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Ion-acoustic waves in a complex plasma with negative ions

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A self-consistent theory of linear waves in complex laboratory plasmas containing dust grains and negative ions is presented. A comprehensive model for such plasmas including source and sink effects associated with the presence of dust grains and negative ions is introduced. The stationary state of the plasma as well as the dispersion and damping characteristics of the waves are investigated. All relevant processes, such as ionization, diffusion, electron attachment, negative-positive ion recombination, dust charge relaxation, and dissipation due to electron and ion elastic collisions with neutrals and dust particles, as well as charging collisions with the dusts, are taken into consideration.

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I. INTRODUCTION

Negative ions and negatively charged micrometer to nanometer sized dust grains are ubiquitous in astrophysical as well as industrial processing plasmas [1–7]. The negative ions can appear in electronegative plasmas as a result of elementary processes such as dissociative or nondissociative electron attachment to neutrals. They are usually rather small in number, and in general do not affect the overall plasma behavior. On the other hand, since the dust grains are almost always highly negative [1,3,4,6,8,9], even in small numbers they can take up a considerable proportion of the total negative charge in the system. The presence of dust grains can affect the characteristics of most collective processes of the plasma since the charge balance in both the steady and dynamic states can be significantly altered. Another situation that often occurs is that the electron number density becomes small because of their absorption by the dust grains or the discharge walls. In this case the negative ions in the plasma can play a very important role.

Dependence of the dust charge on the local electrostatic potential can lead to aperiodic modes associated with dust charging. The effect of the latter on collective processes in space and laboratory plasmas has been investigated by many authors [10–21]. However, most theoretical models of dusty plasmas do not self-consistently take into account the loss of the plasma particles to the dust grains and walls by collisions and their replenishment. Nonstationary effects such as plasma particle absorption by dusts can lead to instabilities of waves propagation in such a plasma [22]). In most earlier studies, unspecified sources and sinks that somehow exactly balance the total particle numbers are implicitly invoked, yet not taken into account in the conservation equations. The loss of plasma electrons and ions resulting from their capture by the dust grains or walls can be included by treating the complex plasma system as thermodynamically open and introducing the corresponding capture processes into the conservation and dust charging equations [11]. However, just accounting for plasma particle capture by the dust grains is clearly insufficient, since without a source of these particles a stationary state could not be established. In a real laboratory device such as the gas discharge, plasma particles are continuously being created by ionization. They are also lost by volume recombination as well as by diffusion and convection, which can take place at the same time scale as that of ionization. Since ionization, recombination, and particle transport are all density dependent, collective processes in the system are strongly affected. Thus, dust particles can modify the entire plasma system by altering the ionization-recombination-diffusion balance in both the steady and dynamic states.

The effect of variation of the dust charge on electron plasma and ion acoustic waves in a low-temperature plasma was investigated [23,24] with self-consistent particle balance taken into consideration. It was shown that dust-charge relaxation is significantly affected by ionization and recombination. These processes self-consistently maintain the equilibrium state of the discharge as well as the particle number densities during the perturbations. It was also shown that dissipation due to interplasma particle collisions as well as elastic Coulomb and the inelastic dust-charging collisions can lead to strong damping of the waves in a typical laboratory plasma [23,24]. In contrast, instability [17] could appear if the sources and sinks are omitted in the conservation equations.

When the plasma contains negative ions, the magnitudes of many characteristic parameters, such as the plasma frequencies can become quite different from that of a normal dusty plasma. These modifications alone can, in certain situations, already radically affect the properties of the natural

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modes, the near-wall sheaths, as well as the dust charging process [25,26]. In the present paper, we consider wave propagation in a complex dusty plasma with negative ions. We shall take into account self-consistently the relevant processes such as ionization, electron attachment, diffusion, positive-negative ion recombination, plasma particle collisions, as well as elastic Coulomb and inelastic dust-charging collisions. It is found that the equilibrium of the plasma as well as the propagation of ion waves are modified to various degrees by these effects.

II. FORMULATION

We consider linear wave propagation in a complex multicomponent weakly ionized ($\Sigma n_j/\Sigma N_j \ll 1$) plasma with finite effective electron (T_e), ion (T_i), and negative-ion (T_-) temperatures, where Σn_j and ΣN_j are the combined number densities of the ionized and neutral species. The size of the dust grains is assumed to be much less than the intergrain distance, the Debye length, as well as the wavelength, so that they can be treated as heavy point masses. The charge of a dust grain varies because of the microscopic electron and ion currents flowing into it according to the potential difference between its surface and the adjacent plasma. The dust grains are treated as an immobile background since the time scales of charge variation and ion wave dynamics are both much less than that of the dust motion.

For simplicity, we consider a one-dimensional (1D) planar geometry with singly charged negative and positive ions. All heavy particle collisions, except that for n_i-n_- recombination (since it is the major sink of negative ions), are neglected. The negative ions are assumed to be due to electron attachment, although other processes can also be easily included. Electron-(positive) ion recombination as well as stepwise ionization involving excited states of the neutrals are neglected.

Accordingly, the conservation equations are

$$\partial_t n_e + \boldsymbol{\nabla}(n_e v_e) = S_e, \tag{2.1}$$

$$\partial_t v_e + \nu_e^{\text{eff}} v_e + \frac{T_e}{m_e n_e}\partial_x n_e = -\frac{e}{m_e}E, \tag{2.2}$$

$$\partial_t n_i + \boldsymbol{\nabla}(n_i v_i) = S_i, \tag{2.3}$$

$$\partial_t v_i + \nu_i^{\text{eff}} v_i + \frac{T_i}{m_i n_i}\partial_x n_i = \frac{e}{m_i}E, \tag{2.4}$$

$$\partial_t n_- + \boldsymbol{\nabla}(n_- v_-) = S_-, \tag{2.5}$$

$$\partial_t v_- + \nu_-^{\text{eff}} v_- + \frac{T_-}{m_- n_-}\partial_x n_- = -\frac{e}{m_-}E, \tag{2.6}$$

$$d_t\tilde{q}_d + \nu_d^{\text{ch}}\tilde{q}_d = -|I_{e0}|\tilde{n}_e/n_{e0} + |I_{i0}|\tilde{n}_i/n_{i0} - |I_{-0}|\tilde{n}_-/n_{-0}, \tag{2.7}$$

and

$$\nabla^2\varphi = -4\pi e(n_i - n_- - n_e - |Z_d|n_d), \tag{2.8}$$

where $E = -\nabla\varphi$ is the electric field of the electrostatic waves, φ is the electrostatic potential, m_j, $n_j = n_{j0} + \tilde{n}_j$, and v_j are the mass, density, and fluid velocity of the species $j = e, i, -, d$ for electron, ion, negative ion, and dust, respectively. The average charge of the dust particles is $q_d = -|Z_d|e = q_{d0} + \tilde{q}_d$. The effective dust charging rate ν_d^{ch} is given in the Appendix. The microscopic currents of the electrons and positive and negative ions at a dust grain are

$$I_e = -\pi a^2 e(8T_e/\pi m_e)^{1/2} n_e \exp(e\Delta\varphi_g/T_e), \tag{2.9}$$

$$I_i = \pi a^2 e(8T_i/\pi m_i)^{1/2} n_i (1 - e\Delta\varphi_g/T_i), \tag{2.10}$$

and

$$I_- = -\pi a^2 e(8T_-/\pi m_-)^{1/2} n_- \exp(e\Delta\varphi_g/T_-), \tag{2.11}$$

where $\Delta\varphi_g$ is the steady-state potential difference between the dust grain and the adjacent plasma. The currents $I_{(e,i,-)}$ arise because of this local potential difference. In a dusty plasma, the effective rates of the electron and ion collisions $\nu_{(e,i,-)}^{\text{eff}}$ include those for elastic and inelastic collisions of electrons and ions with neutrals and charged dust grains. The corresponding rates are discussed in the Appendix.

The source terms $S_{(e,i,-)}$ for the electrons, and the positive and negative ions, are

$$S_e = \nu_{\text{ion}} n_e - \nu_{ed} n_e - \nu_{\text{att}} n_e - \nu_{\text{wall}}^e n_e, \tag{2.12}$$

$$S_i = \nu_{\text{ion}} n_e - \nu_{id} n_i - \nu_{\text{rec}} n_i n_- - \nu_{\text{wall}}^i n_i, \tag{2.13}$$

and

$$S_- = \nu_{\text{att}} n_e - \nu_{-d} n_- - \nu_{\text{rec}} n_i n_- - \nu_{\text{wall}}^- n_-, \tag{2.14}$$

where ν_{ion} is the rate of electron impact ionization of the neutral particles, $\nu_{(e,i,-)d}$ are the rates of collection of plasma species by the dust grains, ν_{att} is the rate of the electron attachment to the neutrals resulting in negative ion production, and ν_{rec} is the rate of recombination of the positive and negative ions in the plasma bulk. The last terms in Eqs. (2.12)-(2.14) represent particle loss at the walls of the discharge, namely, $\nu_{\text{wall}}^e \propto S_{\text{surf}} V_{Te} n_{eS} \exp(-\phi_{\text{wall}}/T_e)$, $\nu_{\text{wall}}^- \propto S_{\text{surf}} V_{T-} n_{eS} \exp(-\phi_{\text{wall}}/T_-)$, and $\nu_{\text{wall}}^i \propto S_{\text{surf}} V_B n_{iS}$. Here, S_{surf} is the effective surface area of particle loss at the walls, V_B, V_{Te}, and V_{T-} are the Bohm, electron, and negative-ion thermal velocities, respectively. Furthermore, ϕ_{wall} is the potential difference between the wall and the plasma bulk, and $n_{(e,i,-)S}$ are the densities of the plasma species near the wall. The wall potential ϕ_{wall}, typically few tens of volts in low-pressure low-temperature discharges, can be taken to be an external parameter, which can be estimated by balancing the electron, and the positive- and negative-ion currents at the wall. A more detailed formulation of the diffusion process would require accounting for the nonuniform plasma density distribution (which is typically ambipolar cosinelike for planar discharges). However, here we are interesting in local wave propagation in a region of fairly uniform plasma far from the walls.

III. EQUILIBRIUM STATE

We shall first obtain the self-consistent equilibrium state. In equilibrium, the system is charge neutral, so that $n_{i0} = n_{e0} + n_{-0} + |Z_{d0}| n_{d0}$. The stationary dust charge is obtained by setting equal the lowest order microscopic electron and ion currents flowing onto the dust particles, or $I_{i0} = |I_{e0}| + |I_{-0}|$.

The equilibrium number densities of the plasma particles can be obtained from

$$\nu_{\rm ion} n_{e0} - \nu_{ed} n_{e0} - \nu_{\rm att} n_{e0} - \nu_{\rm wall}^{e} n_{e0} = 0, \tag{3.1}$$

$$\nu_{\rm ion} n_{e0} - \nu_{id} n_{i0} - \nu_{\rm rec} n_{i0} n_{-0} - \nu_{\rm wall}^{i} n_{i0} = 0, \tag{3.2}$$

and

$$\nu_{\rm att} n_{e0} - \nu_{-d} n_{-0} - \nu_{\rm rec} n_{i0} n_{-0} - \nu_{\rm wall}^{-} n_{-0} = 0. \tag{3.3}$$

From Eq. (3.1), after cancellation of n_{e0}, one finds that the stationary state can exist only if the creation of electrons due to ionization of neutrals balances or exceeds their loss due to collection by the dusts, negative ion formation due to electron attachment, and flow to the chamber walls. That is, the condition $\nu_{\rm ion} > \nu_{ed} + \nu_{\rm att} + \nu_{\rm wall}^{e}$ is a prerequisite for the existence of a stationary state of the 1D electronegative dust-contaminated discharge.

In addition, for low-pressure diffusion equilibrium [27], the flux of positive ions to the discharge walls is to be balanced by the electron and negative ion fluxes, or

$$\nu_{\rm wall}^{e} n_{e0} + \nu_{\rm wall}^{-} n_{-0} = \nu_{\rm wall}^{i} n_{i0},$$

where we have assumed that the plasma particle densities are spatially uniform. From Eqs. (3.1)–(3.3) it follows that

$$\nu_{ed} n_{e0} + \nu_{-d} n_{-0} = \nu_{id} n_{i0}, \tag{3.4}$$

which is the basic relation between the equilibrium densities of the plasma particles and the rates of electron and ion collection by the grains.

Equation (3.4), together with the neutrality condition $n_{i0} = n_{e0} + n_{-0} + |Z_d| n_{d0}$, allows one to calculate the stationary values of the number densities of the electrons

$$n_{e0} = n_{i0} \frac{\nu_{id}/\nu_{ed}}{1 - \nu_{-d}/\nu_{ed}} \left[1 - \frac{\nu_{-d}}{\nu_{id}} (1 - \kappa_0) \right], \tag{3.5}$$

and negative ions

$$n_{-0} = \frac{\nu_{i0}}{1 - \nu_{-d}/\nu_{ed}} \left[1 - \frac{\nu_{id}}{\nu_{ed}} - \kappa_0 \right], \tag{3.6}$$

where $\kappa_0 = |Z_{d0}| n_{d0}/n_{i0}$ denotes the proportion of negative charge residing on the dust particles. The expressions (3.5) and (3.6) are also consistent with the case when the negatively charged ions are absent. Indeed, in that case we have $n_{-0} = 0$ and $n_{e0} = n_{i0} \nu_{id}/\nu_{ed}$ [24,28].

From Eq. (3.2), we obtain

$$n_{i0} = \frac{\nu_{\rm ion}[\nu_{id} - \nu_{-d}(1-\kappa_0)] - (\nu_{id} - \nu_{\rm wall}^{i})(\nu_{ed} - \nu_{-d})}{\nu_{\rm rec}\nu_{ed}}, \tag{3.7}$$

for the equilibrium value of the ion number density in 1D dust-contaminated electronegative discharge in the low-pressure diffusion equilibrium regime. Equation (3.7) imposes the following restriction on the minimum ionization rate:

$$\nu_{\rm ion} > \nu_{\rm ion}^{\rm min} = (\nu_{id} - \nu_{\rm wall}^{i})(\nu_{ed} - \nu_{-d})/[\nu_{id} - \nu_{-d}(1-\kappa_0)]$$

for the existence of the discharge in question. Otherwise, the rate of new particle production will not be sufficient to compensate the losses at the discharge walls and to the dust grains. One can also clearly see from Eq. (3.7) that the ion density diminishes when positive-negative ion recombination becomes more intense.

Equations (3.5)–(3.7), together with the overall charge neutrality condition and the equation for the balance of the electron and positive- and negative-ion currents on the dust grains, self-consistently determine the stationary number densities in the complex discharge plasma. The stationary state discussed here is quite general and can also be used for investigating other dynamic processes in complex plasmas.

IV. DISPERSION RELATION

We are interested in waves on the ion time scale. Assuming that perturbations are of the form $\exp[i(kz - \omega t)]$, where k is the wave number, we linearize Eqs. (2.1)–(2.8) and obtain for the dust charge perturbation

$$\tilde{q}_d = \frac{i}{\omega + i\nu_d^{\rm ch}} \left(\frac{|I_{e0}|}{n_{e0}} \tilde{n}_e + \frac{|I_{i0}|}{n_{i0}} \tilde{n}_i - \frac{|I_{-0}|}{n_{-0}} \tilde{n}_- \right). \tag{4.1}$$

The electron fluid velocity is then given by

$$v_e = -\frac{kT_e}{m_e(\omega + i\nu_e^{\rm eff})} \left(\frac{e\varphi}{T_e} - \frac{\tilde{n}_e}{n_{e0}} \right), \tag{4.2}$$

and the perturbed electron density is

$$\tilde{n}_e = -\frac{ek^2 n_{e0}\varphi}{\eta_e m_e(\omega + i\nu_e^{\rm eff})}, \tag{4.3}$$

where $\eta_e = \omega - i(\nu_{\rm ion} - \nu_{ed} - \nu_{\rm att} - \nu_{\rm wall}^{e}) - k^2 V_{Te}^2/(\omega + i\nu_e^{\rm eff})$ and $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity. The ion fluid velocity is written as

$$v_i = \frac{kT_i}{m_i(\omega + i\nu_i^{\rm eff})} \left(\frac{e\varphi}{T_i} + \frac{\tilde{n}_i}{n_{i0}} \right), \tag{4.4}$$

and the perturbed ion density is given by

$$\tilde{n}_i = \frac{ek^2 n_{i0}\varphi}{\eta_i m_i(\omega + i\nu_e^{\rm eff})} (1 - i\mathcal{G}_i)\xi_i^{\rm rec}, \tag{4.5}$$

where

$$\mathcal{G}_i=\frac{n_{e0}}{n_{i0}}\frac{m_i\omega+i\nu_i^{\mathrm{eff}}}{m_e\omega+i\nu_e^{\mathrm{eff}}}\frac{\nu_{\mathrm{ion}}}{\eta_e},$$

and $\eta_i=\omega+i(\nu_{id}+\nu_{\mathrm{rec}}n_{-0}+\nu_{\mathrm{wall}}^i)-k^2V_{Ti}^2/(\omega+i\nu_i^{\mathrm{eff}})$. The factor ξ_i^{rec} describes the interdependence of the positive- and negative-ion densities given by the nonlinear recombination term $\nu_{\mathrm{rec}}n_in_-$ in Eqs. (2.13) and (2.14). The negative ion fluid velocity is

$$v_-=-\frac{kT_-}{m_-(\omega+i\nu_-^{\mathrm{eff}})}\left(\frac{e\varphi}{T_-}-\frac{\tilde{n}_-}{n_{-0}}\right), \tag{4.6}$$

and the perturbed negative ion density is

$$\tilde{n}_-=\frac{ek^2n_{-0}\varphi}{\eta_-m_-(\omega+i\nu_e^{\mathrm{eff}})}(1-i\mathcal{G}_-)[1+i\xi_-^{\mathrm{rec}}], \tag{4.7}$$

where

$$\mathcal{G}_-=\frac{n_{e0}}{n_{-0}}\frac{m_-}{m_e}\frac{\omega+i\nu_-^{\mathrm{eff}}}{\omega+i\nu_e^{\mathrm{eff}}}\frac{\nu_{\mathrm{att}}}{\eta_e},$$

and $\eta_-=\omega+i(\nu_{-d}+\nu_{\mathrm{att}}n_{i0}+\nu_{\mathrm{wall}}^-)-k^2V_{T-}^2/(\omega+i\nu_-^{\mathrm{eff}})$. The terms in Eqs. (4.5) and (4.7) attributed to the positive-negative ion recombination are

$$\xi_i^{\mathrm{rec}}=\frac{1+i\gamma_1(1-i\mathcal{G}_-)/(1-i\mathcal{G}_i)}{1+\nu_{\mathrm{rec}}^2n_{i0}n_{-0}/(\eta_-\eta_i)}$$

and

$$\xi_-^{\mathrm{rec}}=\frac{\gamma_2[1-i\mathcal{G}_i+i\gamma_1(1-i\mathcal{G}_-)]}{1-i\mathcal{G}_-},$$

where

$$\gamma_1=\frac{m_i}{m_-}\frac{\omega+i\nu_i^{\mathrm{eff}}}{\omega+i\nu_-^{\mathrm{eff}}}\frac{\nu_{\mathrm{rec}}n_{-0}}{\eta_-}$$

and

$$\gamma_2=\frac{m_-}{m_i}\frac{\omega+i\nu_-^{\mathrm{eff}}}{\omega+i\nu_i^{\mathrm{eff}}}\frac{\nu_{\mathrm{rec}}n_{i0}\eta_-}{\eta_i\eta_--\nu_{\mathrm{rec}}^2n_{i0}n_{-0}},$$

where we have neglected the variation of the rates of electron and ion capture with the dust charge. The validity of this assumption will be discussed later. We note that $\xi_i^{\mathrm{rec}}(\nu_{\mathrm{rec}}=0)=1$ and $\xi_-^{\mathrm{rec}}(\nu_{\mathrm{rec}}=0)=0$.

Combining Eqs. (4.1)–(4.7), we obtain

$$\begin{aligned}1&-\frac{\omega_{pe}^2}{\eta_e(\omega+i\nu_e^{\mathrm{eff}})}\left(1+i\frac{\tilde{\nu}_{ed}}{\omega+i\nu_d^{\mathrm{ch}}}\right)-\frac{\omega_{pi}^2\xi_i^{\mathrm{rec}}(1-i\mathcal{G}_i)}{\eta_i(\omega+i\nu_i^{\mathrm{eff}})}\\&\times\left(1+\frac{\tilde{\nu}_{id}}{\omega+i\nu_d^{\mathrm{ch}}}\right)-\frac{\omega_{p-}^2(1-i\mathcal{G}_-)(1+\xi_-^{\mathrm{rec}})}{\eta_-(\omega+i\nu_-^{\mathrm{eff}})}\\&\times\left(1+i\frac{\tilde{\nu}_{-d}}{\omega+i\nu_d^{\mathrm{ch}}}\right)\\&=0,\end{aligned} \tag{4.8}$$

which is the general dispersion relation for the electrostatic waves in a typical laboratory dusty plasma. Here, $\tilde{\nu}_{(e,i,-)d}=|I_{(e,i,-)0}|n_{d0}/en_{(e,i,-)0}$.

For $\omega\ll\omega_{pe}$, $T_i\ll T_e$, n_{e0} and n_{i0} of the same order, and in the absence of negative ions, dissipation, and dust-charge variation, we recover from Eq. (4.8) the ion acoustic waves propagating at the frequency $\omega=kV_S/\sqrt{1+\lambda_e^2}$, where $V_S=\sqrt{(T_e/m_i)(n_{i0}/n_{e0})}$ is the dust-modified ion acoustic speed and $\lambda_e=V_{Te}/\omega_{pe}$. Clearly, in the presence of dust grains (so that n_{i0} and n_{e0} can be quite different), the order of magnitude of the characteristic parameters of an electron-ion plasma can be radically modified. In the absence of negative ions, we can recover the dispersion relation of the dust-modified ion-acoustic waves (IAWs) in collisional dusty plasmas with variable-charge dusts [24].

V. PROPAGATION AND DAMPING OF ACOUSTIC WAVES

When $k^2V_{Te}^2\gg[\omega^2,\omega\nu_e^{\mathrm{eff}},\omega(\nu_{\mathrm{ion}}-\nu_{ed}-\nu_{\mathrm{att}}-\nu_{\mathrm{wall}}^e)]$, $\omega\sim\nu_i^{\mathrm{diss}}$, where ν_i^{diss} is any of the ion dissipation terms in expression (2.13), and $\omega^2\gg k^2V_{Ti}^2$ [24], we obtain from Eq. (4.8),

$$\mathcal{D}_{\mathrm{ch}}(\omega)\mathcal{D}_{\mathrm{IAW}}(\omega,k)=i\beta_{\mathrm{coupl}}(\omega), \tag{5.1}$$

where

$$\mathcal{D}_{\mathrm{IAW}}=-(\omega_{pe}/kV_{Te})^2+\omega_{pi}^2/w_i^{\mathrm{eff}}w_i^*+\omega_{p-}^2/w_-^{\mathrm{eff}}w_-^*,$$

$$\beta_{\mathrm{coupl}}(\omega)=-\tilde{\nu}_{de}(\omega_{pe}/kV_{Te})^2+\tilde{\nu}_{di}\omega_{pi}^2/w_i^{\mathrm{eff}}w_i^*+\tilde{\nu}_{d-}\omega_{p-}^2/w_-^{\mathrm{eff}}w_-^*,$$

and $\mathcal{D}_{\mathrm{ch}}(\omega)=\omega+i\nu_d^{\mathrm{ch}}$, $w_i^{\mathrm{eff}}=\omega+i\nu_i^{\mathrm{eff}}$, $w_i^*=\omega+i\nu_i^*$, $w_-^{\mathrm{eff}}=\omega+i\nu_-^{\mathrm{eff}}$, $w_-^*=\omega+i\nu_-^*$, $\nu_i^*=\nu_{id}+\nu_{\mathrm{rec}}n_{-0}+\nu_{\mathrm{wall}}^i$, and $\nu_-^*=\nu_{-d}+\nu_{\mathrm{rec}}n_{i0}+\nu_{\mathrm{wall}}^-$. Equation (5.1) couples the ion acoustic waves [24] given by the dispersion $\mathcal{D}_{\mathrm{IAW}}(\omega,k)=0$ with the dust charge relaxation mode given by the relation $\mathcal{D}_{\mathrm{ch}}(\omega)=0$. In the derivation of the dispersion relation (5.1), recombination effects have been neglected. This is valid for $|\eta_i\eta_e|\gg\nu_{\mathrm{rec}}^2n_{i0}n_{-0}$ and $|\eta_i|\gg\nu_{\mathrm{rec}}^2n_{i0}$, and will be discussed in more detail in Sec. VI.

If in Eq. (5.1) we retain only the dust-charging terms and the usual electron- and ion-neutral collisions, we obtain an equation similar to that for ion-acoustic surface waves [19]. If only the negative ion effects are ignored, a dispersion relation similar to Eq. (19) of Ref. [24] is obtained. The minor difference is due to different dominant particle loss mechanisms, namely, the diffusion loss to the discharge walls here and the volume recombination loss in Ref. [24].

When the coupling between the ion acoustic and the charge relaxation modes is weak ($\omega \gg \tilde{\nu}_{(e,i,-)}$), the two modes are independent. However, in this case the eigenfrequency of the ion acoustic wave is still modified such that $\omega_{\mathrm{IAW}} = \omega_{\mathrm{IAW0}} + \delta\omega_{\mathrm{IAW}}^{\mathrm{coupl}}$, where

$$\delta\omega_{\mathrm{IAW}}^{\mathrm{coupl}} = i\beta_{\mathrm{coupl}}/[\partial_\omega \mathcal{D}_{\mathrm{IAW}}(\omega,k)]|_{\omega=\omega_{\mathrm{IAW}_0}},$$

so that the ion wave frequency is down-shifted and the damping decrement increased. This is a typical consequence of the coexistence of ion acoustic waves and dust charge relaxation in dust-contaminated plasmas [19,24].

The dust charge relaxation rate is $\omega_d^{\mathrm{relax}} = -i\nu_d^{\mathrm{ch}} + \delta\omega_d^{\mathrm{relax}}$, where

$$\delta\omega_d^{\mathrm{relax}} = i\beta_{\mathrm{coupl}}|_{\omega=-i\nu_d^{\mathrm{ch}}}/\mathcal{D}_{\mathrm{IAW}}(-i\nu_d^{\mathrm{ch}},k),$$

so that the dust charge relaxation rate diminishes because of coupling to the ion acoustic waves. Frequency down-shift of the plasma modes accompanied by diminishing of the dust charge relaxation rates is quite common to electropositive (negative-ion free) complex plasmas and is attributed to additional dissipative wave energy transfer to the purely damped dust charge relaxation mode followed by the self-organization of the complex plasma system to minimize the dissipative loss [17,19,24].

The expressions (4.8)–(5.1) describe the propagation of ion acoustic waves in a plasma contaminated by variable-charge dust grains for a wide range of parameters of practical interest. The frequency of the ion acoustic waves should not be too low, otherwise inclusion of temperature fluctuations and use of the full Braginski transport equations [29] for strongly collisional plasmas would be necessary [30]. This is especially the case if the frequency of the ion acoustic waves is less than the effective rate of the ion collisions.

We now investigate the dispersion relation (5.1) numerically for real wave frequency and complex wave number. The computations are carried out for the representative parameters of 10% C_4F_8–90% Ar fluorocarbon plasmas often used for ultrafine and highly selective etching of polysilicons [31,32]. The parameters used in computations are summarized in Table I. The dispersion relation (5.1) is solved numerically for three different regimes of dissipative loss and four different values of the proportion of charge on the dust particles and negative ions. In Table II, we present the typical dust charge $|Z_d|$, the (negative) charge proportions $\kappa_0 = |Z_d| n_{d0}/n_{i0}$ and n_{e0}/n_{i0} on the dust grains and electrons, the ion plasma frequency ω_{pi} normalized by $\omega_{\mathrm{IAW}}^{\mathrm{lim}} = \sqrt{\omega_{pi}^2 + \omega_{p-}^2}$, which is the upper limiting frequency of the ion acoustic waves in the low dissipation limit, and the electrostatic parameter $\Theta = e^2|Z_d|/aT_e$ (the ratio of the electrostatic energy of a dust grain with charge Z_d and radius a to the electron thermal energy [14]), computed for four cases of negative ion content. Cases (i)–(iv) correspond to the following values of the dust size a and the ratio n_{-0}/n_{i0} representing negative ion content: (i) 100 nm, 0.1; (ii) 100 nm, 0.4; (iii) 1 μm, 0.1; and (iv) 1 μm, 0.4, respectively.

TABLE I. The main plasma and particulate parameters in the computation.

Parameter	Notation	Value
Electron temperature	T_e	2.0 eV
Grain size	a	0.1, 1.0 μm
Dust mass density	ρ	1.5 g/cm^3
Positive ion density	n_0	4×10^{11} cm^{-3}
Dust number density	n_{d0}	4×10^{7} cm^{-3}
Positive ion mass (Ar^+)	m_{Ar^+}	$1836\times40\times m_e$
Negative ion mass (F^-)	m_F	$1836\times19\times m_e$
Positive ion temperature	T_i	0.2 eV
Negative ion temperature	T_-	0.1 eV
Temperature of neutrals	T_n	0.026 eV
Wall potential	ϕ_{wall}	$-10,\ldots,-20$ V
Negative ion proportion (F^-)	n_{-0}/n_{i0}	0.1, 0.4
Elastic ion collisions (+)	$\nu_i^{\mathrm{eff}}/\omega$	0.025–0.55
Inelastic ion collisions (+)	ν_i^*/ω	0.03–0.6
Elastic ion collisions (−)	$\nu_-^{\mathrm{eff}}/\omega$	0.0225–0.5
Inelastic ion collisions (−)	ν_-^*/ω	0.0275–0.525

One can show that in the absence of all dissipative effects, the wave number is real definite and the frequency approaches asymptotically the upper limiting frequency $\omega_{\mathrm{IAW}}^{\mathrm{lim}}$. The low-dissipation case (Fig. 1) also reflects this tendency. From Fig. 1 and Table II, it is seen that the effect of the negative ions is to increase the upper limit of the ion acoustic wave frequency (as compared to ω_{pi} for simple electron-ion plasmas). For example, $\omega_{\mathrm{IAW}}^{\mathrm{lim}}/\omega_{pi} = 1.1$ in the cases (i) and (iii), and 1.357 for (ii) and (iv). However, in practice the upper limiting frequency is only defined at relatively low levels of dissipation. At higher levels, dissipative effects lead to an increase of the domain of negative dispersion, with no clear upper limit, as can be seen in Figs. 2 and 3.

For low dissipation, we can see in Figs. 1(a) and 1(b) that $|k_i| < |k_r|$, where k_i and k_r are the imaginary (Im k) and real (Re k) parts of the wave number, respectively. The dissipation in the system results in the appearance of domains of negative dispersion ($dk_r/d\omega < 0$), which is a common feature for many wave processes in plasmas [33]. From Fig. 1(c), it is seen that the wave phase velocity V_{ph} can be either larger or smaller than V_S. In the sub-ω_{pi} frequency range, it first decreases and then gets an upturn due to negative dispersion effects at $\omega \sim \omega_{\mathrm{lim}}$. We note that in the presence of dissipation the phase velocity does not vanish at $\omega = \omega_{\mathrm{lim}}$, as is in the dissipationless case. Furthermore, V_{ph} is larger when the proportion of the negative ions is higher, as shown in Fig. 1(c). The curves 1 and 3 in Fig. 1(c) show that the extra negative charge on larger dust grains increases the wave phase velocity, as is the case of the ion acoustic surface wave [19].

In the intermediate particle-loss case, one can no longer observe clearly resolved upper limiting frequencies as was in the no- or low-dissipation case [Fig. 2]. Likewise, the imaginary part of the wave number becomes comparable to k_r

TABLE II. Representative characteristics of dust particles and electronegative plasmas.

Parameter	Notation	Case (i)	Case (ii)	Case (iii)	Case (iv)
Dust charge	$\|Z_d\|$	459	414	4016	3365
Dust charge proportion	κ_0	0.046	0.041	0.402	0.263
Electron charge proportion	n_{e0}/n_{i0}	0.854	0.56	0.498	0.337
Upper limiting frequency	$\omega_{\mathrm{IAW}}^{\mathrm{lim}}/\omega_{pe}$	1.1	1.357	1.1	1.357
Electrostatic parameter	Θ	3.3	2.98	2.89	2.42

[Fig. 2(b)]. In the frequency range $\omega<\omega_{pi}$, the phase velocity becomes less frequency dependent [Fig. 2(c)]. Even though V_{ph} decreases with ω in this range, but it does not vanish in the limiting frequency region. Moreover, in remarkable contrast to Fig. 1(c), V_{ph} always remains larger than V_S in the entire frequency range of interest. One can also see that the phase velocity increases strongly in the region ($\omega>0.9\omega_{pi}$) of the negative dispersion. Figure 2(a) also reflects a general tendency that the domain of negative dispersion increases for smaller values of k_r when there is more negative charge in the plasma. This phenomenon can be attributed to the additional dissipation (both elastic and inelastic losses) arising from the negatively charged dust grains and negative ions.

Finally, in the higher-loss dissipative case shown in Fig. 3, the domain of negative dispersion appears even at $k_rV_S/\omega_{pi}\sim0.3$–$0.45$. In the intermediate-loss case, similar domains appear at higher values of k_rV_S/ω_{pi}, typically ~0.55–0.45 [Fig. 2(a)]. Figure 3(b) suggests that here dissipative effects (k_i) are more pronounced. The phase velocity, being somewhat higher than that in Fig. 2, also exceeds V_S in the entire frequency range of interest. However, the difference between the phase velocities in cases (ii) and (iii), very pronounced in the low- and intermediate-loss cases, becomes less resolved, as shown in Fig. 3(c).

VI. DISCUSSION AND CONCLUSION

We shall now discuss in more detail some of the subtle points in our results, as well as the validity of the assumptions used in this paper.

Electron deficiency in dusty plasmas. Electron loss to the dust grains has to be compensated by additional ionization. If the rate of ionization is less than that of recombination and electron capture by the dust grains, the number of electrons will rapidly reduce. That is, n_{e0} can be very small if $\nu_{\mathrm{ion}}\sim\nu_{ed}$ and/or ν_{wall}^{e}, corresponding to insufficient ionization and too rapid recombination and capture, respectively. Both of the latter situations can occur in dusty plasmas, leading to a deficiency of electrons. In this case, the negative charge of the system resides mainly in the highly charged dust grains and the negative ions. It should be noted that even in a dusty plasma of very low electron density the contribution of the electrons in maintaining the charge balance of the equilibrium state remains important.

Relation between particle densities in electronegative plasmas. Equation (3.4) reflects the fundamental link between the equilibrium number densities of the electrons and positive-negative ions via the rates of their collection by the dust grains. Physically, this equation means that the surfaces of the dust particles act as small walls, and the flows of the plasma particles must also be balanced, in a manner similar to the balance of particle wall flows in the low-pressure diffusion equilibrium. In the absence of negative ions, Eq. (3.4) yields the conventional relation $n_{e0}=(\nu_{id}/\nu_{ed})n_{i0}$ between the equilibrium positive ion and electron densities and the rates of ion and electron collection by the dust grains [28]. It turns out that this relation is critical for the existence of an equilibrium dust-plasma system even for conditions other than that of the low-pressure diffusion regime. In fact, it is also valid in the intermediate pressure regime if volume electron-ion recombination is a dominant particle loss mechanism [34,35].

Neglect of fluctuations in the electron and ion capture frequencies. We have neglected the wave-induced fluctuations in the electron and ion capture frequencies arising from the variation of the average dust charge. This assumption is valid if the conditions

$$\partial_{Z_d}\nu_{(e,i,-)d}\ll\nu_{(e,i,-)d} \tag{6.1}$$

are met. The largest value of the ratio $\partial_{Z_d}\nu_{(e)d}/\nu_{(e)d}$ is typically one order of magnitude less than unity [28], so that the conditions (6.1) are indeed satisfied. However, for larger (10 μm or more) dust particles the conditions could be violated [24].

Plasma uniformity and charge neutrality. In this paper, local rather than global waves in a quiescent electronegative plasma are of interest. Specifically, the wavelengths involved are much smaller than a characteristic discharge dimension (e.g., chamber size, interelectrode spacing, etc.) and much larger than the electron-ion Debye length. Thus, phenomena such as breach of charge neutrality in the regions near the walls, electrodes, or dust grains are not explicitly included. This "local" approach is widely adopted in existing theories of waves in dusty plasmas. Specific phenomena such as global wave structure or wave propagation in the sheath, where the plasma is not charge neutral, are outside the scope of this work. Hence, the details (e.g., nonuniformity of the electron-ion number densities) of the sheaths near the dust grains and discharge walls are ignored since we are mainly concerned with processes occurring in the plasma bulk and wavelengths larger than the Debye length. On the other hand, the rates of electron-ion loss to the walls and dust grains (see Sec. II and Appendix) do include certain features of the sheath regions, via the potential difference between the wall or grain surface

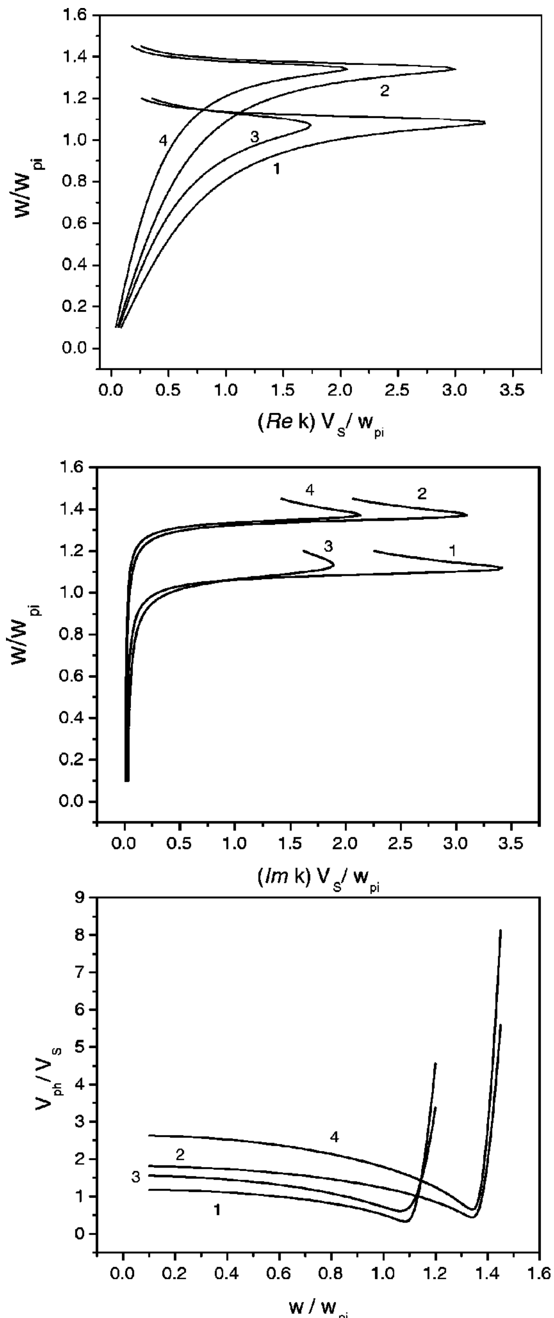

FIG. 1. The normalized frequency ω/ω_{pi} of the ion-acoustic waves versus the normalized real (top figure) and imaginary (middle figure) wave numbers $\mathrm{Re}\,kV_S/\omega_{pi}$ and $\mathrm{Im}\,kV_S/\omega_{pi}$, respectively; and the normalized phase velocity V_{ph}/V_S versus the normalized frequency ω/ω_{pi}. The curves 1–4 are for the following values of the grain size a and proportion of negative ions n_{-0}/n_{i0}: 100 nm, 0.1; 100 nm, 0.4; 1 μm, 0.1; and 1 μm, 0.4, respectively. The values of the dissipation parameters are $\nu_i^{\mathrm{eff}}/\omega = 0.025$, $\nu_-^{\mathrm{eff}}/\omega = 0.03$, $\nu_i^*/\omega = 0.225$, and $\nu_-^*/\omega = 0.0275$.

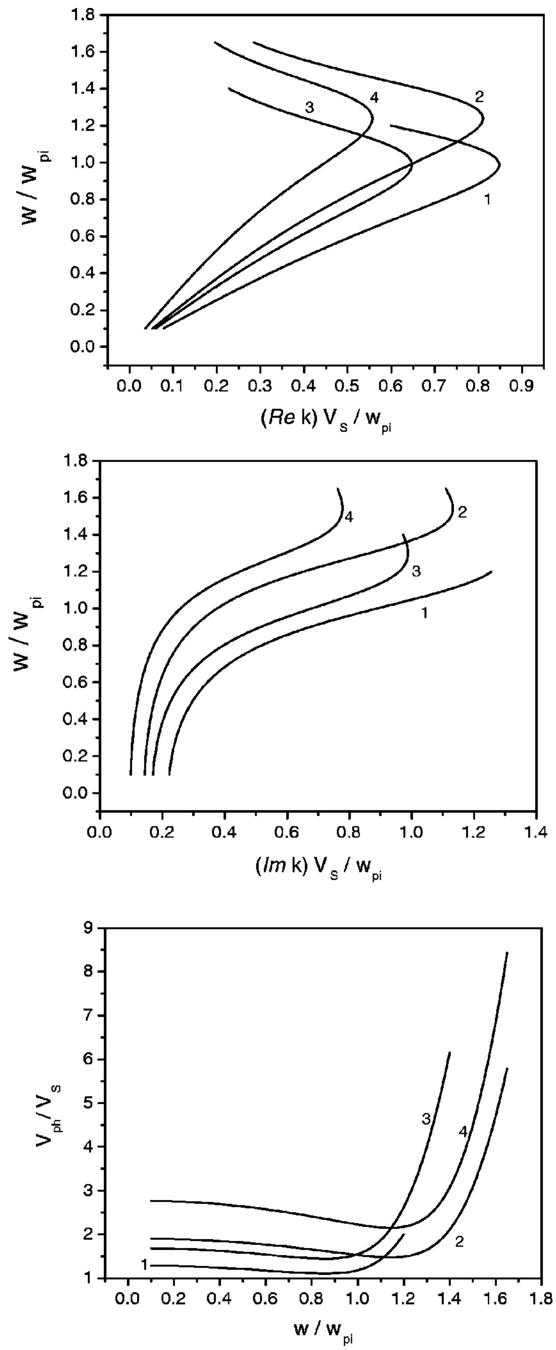

FIG. 2. Same as in Fig. 1, but for the dissipation parameters $\nu_i^{\mathrm{eff}}/\omega = 0.25$, $\nu_-^{\mathrm{eff}}/\omega = 0.3$, $\nu_i^*/\omega = 0.225$, and $\nu_-^*/\omega = 0.275$.

and the plasma bulk. For completeness, we note that the dust sheath width must be small compared to the grain size and the intergrain spacing.

Role of neutral particles in electronegative plasmas. The density n_n of the neutral particles enters the expressions for the electron-neutral and ion-neutral collisions as well as the ionization rates. In plasmas containing negative ions, the neutrals can thus be important in the building up of the steady state as well as the dynamics of the waves. Physically, this is expected since the neutrals are involved not only in the production of the positive ions via electron impact ionization, but also in production of negative ions via electron attachment. In fact, the corresponding rate coefficients are directly proportional to the density of neutrals. On the other hand, the variation of n_n is only due to ionization and electron attachment, since the neutrals do not respond to the electrostatic waves. One can easily include the fluctuation of n_n (due to ionization and electron attachment) in ν_{en}, ν_{in},

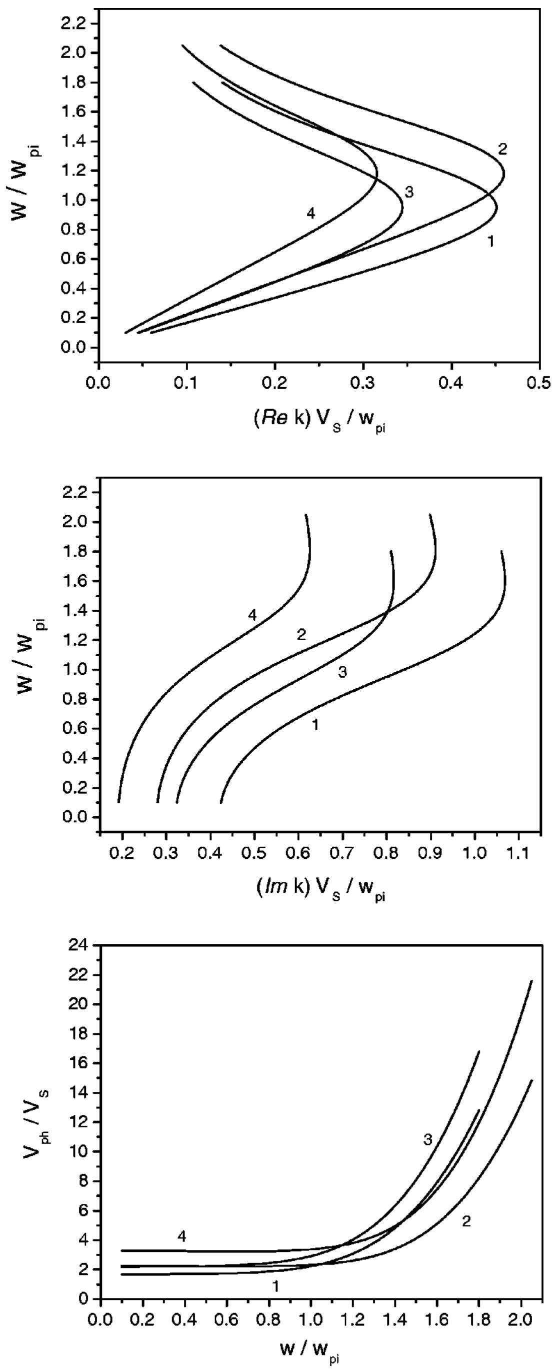

FIG. 3. Same as in Fig. 1, but for the dissipation parameters $\nu_i^{\rm eff}/\omega = 0.55$, $\nu_-^{\rm eff}/\omega = 0.6$, $\nu_i^*/\omega = 0.5$, and $\nu_-^*/\omega = 0.525$.

ν_{-n}, and $\nu_{\rm ion}$. But since our plasma is weakly ionized, and the wave perturbations are of the order of $\tilde{n}_e$, one has $\tilde{n}_n/n_{n0} \ll \tilde{n}_e/n_{e0}$. Thus the variation of n_n itself does not contribute significantly to the wave motion.

Existence of a stationary state. It is necessary to verify the condition for the existence of a stationary state of the dusty discharge under consideration. That is, the ionization rate must exceed the combined rates of electron capture by the dust grains, electron attachment to the neutrals, and electron loss to the walls (assuming that there is no secondary emission or surface ionization of the dust). In a hydrogen plasma (containing no negative ions) with $n_{i0} \sim 10^{10}$ cm^{-3}, $n_n \sim 5 \times 10^{14}$ cm^{-3}, $a \sim 1$ μm, and $n_{i0}/n_{e0} = 2$, the ionization rate would be sufficiently high at electron temperatures exceeding 1.5 eV [24]. For electronegative (e.g., silane or fluorocarbon) plasmas similar results can be expected, although the corresponding expressions for the ionization and electron attachment rates depend on several factors and are thus more complicated [36].

It should be emphasized that maintaining a stationary state in real low-pressure plasma discharges involves a dynamic process (on a longer time scale than that of the waves) that includes continuous creation of electrons and ions by ionization and their continuous transport and absorption to/by the walls and dust grains. Analogously to dust-free plasma discharges (see, e.g., Ref. [27]), the highly mobile electrons are transported to the walls and dust grains (acting as multiple small "walls"), making their surface potential (the order of T_e) negative. The positive ions, driven by the negative potential to the walls, recombine with the electrons on the wall-dust surfaces [at rates given by Eqs. (2.12)–(2.14)] and reappear to the plasma bulk as neutrals. Hence, the ionization source continuously compensates the electron and ion loss mentioned above.

In the more complex case involving negative ions, electrons also attach to the high-electron-affinity neutrals, causing an additional electron loss. This is also a dynamic process, since the positive and negative ions recombine within the plasma bulk. The electron-ion pairs lost to this process must also be reinstated by ionization in order that a stationary state exists. In the classical (no dust grains and negative ions) problem of low-frequency plasma discharges, the ionization rate must exceed the rate of the electron-ion ambipolar diffusion loss to the walls [27].

Applicability of the fluid model. Here, we have adopted a warm fluid approximation in a simplified 1D model for describing the propagation of the ion-acoustic waves in electronegative complex plasmas. Alternatively, a kinetic approach would account for collisionless wave damping and plasma heating, particle acceleration and trapping, etc. However, most existing kinetic models are applicable in the collisionless or near collisionless regimes [37]. The 1D discharge model adopted here would be most appropriate in the pressure range $\gtrsim 50$ mTorr typical for most of the laboratory dusty plasma experiments [13,38]. In this pressure range collisionless power absorption from nonlocal electron kinetic effects is weak compared to that of inelastic electron-neutral collisions [39,40]. In any case kinetic effects can only marginally affect the dispersion and field pattern of the ion acoustic waves [37], since the wave phase velocity here satisfies $V_{Ti} \ll v_{ph} \ll V_{Te}$. Thus, accounting for kinetic effects in the present problem would result in corrections that are of order $v_{Ti}/V_{ph} \sim V_{ph}/V_{Te} \ll 1$. However, one should exercise a certain degree of caution in applying our results for the IAWs' dispersion properties in the immediate vicinity of the ion plasma frequency ω_{pi}, where the fluid model is usually invalid. In a dissipative complex plasma system, the wave frequency should thus satisfy $|\omega_{pi} - \omega| \gtrsim \nu_i^{\rm diss}$ for our results to remain accurate.

Effect of recombination. It is of interest to point out another effect of the presence of negative ions in collisional plasmas. We recall that recombination effects can be neglected if $|\eta_i \eta_e| \gg \nu_{\rm rec}^2 n_{i0} n_{-0}$ and $|\eta_i| \gg \nu_{\rm rec}^2 n_{i0}$ (Sec. V). For the frequency regime ($\omega < \omega_{pi}$) of interest, these conditions can be simplified to $\omega \gg \nu_{\rm rec} n_{i0}$. Physically, this means that the positive ions, the main driver of the ion waves, must not vanish by recombination with the negative ions before an oscillation period is complete. Otherwise, the process can become aperiodic. This means that too many negative ions can destroy the natural ion acoustic waves in low-temperature gas discharge plasmas.

To conclude, we have presented in this paper a self-consistent theory of ion acoustic waves in a low-temperature plasma containing dust particles and negative ions. The relevant sources and sinks such as ionization and recombination of the particles are taken into consideration. The theory accounts for the variation of the average dust charge, the elastic and charging collisions of the electrons and ions with the dust grains. Accounting for the ionization and recombination processes also leads to a self-consistent determination of the equilibrium state of the dust-contaminated discharge.

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APPENDIX: COLLISION AND DUST CHARGING RATES

In Eq. (2.2) the effective rate of electron collisions is $\nu_e^{\rm eff} = \nu_e + \nu_e^{\rm el} + \nu_e^{\rm ch}$, and in Eq. (2.4) that of ion collisions are $\nu_i^{\rm eff} = \nu_i + \nu_i^{\rm el} + \nu_i^{\rm ch}$, and $\nu_-^{\rm eff} = \nu_- + \nu_-^{\rm el} + \nu_-^{\rm ch}$. Here, $\nu_e = \nu_{en} + \nu_{ei} + \nu_{e-}$, $\nu_i = \nu_{in} + \nu_{i-} + \nu_{ie}$, and $\nu_- = \nu_{-n} + \nu_{-i} + \nu_{-e}$ are the rates of electron and ion collisions with the neutrals and plasma particles. Furthermore, $\nu_{(e,i,-)}$ denotes the rates of elastic collisions between the plasma particles, $\nu_{(e,i,-)}^{\rm el}$ and $\nu_{(e,i,-)}^{\rm ch}$ stand for the rates of elastic (electrostatic) scattering and inelastic charge capture of electrons and ions by the dust grains, respectively.

In the dust-charging equation (2.7), the dust charging rate [15] is $\nu_d^{\rm ch} = a\omega_{pi}^2 \mathcal{A}/\sqrt{2\pi} V_{Ti}$, where $\mathcal{A} = 1 + (T_i/T_e) + (\varphi_d^{\rm el}/\varphi_e^{\rm th})$, $\varphi_d^{\rm el} = Z_d e/a$, $\varphi_e^{\rm th} = T_e/e$, $V_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal velocity, ω_{pi} is the ion plasma frequency, and a is the average dust grain radius.

For the dust charging collisions we have [11,41]

$$\nu_e^{\rm ch} = \frac{3}{2} \nu_i^{\rm ch} \frac{n_{i0}}{n_{e0}} \frac{\alpha\sigma}{T_i/T_e + \alpha} = \nu_d^{\rm ch} \frac{\alpha\gamma\sigma}{\mathcal{A}}, \qquad \text{(A1)}$$

and the rate of electron and ion capture by the dust grain is

$$\nu_{ed} = \frac{n_{i0}}{n_{e0}} \nu_{id} = \nu_d^{\rm ch} \frac{\alpha\gamma}{\mathcal{A}}, \qquad \text{(A2)}$$

where $\alpha = \mathcal{A} - 1$, $\gamma = (Z_{d0} n_{d0}/n_{e0}) \varphi_e^{\rm th}/\varphi_d^{\rm el}$, and $\sigma = 4 + \varphi_d^{\rm el}/\varphi_e^{\rm th}$.

The rates of elastic electron- and ion-dust Coulomb collisions are

$$\nu_e^{\rm el} = \alpha \nu_i^{\rm el} \frac{T_i}{T_e} \frac{n_{i0}}{n_{e0}} \exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right) = \frac{2}{3} \nu_d^{\rm ch} \frac{\alpha\gamma\Lambda}{\mathcal{A}} \exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right), \qquad \text{(A3)}$$

where Λ is the Coulomb logarithm. The rates of elastic and inelastic collisions of ions with the negatively charged dust grains have forms similar to the electron-dust collision rates. Moreover, due to the much larger mass and lower temperature of the dust grains, these rates are usually negligible compared to the effective rates of collisions between the negative ions and the neutrals and the positive ions, and are thus neglected in the present study. It should, however, be noted that for complex plasmas with positively charged dust grains, the rate of collisions between the negative ions and the negatively charged dust grains can become significant and may have to be accounted for.

The electron- and ion-neutral collision frequencies are $\nu_{en} = n_n \sigma_{en} V_{Te}$, $\nu_{in} = n_n \sigma_{in} V_{Ti}$, where n_n is the neutral gas density, σ_{en} and σ_{in} are the electron (ion)- neutral collision cross sections, V_{Te} and V_{Ti} are electron and ion thermal velocities [42]. The corresponding rate of negative ion-neutral collisions can be expressed as $\nu_{-n} = n_n \sigma_{-n} V_{T-}$, where σ_{-n} is the negative ion-neutral collision cross section [27,42]. The expressions for the electron-ion and ion-electron collision frequencies can be found in Ref. [33]. The collision terms involving the dust grains are also similar. The rates of dust-charge variation have been calculated using kinetic theory [11,41]. The fact that the dissipative loss in dusty plasmas is generally higher than that in dust-free plasmas (due to Coulomb collisions with and capture by the dust grains) has been confirmed experimentally [13,43].

The rates of ionization of the neutrals are usually assumed to be of the Arrhenius form, or $\nu_{\rm ion} \propto n_n \nu_0(T_e) \exp(-U_i/T_e)$, where $\nu_0(T_e)$ is a slowly varying function of the temperature, and U_i is the ionization potential [27]. The rates of electron attachment and positive-negative ion recombination have been tabulated in the literature for many electronegative gases (see, e.g., Ref. [36]).

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