IGDK Munich – Graz

Optimization and Numerical Analysis for Partial Differential Equations with Nonsmooth Structures

Invisible Control of Self-Organizing Agents Leaving Unknown Environments

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Problem and Goals

- We are concerned with the multiscale modeling, control and simulation of self-organizing agents leaving an unknown area.
- In [1] we explore the possibility of sparsely controlling these systems with a *bottom-up* approach, where control on the crowd (followers) is obtained by means of very few aware agents (leaders), *hidden in the crowd and not recognized by followers*.
- In [3] we compute optimality conditions for optimal controls when the interaction among agents is of mean-field type.

The microscopic model

We propose a microscopic model for a human crowd leaving an unknown environment under limited visibility. Due to their lack of information about the positions of exits, agents need to explore the environment first.

Let Ω denote the area to be evacuated, x^τ be the exit and Σ be its visibility zone (i.e., followers can see the exit inside Σ). The microscopic dynamics described by N^{F} followers and N^{L} leaders is: for $i = 1, \ldots, N^{\mathrm{F}}$ and $k = 1, \ldots, N^{\mathrm{L}}$,

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = A(x_i, v_i) + \sum_{j=1}^{N^{\mathrm{F}}} H(x_i, v_i, x_j, v_j; \mathbf{x}, \mathbf{y}) + \sum_{\ell=1}^{N^{\mathrm{L}}} H(x_i, v_i, y_\ell, w_\ell; \mathbf{x}, \mathbf{y}), \\ \dot{y}_k = w_k = \sum_{j=1}^{N^{\mathrm{F}}} K(y_k, x_j) + \sum_{\ell=1}^{N^{\mathrm{L}}} K(y_k, y_\ell) + u_k. \end{cases} \tag{1}$$

- A is a self-propulsion term of the form

$$A(x, v) := (1 - \chi_\Sigma(x)) C_z (z - v) + \underbrace{\chi_\Sigma(x) C_\tau \left(\frac{x^\tau - x}{|x^\tau - x|} - v \right) + C_s (s^2 - |v|^2) v}_{:= S(x,v)},$$

  where $s \geq 0$ is a given characteristic cruise speed, z is a 2-dimensional random vector with normal distribution $\mathcal{N}(0, \sigma^2)$, and $C_z, C_\tau, C_s > 0$.
- The interactions follower-follower and follower-leader are given by

$$H(x, v, y, w; \mathbf{x}, \mathbf{y}) := -C_r^{\mathrm{F}} R_{\gamma,r}(x, y) + (1 - \chi_\Sigma(x)) \chi_{B_{\mathcal{N}(x;\mathbf{x},\mathbf{y})}}(y) \frac{C_a}{\mathcal{N}^*}(w - v),$$

  for given positive constants C_r^{F}, C_a, r, and γ, and where

$$R_{\gamma,r}(x, y) = \begin{cases} e^{-|y-x|^\gamma} \frac{y-x}{|y-x|} & \text{if } y \in B_r(x) \setminus \{x\}, \\ 0 & \text{otherwise.} \end{cases}$$

  The function $R_{\gamma,r}$ models a **metrical** repulsive force, while the second term accounts for the **topological** alignment force, which vanishes inside Σ.

  Followers do not distinguish between other followers and leaders!
- $\mathcal{B}_{\mathcal{N}}(x; \mathbf{x}, \mathbf{y})$ is the *minimal* ball centered at x encompassing at least $\mathcal{N}$ agents, and $\mathcal{N}^*$ is the actual number of agents in $\mathcal{B}_{\mathcal{N}}(x; \mathbf{x}, \mathbf{y})$. Computing $\mathcal{B}_{\mathcal{N}}(x; \mathbf{x}, \mathbf{y})$ requires the knowledge of the positions of all agents, given by $\mathbf{x}$ and $\mathbf{y}$.
- The interactions leader-follower and leader-leader reduce to a mere (metrical) repulsion, i.e., $K(x, y) = C_r^{\mathrm{L}} R_{\zeta,r}(x, y)$, where $C_r^{\mathrm{L}}, \zeta > 0$ are in general different from C_r^{F} and γ. Here the repulsion force is a velocity field, while for followers it is an acceleration.
- u_k is the control chosen in two ways: as the unit vector pointing towards the exit (*fixed strategy*), or as a solution in the set of admissible controls U_{adm} of

$$\min_{u(\cdot) \in U_{\mathrm{adm}}} \{ t > 0 \mid x_i(t) \notin \Omega, \quad \forall i = 1, \ldots, N^{\mathrm{F}} \}, \quad \text{subject to (1).} \tag{2}$$

The mesoscopic model

- Our interest in (1) lies in the case $N^{\mathrm{L}} \ll N^{\mathrm{F}}$, that is the population of followers exceeds by far the one of leaders.
- When N^{F} is so large, a microscopic description of both populations is no more a viable option, thus we consider the evolution of the distribution of followers, denoted by $f(x, v)$, together with the microscopic equations for the leaders (whose number is still small), with distribution $g(x, v)$.
- Their evolution is described by the following system (studied in [3])

$$\begin{cases} \frac{\partial}{\partial t} f + v \cdot \nabla_x f = -\nabla_v \cdot (\mathcal{G}[f, g] f) + \frac{1}{2} \sigma^2 C_z^2 (1 - \chi_\Sigma)^2 \Delta_v f, \\ \dot{y}_k = w_k = \int_{\mathbb{R}^{2d}} K(y_k, x) f(x, v) dx dv + \sum_{\ell=1}^{N^{\mathrm{L}}} K(y_k, y_\ell) + u_k, \end{cases} \tag{3}$$

  where

$$\mathcal{G}[f, g](x, v) = S(x, v) + \int_{\mathbb{R}^{2d}} H(x, v, \hat{x}, \hat{v}; \pi_1 f, \pi_1 g) (f(\hat{x}, \hat{v}) + g(\hat{x}, \hat{v})) \, d\hat{x} d\hat{v}.$$

- To simulate the above coupled ODE-PDE system we first derive a Boltzmann-type dynamics obtained by the binary interactions follower-follower and follower-leader, then we recover the Fokker-Planck operator in (3) by means of a *grazing interaction limit*, as in [8], and we use a Monte Carlo-based method, see [7].

Simulations of the microscopic model

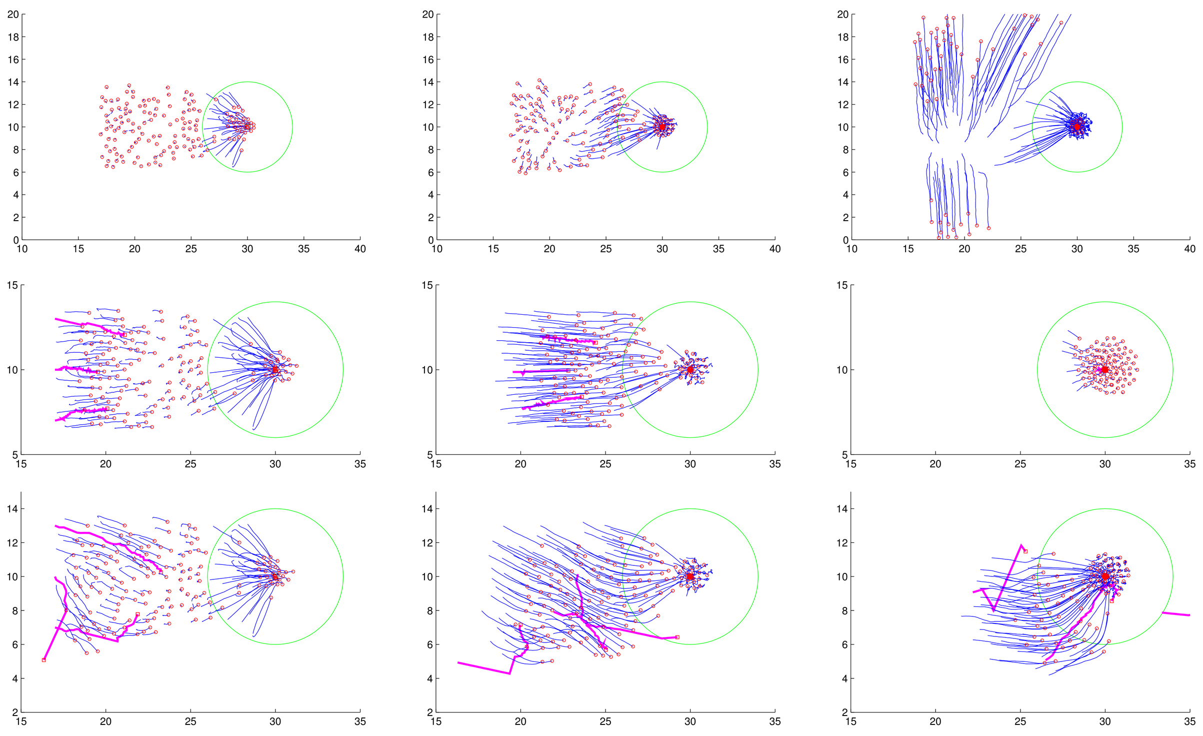

Figure: First row: no leaders. Second row: three leaders, fixed strategy. Third row: three leaders, optimal strategy (2) (computed via compass search methods).

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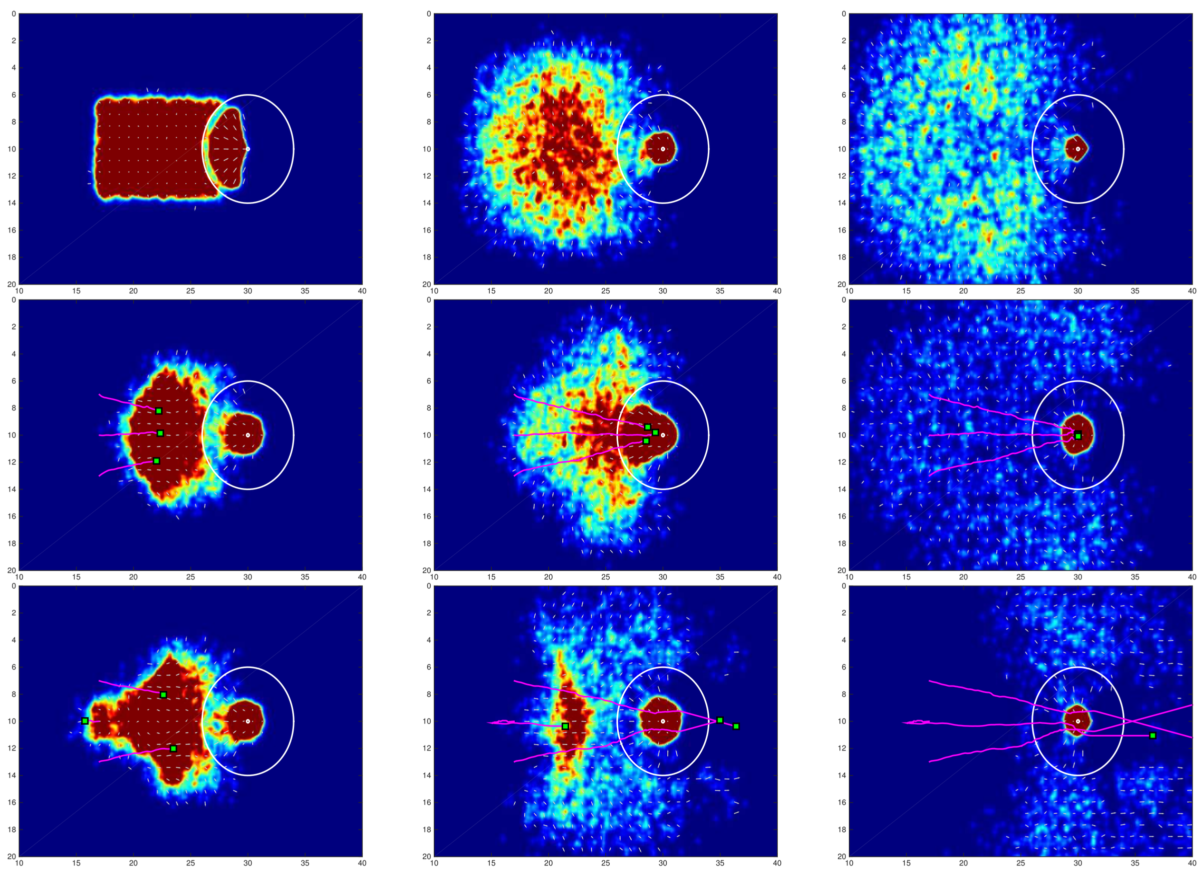

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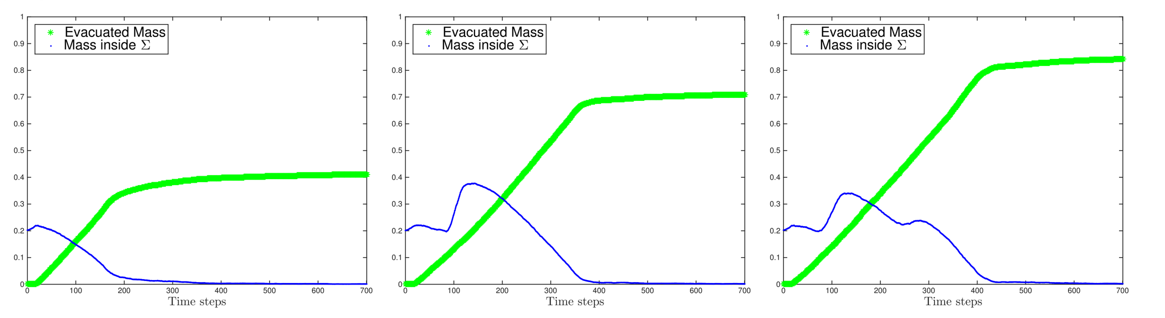

Figure: Occupancy of the exit's visibility zone Σ, dotted lines, and percentage of evacuated mass, star lines, as function of time. Left figure, histograms for the case without leaders, (percentage of evacuated mass 41.2%). Central figure, histograms for leaders moving with a fixed strategy (percentage of evacuated mass 71.3%). Right figure, histograms for leaders with an optimal strategy (percentage of evacuated mass 85.2%). Optimal strategies avoid congestions at exits.

Individual publications

[1] G. Albi, M. Bongini, E. Cristiani, and D. Kalise. Invisible control of self-organizing agents leaving unknown environments. *Preprint No. IGDK-2015-09*, 2015.

[2] M. Bongini, M. Fornasier, and D. Kalise. (Un)conditional consensus emergence under feedback controls. *Discrete Contin. Dyn. Syst. Ser. A*, 35(9):4071–4094, 2015.

[3] M. Bongini, M. Fornasier, F. Rossi, and F. Solombrino. Mean-field Pontryagin maximum principle. *Preprint No. IGDK-2015-10*, 2015.

[4] M. Bongini and M. Fornasier. Sparse stabilization of dynamical systems driven by attraction and avoidance forces. *Netw. Heterog. Media*, 9(1):1–31, 2014.

[5] M. Bongini, M. Fornasier, F. Frölich, and L. Hagverdi. Sparse control of force field dynamics. In *International Conference on NETwork Games, COntrol and OPtimization*. October 2014.

[6] M. Bongini, M. Fornasier, O. Junge, and B. Scharf. Sparse control of alignment models in high dimension. *Netw. Heterog. Media*, 10(3):647–697, 2014.

Other references

[7] G. Albi and L. Pareschi. Binary interaction algorithms for the simulation of flocking and swarming dynamics. *Multiscale Model. Simul.*, 11(1):1–29, 2013.

[8] C. Villani. A review of mathematical topics in collisional kinetic theory. *Handbook of mathematical fluid dynamics*, 1:71–74, 2002.

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