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Invited Paper

Numerical simulation of coastal changes

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Abstract

Two-dimensional changes of beach profile are computed by the continuity equation of sediment. Wave transformation used in the continuity equation is obtained by solving the shallow water equations numerically. The application of this method for different wave height, period, and sand particle diameters shows that this approach is reasonable.

Introduction

In Japan, most of the beautiful sand beaches which represent typical Japanese scenary were lost because of the failure of river sediment control. Therefore, keeping remaining sand beaches is the most important work for coastal engineers. Coastal changes are usually predicted by hydraulic model tests or numerical computation. Recently the principle tool for the prediction of coastal changes becomes the numerical computation because of its cost and easiness. As the first step of this research, the coastal changes are computed when on-offshore sand transport is dominated. In this case longshore movement of sand transport is neglected. This computation consists of two parts. One is the computation of wave transformation by using shallow wave equations. This computation gives the correct wave run up and back wash process. The other is the computation of water depth change due to sand movement. This study investigates the method of computation of the beach profile change due to on-offshore drift of sand.

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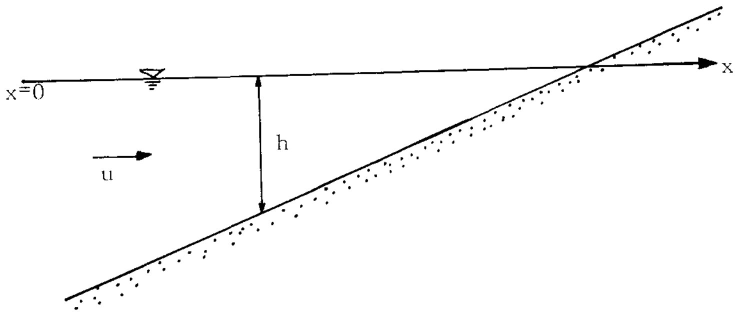

Figure 1: Coordinate System

Governing Equations

To compute beach profile change, wave transformation must be known. This is described by the following shallow water equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = g(S_o - S_f) \tag{1}$$

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0 \tag{2}$$

in which u= depth averaged velocity; h= water depth; g= the gravitational acceleration; t= time; x= coordinate system(see Fig.1); S_o = sea bottom slope; and S_f = friction slope. To solve Eqs.(1) and (2) numerically, Lax and Wendroff numerical scheme[2] is used. Eqs.(1) and (2) are transformed into the following equation[1]:

$$\boldsymbol{U}_{j,n+1} - \lambda\{\frac{1}{2}[\boldsymbol{F}_{j+1,n} - \boldsymbol{F}_{j-1,n}] + \Delta x\boldsymbol{G}_{j,n}\} + \frac{\lambda^2}{2}\{g_{j,n} - g_{j-1,n} - \Delta x\boldsymbol{S}_{j,n}\} \tag{3}$$

in which

$$\boldsymbol{U} = \begin{bmatrix} m \\ h \end{bmatrix}, \quad m = uh, \quad \lambda = \frac{\Delta t}{\Delta x},$$

$$g_{j,n} = \frac{1}{2}[\boldsymbol{A}(\boldsymbol{U}_{j+1,n}) + \boldsymbol{A}(\boldsymbol{U}_{j,n})][\boldsymbol{F}_{j+1,n} - \boldsymbol{F}_{j,n} + \frac{\Delta x}{2}(\boldsymbol{G}_{j+1,n} + \boldsymbol{G}_{j,n})],$$

$$\boldsymbol{A} = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{U}} = \begin{bmatrix} 2m/h & gh - m^2/h^2 \\ 1 & 0 \end{bmatrix},$$

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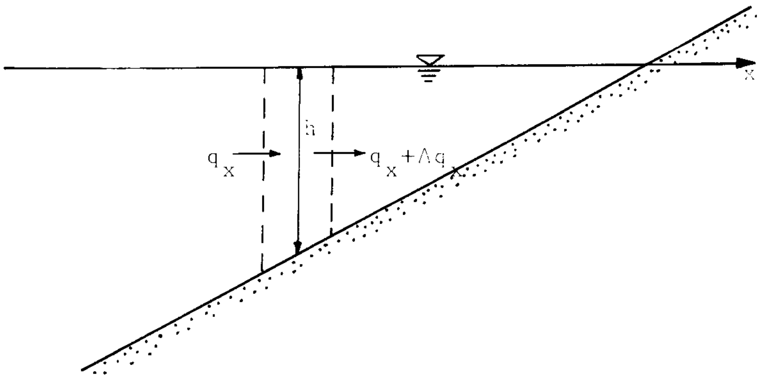

Figure 2: Derivation of Sand Continuity Equation

$$\boldsymbol{F} = \begin{bmatrix} m^2/h + gh^2/2 \\ m \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} -gh(S_o - S_f) \\ 0 \end{bmatrix},$$

$$\boldsymbol{S}_{j,n} = \Delta x \frac{\partial \boldsymbol{G}_{j,n}}{\partial t} = \Delta x \begin{bmatrix} -g(S_o - S_f)\frac{\partial h}{\partial t} + gh\{\frac{\partial S_f}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial S_f}{\partial u}\frac{\partial u}{\partial t}\} \\ 0 \end{bmatrix}_{j,n}$$

The computation of the run-up and back-wash are done according to Hibberd and Peregrine[1]. The continuity equation of sediment(see Fig.2) is given by

$$\frac{\partial h}{\partial t} = \frac{\partial q_x}{\partial x} + \epsilon \frac{\partial}{\partial x}\{\mid q_x \mid \frac{\partial h}{\partial x}\} \tag{4}$$

The empirical expression of sediment transport is given by[3]

$$q_x = \frac{B_w w_o \sqrt{f}}{(1-\lambda)s\sqrt{sgd}} \frac{U_*^2 - U_{*c}^2}{g} u \tag{5}$$

in which λ = void ratio; q_x= rate of sediment transport; ϵ= constant parameter to include the effect of beach profile change; $s = \rho_s/\rho - 1$; ρ= water density; ρ_s= sand density; d= mean diameter of sand particle; $f = g\overline{n}^2/h^{1/3}$; $\overline{n}$= Manning's roughness coefficient; $B_w = 7.0$ (constant); $U_* = \sqrt{f}u$; $U_{*c} = \sqrt{\psi_c(\rho_s - \rho)gd}$; ψ_c= the critical Shields number; and w_o = the falling velocity of sand particle. The value of q_x is zero if U_* is less than U_{*c}. When the incident waves come from the offshore, sand particles in the deeper region than h_i do not move. But if the water depth becomes shallower than h_i, sand particles move as the wave passes. The limiting water depth h_i is expressed by[4]

$$\frac{H}{L} = 2.4(\frac{d}{L})^{1/3} \sinh \frac{2\pi h_i}{L}$$

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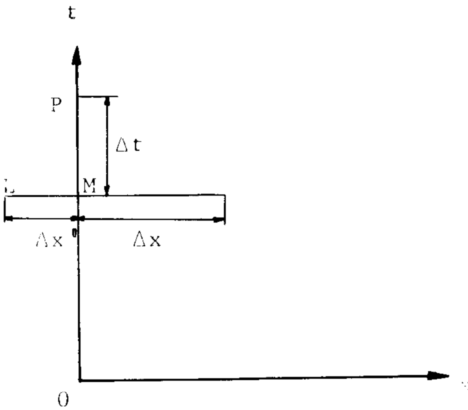

Figure 3: Coordinate System at Offshore Boundary

in which L= wave length. The continuity equation of sand is applied in the shallower water-depth region than h_i.

Boundary and Initial Conditions

The boundary conditions with respect to wave motion are located at the up-wave side (offshore boundary) and the down-wave side (onshore boundary). At the up-wave side the incident wave is given by

$$\zeta = \frac{H}{2} \sin \omega t \tag{6}$$

in which H= wave height and ω=wave frequency. The reflected wave from beach is transmitted through this offshore boundary. Due to Hino's open boundary condition assuming the velocity is zero,

$$\frac{h'_p - h_M}{\Delta t} + \frac{u_M h_M}{\Delta x'} = 0 \tag{7}$$

The value of $\Delta x'$ is almost equal to$\sqrt{gh_o}$(see Fig.3). The water depth on the offshore boundary is given by

$$h_p = \frac{h'_p - h_o}{2} + \zeta \tag{8}$$

in which h_o= the stationary water depth on the offshore boundary and $\Delta x'$ = $\Delta t\ \sqrt{gh_o}$. The value of 2 of the denominator on the right hand side of Eq.(8) means that the reflected wave height by a vertical wall is twice that on the open boundary. The water depth is stationary and velocity is zero at the initial time. The initial condition for sediment transport is given by the initial water depth. The boundary conditions for sediment transport at the up-wave and down-wave sides are given by

$$q_x = 0 \tag{9}$$

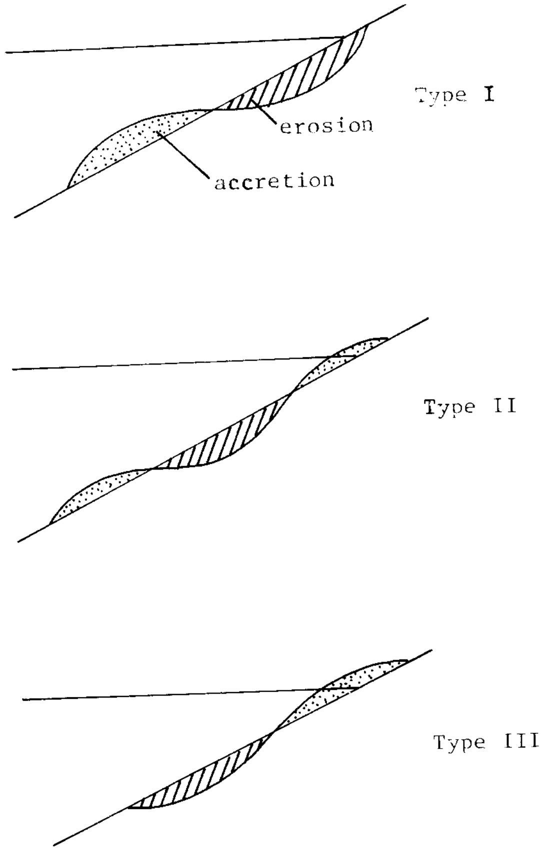

Figure 4: Classification of Beach Profile Changes

Beach Profile Changes

The two-dimensional changes of beach profile which start from the uniform slope and are caused by on-offshore sediment transport are classified into the following three types[4]: Type I is characterized by sediment transport from onshore to offshore. The direction of the sediment transport of Type III is opposite to Type I. For Type II sediment moves to the onshore and offshore. Their sketches are give in Fig.4. Considering the direction of sediment transport, Type I and III correspond to the bar and step type of beach profile. The empirical equation to classify these three types are given by

$$\frac{H/L}{(d/L)^{0.67}/(\tan\beta)^{0.27}} = C \tag{10}$$

in which H= wave height; L= wave length; d= sand particle diameter; and $\tan\beta$ = initial slope of sea bottom. The constant value C is a parameter to discriminate these types of beach profile. For hydraulic model beach, the value is suggested as follows:

$$\begin{array}{ll} C \geq 8 & \text{Type I} \\ 8 \geq C \geq 4 & \text{Type II} \\ C \leq 4 & \text{Type III} \end{array} \tag{11}$$

For the prototype beach, the value is recommended by

$$\begin{array}{ll} C \geq 18 & \text{Type I} \\ 18 \geq C \geq 9 & \text{Type II} \\ C \leq 9 & \text{Type III} \end{array} \tag{12}$$

Considering momentum equation on a sand particle, the theoretical derivation of this kind of the equation is given by Hayasaka and Mizumura[5]. Herein, we apply the numerical result to Eq.(10) and check whether the numerical result is appropriate or not.

Numerical Result

By solving Eqs.(3) and (4) numerically, time-varying beach profiles are computed. The result is plotted in Fig.5. The pattern of this beach profile change corresponds to Type I. That is, sand in the onshore moves to the offshore. Fig.6 compares the numerical result with Eqs.(11) and (12). This result shows that this numerical computation estimates the experimental result very well.

Concluding Remarks

The wave transformation is expressed by the shallow water equations and wave run-up and back-wash are also strictly investigated. Beach profile change is computed by using the continuity equation of sediment. Sediment transport formula is described by the experimental equation. As a result, two-dimensional coastal change is numerically modelled. The determination of several parameters is not fixed, but this model predicts beach profile changes very well.

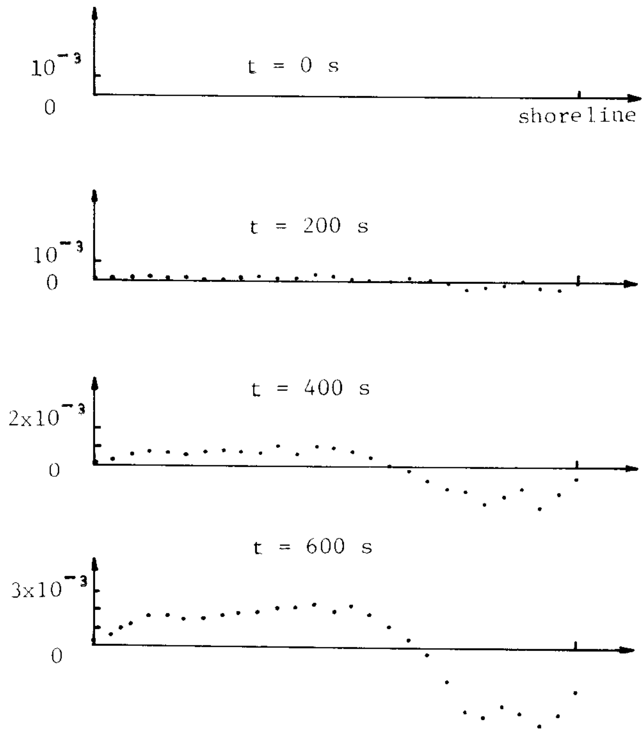

Figure 5: Numerical Beach Profile Changes

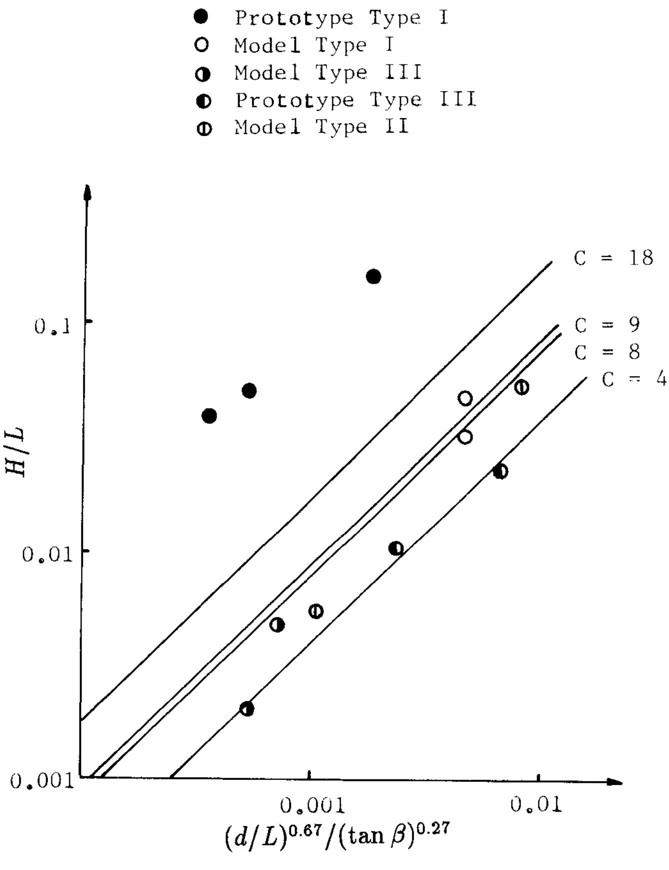

Figure 6: Classification of Beach profile Changes

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