^*)\frac{\partial p_1^*}{\partial k_1}\frac{\partial p_1^*}{\partial a} + (p_1^* - c - t)\left(q'(p_1^*)\frac{\partial p_1^*}{\partial a} + k_1 q''(p_1^*)\frac{\partial p_1^*}{\partial k_1}\frac{\partial p_1^*}{\partial a} + k_1 q'(p_1^*)\frac{\partial^2 p_1^*}{\partial k_1 \partial a}\right).$$

Then, it results:

$$\left.\frac{\partial^2 s_1}{\partial a \partial k_1}\right|_{a=t} = 0\,.$$

Similarly $\left.\frac{\partial^2 s_2}{\partial a \partial k_1}\right|_{a=t} = 0$, implying that $\left.\frac{\partial^2 \alpha_1^*}{\partial a \partial k_1}\right|_{a=t} = 0$. It is also immediate to show that $\left.\frac{\partial^2 \alpha_1^*}{\partial k_1^2}\right|_{a=t} = 0$. As a result at $a = t$, eq. (a7) simplifies to:

$$\tfrac{2}{9}\sigma v(c+t)^2 \mathrm{d}k_1 - \alpha_1^* \left[1/2 + \sigma v(c+t)(k_1 - k_2)\right] q(c+t)\mathrm{d}a - I''(k_1)\mathrm{d}k_1 = 0 .$$

Implying:

$$\left.\frac{\mathrm{d}k_1}{\mathrm{d}a}\right|_{a=t} = \frac{\alpha_1^* \left[1/2 + \sigma v(c+t)(k_1 - k_2)\right] q(c+t)}{\tfrac{2}{9}\sigma v(c+t)^2 - I''(k_1)} .$$

Proof of Proposition 7.
Rearranging condition (14), we can write:

$$\alpha_1 = \alpha(p_1, p_2, F_1, F_2) = \frac{M_1^q + \sigma(F_2 - F_1)}{M_1^q + M_2^q} \tag{a8}$$

where

$$M_1^q = \frac{1}{2} + \sigma k_2 \left(v(p_1) - v(p_2)\right)$$
$$M_2^q = \frac{1}{2} + \sigma \left(k_2 v(p_2) - k_1 v(p_1)\right)$$

which, following Laffont *et al.* (1998), can be interpreted, respectively, as a "measure" of market share of network 1 related only to off-net calls (M_1^q) and of market share of network 2 related to on-net calls (M_2^q) according to the different levels of quality that characterize the two services.[13] After maximization, we obtain at the equilibrium the following conditions for network 1:

$$p_1^* = c + t + (a-t)(1-\alpha_1^*)\frac{k_2}{\bar{k}}, \text{ with } \bar{k} = \alpha_1^* k_1 + (1-\alpha_1^*)k_2 , \tag{a9}$$

$$F_1^* = \frac{\alpha_1^*}{\sigma}(M_1^q + M_2^q) + f - (p_1^* - c - t)\left[\bar{k} + \alpha_1^*(k_1 - k_2)\right] q(p_1^*) - \\ -(1-2\alpha_1^*)(a-t)k_2\left(q(p_2^*) - q(p_1^*)\right) \tag{a10}$$

while for network 2 it results:

$$p_2^* = c + t + \alpha_1^*(a-t) , \tag{a11}$$

13 This interpretation would also hold if the fixed components of the two-part tariffs are taken into account. Equivalently, M_i^q can be interpreted as a measure of network *i*'s share when consumers formulate the most pessimistic assumption about that market share (i.e. they expect everybody to join network *j*).

$$F_2^* = \frac{(1-\alpha_1^*)}{\sigma}(M_1^q + M_2^q) + f - (p_2^* - c - t)k_2 q(p_2^*) + (1-2\alpha_1^*)(a-t)k_2\left(q(p_1^*) - q(p_2^*)\right) \quad \text{(a12)}$$

where α_1^* is the market share of network 1 at the equilibrium. Substituting conditions (a9), (a10), (a11) and (a12) into (14), market share in equilibrium is given by:

$$\alpha_1^* = \frac{1}{3} + \frac{1}{3}\frac{M_1^q + \sigma\Theta}{M_1^q + M_2^q} \quad \text{(a13)}$$

where M_1^q and M_2^q are now evaluated in p_i*, with i = 1, 2, and

$$\Theta = (p_1^* - c - t)q(p_1^*)\left[\bar{k} + \alpha_1^*(k_1 - k_2)\right] - (p_2^* - c - t)k_2 q(p_2^*) .$$

Evaluating the equilibrium market share in a = t, since $\Theta\big|_{a=t} = 0$, $M_1^q\big|_{a=t} = 1/2$ and $M_2^q\big|_{a=t} = 1/2 + \sigma v(c+t)(k_2 - k_1)$ and rearranging, we have eq. (17).

To prove Proposition 7, from eq. (a13) we have that $\alpha_1^* > 1/2$ if and only if $M_1^q + 2\sigma\Theta > M_2^q$, which can be rearranged to obtain:

$$2k_2 W(p_1^*) + (k_1 - k_2)\left[v(p_1^*) + (p_1^* - c - t)q(p_1^*)4\alpha_1^*\right] > 2k_2 W(p_2^*) , \quad \text{(a14)}$$

where $W(\cdot)$ is total welfare generated at certain price by a network of unit quality. We also know that the difference in usage fees is given by the following condition:

$$p_1^* - p_2^* = (a-t)\frac{(1-\alpha_1^*)^2 k_2 - \alpha_1^{*2} k_1}{\bar{k}} . \quad \text{(a15)}$$

Let us first conjecture that firm 1 has a bigger market share than the rival (remember that $k_1 > k_2$), then, if $a > t$, the RHS of eq. (a15) is negative, both prices are above marginal cost and inequality (a14) is satisfied. If $a < t$ we can work along the same line; however we also need an additional sufficient condition to ensure that the square bracket in (a14) is positive: $v(p_1^*) + (p_1^* - c - t)q(p_1^*)4\alpha_1^* = v(p_1^*) + (a-t)q(p_1^*)4\alpha_1^*(1-\alpha_1^*)k_2/\bar{k} > v(p_1^*) + (a-t)q(p_1^*) > 0$ which is ensured when $v(p_1^*)$ is sufficiently high. If we now conjecture that firm 1 is the smaller firm, we would always get a contradiction. Imagine $a > t$. If a is close to t, the price difference is small but (a14) would then be violated. If products are differentiated, then market shares would be close to ½, but then the price difference would be negative and (a14) would be violated. If demand is inelastic, (a14) is immediately violated. Similar contradictions would arise if $a < t$.

To conclude the characterization of stage II, equilibrium gross profits are given by the following expressions:

$$\pi_1 = \frac{\alpha_1^{*2}}{\sigma}(M_1^q + M_2^q) + \alpha_1^{*2}(a-t)k_2\left(q(p_2^*) - q(p_1^*)\right) + \alpha_1^{*2}(p_1^* - c - t)q(p_1^*)(k_2 - k_1) \quad \text{(a16)}$$

$$\pi_2 = \frac{(1-\alpha_1^*)^2}{\sigma}(M_1^q + M_2^q) + (1-\alpha_1^*)^2 (a-t)k_2\left(q(p_1^*) - q(p_2^*)\right). \qquad \text{(a17)}$$

Proof of Proposition 8.

From (a9) and (a11) we have the following comparative statics on prices:

$$\frac{\partial p_1^*}{\partial a} = (1-\alpha_1^*)\frac{k_2}{\bar{k}} - \frac{\partial \alpha_1^*}{\partial a}(a-t)\frac{k_2}{\bar{k}} \text{ and } \left.\frac{\partial p_1^*}{\partial a}\right|_{a=t} = (1-\alpha_1^*)\frac{k_2}{\bar{k}}$$

$$\frac{\partial p_2^*}{\partial a} = \alpha_1^* + \frac{\partial \alpha_1^*}{\partial a}(a-t) \text{ and } \left.\frac{\partial p_2^*}{\partial a}\right|_{a=t} = \alpha_1^*$$

$$\left.\frac{\partial p_1^*}{\partial k_1}\right|_{a=t} = \left.\frac{\partial p_1^*}{\partial k_2}\right|_{a=t} = 0 \text{ .}$$

Let us now analyze the effect of *a* on the equilibrium market share given by (a13). We have:

$$\frac{\partial \alpha_1^*}{\partial a} = \left[\left(\frac{\partial M_1^q}{\partial a} + \sigma\frac{\partial \Theta}{\partial a}\right)(M_1^q + M_2^q) - \frac{\partial (M_1^q + M_2^q)}{\partial a}(M_1^q + \sigma\Theta) \right] / \left[3(M_1^q + M_2^q)^2\right] \qquad \text{(a18)}$$

Evaluating all the above terms in $a = t$, it results:

$$\left.\frac{\partial M_1^q}{\partial a}\right|_{a=t} = \sigma k_2 q(c+t)\left[\alpha_1^* - (1-\alpha_1^*)k_2/\bar{k}\right]$$

$$\left.\frac{\partial M_1^q}{\partial a} + \frac{\partial M_2^q}{\partial a}\right|_{a=t} = \sigma q(c+t)(1-\alpha_1^*)\frac{k_2}{\bar{k}}(k_1 - k_2)$$

$$\left.\frac{\partial \Theta}{\partial a}\right|_{a=t} = (1-\alpha_1^*)k_2 q(c+t)\left[1 + \alpha_1^*(k_1 - k_2)/\bar{k}\right] - \alpha_1^* k_2 q(c+t) \text{ .}$$

where α_1^* is the equilibrium market share evaluated in $a = t$, given by eq. (17).
Given that $\Theta|_{a=t} = 0, M_1^q|_{a=t} = 1/2, M_2^q|_{a=t} = 1/2 + \sigma v(c+t)(k_2 - k_1)$, substituting in (a18) and rearranging it results at an interior equilibrium:

$$\left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t} = \frac{\sigma(1-\alpha_1^*)k_2 q(c+t)\frac{k_1 - k_2}{\bar{k}}\left(2\alpha_1^* M_2^q|_{a=t} + \alpha_1^* - 1/2\right)}{3\left[1 + \sigma v(c+t)(k_2 - k_1)\right]^2} = \frac{\sigma(1-\alpha_1^*)^2 k_2 q(c+t)\frac{k_1 - k_2}{\bar{k}}}{3\left[1 - \sigma v(c+t)(k_1 - k_2)\right]} > 0 \text{ .}$$

It is immediate to obtain:

$$\left.\frac{\partial \alpha_1^*}{\partial k_1}\right|_{a=t} = -\left.\frac{\partial \alpha_1^*}{\partial k_2}\right|_{a=t} = \tfrac{\sigma}{6} v(c+t)/\left[1 - \sigma v(c+t)(k_1 - k_2)\right]^2 \text{.}$$

We can finally analyze the effects of access charges on profits. From equations (a16) and (a17) we have:

$$\frac{\partial \pi_1}{\partial a} = 2\alpha_1^* \frac{\partial \alpha_1^*}{\partial a}\left(M_1^q + M_2^q\right) + \frac{\alpha_1^{*2}}{\sigma}\left(\frac{\partial M_1^q}{\partial a} + \frac{\partial M_2^q}{\partial a}\right) + (a-t)\frac{\partial\left(\alpha_1^{*2} k_2 (q(p_2) - q(p_1))\right)}{\partial a} +$$
$$+\alpha_1^{*2} k_2 (q(p_2) - q(p_1)) + \alpha_1^{*2} q(p_1)(k_2 - k_1)\frac{\partial p_1}{\partial a} + (a-t)\frac{\partial\left(\alpha_1^{*2}(1-\alpha_1^*)\frac{k_2}{\bar{k}} q(p_1)(k_2 - k_1)\right)}{\partial a}.$$

Since in $a = t$, $q(p_1) = q(p_2) = q(c + t)$, it results:

$$\left.\frac{\partial \pi_1}{\partial a}\right|_{a=t} = 2\alpha_1^* \left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t}\left(M_1^q + M_2^q\right) + \frac{\alpha_1^{*2}}{\sigma}\left.\left(\frac{\partial M_1^q}{\partial a} + \frac{\partial M_2^q}{\partial a}\right)\right|_{a=t} + \alpha_1^{*2} q(c+t)(k_2 - k_1)\left.\frac{\partial p_1^*}{\partial a}\right|_{a=t}.$$

where α_1^* is given by eq. (17). Given the above conditions and rearranging, we have:

$$\left.\frac{\partial \pi_1}{\partial a}\right|_{a=t} = \frac{2}{3}\alpha_1^*(1-\alpha_1^*)^2 \frac{k_2}{\bar{k}} q(c+t)(k_1 - k_2) > 0 \text{ for } k_1 > k_2.$$

For $k_1 = k_2$ we obtain the "neutrality result" of ALRT. For network 2 we have:

$$\frac{\partial \pi_2}{\partial a} = -2(1-\alpha_1^*)\frac{\partial \alpha_1^*}{\partial a}\left(M_1^q + M_2^q\right) + \frac{(1-\alpha_1^*)^2}{\sigma}\left(\frac{\partial M_1^q}{\partial a} + \frac{\partial M_2^q}{\partial a}\right) + (1-\alpha_1^*)^2 k_2 (q(p_1) - q(p_2)) +$$
$$+(a-t)\frac{\partial\left((1-\alpha_1^*)^2 k_2 (q(p_1) - q(p_2))\right)}{\partial a}.$$

In $a = t$, the above expression reduces to:

$$\left.\frac{\partial \pi_2}{\partial a}\right|_{a=t} = -2(1-\alpha_1^*)\left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t}\left(M_1^q + M_2^q\right) + \frac{(1-\alpha_1^*)^2}{\sigma}\left.\left(\frac{\partial M_1^q}{\partial a} + \frac{\partial M_2^q}{\partial a}\right)\right|_{a=t} =$$
$$= \frac{1}{3}(1-\alpha_1^*)^3 \frac{k_2}{\bar{k}} q(c+t)(k_1 - k_2) > 0.$$

Proof of Proposition 9.
We want to analyze what happens to investments when a is increased slightly above t. If we can prove the impact is negative, then Proposition 9 is proven since in a symmetric equilibrium gross profits are fixed and firms would benefit from lower investment costs. Moreover, firms would depart even further away from the socially optimal level.

Recall from Section 5.3 that in a symmetric equilibrium with $a = t$, there is a continuum of equilibria. When a is increased by an infinitesimal amount, such continuum of equilibria would still exists, where the lowest level is determined by the exogenously set $\underline{k}$ while the highest level is determined by the FOC of the firm with the lower quality (best reply from below, see eq. (21)). Hence our analysis impinges upon determining the impact that a small increase in a has on the interior solution of the best reply of the operator with the lower quality level, that we assume w.l.o.g. to be firm 2: $k_2 < k_1$. Then the profit of firm 2 is given by eq. (a17) and the effect we want to study is given by the following condition:

$$\left.\frac{dk_2}{da}\right|_{\substack{a=t\\k_2=k_1}} = -\left.\frac{\partial^2\pi_2}{\partial k_2\partial a}\right|_{\substack{a=t\\k_2=k_1}} \Bigg/ \left.\frac{\partial^2\pi_2}{\partial k_2^2}\right|_{\substack{a=t\\k_2=k_1}} \qquad \text{(a19)}$$

Differentiating (a17) w.r.t. k_2, we have:

$$\frac{\partial\pi_2}{\partial k_2} = \frac{1}{\sigma}\left[-2(1-\alpha_1^*)\frac{\partial\alpha_1^*}{\partial k_2}(M_1^q+M_2^q)+\frac{\partial(M_1^q+M_2^q)}{\partial k_2}(1-\alpha_1^*)^2\right]+ \\ +(a-t)\frac{\partial\left((1-\alpha_1^*)^2k_2\left(q(p_1^*)-q(p_2^*)\right)\right)}{\partial k_2}-I'(k_2) \qquad \text{(a20)}$$

Since $\left.\frac{\partial(M_1^q+M_2^q)}{\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = \sigma v$, $\left.\frac{\partial^2(M_1^q+M_2^q)}{\partial k_2^2}\right|_{\substack{a=t\\k_1=k_2}} = 0$, and $\left.\frac{\partial\Theta}{\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = \left.\frac{\partial^2\Theta}{\partial k_2^2}\right|_{\substack{a=t\\k_1=k_2}} = 0$ implying $\left.\frac{\partial^2\alpha_1^*}{\partial k_2^2}\right|_{\substack{a=t\\k_1=k_2}} = \frac{\sigma^2v^2}{3} > 0$, then the denominator of (a19) becomes:

$$\left.\frac{\partial^2\pi_2}{\partial k_2^2}\right|_{\substack{a=t\\k_1=k_2}} = \frac{\sigma v(c+t)^2}{18} - I''(k_2),$$

which is assumed to be negative for SOC to hold.

Now we must calculate:

$$\frac{\partial^2\pi_2}{\partial k_2\partial a} = -\frac{2}{\sigma}\left\{-\frac{\partial\alpha_1^*}{\partial k_2}\frac{\partial\alpha_1^*}{\partial a}(M_1^q+M_2^q)+(1-\alpha_1^*)\frac{\partial^2\alpha_1^*}{\partial k_2\partial a}(M_1^q+M_2^q)+(1-\alpha_1^*)\frac{\partial\alpha_1^*}{\partial k_2}\frac{\partial(M_1^q+M_2^q)}{\partial a}\right\} \\ +\frac{\partial^2(M_1^q+M_2^q)}{\partial k_2\partial a}\frac{(1-\alpha_1^*)^2}{\sigma}-2\frac{(1-\alpha_1^*)}{\sigma}\frac{\partial\alpha_1^*}{\partial a}\frac{\partial(M_1^q+M_2^q)}{\partial k_2} \\ +(a-t)\left\{\frac{\partial^2\left((1-\alpha_1^*)^2k_2\left(q(p_1^*)-q(p_2^*)\right)\right)}{\partial k_2\partial a}\right\}+\frac{\partial\Gamma}{\partial k_2}. \qquad \text{(a21)}$$

where $\Gamma = \frac{\partial\left((1-\alpha_1^*)^2k_2\left(q(p_1^*)-q(p_2^*)\right)\right)}{\partial k_2}$. We have $\left.\frac{\partial\Gamma}{\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = 0$, $\left.\frac{\partial\alpha_1^*}{\partial a}\right|_{\substack{a=t\\k_1=k_2}} = 0$, $\left.\frac{\partial p_i^*}{\partial a}\right|_{\substack{a=t\\k_1=k_2}} = \frac{1}{2}$, $\left.\frac{\partial^2(M_1^q+M_2^q)}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{\sigma q}{2}$, $\left.\frac{\partial(M_1^q+M_2^q)}{\partial a}\right|_{\substack{a=t\\k_1=k_2}} = 0$. Hence (a21) simplifies to:

$$\left.\frac{\partial^2\pi_2}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{1}{\sigma}\left.\frac{\partial^2\alpha_1^*}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} - \frac{q(c+t)}{8} \tag{a22}$$

The last term we have to analyze is:

$$\frac{\partial^2\alpha_1^*}{\partial k_2\partial a} = \frac{1}{3\left(M_1^q+M_2^q\right)^4}\left\{\left[\left(\frac{\partial^2 M_1^q}{\partial k_2\partial a}+\sigma\frac{\partial^2\Theta}{\partial k_2\partial a}\right)\left(M_1^q+M_2^q\right)+\left(\frac{\partial M_1^q}{\partial a}+\sigma\frac{\partial\Theta}{\partial a}\right)\frac{\partial\left(M_1^q+M_2^q\right)}{\partial k_2}+\right.\right.$$
$$\left.-\frac{\partial^2\left(M_1^q+M_2^q\right)}{\partial k_2\partial a}(M_1^q+\sigma\Theta))-\frac{\partial\left(M_1^q+M_2^q\right)}{\partial a}\left(\frac{\partial M_1^q}{\partial k_2}+\sigma\frac{\partial\Theta}{\partial k_2}\right)\right]\left(M_1^q+M_2^q\right)^2+$$
$$\left.-2\frac{\partial\left(M_1^q+M_2^q\right)}{\partial k_2}\left(\left(\frac{\partial M_1^q}{\partial a}+\sigma\frac{\partial\Theta}{\partial a}\right)\left(M_1^q+M_2^q\right)-\frac{\partial\left(M_1^q+M_2^q\right)}{\partial a}(M_1^q+\sigma\Theta)\right)\right\}$$

Since $\left.\frac{\partial M_1^q}{\partial a}\right|_{\substack{a=t\\k_1=k_2}} = 0$, $\left.\frac{\partial^2 p_1^*}{\partial a\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = \frac{\sigma v}{6}+\frac{1}{4k_2}$, $\left.\frac{\partial^2 p_2^*}{\partial a\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = \left.\frac{\partial\alpha_1^*}{\partial k_2}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{\sigma v}{6}$, $\left.\frac{\partial\Theta}{\partial a}\right|_{\substack{a=t\\k_1=k_2}} = 0$, $\left.\frac{\partial M_1^q}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = -q\left(\sigma v k_2/3+\sigma/4\right)$, $\left.\frac{\partial^2\Theta}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = q\left(\sigma v k_2/3-1/4\right)$, finally we have:

$$\left.\frac{\partial^2\alpha_1^*}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{\sigma q(c+t)}{12}$$

and condition (a22) becomes:

$$\left.\frac{\partial^2\pi_2}{\partial k_2\partial a}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{q(c+t)}{24}$$

In conclusion, we have:

$$\left.\frac{dk_2}{da}\right|_{\substack{a=t\\k_1=k_2}} = -\frac{-q(c+t)/24}{\frac{1}{18}\sigma v(c+t)^2-I''(k_2)} < 0.$$